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Multi-objective design optimization of the Delthaptic, a new 6-DOF haptic device

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***Abstract*—The optimization of the geometric parameters of a mechanism according to some desired objectives is an important aspect of design. The paper presents the multi-objective optimization of a new 6DOF haptic device, the Delthaptic. The design requirements are classified into constraints or objectives according to their priority and their assumed influence. It allows to simply formulate the optimization problem by using a Penalty method. An optimal solution is chosen into the resulting Pareto front and geometric parameters are fixed.**

I. Introduction

The design of a versatile haptic device is a current major research concern. Different types of strutures can be found in the literature as well as commercially available. The first developped haptic devices are based on serial mechanisms such as the Virtuose 6D of Haption, the PHANToM 6DOF of SensAble Technologies [1], or the Haptic Master [2]. To get higher rigidity, lower inertia and better positioning accuracy some authors prefere 6-DOF fully parallel mechanisms. Among them, Yoon et al. [3] use a pantograph linkage mechanism, Lee et al. [4] develop a double-chain leg structure, and Lambert et al. add a grasping DOF to their parallel structure with the PentaG [5] and its 7-DOF new evolution. However parallel manipulators meet singularity issues and have limited workspaces, particularly for rotational motions. Hybrid structures combining parallel and serial mechanisms are chosen as a compromise in several multi-purpose interfaces. Among the most advanced hybrid interfaces one finds the Sigma.7 of Force Dimension, the Falcon of Novint Technologies, or the Delta-R [6], which are all based on a Delta robot for the translational motions and a serial wrist for rotational movements. Numerous other hybrid structures are proposed as haptic devices as the well-known Freedom-7 [7] or the 6-DOF master of Ryu et al. [8] dedicated to mobile manipulation.

The proposed 6-DOF haptic device couples two Delta robots through a handle to obtain rotational and translational motions. In that way the Delthaptic can conserve the benefits of parallel structures while providing a large and singularity-free workspace. This type of robots has been introduced by Lallemand et al. [9] with the 2-Delta. It consists in 6-DOF manipulators pairing two independent well-known parallel structures. Several paired parallel robots can be found in the literature such as the tele-surgery haptic master of Gosselin et al. [10] build by pairing two 5-DOF parallelogram structures, the Heli4 parallel robot [11] introducing a ball screw in a 4-DOF Delta mechanism, or the Dual4 [12] allowing a high rotational range for a Schoenflies motion generator.

Performances of haptic devices are highly dependent on the geometric parameters of the structure. Dimensional synthesis of robots are achieved by many authors in the literature. They developp optimization methods to choose the design parameters according to different requirements. Plenty of optimizations are based on the prescribed workspace as in [13] for the Delta robot. A lot of optimizations also consider dexterity indexes such as the maximum dexterity in [14] and [15] or a dexterous workspace [16]. But other less common criteria are proposed according to specific requirements of the designed structures : stiffness objectives [17], actuator forces [18], mass/inertia [19], or Dynamic Conditioning Index [20]. However the challenge is to find the optimal design that makes the best compromise between all these objectives. Several methods can be used to deal with a multi-objective optimization problem. A first approach consists in treating successively the optimization of the different objectives and compare the best solutions. But the choice of the optimal solution remains a subjective decision. An other technique is to minimize a cost function defined as a weighted sum of the objective functions of the requirements. But cost-function approaches may converge toward a local minima. It also needs to properly choose the weights with methods such as the experimental plans method. A suitable approach of the multi-objective synthesis of a mechanism is to search the domain of optimal solutions that represents the compromise between all the objectives. The Pareto front is a usual representation of this domain of optimal solutions used in several papers [21] [22]. Hao et al. [23] use interval analysis to determine the feasible parameters boxes that satisfy one requirement. The intersection of these boxes is the domain of optimal solutions.

The paper proposes to plot the Pareto front as domain of optimal solutions of the multi-objective optimization. The proposed approach classifies the requirements into functional constraints or optimal objectives. A penalty method is used to transform the constrained problem into an unconstrainted optimization process. The synthesis of the haptic device is carried out with this method. The Pareto frontier of optimal solutions is obtained and a set of geometric parameters is selected to design the Delthaptic.

The paper is organized as follows. In Section II the structural design of the interface is introduced. Then the geometric and kinematic models of the Delthaptic are described. Section III details the dimensional synthesis of the device through the proposed multi-objective optimization.

II. THE DELTHAPTIC DEVICE

A. Structural Design

The Delthaptic is a versatile haptic device with 6 active DOFs designed to ensure a large and singularity-free workspace. To be suitable for various applications its prescribed workspace has been defined in order to reach the human hand motion ranges. In that way, the prescribed translational workspace is chosen within a cube of $200 \times 200 \times 200mm$ in which the rotational workspace is a half-sphere around the handle axis with a self-rotation between $\pm 180°$. The prescribed workspace is discretized in 1716 points P_k for the optimization. This wide rotational workspace represents a challenge for the design of the Delthaptic that traditional parallel manipulators can not achieve. The proposed structure is composed by two Delta robots connected through a handle. In that way it conserves the benefits of parallel manipulators, by having grounded actuators and ensuring high dynamics, high stiffness and high accuracy. Thanks to the paired structure it is able to provide the full prescribed workspace.

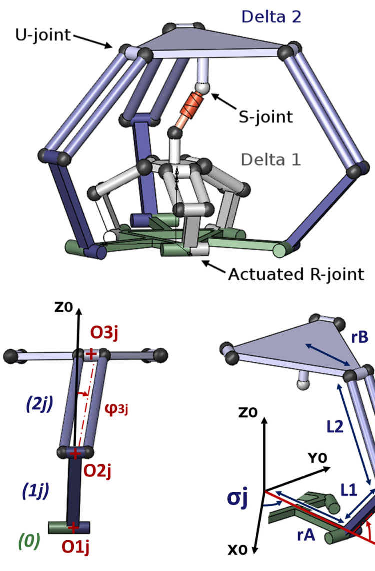

Fig. 1. Delthaptic schematic drawing and parameterization

The mechanism is described in Fig. 1. The both Delta robots i are linked to the same fixed base through their actuated joints driving the angles $\boldsymbol{q_i} = [\varphi_{11,i} \ \varphi_{12,i} \ \varphi_{13,i}]^T$. Their two mobile platforms are connected to the extremities of the handle. The (X, Y, Z) translational movements are obtained by the motions of the Delta 1. The relative displacements of the Delta 2 with respect to the Delta 1 generate the (ϕ, θ) tilt movements of the handle. A ball screw system is added inside the handle to transform the relative motion along the handle axis to the self-rotation (ψ). Its thread is $p = 20mm$ to get a full reversibility of the self-rotation and to be able to transmit the torques. Rotational movements are described by using the tilt-and-torsion modified Euler angles (ϕ, θ, ψ) [24] as shown in Fig. 2.

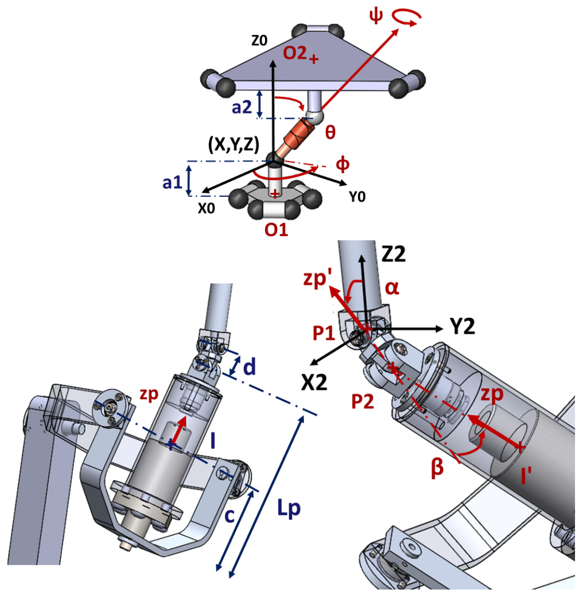

Fig. 2. Delthaptic handle design and parameterization

The design of the handle is detailed in Fig. 2. The geometric parameters of each Delta robot i are $(L_{1,i}, L_{2,i}, r_{A,i}, r_{B,i}, \sigma_{1i}, a_i)$. These parameters are obtained by the optimization process. Additional parameters of the handle such as (c, d) are fixed to be easily handled by an operator. These parameters will be used to define the geometric and kinematic models of the device in the following subsections. The handle length is deduced from the self-rotation according to the ball screw equation: $L_p = L_{ini} + \psi \times \dfrac{p}{2\pi}$ with $L_{ini} = 135mm$.

B. Geometric modeling

The geometric modeling of the Delthaptic allows to analyze its permissible workspace according to the design parameters.

1) Geometric model of the Delta robot: The geometric analysis of the Delta has been widely studied in the literature.

The mobile platform position $\boldsymbol{X_i} = [X_i \ Y_i \ Z_i]^T$ of the i^{th} Delta robot can be calculated from its actuated angles $\boldsymbol{q_i}$ thanks to the Direct Geometric Model (DGM). Each leg j of the Delta leads to the equation of a sphere of radius $L_{2,i}$, centered in $(x_{j,i}, y_{j,i}, z_{j,i})$:

$$(X_i - x_{j,i})^2 + (Y_i - y_{j,i})^2 + (Z_i - z_{j,i})^2 = L_2^2 \quad (1)$$

$$with \begin{cases} x_{j,i} = (r_{A,i} - r_{B,i} + L_{1,i}cos(\varphi_{1j,i}))cos(\sigma_{ji}) \\ y_{j,i} = (r_{A,i} - r_{B,i} + L_{1,i}cos(\varphi_{1j,i}))sin(\sigma_{ji}) \\ z_{j,i} = -L_{1,i}sin(\varphi_{1j,i})) \end{cases}$$

The Delta DGM solution $\boldsymbol{X_i}$ is the intersection of the three spheres. The resolution of the DGM can have two solutions which match the both assembly modes of the Delta.

By transforming this equation for each leg j the Inverse Geometric Model (IGM) can be expressed as follows.

$$\alpha_{j,i}cos(\varphi_{1j,i}) + \beta_{j,i}sin(\varphi_{1j,i}) = \gamma_{j,i} \quad (2)$$
$$\begin{cases} \alpha_{j,i} = 2L_{1,i}(r_{A,i} - r_{B,i} - X_i cos(\sigma_{ji}) - Y_i cos(\sigma_{ji})) \\ \beta_{j,i} = 2L_{1,i}Z_i \\ \gamma_{j,i} = 2(r_{A,i} - r_{B,i})(X_i cos(\sigma_{ji}) + Y_i cos(\sigma_{ji})) - \\ \quad (r_{A,i} - r_{B,i})^2 - L_{1,i}^2 + L_{2,i}^2 - X_i^2 - Y_i^2 - Z_i^2 \end{cases}$$

A solution for IGM exists if $h_{j,i} = \gamma_{j,i}^2 - \alpha_{j,i}^2 - \beta_{j,i}^2$ is negative or nil. By rewriting the constraint equation $h_{j,i}$ into local coordinates of each leg one obtains the equation of a torus. The Delta workspace is given by the intersection volume of the three torus. The distance between the desired platform position $\boldsymbol{X_i}$ and the Delta workspace frontier can be represented by $|h_{j,i}|$. If the condition $h_{j,i} \leq 0$ is verified then the desired position is in the torus intersection, and so is reachable by the Delta.

2) Geometric model of the Delthaptic: As the geometric model of the Delta robot is known, the relationships between the mobile platforms and the handle lead to the Delthaptic models. The link with the platform of the Delta 2 through the spherical joint allows to express the handle axis $\boldsymbol{z_p}$ as a function of the angles α and β defined in Fig. 2.

$$\boldsymbol{z_p} = \begin{vmatrix} \mathbf{s}\beta\ \mathbf{c}\sigma_{12} + \mathbf{s}\alpha\ \mathbf{c}\beta\ \mathbf{s}\sigma_{12} \\ \mathbf{s}\beta\ \mathbf{s}\sigma_{12} - \mathbf{s}\alpha\ \mathbf{c}\beta\ \mathbf{c}\sigma_{12} \\ \mathbf{c}\beta\ \mathbf{c}\alpha \end{vmatrix} = \begin{vmatrix} \mathbf{s}\theta\ \mathbf{c}\phi \\ \mathbf{s}\theta\ \mathbf{s}\phi \\ \mathbf{c}\theta \end{vmatrix} \quad (3)$$

The handle axis can be also expressed by:

$$(L_p - c)\boldsymbol{z_p} = \boldsymbol{IO_2} - d\boldsymbol{z'_p} - a_2\boldsymbol{z_0} \quad (4)$$

On one hand, the Equation (4) expresses α and β with respect to the positions of the both Delta robots $\boldsymbol{X_i}$. The Equation (3) constitutes the DGM of the Delthaptic. It allows to evaluate the handle position and orientation in function of the actuated angles $\boldsymbol{q_i}$ of the two Delta. On the other hand the combination of the two equations leads to the IGM of the haptic device.

$$\begin{cases} X_1 = X - (L_p - L_{ini})\mathbf{s}\theta\ \mathbf{c}\phi \\ Y_1 = Y - (L_p - L_{ini})\mathbf{s}\theta\ \mathbf{s}\phi \\ Z_1 = Z - a_1 - (L_p - L_{ini})\mathbf{c}\theta \\ X_2 = (L_p - c)\mathbf{c}\phi\ \mathbf{s}\theta + d\ \boldsymbol{z'_p}(1) + X1 \\ Y_2 = (L_p - c)\mathbf{s}\phi\ \mathbf{s}\theta + d\ \boldsymbol{z'_p}(2) + Y1 \\ Z_2 = (L_p - c)\mathbf{c}\theta + a_1 + a_2 + d\ \boldsymbol{z'_p}(3) + Z1 \end{cases} \quad (5)$$
$$and \ \ L_p = \psi\frac{p}{2\pi} + L_{ini}$$

C. Kinematic analysis

The kinematic analysis of the Delthaptic is essential to ensure its proper functioning with respect to its singular configurations.

1) Delta robot kinematics: The kinematic model of the Delta robot is fully detailed in numerous papers. The platform velocity of the Delta i is considered as the sum of the contributions of each bar of the leg j. Then the equation is transformed into:

$$\boldsymbol{\dot{X}_i} \cdot \boldsymbol{O_{2j}O_{3j}} = \boldsymbol{\dot{q}_{j,i}} \cdot (\boldsymbol{O_{1j}O_{2j}} \times \boldsymbol{O_{2j}O_{3j}}) \quad (6)$$

The Delta robot kinematic model is obtained by applying (6) to the three legs. In (7), $\boldsymbol{J_i}$ is the Delta i Jacobian matrix, $\boldsymbol{\dot{X}_i}$ the mobile platform velocity vector and $\boldsymbol{\dot{q}_i}$ the articular velocities.

$$\boldsymbol{A_i\dot{X}_i} = \boldsymbol{B_i\dot{q}_i} \ \ with \ \ \boldsymbol{J_i} = \boldsymbol{A_i^{-1}B_i} \quad (7)$$

The Equation (7) demonstrates that the Delta robot has 2 assembly modes (solutions of the direct kinematic model), the platform upper position when $det(\boldsymbol{J_i}) < 0$ and the lower one ($det(\boldsymbol{J_i}) > 0$). It also admits 8 working modes (solutions of the inverse kinematics) with respect to each leg j posture: internal when $B_{ji} > 0$ and external when $B_{ji} < 0$ ($\boldsymbol{B_i} = diag(B_{1i}, B_{2i}, B_{3i})$).

2) Delthaptic kinematics: The link between the handle and the Delta 2 mobile platform leads to (8). The relationship between the handle and the Delta 1 through the gimbal mechanism is expressed by (9), where h represents the handle and s the screw.

$$\begin{aligned} \boldsymbol{V_{I',h/0}} &= \boldsymbol{V_{P_2,h/0}} + \boldsymbol{\omega_{h/0}} \times \boldsymbol{P_2I'} \quad (8) \\ &= \boldsymbol{v_2} + \dot{\alpha}\boldsymbol{x_{1,2}} \times \boldsymbol{P_1P_2} + \boldsymbol{\omega_{h/0}} \times \boldsymbol{P_2I'} \\ \boldsymbol{V_{I',h/0}} &= \boldsymbol{V_{I',s/0}} + \frac{p}{2\pi}\boldsymbol{\omega_{h/s}} \quad (9) \\ &= \boldsymbol{v_1} + \boldsymbol{\omega_{s/0}} \times \boldsymbol{II'} + \frac{p}{2\pi}\boldsymbol{\omega_{h/s}} \end{aligned}$$

The translational velocity of the handle $\boldsymbol{V_{I',h/0}}$ and its angular velocity $\boldsymbol{\omega_{h/0}}$ are deduced from these equations.

The forward kinematic model of the Delthaptic is then written by integrating the two individual Delta robot models.

$$\dot{\boldsymbol{X}} = \boldsymbol{J}\ \dot{\boldsymbol{q}}\ ;\ \dot{\boldsymbol{X}} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}\ ;\ \dot{\boldsymbol{q}} = \begin{bmatrix} \dot{\varphi}_{11,1} \\ \dot{\varphi}_{12,1} \\ \dot{\varphi}_{13,1} \\ \dot{\varphi}_{11,2} \\ \dot{\varphi}_{12,2} \\ \dot{\varphi}_{13,2} \end{bmatrix} \tag{10}$$

$$\boldsymbol{J} = \underbrace{\begin{bmatrix} \begin{bmatrix} \boldsymbol{J_{x1}} \\ \boldsymbol{J_{y1}} \\ \boldsymbol{J_{z1}} \end{bmatrix} & \begin{bmatrix} \boldsymbol{J_{x2}} \\ \boldsymbol{J_{y2}} \\ \boldsymbol{J_{z2}} \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{J_{\omega x1}} \\ \boldsymbol{J_{\omega y1}} \\ \boldsymbol{J_{\omega z1}} \end{bmatrix} & \begin{bmatrix} \boldsymbol{J_{\omega x2}} \\ \boldsymbol{J_{\omega y2}} \\ \boldsymbol{J_{\omega z2}} \end{bmatrix} \end{bmatrix}}_{\boldsymbol{J_c}} \times \begin{bmatrix} \boldsymbol{J_1} & 0 \\ 0 & \boldsymbol{J_2} \end{bmatrix}$$

The proposed approach allows to simply expressed the Delthaptic forward kinematic model as a function of the models of each Delta robot and a linkage matrix $\boldsymbol{J_c}$. The singular configurations are reached when the determinant of the Jacobian matrix $\boldsymbol{J}$ vanishes. The Equation (10) demonstrates that the haptic device singularities are those of the both Delta robots (serial singularities when $det(\boldsymbol{B_i})$ and parallel ones when $det(\boldsymbol{A_i})$).

III. Dimensional synthesis of the Delthaptic for a prescribed workspace

This section describes the multi-objective optimization process of the Delthaptic.

A. *Multi-objective optimization*

1) Method of optimization: The main difficulty of the multi-objective synthesis of mechanisms is to properly formulate the optimization problem. Few studies deal with the choice of the objectives according to their importance and their technological impact. A such analysis of the objectives should yet lead to a better formulation of the optimization problem.

Hao et al. [23] classify the objectives in two groups. The compulsory requirements are essential for a suitable functioning of the mechanism. The other requirements are qualified as relaxable. The classification of the objectives between compulsory or relaxable requirements constitutes the first step of the problem formulation proposed in this paper.

The compulsory requirements represent the constraints that need to be verified. A penalty method is adopted to transform the constrained problem into an unconstrained optimization. In that way, the compulsory requirements are added as penalty components F_{Ci} in each objective function of the optimization problem. Thus, the penalty function becomes $F_C = \alpha_1 F_{C1} + \alpha_2 F_{C2} + ...$. The penalty coefficients $\alpha_1 > \alpha_2 > ...$ are chosen according to the priority order of the compulsory requirements with regard to the mechanism functioning. They must be much more higher than the order of magnitude of the functions of the relaxable requirements. When all the compulsory requirement are checked the penalty function F_C is set to zero.

The relaxable requirements are objectives that should be optimized to have a better design. They lead to the fitness functions F_{Ri} of each objective functions F_i. Thanks to the choice of high penalty coefficient, the fitness functions are minimized when the penalty function is null and so the compulsory requirements verified. Then, the objective function is $F_i = F_c + F_{Ri}$.

The scheme on Fig. 3 summarizes the way to establish the objective functions of the optimization from the different requirements.

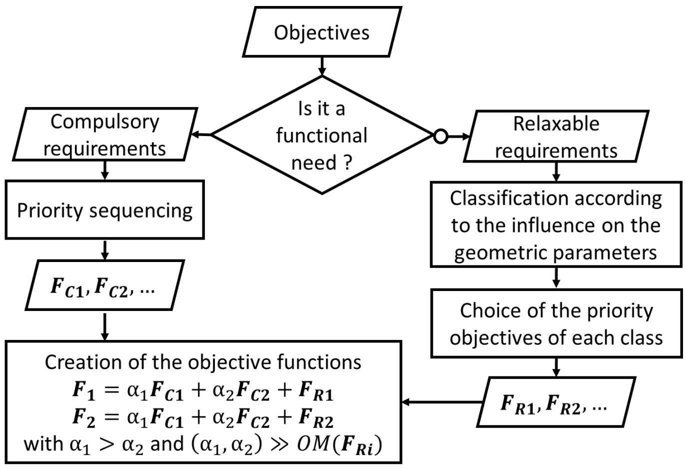

Fig. 3. Construction of the objective functions

A large number of relaxable objectives can be considered, but the optimization of all these requirements would lead to an unclear multi-dimensional domain of optimal solutions. To deal with this issue, the relaxable objectives are classified with respect to their presumed influence on the geometric parameters. Indeed, the minimization of different objectives can lead to the same changes in the design vector. Then, it is sufficient to minimize only one requirement of each class in the optimization process. A priority requirement is arbitrarily selected to constitute the class fitness function F_{Ri}. This classification allows to highly reduce the number of objective functions without changing the optimization result based on technological knowledge. A simpler domain of optimal solutions is obtained which verifies the compulsory requirements and minimizes the relaxable objectives.

2) Formulation of the problem for the Delthaptic: The unknown vector of design parameters for the Delthaptic is $\boldsymbol{I} = [L_{1,1}, L_{1,2}, L_{2,1}, L_{2,2}, r_1, r_2, a_1, a_2, H]$, where $L_{1,i}$ and $L_{2,i}$ are the leg lengths, $r_i = r_{B,i} - r_{A,i}$ is the difference between the both platform radius, a_i is the distance between the mobile platform and the handle joint and H represents the height of the center of the prescribed workspace from the base platform. The orientations of the Delta robots are fixed to $\sigma_{11} = 60$ and $\sigma_{12} = 0$ to avoid collision between the both parallel manipulators.

Several criteria can be taken into account to optimize the

Delthaptic. The studied requirements are the following:
- Access to the prescribed workspace
- Minimize the distance to the prescribed workspaces
- Avoid change of assembly mode
- Avoid change of working mode
- Avoid singular configuration
- Maximize the distance to singular configurations
- Minimize the mass/inertia of the system
- Get a compact device
- Minimize the maximal torque need for the haptic feedback
- Maximize the structure stiffness

Among these objectives, 4 compulsory requirements need to be verify for each point P_k of the discretized prescribed workspace (PW), defined in section II-A. Constraints are expressed as follows.
- Accessibility of PW : $h_{j,i}(P_k) \leq 0$ for the leg j of Delta i.
- No singularity in PW : $det(\boldsymbol{J_i}(P_k)) \neq 0$.
- No assembly mode change in PW: $det(\boldsymbol{J_i}(P_k)) < 0$. The assembly mode with upper position of the mobile platform is chosen because it leads to a larger singularity-free workspace for the Delta.
- No working mode change in PW: $B_{ji}(P_k) < 0$. The working mode with all legs in external posture is chosen because it leads to a higher stiffness of the structure.

By combining the above constraints three penalty components are considered. F_{C1} would lead to the accessibility of PW, F_{C2} to avoid singularity and change of assembly mode, and F_{C3} to fix the working mode. The penalty coefficients are chosen in order to verify them successively. The penalty function F_C is calculated as follows.

$$\left.\begin{aligned} F_{C1} &= \sum_{k=1}^{n}\sum_{i=1}^{2}\sum_{j=1}^{3}(h_{j,i}(P_k) > 0) \\ F_{C2} &= \sum_{k=1}^{n}\sum_{i=1}^{2}(det(\boldsymbol{J_i}(P_k)) \geq 0) \\ F_{C3} &= \sum_{k=1}^{n}\sum_{i=1}^{2}\sum_{j=1}^{3}(B_{j,i}(P_k) \geq 0) \end{aligned}\right\} \; for \; P_k \in PW$$

$$F_C = F_{C1} \times 10^{15} + F_{C2} \times 10^{10} + F_{C3} \times 10^{5} \qquad (11)$$

The remaining objectives are relaxable requirements. It is pretty simple to assume the influence of these objectives on the vector of design parameters I for the haptic device. Indeed a first class of relaxable requirements would tend to reduce the values of the geometric parameters: get a compact device, maximize the structure stiffness, minimize the mass/inertia, be as close as possible to the prescribed workspace. While the other relaxable requirements (maximize the distance to singular configuration, minimize the maximal torque) would induce the global increase of the geometric parameters. These assumptions can be easily proved.

As the relaxable requirements can be classified into two classes of presumed impacts on the design vector, two objectives functions are sufficient to accurately optimize the mechanism. In that way, the first fitness functions is chosen to be F_{R1} that minimizes the distance to the prescribed workspace $\mid h_{j,i}(P_k) \mid$. The second one F_{R2} maximizes the distance to singularities, which can be assimilated to the dexterity of the both Delta robots $\eta_i(P_k) = \dfrac{1}{cond(\boldsymbol{J_i}(P_k))}$ according to (10). It is worth noting that different fitness functions could be selected for each class but would lead to similar results of optimization. The two objective functions $F_i = F_C + F_{Ri}$ are thus defined in (12).

$$F_1 = F_C + \sum_{k=1}^{n}\sum_{i=1}^{2}\sum_{j=1}^{3} \mid h_{j,i}(P_k) \mid \qquad (12)$$

$$F_2 = F_C + \sum_{k=1}^{n}\sum_{i=1}^{2} cond(\boldsymbol{J_i}(P_k))$$

B. Results of the optimization

The geometric parameters I are bounded to limit the optimization to an acceptable global size of the haptic device. The multi-objective optimization is carried out by minimizing the both objective functions F_1 and F_2 with a Genetic Algorithm under MATLAB. The optimal solutions are represented on the Pareto front on the Fig. 4 bellow.

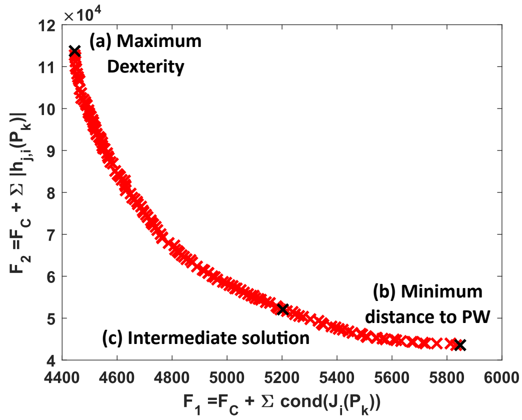

Fig. 4. Pareto front of the multi-objective optimization

The Pareto front represents the domain of optimal mechanisms which verify the compulsory requirements ($F_C = 0$) and minimize the two relaxable objectives (F_{R1},F_{R2}).

TABLE I
GEOMETRIC PARAMETERS OF THREE OPTIMAL DESIGNS

	$L_{1,1}$	$L_{1,2}$	$L_{2,1}$	$L_{2,2}$	r_1	r_2	a_1	a_2	H
a	316	423	406	596	-23	-91	130	127	404
b	277	253	364	421	-34	-98	165	127	387
c	288	355	374	448	-25	-98	128	126	377

Different solutions can be selected according to the major needs of the designer. Three design solutions are compared on the Table I where **(a)** has the maximum dexterity, **(b)** is closer to the prescribed workspace (PW) and **(c)** is an intermediate solution. The solution with the maximum dexterity **(a)** has important leg lengths and so a lower compacity than the two other designs.

The singular configurations are reached when $det(\boldsymbol{J}) = 0$. The Jacobian matrix determinant is plotted in Fig. 5 over the constant orientation workspace at $[\phi, \theta, \psi] = [0, 0, 0]$. Singular configurations are close to the prescribed translational workspace for the design **(b)**. The intermediate solution **(c)** allows to take away the singularity without significant increase of the size of the haptic device. It represents the selected optimal design according to the desired objectives.

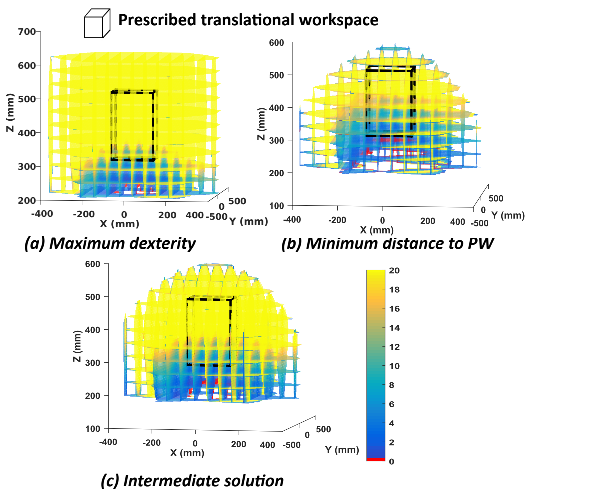

Fig. 5. $\mid det(\boldsymbol{J}) \mid$, constant orientation workspace $[\phi, \theta, \psi] = [0, 0, 0]$

IV. Conclusion

An approach of the multi-objective synthesis of mechanisms is presented in this paper. A penalty method is described to formulate the objective functions. It is proposed to consider the requirements with respect to their priority as penalty functions or as fitness functions. The relaxable requirements are classified by assuming their impact on the design parameters. It is then sufficient to consider only one fitness function by class of objectives. This approach simplifies the problem formulation. The multi-objective optimization is carried out on a new 6-DOF haptic device, the Delthaptic. An optimal solution is chosen and geometric parameters are fixed.

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References

[1] T. H. Massie and J. K. Salisbury, "The phantom haptic interface: A device for probing virtual objects," in *Proceedings of the ASME winter annual meeting, symposium on haptic interfaces for virtual environment and teleoperator systems*, vol. 55, no. 1, 1994, pp. 295–300.

[2] R. Q. Van der Linde, P. Lammertse, E. Frederiksen, and B. Ruiter, "The hapticmaster, a new high-performance haptic interface," in *Proc. Eurohaptics*, 2002, pp. 1–5.

[3] J. Yoon and J. Ryu, "Design, fabrication, and evaluation of a new haptic device using a parallel mechanism," *IEEE/ASME Transactions on Mechatronics*, vol. 6, no. 3, pp. 221–233, 2001.

[4] J. Lee and K. Eom, "Design of a new 6-dof parallel haptic device," in *IEEE International Conference on Robotics and Automation, ICRA 2001*, vol. 1, 2001, pp. 886–891.

[5] P. Lambert, H. Langen, and R. M. Schmidt, "A novel 5 dof fully parallel robot combining 3t1r motion and grasping," in *ASME 2010 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2010, pp. 1123–1130.

[6] J. Arata, H. Kondo, N. Ikedo, and H. Fujimoto, "Haptic device using a newly developed redundant parallel mechanism," *IEEE Transactions on Robotics*, vol. 27, no. 2, pp. 201–214, 2011.

[7] V. Hayward, P. Gregorio, O. Astley, S. Greenish, M. Doyon, L. Lessard, J. McDougall, I. Sinclair, S. Boelen, X. Chen *et al.*, "Freedom-7: A high fidelity seven axis haptic device with application to surgical training," in *Experimental Robotics V*, 1998, pp. 443–456.

[8] D. Ryu, J.-B. Song, C. Cho, S. Kang, and M. Kim, "Development of a six dof haptic master for teleoperation of a mobile manipulator," *Mechatronics*, vol. 20, no. 2, pp. 181–191, 2010.

[9] J. Lallemand, A. Goudali, and S. Zeghloul, "The 6-dof 2-delta parallel robot," *Robotica*, vol. 15, no. 04, pp. 407–416, 1997.

[10] F. Gosselin, C. Bidard, and J. Brisset, "Design of a high fidelity haptic device for telesurgery," in *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*, 2005, pp. 205–210.

[11] S. Krut, O. Company, V. Nabat, and F. Pierrot, "Heli4: a parallel robot for scara motions with a very compact traveling plate and a symmetrical design," in *Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on*, 2006, pp. 1656–1661.

[12] O. Company, F. Pierrot, V. Nabat, and M. de la O Rodríguez, "Schoen flies motion generator: A new non redundant parallel manipulator with unlimited rotation capability," in *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*, 2005, pp. 3250–3255.

[13] M. Laribi, L. Romdhane, and S. Zeghloul, "Analysis and dimensional synthesis of the delta robot for a prescribed workspace," *Mechanism and machine theory*, vol. 42, no. 7, pp. 859–870, 2007.

[14] S. Bai, "Optimum design of spherical parallel manipulators for a prescribed workspace," *Mechanism and Machine Theory*, vol. 45, no. 2, pp. 200–211, 2010.

[15] M. Arsenault and R. Boudreau, "The synthesis of three-degree-of-freedom planar parallel mechanisms with revolute joints (3-rrr) for an optimal singularity-free workspace," *Journal of Robotic Systems*, vol. 21, no. 5, pp. 259–274, 2004.

[16] Z. Wang, S. Ji, Y. Wan, C. Ou, J. Sun, and G. Wang, "Optimal design of parallel robots for the prescribed regular dexterous workspace," in *IEEE International Conference on Automation and Logistics*, 2007, pp. 563–568.

[17] E. Courteille, D. Deblaise, and P. Maurine, "Design optimization of a delta-like parallel robot through global stiffness performance evaluation," in *Intelligent Robots and Systems, 2009. IROS 2009. IEEE/RSJ International Conference on*, 2009, pp. 5159–5166.

[18] Y. Zhao, "Dynamic optimum design of a three translational degrees of freedom parallel robot while considering anisotropic property," *Robotics and Computer-Integrated Manufacturing*, vol. 29, no. 4, pp. 100–112, 2013.

[19] R. Unal, G. Kiziltas, and V. Patoglu, "A multi-criteria design optimization framework for haptic interfaces," in *Haptic interfaces for virtual environment and teleoperator systems, 2008. haptics 2008. symposium on*, 2008, pp. 231–238.

[20] O. Ma and J. Angeles, "Optimum design of manipulators under dynamic isotropy conditions," in *IEEE International Conference on Robotics and Automation, ICRA*, 1993, pp. 470–475.

[21] R. Kelaiaia, O. Company, and A. Zaatri, "Multiobjective optimization of a linear delta parallel robot," *Mechanism and Machine Theory*, vol. 50, pp. 159–178, 2012.

[22] Z. Gao, D. Zhang, and Y. Ge, "Design optimization of a spatial six degree-of-freedom parallel manipulator based on artificial intelligence approaches," *Robotics and Computer-Integrated Manufacturing*, vol. 26, no. 2, pp. 180–189, 2010.

[23] F. Hao and J.-P. Merlet, "Multi-criteria optimal design of parallel manipulators based on interval analysis," *Mechanism and machine theory*, vol. 40, no. 2, pp. 157–171, 2005.

[24] I. Bonev, D. Zlatanov, and C. Gosselin, "Advantages of the modified euler angles in the design and control of pkms," in *2002 International Conference of Parallel Kinematic Machines*, 2002, pp. 171–188.