Differential Evolution Aplication in Portfolio Optimization for Electricity Markets

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***Abstract*—Smart Grid technologies enable the intelligent integration and management of distributed energy resources. Also, the advanced communication and control capabilities in smart grids facilitate the active participation of aggregators at different levels in the available electricity markets. The portfolio optimization problem consists in finding the optimal bid allocation in the different available markets. In this scenario, the aggregator should be able to provide a solution within a timeframe. Therefore, the application of metaheuristic approaches is justified, since they have proven to be an effective tool to provide near-optimal solutions in acceptable execution times. Among the vast variety of metaheuristics available in the literature, Differential Evolution (DE) is arguably one of the most popular and successful evolutionary algorithms due to its simplicity and effectiveness. In this paper, the use of DE is analyzed for solving the portfolio optimization problem in electricity markets. Moreover, the performance of DE is compared with another powerful metaheuristic, the Particle Swarm Optimization (PSO), showing that despite both algorithms provide good results for the problem, DE overcomes PSO in terms of quality of the solutions.**

Keywords—Differencial Evolution; Portfolio Optimization; Electricity Markets.

I. Introduction

Electric systems in developed countries have been evolving to what is known as Smart Grids (SG). The concept of SG is typically used generically, although many definitions are found in the literature. An SG is defined as an "*electricity network that can intelligently integrate the actions of all users connected to it: generators, consumers and those that do both in order to efficiently deliver sustainable, economic and secure electricity supplies. A smart grid employs innovative products and services together with intelligent monitoring, control, communication, and self-healing technologies*" [1]. Given this definition, it can be noticed that an SG is not limited to the technical aspects of the intelligent grid, but also to a set of market solutions that can lead to the construction of a market into the grid [2].

In the context of SGs, there is a so-called aggregator entity that might act as a single entity on behalf of other SG players, with the aim of providing them with aid for the electricity negotiations [3]. The aggregators will try to negotiate the energy needed to suppress the energy needs of their households. To this end, aggregators try to carry out the negotiation actions in order to take profit from them. In this way the aggregators can be classified as sellers, buyers, or both (when they sell and buy). The aggregators also have the possibility to make multiple offers of purchase and sale in the market in order to obtain the best desired option.

The theory of portfolios, although it has appeared in the field of economics and finance, is a technique that enables analyzing the best combination between existing assets and markets, in order to obtain the best results by making diverse offers in multiple markets [4]. Nowadays, portfolio theory has been applied to electric system problems, and is widely used by private investors in order to evaluate the viability of their investment (e.g., the technology to be used, location and diversification of the sale of production, and so on) [5]. This theory is a very effective tool for dealing with the possible uncertainties that may exist, e.g., renewables, price expected for electricity, the electricity produced/consumed, and so on.

Roques et al. [5] applied the portfolio theory to identify the efficient combination of its investment between generation technologies (e.g., coal, nuclear, and combined cycle power plants). In this study, the authors considered the impact that the uncertainties (as is the case of the price of fuel, electricity and CO_2) might cause in the portfolio solution. The same study is carried out by Muñoz et al. [6], although in this case the investment refers to renewable production projects with different results for the generation of the final product. Both studies, [5] and [6], intend to inform the investor how to proceed to obtain the best return.

Considering the approach to the problem from the perspective of the aggregator, there are also some studies, such as the work done by Liu and Wu [7], where the allocation of electricity by a producer is discussed, allowing different types of commercial negotiations in the risk-free contracts, riskier contracts and the spot market. In this approach, it is possible to plan for different time horizons (one day, one week, one year, or serval years). Lorca and Prina [8] also address the problem from a manager perspective considering an energy producer that has a generation of thermal units in which it is possible to sell its generation in spot markets, and bilateral contracts (forwards contracts and contract for difference). The optimization problem is solved using a stochastic optimization model.

Another approach from the aggregator's point of view is presented in [9]. In this work, the authors consider an aggregator that can perform both actions on the market: purchases and sales.

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In the example presented by the authors, it is possible to sell electricity in five different markets and it is possible to buy in a market only. In order to optimize the problem, the authors use the Particle Swarm Optimization (PSO) and conclude that this method proved to be effective in solving the problem. The solution to the problem of portfolios using metaheuristics is quite popular. For instance, in [9], a PSO algorithm is used to solve the problem of portfolios applied to the problem of allocation of electricity in the electricity markets. Another application of metaheuristics in the resolution of the problem of portfolios in the electricity markets is present in [10], where the author uses the Genetic Algorithms (GA) to respond to the problem of the allocation of electricity in different markets.

In this paper, the application of portfolios will be used as a decision and support tool in an aggregator perspective, where special attention is paid to market prices to achieve profits in the sale and purchase of electricity. In order to solve the problem of portfolio optimization, we propose the use of a metaheuristic technique, namely Differential Evolution (DE). DE is also compared with the PSO, showing that, even when both heuristics present a satisfactory performance, DE can overcome the results of PSO.

The paper is organized into six different sections, starting with the introduction section where a short review of the current state of art is made, as well as the contextualization of the portfolios problem to the electricity markets. The second section presents the problem description. In the third section, we present the metaheuristics used to solve the problem as well as all the strategies used. In the fourth section the case study is presented, followed by the results in the fifth. Finally, conclusions are addressed in the sixth section.

II. Problem Description

In this paper, the allocation of electricity in different markets to achieve the maximum possible profits for the energy aggregator is studied. It is assumed that a Virtual Power Player (VPP) [3] that has an aggregation of production and the possibility to buy and sell electricity in the market. Eq. (1) represents the profits or return expected by the VPP after market transactions. We divided the equation into three lines for better understanding. The initial line refers to the return of sales of electricity in the different markets; the second line to the purchase cost in the respective markets; and the third line to the cost of production of the generation of energy (i.e., by renewable generation):

$$Max\ (Return) = \left[\begin{array}{c} \sum_{M=1}^{N_M} \left(Spow_{(M,d,p)} \times ps_{(M,d,p)} \times binS_{(M,d,p)} \right) - \\ \sum_{M=1}^{N_M} \left(Bpow_{(M,d,p)} \times pb_{(M,d,p)} \times binB_{(M,d,p)} \right) - \\ \sum_{G=1}^{N_G} \left(Prod_{(G,d,p)} \times pg_{(G,d,p)} \right) \end{array} \right] \quad (1)$$

$$\forall\, d \in Nday, \forall\, p \in Nper, binS_M \in \{0,1\}, binB_M \in \{0,1\}$$

The index M represents the market and can range from 1 up to N_M; G represents the considered generator and can range from 1 up to N_G; index d represents the day and can range from 1 up to N_{day} (total number of days); p represents the referred period, and can range from 1 up to $Nper$ (number total of periods). The variable $Spow$, represent the power selling, ps represent the selling price, $binS$ represent the binary variable which is active if the sale is execute, $Bpow$ represent the power buying, pb the price for buying, $binB$ represent the binary variable which is active if the buy is execute, $Prod$ represent the production and pg represent the generation price.

Market prices, as in the case of Eq's. (2) and (3), are obtained by forecasts techniques that allows to reach a value by means of a historical price.

$$ps_{(M,d,p)} = Price\left(Spow_{(M,d,p)}\right) \quad (2)$$

$$pb_{(M,d,p)} = Price\left(Bpow_{(M,d,p)}\right) \quad (3)$$

Eq. (2) and Eq. (3) represent the prices for selling and buying energy respectively. Two methods for obtaining the prices have been used in this paper. In the first method, the prices are indifferent to the quantity traded, while in the second method the prices can be affected by the amount. The specifics about such methods can be found in [11].

Eq. (4) represents the cost of production for a thermoelectric type generator:

$$pg_{(G,d,p)} = a \times Prod_{G,d,p}^2 + b \times Prod_{G,d,p} + c \quad (4)$$

where indices a, b and c represent the coefficient of total cost production.

Additionally, the problem is also subject to some constraints, namely:

Balance constraint: states that the sum of the electricity sale into the markets must be equal to the electricity purchased plus the electricity generated:

$$\sum_{M=1}^{NumM} Spow_{(M,d,p)} = \sum_{M=1}^{NumM} Bpow_{(M,d,p)} + \sum_{G=1}^{NumG} Prod_{(G,d,p)} \quad (5)$$

-- Restrictions to buy and sell simultaneously in some markets:

$$Bpow_{(M,d,p)} = \begin{cases} 0 & if\ M = 1 \\ Bpow_{(M,d,p)} & o.w. \end{cases} \quad (6)$$

$$binS_{(M,d,p)} + binB_{(M,d,p)} \leq 1; \qquad M = \{2,3\} \quad (7)$$

In this restriction, Eq. (6) refers to the impossibility of purchasing electricity in market number 1 (Particularly for this case). Eq. (7) refers to markets 2 and 3, where in this case the sum of the respective binary must be less than or equal to 1.

To solve the optimization problem, we will use the DE metaheuristics, which will be compared to a resolution using another metaheuristic, the PSO.

III. Differential Evolution

DE is a popular search strategy used to optimize functions of the form $f(x_1, x_2, \ldots, x_D)$, where D is the dimension of the problem (i.e., number of variables). The basic version of DE uses a population of individuals (Pop) in which each induvial represents a solution to the problem encoded as $\vec{x}_{i,G} = [x_{1,i,g}; \dots; x_{D,i,g}]$, where g is the generation number, $i = [1, \dots, NP]$ is the index of a given individual, and NP is the size of the population. DE iterates by creating new solutions using mutation and crossover operators. After that, the algorithm keeps the individuals with better fitness by evaluating them in an objective function, and replacing the worst individuals in Pop. The process is repeated for a fixed number of generations (GEN) or until a computational condition is reached. The reader can be referred to [12] for a detailed explanation of the algorithm.

A. Encoding of the Individuals

The encoding of the solutions is crucial for the success of the algorithm. Therefore, for this problem the solutions are encoded as vectors of the form:

$$\vec{x} = [\{Bpow_1, \dots, Bpow_{N_M}\}, \{Spow_1, \dots, Spow_{N_M}\}, \{binB_1, \dots, binB_{N_M}\}, \{binS_1, \dots, binS_{N_M}\}, \{Prod_1, \dots, Prod_{N_G}\}] \tag{8}$$

where $\{Bpow_1, \dots, Bpow_{N_M}\}/\{Spow_1, \dots, Spow_{N_M}\}$ is a group of continuous variables representing the amount of energy to buy/sell in each market, $\{binB_1, \dots, binB_{N_M}\}/\{binS_1, \dots, binS_{N_M}\}$ are binary variables to enable the possibility to trade in specific markets, and $\{Prod_1, \dots, Prod_{N_G}\}$ are the power of available dispatchable generators. Therefore, an individual $\vec{x}$ has a dimension $D = 4 \cdot N_M + N_G$. This encoding allows a direct evaluation of each individual in Eq. (1)

B. Initialization

There are diverse ways in which the initial population can be generated, however, random initialization is the preferred one due to its simplicity. According to that, for each individual, each variable is initialized randomly into the allowed bounds as:

$$\vec{x}_{j,i,G} = rand_j[xlb_j, xub_j] \quad \forall j \tag{9}$$

where $rand_j[x_{lb,j}, x_{ub,j}]$ is a random number within the lower ($x_{lb,j}$) and upper ($x_{ub,j}$) bounds of variable jth of any individual $i \in Pop$.

In order to start the search, it is necessary an initial starting solution for the metaheuristic. The initial solution is created based on random variables, but ensuring that it will never go outside the search limits. We call this process *"direct-repair"* generation. The initial solution is created applying the following rules:

Step 1: a random value is assigned to the generation according to:

$$Gen_p = rand \times Max(Prod_{Therm}) + Prod_p^{PV} \tag{10}$$

Since a rand is a random value between [0,1], Eq. 10 guarantees that this value lies into the limits of thermal generation.

Step 2: four binary variables are generated for the purchase:

$$binB_p = \begin{bmatrix} rand([0,1]) \\ rand([0,1]) \\ rand([0,1]) \\ rand([0,1]) \end{bmatrix} \tag{11}$$

Step 3: We calculate randomly the value of energy to buy into the markets according to the limits:

$$Bpow_p = rand(1{,}4).binB_p.Max(Bpow) \tag{12}$$

Step 4: With the definition of energy to buy, we calculate the required energy to buy to meet the balanced constraint:

$$Pow\ Sell_p = sum(Bpow_p) + Gen_p \tag{13}$$

Step 5: We create the binary values for the selling markets:

$$binS_p = \begin{bmatrix} rand([0,1]) \\ 1 - Bin_p^{Buy}(1) \\ 1 - Bin_p^{Buy}(2) \\ rand([0,1]) \\ rand([0,1]) \end{bmatrix} \tag{14}$$

It is imperative to ensure that at least one of the binary markets takes the value of one, in position 3 and 4 an operation is performed that will prevent both binary buying and selling of these positions take the same value, so it is only possible to make purchase or sale or any of the options.

Step 6: a vector with random numbers is multiplied by the binary variable of each market:

$$Spow_p = rand(1{,}4).binS_p \tag{15}$$

Step 7: a distribution of the total available sales quantity is carried out for the different markets, where the quantity of each market is divided by the sum of all the quantities.

$$Spow_p = \begin{bmatrix} Spow_p(1)/sum(Spow_p) \\ Spow_p(2)/sum(Spow_p) \\ Spow_p(3)/sum(Spow_p) \\ Spow_p(4)/sum(Spow_p) \\ Spow_p(5)/sum(Spow_p) \end{bmatrix} \tag{16}$$

$$Spow_p = Spow_p\,.Pow\ Sell_p \tag{17}$$

Step 8: finally, the initial solution is constructed:

$$x_{init} = [Spow_p, Bpow_p, Gen_p - Prod_p^{PV}, Prod_p^{PV}] \tag{18}$$

where it is constituted by 5 variables of sale of electricity, 4 of purchase, one of thermal generation and another one of PV generation.

C. Mutation DE Strategies

One of the key elements of the success of DE relays in its easy mutation function used to generated new individuals. The basic DE operator is known as DE/rand/1 and is defined as:

$$\vec{m}_{i,g} = \vec{x}_{r1,g} + F(\vec{x}_{r2,g} - \vec{x}_{r3,g}) \tag{19}$$

where $r1 \neq r2 \neq r3 \neq i \in \{1,2, \dots, NP\}$ are randomly chosen indices from the population mutually different from each other and from current target vector i, and F (mutation parameter) is a real number in the range [0,1] that scales the difference between vectors $(\vec{x}_{r2,G} - \vec{x}_{r3,G})$ having a direct impact in the exploitation/exploration capabilities of the algorithm. This function gives place to the mutant vector $\vec{m}_{i,G}$.

Additionally, different works have proposed modifications to the basic DE operator (i.e., Eq. (19)) to tackle problems with distinctive characteristics. Those modifications have given place to the so-called DE strategies analyzed in this work [13]. In this paper, additionally to the application of DE/rand/1, we explore the effectiveness of three more DE strategies:

1) DE/target-to-best/1

This strategy has similar convergence properties as the PSO algorithm and its name, "target-to-best", is given due to the base vectors are chosen to lie on the line defined by the target vector $\vec{x}_{i,g}$ and the best-so-far vector $\vec{x}_{best,g}$ as follows:

$$\vec{m}_{i,g} = \vec{x}_{i,g} + F(\vec{x}_{best} - \vec{x}_{i,g}) + F(\vec{x}_{r1,g} - \vec{x}_{r2,g}) \quad (20)$$

2) DE/rand/1 with Dither

For this strategy, a simple random perturbation on the F parameter, known as dither in the literature, is incorporated into the standard mutation operator as follows:

$$\vec{m}_{i,g} = \vec{x}_{r1,g} + rand(F,1) * (\vec{x}_{r2,g} - \vec{x}_{r3,g}) \quad (21)$$

where $rand(F,1)$ is a random number in the range $[F,1]$. The so-called dither variation has proved to improve the performance of DE in different problems [14].

3) DE/rand/1/either-or algorithm

This strategy creates the mutant individual either by a DE/rand/1 scheme with probability PF, or as a randomly recombinant scheme with probability$1 - PF$:

$$\vec{m}_{i,g} = \begin{cases} \vec{x}_{r1,g} + F(\vec{x}_{r2,g} - \vec{x}_{r3,g}) & if\,(rand < PF) \\ \vec{x}_{r1,g} + k(\vec{x}_{r2,g} + \vec{x}_{r3,g} - 2\vec{x}_{r1,g}) & o.w. \end{cases} \quad (22)$$

where k is a scale factor similar to parameter F. Price et al. recommended a value of $k = 0.5(F+1)$ and $PF = 0.4$ [11]. It is worth to notice that this strategy has shown competitive results against classical DE strategies [14].

D. Crossover

The crossover operator is applied to create the trial vector $\vec{t}_{i,G}$ according to:

$$\vec{t}_{j,i,g} = \begin{cases} \vec{m}_{j,i,g} & \text{if } (rand < Cr \vee (j = \text{Rnd}) \\ \vec{x}_{j,i,g} & \text{o.w.} \end{cases} \quad (23)$$

where Cr represents the probability of choosing the jth element of $\vec{m}_{i,g}$ otherwise from the target vector $\vec{x}_{i,g}$. A random integer value Rnd is chosen in the interval [1, D] to guarantee that at least one element is taken from $\vec{x}_{i,g}$.

E. Boundary Constraints

Mutation strategies (Sec. IVB) might generate individuals with values that violate the variables' boundary constrains. To address this issue, boundary control strategies are used to repair infeasible individuals. In this paper, we use a boundary control technique known as bounce-back [15]. In contrast to random reinitialization (the most used control technique), bounce-back uses the information on the progress towards the optimum region by reinitialized the variable value between the base variable value and the bound being violated as follows:

$$\vec{t}_{j,i,g} = \begin{cases} \text{rand}(xlb_j, \vec{x}_{j,i,g}) & \text{if } \vec{t}_{j,i,g} < xlb_j \\ \text{rand}(\vec{x}_{j,i,g}, xub_j) & \text{if } \vec{t}_{j,i,g} > xub_j \\ \vec{t}_{j,i,g} & o.w. \end{cases} \quad (24)$$

This boundary control method showed to be effective for the application studied in this paper. Other repair methods can be analyzed in future work.

F. Fitness and Selection

The individuals should be evaluated according a fitness function including objective function and constraints violations. For this reason, the true fitness of an individual is calculated using a constraint-handled method based on penalties as:

$$f'(\vec{X}) = f(\vec{X}) + \sum_{i=1}^{N_c} \max[0, g_i] \cdot \rho_i \quad (25)$$

where $f(\vec{X})$ is the value of the individual evaluated in Eq. (1), while g_i are the values obtained from Eq. (5-7) when those constraints are violated. ρ_i are weighted values to modify the importance of the constraints under analysis. In addition to this constraints-handling method, we introduce a direct repair method (see Sect. IVG) that reset the individual to a feasible zone when a constraint violation is found.

Selection follows a simple rule of elitist done by comparing the fitness between the trial matrix $\vec{t}_{i,g}$, and the target matrix $\vec{x}_{i,g}$ in the objective function:

$$Pop_{i,g+1} = \begin{cases} \vec{t}_{i,g} & \text{if } f'(\vec{t}_{i,g}) \le f'(\vec{x}_{i,g}) \\ \vec{x}_{i,g} & \text{o.w.} \end{cases} \quad (26)$$

where $Pop_{i,g+1}$ is the next generation population, that changes by accepting or rejecting new individuals, and $f'(.)$ is the fitness function used to measure the performance of an individual (i.e., Eq. (1) plus the constraints violation values).

G. Constraint-Handling and Direct-Repair methods

As it was explained in the previous subsection, a constraint-handling method based on a penalty function was used due to its simplicity. However, using domain knowledge about the problem, we propose a different constraint-handling method based on direct-repair. This method is very simple to implement and is based on the ad-hoc initialization procedure presented in Sect. IVB.

ALGORITHM 1: DE PSEUDOCODE

```
INITIALIZE
Set control parameters F ∈ [0,1], Cr ∈ [0,1], and NP.
Create initial Pop with heuristic method (Sect. IV.B.).
Evaluate fitness of Pop (Eq. (1)).
IF Direct-repair is used THEN
   Apply direct-repair to unfeasible individuals Sect. IVG)
   Evaluate fitness of the repaired individual
END IF
FOR g = 1 to GEN
    FOR i = 1 to NP
       Select individuals from Pop
       Mutation strategy (any from Eq. (19-22))
       Recombination (Eq. (23))
       Verify boundary constraints
       IF Boundary constraints are violated THEN
          Apply boundary control
       END IF
       Evaluate fitness of t_{i,g} (Eq. (1)).
       IF Direct-repair is used THEN
         Apply direct-repair (Sect. IVG)
        END IF
       Apply selection operator (Eq. (26))
       Update Pop (and x_best for DE/target-to-best)
    END FOR
END FOR
```

Basically, if in the evaluation process constraints violations are identified, the individual is randomly repaired using the initialization process from IVB. Such initialization process guarantee that the individual does not present any constraint violations, resulting in the convergence of individuals to the feasible zone. A pseudocode of DE algorithm, including in red the direct-repair method, is presented in algorithm 1.

IV. Case Study

This section presents the case study. The optimization problem was solved using DE metaheuristic and compared with PSO to prove the veracity of the results. All approaches have been implemented in MATLAB software (version – R2016a), on a computer with 1 processor Intel® w3565 3.2GHz, with 4 Cores, 8 GB of RAM and operating system Windows 10 64bites.

The portfolio to be optimized refers to a VPP (with prosumer capabilities in the market), which has a thermal production generator and an aggregate of photovoltaic energy production. The historical price that was used to train the forecasting methods was obtained from the historical data of the MIBEL market [16]. TABLE 1 defines the problem parameters used to solve the problem.

TABLE 1. Input data

Variable	**Value**
$NumM$ – Number of Markets	5
$Nper$ – Number of periods	24
$Nday$ – Number of days	1
$Max(Prod)$ – Maximum of production (kW)	172.57
$Max(Prod_{Therm})$ – Maximum of thermoelectric production (kW)	50.00
$Max(Prod_{PV})$ – Maximum of photovoltaic production (kW)	122.57
$Max\left(Bpow_{(M)}\right)$ – Maximum of purchase in each market (kW)	25.00
a	4.45
b	0.207
c	0.024

It is imperative to define a maximum purchase per market, otherwise the model will buy infinitely in the cheapest market to sell where the price is maximum. The quantity for selling is always limited due to the restriction Eq. (5). With the number of markets equal to five, by restriction number (6), in the market number 1 is impossible to sell, by equation number (7) is only possible the sale or the purchase or no action. In the market 1,2,3 prices do not depend on the quantity traded, in the market 4,5 prices are influenced by the quantity traded.

V. Results

This section is divided into two parts. The first part is devoted to the tuning and performance of DE strategies for this particular problem (Sect. VI.A). In the second part, a comparison between DE and PSO is presented.

A. DE parameter tuning

For conducting the parameter tuning, we performed an analysis inspired in [13]. In the first stage, we fix the number of generations to $GEN = \left\lfloor \frac{10000}{30} \right\rfloor = 333$, and the population size to $NP = 30$. After that, we performed a swept of the parameters F and Cr, in the range [0.1,1] and steps of 0.1 for both parameters. Such swept allow us to test all possible combinations of parameters and determine the most suitable combination of F and Cr that leads to good performance. We did ten trials for the standard DE/rand/1 algorithm and the three DE strategies introduced in Sect. IVC. Moreover, we also tested the DE strategies using constraint-handling and the direct-repair method from Sect. IVG.

Figure 1 shows heatmaps resulting for the experimental swept test. Lighter parts represent better average profits for the aggregator, while darker zones represent poor performance of the algorithms.

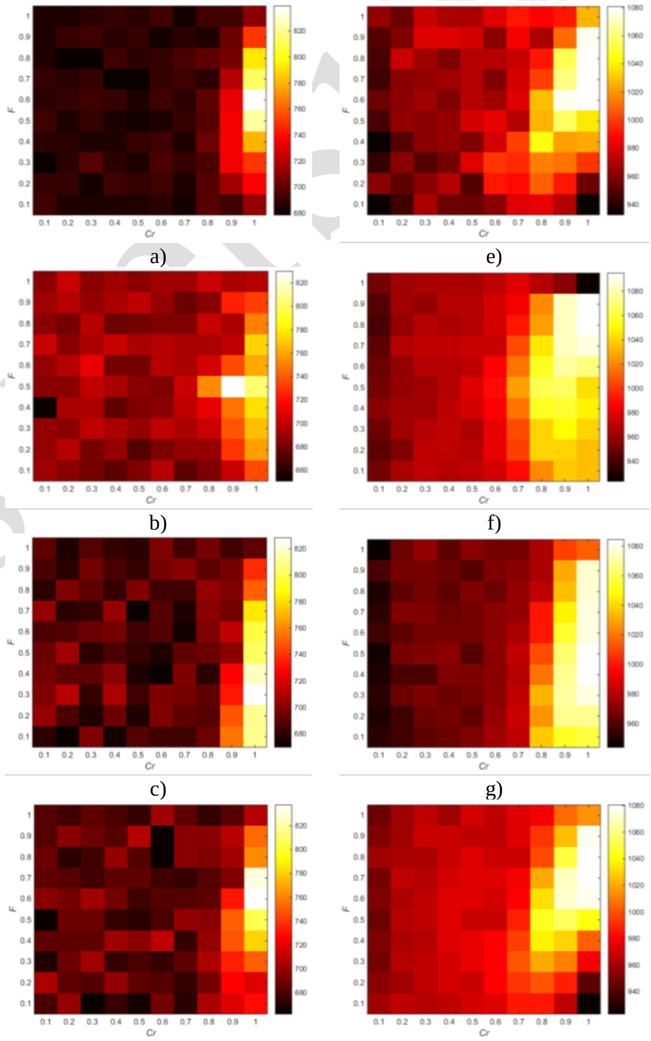


Fig. 1. Swet testing of four DE strategies using constraint handling penalties and direct-repair method. Ten trials and a fixed value of NP=30, GEN=10000/30, was considered. Constraint handling: a) DE/rand/1, b) DE/target-to-best/1, c) DE/rand/1 with dither, d) DE/rand/1/either-or. Direct-repair: e) DE/rand/1, f) DE/target-to-best/1, g) DE/rand/1 with dither, h) DE/rand/1/either-or.

From Fig. 1a)-d), we can appreciate the effect of parameters F and Cr when constraints handling is used. It can be observed that, despite the selected DE strategy, a high value of Cr results in better profits. The algorithms are less sensitive to parameter F, since it can be appreciated that when the value of

Cr is well-chosen, F parameter can take a wider range of values without affecting the performance of the algorithm. Similar behavior can be observed in Fig. 1e)-h), where the DE strategies with direct-repair method where tested. A wider light area can be appreciated, with higher values of profits compared with DE strategies using constraint handling technique.

Table 2 presents the range of values for parameters F and Cr where DE strategies showed good performance. As additional information, the fitness for the most suitable set of parameters when constraint handling was used was around 840 m.u., while by using the direct-repair method the fitness was around 1080 m.u. (not showed in the table). This gives us some insides on the effectiveness of the direct-repair method.

TABLE 2. RECOMMENDED RANGES OF VALUES FOR PARAMETERS *F* AND *CR*

Strategy	**Constraint Handling**		**Direct repair**	
	$\boldsymbol{F}$	$\boldsymbol{Cr}$	$\boldsymbol{F}$	$\boldsymbol{Cr}$
DE/rand/1	[0.4,0.8]	[0.9-1]	[0.6,0.9]	[0.8,1]
DE/target-to-best/1	[0.2,0.7]	[0.9-1]	[0.1,0.9]	[0.8,1]
DE/rand/1 with dither	[0.1,0.7]	1	[0.1,0.9]	[0.8,1]
DE/rand/1/either or	[0.4,0.7]]	1	[0.3,0.9]	[0.8,1]

In the second stage of parameter tuning, we fixed the best values for F and Cr for each strategy found at stage one, and we varied NP parameter and the number of generations to see the effect on DE strategies performance. Tables 3 and 4 show the profits when NP is varied in the range [10,50] in steps of 10. It is important to clarify that for this test, the number of fitness function evaluations have been fixed to 10,000, so that the number of generations was set to *GEN* = 10,000/*NP*

Tables 3 and 4 shows the average profits of ten trials with constraint-handling method and direct-repair method. Notice that the major the number of individuals, the best the performance of the strategies. We have highlighted (in bold) the best strategy for each NP value. It is worth to noticed that, when direct-repair method is used, all the strategies found higher profits compared to constraint-handling method despite the number of individuals used. This highlights the importance of the incorporation of domain knowledge in the metaheuristic frameworks, leading to best results.

TABLE 3. CONSTRAINT-HANDLING NP TUNING

DE Strategy	**NP=10**	**NP=20**	**NP=30**	**NP=40**	**NP=50**
DE/rand/1	639.03	746.39	**839.27**	869.85	887.07
DE/target-to-best/1	**663.57**	748.36	829.93	876.36	**905.96**
DE/rand/1 with dither	650.43	**767.67**	828.29	882.60	903.31
DE/rand/1/either or	647.38	741.93	837.33	**892.80**	891.75

TABLE 4. DIRECT-REPAIR NP TUNING

Strategy	**NP=10**	**NP=20**	**NP=30**	**NP=40**	**NP=50**
DE/rand/1	**1014.4**	1071.7	1080.8	1088.7	1079.0
DE/target-to-best/1	1009.9	1073.3	**1094.9**	**1098.8**	**1095.8**
DE/rand/1 with dither	1002.0	**1076.2**	1084.5	1082.6	1082.1
DE/rand/1/either or	993.9	1073.7	1080.5	1082.7	1079.5

To summarize, the information obtained from the parameter tuning analysis, Table 5 presents our selection of parameters to compete against PSO. Since the best results were found with DE/target-to-best/1 strategy (overall), we decided to use such strategy with different combinations of parameters in the comparison against PSO. We label the selected combination as shown in Table 5.

TABLE 5. DE/TARGET-TO-BEST/1 SELECTED SETTINGS

Label	Boundary Method	**F**	**Cr**	**NP**	**Ave. Fitness**	**Std**
DE1	Constraint Handling	2	10	10	**663.57**	101.88
DE2		5	10	50	**905.96**	114.62
DE3	Direct Repair	9	9	10	1009.87	39.64
DE4		7	10	50	**1095.81**	30.79

B. *Comparison of DE strategies and PSO*

TABLE 6 presents the characteristic of the different methods applied, namely the namely the DE target-to-best strategies and the traditional PSO algorithm. The parameters for PSO are: a linear decreasing inertia, c1=1 and c2=1, resulting from a former study in [9].

TABLE 6. METHODS DESCRIPTION

Method	**Label**	**C.H.**	**D.R.**	**It.**	**Nº Ind./Par.**
DE	DE1	X		1000	50
	DE2		X	1000	50
	DE3	X		5000	10
	DE4		X	5000	10
PSO	PSO1	X		1000	50
	PSO2		X	1000	50
	PSO3	X		5000	10
	PSO4		X	5000	10

* In all versions, 50000 evaluations were performed.

In Table 6, we present different labels according to different considerations, for instance the use of C.H. and D.R. (representing Constraint-handling and Direct-repair respectively). The number of iterations was set to 1000 and 5000, and the number of individuals for DE and number of particles for PSO was fixed to 50 and 10. Despite we vary the size of the population and generations, the number of evaluations was fixed to 50,000. We performed 100 trials for each experiment. The reported results correspond to the average over the 100 trials.

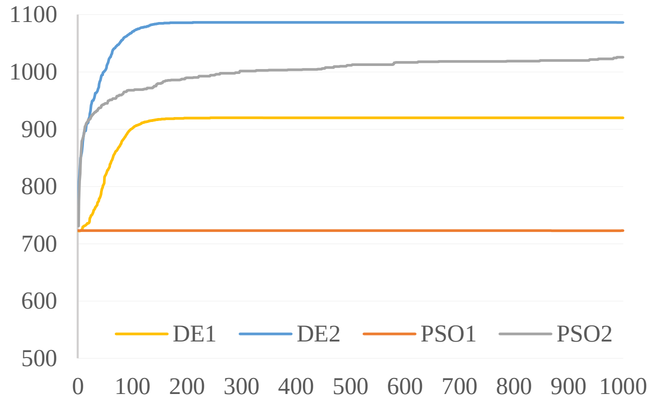
TABLE 7 shows the objective function results for the different methods applied to the problem. In the table are presented the values for maximum, minimum, mean and standard deviation (STD) profits of the 100 simulations.

TABLE 7. OBJECTIVE FUNCTION RESULTS

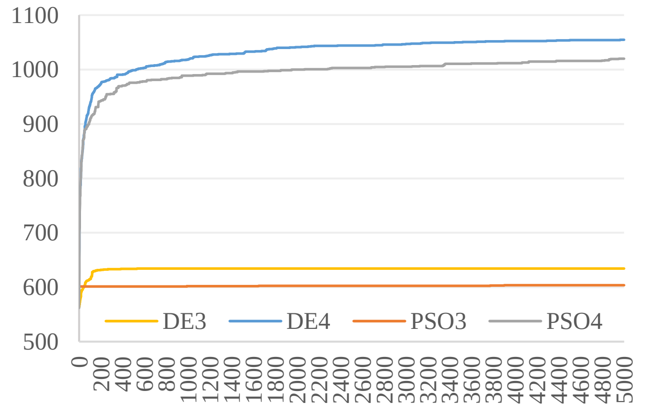
Method	**Objective function (m.u.)**			
	Maximum	**Minimum**	**Mean**	**STD**
DE1	1075.287	681.0024	919.7595	101.9695
DE2	1160.087	1080.934	1086.212	20.08141
DE3	954.8617	390.7376	634.2682	101.5183
DE4	1128.371	987.1924	1054.51	27.94936
PSO1	937.3555	509.7173	722.8751	98.32423
PSO2	1072.948	978.2433	1021.064	24.6777
PSO3	1005.304	391.2469	603.7921	119.2348
PSO4	1057.836	980.0274	1017.561	22.32771

As it is possible to observe, the methods that use direct-repair have higher values in relation to the methods that use constraint-handling. The DE2 method presents the best results in general, while the PSO2 was the one that registers the best results among the PSO tested methods. The STD values for the methods that use constraint-handling are considerably higher than the values resulting from the methods that use direct-repair. This indicates that such methods present less variability, considering that the STD is a measure that indicates the dispersion of the solutions around the mean. The methods that use 50 individuals/particles have better mean results than the methods that use 10 individuals/particles although they did the same number of evaluations.

Fig. 2 shows the average convergence of the method (i.e., the mean value over the 100 simulations for each iteration). Since it is only possible compare the methods with the same numbers of iterations, Fig. 2a presents the convergence of the methods using 1000 iterations and 50 individuals, while Fig. 2b shows the convergence for methods using 5000 iterations and 10 individuals. As it is possible to observe, methods that used the direct-repair have a better convergence overall. DE2 with direct-repair presents the best convergence properties. Despite PSO2 presents worse converge capabilities than DE2, it is better than the methods that use the constraint-handling.



a)



b)

Fig. 2 . Convergence capabilities of the tested methods. a) mean results for 1000 iterations and 50 individuals b) mean results for 5000 itertions and 10 individuals.

Fig. 2 also shows that PSO1 method does not show significant variation throughout the search, which means that the particles get stuck it the point where they initially felt. It can be concluded that the methods are benefited from a larger number of individuals/particles when constraint-handling is used. In general, the DE4 (i.e., using 10 individuals and 5000 iterations) was the method that presents better convergence capabilities among all the tested algorithms.

Fig. 3 shows how the price varies in a trading period depending on the quantity that is traded. It has already been mentioned that there are markets where the price of electricity is constant (1,2 and 3), and variable (4 and 5). These same markets are used to sell and buy electricity.

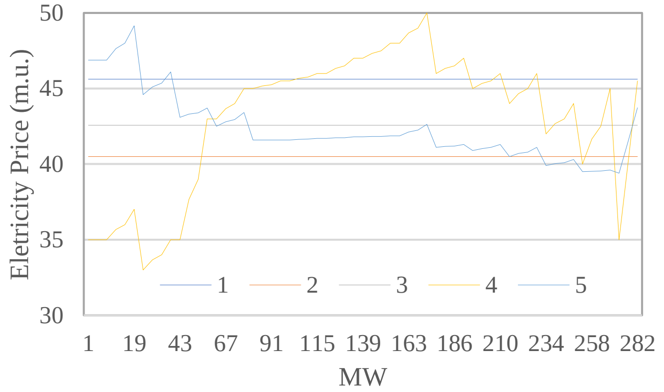


Fig. 3. Eletricity prices of the five considered markets.

TABLE 8, shows the results of the variables that indicate the amount of sales in each market. It is expected that electricity will be sold in the markets where the price is higher. The values presented in the table correspond to the maximum value of the objective function presented in TABLE 7. The method DE2 was the one that presented the highest maximum in the set and makes the sale of the electricity in the market number 4. The same happens with the DE4 that also sells most of the electricity in the market 4, but also sells in market 1 and 4 in smaller quantity. The third is the PSO2, that sells electricity in market 4, although compared to DE2, in less quantity.

TABLE 8. SALES RESULTS IN DIFERENT MAKETS (MW)

Method	Markets Sell				
	1	2	3	4	5
DE1	77.254	0.000	0.000	0.150	18.723
DE2	0.000	0.000	0.000	123.591	0.000
DE3	84.630	0.000	0.000	0.842	3.260
DE4	1.873	0.000	0.000	115.051	2.413
PSO1	61.641	0.000	0.000	0.000	29.132
PSO2	0.000	0.000	0.000	117.459	0.000
PSO3	86.683	0.000	0.000	0.000	0.000
PSO4	0.000	0.000	0.000	112.396	0.000

TABLE 9 shows the results of the purchase of electricity in the different markets as well as the amount of electricity that was generated. As can be observed, in all cases there was electricity generation. In the case of DE2, the total quantity of electricity is bought in all markets and as the possibility to see in TABLE 8, the markets 4 and 5 was used to buy also electricity, that means that both actions were performed in markets 4 and 5. The model sells the electricity in the same market that it bought, although with different price as it is possible to see from Fig. 3. The case of DE4 is similar DE2. DE4 also did purchases in all markets, with the particularity of buying less electricity in market 5, and also generating less electricity. The PSO2 performs similar actions to those of DE2 and DE4, buying electricity in all markets and selling into market 4. It can be observed that, in markets 2, 3, 4 and 5, the best performing methods always seek to buy the maximum amount of electricity because there is a market where the sale of the sum of this electricity is profitable (i.e., market 4). The value of the objective function (presented in TABLE 7) is obtained by performing the balance of the three actions, in which the value of the purchases and generation contributes negatively.

TABLE 9. BUYS RESULTS IN DIFFERENT MARKETS (MW)

Method	**Markets Buy**				**Gen.**
	2	**3**	**4**	**5**	
DE1	25.000	24.935	25.000	0.000	21.191
DE2	25.000	25.000	25.000	25.000	23.590
DE3	12.881	23.873	22.171	9.107	20.699
DE4	25.000	25.000	25.000	23.514	20.822
PSO1	18.297	22.597	23.554	0.000	26.324
PSO2	22.565	24.818	22.957	22.062	25.058
PSO3	24.755	16.244	24.192	0.000	21.492
PSO4	23.239	24.594	22.854	17.170	24.540

The method DE2 shows the highest value in purchase and sale actions. Those values in combination with a medium generation value (compared with the other approaches) gives as a result the highest profits from the tested algorithms.

VI. CONCLUSION

The problem of portfolio optimization that was created in the field of economics and finance has the possibility of being extended to other areas such as the electricity market. With this theory, it is possible to plan the negotiations by the players in order to find solutions that can bring benefits. In this paper, to solve the portfolio optimization problem in electricity markets, two different techniques, the DE and the PSO, are used. Several studies have showed that the performance of the standard DE is highly related to the adoption of a proper set of control parameters, namely the parameters F and Cr. We showed through a systematic parametric analysis such impact in the quality of the solutions, and determined the best set of parameters needed to achieve satisfactory performance of different DE strategies. From the results obtained, the DE algorithms presents better results when comparing to the PSO, reaching the maximum values and relatively higher average profits in the analyzed case study.

As future work, it is intended to build a multi-period model, which allows to obtain the portfolio for all periods. It is also intended to add batteries that will serve to the aggregator to accumulate energy.

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