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Joint Access Point Placement and Power-Channel-Resource-Unit Assignment for IEEE 802.11ax-Based Dense WiFi Network with QoS Requirements*

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***Abstract*—IEEE 802.11ax is the standard for the new generation WiFi networks. In this paper, we formulate the problem of joint access point (AP) placement and power-channel-resource unit assignment for 802.11ax-based dense WiFi. The objective is to minimize the number of APs. Two quality-of-service (QoS) requirements are to be fulfilled: (1) a two-tier throughput requirement which ensures that the throughput of each station is good enough, and (2) a fault tolerance requirement which ensures that the stations could still use WiFi even when some APs fail. We prove that this problem is NP-hard. To tackle this problem, we first develop an analytic model to derive the throughput of each station under the OFDMA mechanism and a widely used interference model. We then design a heuristic algorithm to find high-quality solutions with polynomial time complexity. Simulation results under both fixed-user and mobile-user cases show that: (1) when the area is small (50 $\times$ 50 m^2), our algorithm gives the optimal solutions; when the area is larger (80 $\times$ 60 m^2), our algorithm can reduce the number of APs by 34.9-87.7% as compared to the Random and Greedy algorithms. (2) Our algorithm can always get feasible solutions that fulfill the QoS requirements.**

***Index Terms*—IEEE 802.11ax standard, dense WiFi network, access point placement, power, resource assignment, quality of service, fault tolerance.**

I. Introduction

IN dense WiFi, a set of access points (APs) provide WiFi to many users in a given region (e.g., a stadium), where the distance between any two adjacent APs is small (e.g., $5 \sim 10\ m$ [1]). When a dense WiFi network adopts IEEE 802.11n/ac, stations may suffer from low throughput for two reasons [2]: 1) When many stations use WiFi at the same time, there would be significant chance that their backoff counters would reach zero simultaneously and consequently there would be significant frame collisions. 2) Neighboring APs would cause significant interference among themselves, reducing the number of concurrent transmissions in different basic service sets [3].

IEEE 802.11ax has been developed as the new generation WiFi standard [4] [5]. It is promising for dense WiFi because it possesses the following desirable features: 1) It improves the throughput by a factor of at least 4 [6] as compared to IEEE 802.11n/ac [7]. 2) It adopts Orthogonal Frequency Division Multiple Access (OFDMA) [8] [9] which employs multiple subcarriers and these subcarriers are divided into multiple resource units (RUs) [10] [11], such that the RUs can be properly allocated to stations for concurrent transmission. 3) It adopts Downlink/Uplink Multi-user Multiple-input Multiple-output (DL/UL MU MI-MO), which improves the throughput by using multiple spatial streams [12] [13]. 4) It uses trigger frame such that an AP can coordinate the stations for concurrent transmissions within its basic service set. 5) It adopts both the 2.4 GHz and 5 GHz bands which provide more non-overlapping channels to reduce the signal interference.

When IEEE802.11ax is adopted for dense WiFi, the resulting performance depends on two crucial factors. The first factor is AP placement (i.e., where to place each AP in the given region). The second factor is resource assignment for the APs and the stations, where the resources include: power, channel and resource unit. To the best of our knowledge, there is little research on joint AP placement and power-channel-RU assignment for IEEE 802.11ax-based dense WiFi networks (for details, please refer to Tables I and II and Section II).

There are two application scenarios for dense WiFi:

- **Application Scenario 1:** Users are located at fixed locations (e.g., they are sitting at fixed seats in a stadium, a recital hall or a concert hall).
- **Application Scenario 2:** Users are moving around (e.g., they are moving to visit the booths in an exhibition hall).

Both scenarios are considered in our work. Our problem has the following salient features:

- **Dense users:** We consider a given region where there are many potential users located at known locations (e.g., fixed seats in a stadium, a recital hall or a concert hall).
- **Two-tier throughput requirement:** One simple QoS requirement is to ensure that the throughput of every station is at least equal to a given value. While this requirement is simple, it is costly in dense WiFi scenarios

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TABLE I
DIFFERENCES BETWEEN OUR WORK AND THE EXISTING WORKS

Work	For Dense WiFi?	Joint AP Placement and Power-Channel-RU Assignment?	Support 802.11ax?	Satisfy Two-tier Throughput Requirement?	Tolerate the Failure of n APs?	Minimize Cost?	Remarks
Our work	Yes	Yes	Yes	Yes	Yes	Yes	Minimize the number of APs
[14]	No	No	No	No	No	Yes	Minimize the number of APs
[18]	No	No	No	No	No	No	Maximize the total throughput
[19] [20]	No	No	No	No	No	Yes	Minimize the number of APs
[21]	No	No	No	No	No	Yes	Minimize the total cost of APs
[22] [23]	No	No	No	No	No	No	Maximize the coverage
[24]	No	No	No	No	No	No	Minimize total transmission power of APs
[25]	Yes	No	No	No	Yes	No	Re-allocate resources when n APs fail
[26]	No	No	No	No	Yes ($n = 1$)	No	Re-allocate resources when one AP fails

because there are many stations, some stations may be located at undesirable locations, and it may be necessary to set up many APs in order to ensure large enough throughput for every individual station. To be more cost-effective in dense WiFi environments, we introduce a two-tier throughput requirement as follows: the throughput of at least β% of the stations is at least ρ_H while the throughput of the remaining stations is at least ρ_L (e.g., the throughput of at least 95% of the stations is at least 1 Mbps while the throughput of the remaining stations is 0.5 $\sim$ 1 Mbps). β (called user satisfaction ratio), ρ_H and ρ_L ($\rho_H > \rho_L$) are design choices specified by the network designer. If they have larger values, better WiFi services are provided at higher cost.

- **Fault tolerance requirement:** In a dense WiFi network, many APs are adopted and there would be significant chance that some of these APs fail. To handle this issue, we adopt the following fault tolerance requirement: when n APs fail, each station associated with the failed AP can be re-associated with a functioning AP. n is called the fault tolerance degree and it is a given requirement where a larger n would result in better fault tolerance at higher cost.
- **Joint AP placement and power-channel-RU assignment in the planning phase:** We consider the planning phase and minimize the cost of the dense WiFi (in terms of the number of APs) while fulfilling two QoS requirements (i.e., two-tier throughput requirement and fault tolerance requirement). Both the objective function (i.e., cost) and the constraints (i.e., QoS requirements) depend on AP placement, power assignment, channel assignment and resource unit assignment. For example, the throughput in the two-tier throughput requirement is affected by all these four factors. These four factors are inter-dependent on each other (e.g., if the AP placement is changed, the channel assignment, the power assignment and the resource unit assignment should also be changed accordingly). To solve the dense WiFi deployment problem in the planning phase, it is necessary to jointly perform AP placement, power assignment, channel assignment and resource unit assignment.

The contributions of this paper include the following:

1) **New Problem:** We consider an IEEE 802.11ax-based dense WiFi network in a given region with many potential users at given locations. We jointly optimize the AP placement, power assignment, channel assignment and resource unit assignment, where the objective is to minimize the cost in terms of the number of APs while fulfilling a given two-tier throughput requirement and a given fault tolerance requirement. Table I shows that this new problem is different from the existing ones in the literature. We prove that this problem is NP-hard.

2) **New Solution:** We first develop an analytic model to derive the throughput of each station under the OFDMA mechanism and a widely used interference model. Then we design a heuristic algorithm to find high-quality solutions with polynomial time complexity. We conduct extensive simulations with comprehensive parameter settings to demonstrate that the proposed heuristic algorithm is efficient.

The remainder of this paper is organized as follows. Section II reviews the related work. Section III formulates the problem and Section IV describes how we obtain the throughput of stations and presents our heuristic algorithm. Section V defines the modified problems and solutions. Section VI describes our experiments and presents the performance of our algorithm, and lastly, Section VII concludes this paper.

II. RELATED WORK

A. IEEE 802.11ax-Based Dense WiFi Network

IEEE 802.11ax-based dense WiFi network has been attracting researchers' attention recently. Bellalta et al. [1] described the characteristics of IEEE 802.11ax and presented some of the network-level functionalities that are required to improve the user experience in dense WiFi scenarios. Deng et al. [2] pointed out that IEEE 802.11ax will fuel the future intelligent information infrastructure for big data transfer and various mobile applications. Deng et al. [15] discussed the challenges for IEEE 802.11ax in the design of physical layer and medium access control (MAC) sub-layer. Furthermore, they presented the expected features on the MAC protocol design to provide better QoS support in the IEEE 802.11ax-based dense WiFi network [16]. Afaqui et al. [17] disclosed advanced technological enhancements presented in IEEE 802.11ax to improve the user throughput within a dense WiFi network. The above results show that IEEE 802.11ax-based dense WiFi network will become popular in the near future.

B. AP Placement and Fault Tolerance

AP placement has been intensively investigated. Ling et al. [18] designed an algorithm to jointly solve the two problems of AP placement and channel assignment for better network services. Zheng et al. [19] studied the AP placement problem under the physical interference model. They proposed an algorithm aiming to minimize the number of APs. In [20], an AP placement optimization problem is formulated. Its objective is to determine the optimal placement of APs such that the number of APs is minimized. Zhang et al. [21] addressed the AP placement problem that AP can be equipped with multiple radios. They found out the optimal AP placement such that the total cost of all APs is minimized. Zhang [22] et al. presented an optimization framework of AP placement, whose aim is to maximize the signal coverage. Kiran [23] et al. focused on the optimization of the AP placement and maximized the coverage by optimizing power allocation. Audhya [24] et al. addressed the problem of optimally placing the APs in an ultra-dense 5G network to cover a given region. Their objective is to minimize the total transmission power of APs.

In addition, AP fault tolerance is an important research issue. Zhou et al. [14] studied the problem of enhancing the fault tolerance of a traditional WiFi network in the design stage. They considered the situation that when one AP fails, the stations it serves shall switch to other APs and a certain percentage of traffic demands of these affected stations shall still be met. Liu et al. [25] proposed a self-healing scheme to provide a continuous service for users in ultra-dense network. In their scheme, when one or more APs fail, each normal AP can autonomously perform resource re-assignment for supporting its own users as well as the affected users. Moreover, Lee et al. [26] proposed a resource allocation algorithm to overcome the unforeseen AP failures.

However, none of the above works combine AP placement and AP fault tolerance and take into account the IEEE 802.11ax-based dense WiFi scenarios with the fault tolerance and two-tier throughput requirements.

C. Resource Assignment for WiFi Network

Resource assignment such as power, channel and RU assignment is a key factor that affects the performance of WiFi.

In terms of power control, Guessous et al. [27] designed a power control approach to reduce co-channel interference and maximize transmission opportunities for stations. Besides, to improve the throughput, Kim et al. [28] proposed a power control algorithm in which fine-grained power adjustment is enabled by co-channel interference detection and estimation in the middle of data frame reception. Furthermore, Su et al. [29] and Hoefel et al. [30] presented two power control schemes to improve the throughput or provide fairer transmission mechanism. These studies show that reasonable power control strategy is important in improving the network performance.

In fact, channel assignment also plays a key role in deploying high-performance WiFi network. Ribeiro et al. [31] presented a multi-factor channel assignment approach to reduce the packet loss ratio. Lei et al. [32] took into account both the channel overlapping degree and the distribution of stations and designed a channel selection scheme to improve the network throughput. Raschella et al. [33] presented a channel assignment algorithm to minimize the interference impact and improve the data transmission quality. Moreover, to improve both throughput and fairness, Oh et al. [34] and Kasasbeh et al. [35] studied the co-channel interference problem and proposed efficient channel assignment methods to eliminate the interference between adjacent APs.

In addition to power control and channel allocation, RU assignment also has a great influence on network performance. Karthik et al. [10] proposed a contention-based RU allocation method, where the AP transmits frames by determining the RUs to be allocated to the stations. This method improves the network QoS. Wu et al. [36] presented the High Throughput RU Assignment Scheme (HiTRAS), which allocates RUs to multiple stations so that they can transmit simultaneously and thus high throughput is gained. To reduce frame collisions, Lanante et al. [37] equiped Uplink OFDMA Random Access (UORA) with carrier sensing capability and proposed the hybrid UORA (H-UORA) RU assignment algorithm, which improves the network throughput.

These works, however, do not consider the joint design of AP placement and power-channel-RU assignment, and therefore cannot be applied to solve our problem directly.

D. Optimization for Dense 5G Networks

The existing works on dense 5G networks address cell planning, radio resource control, and power control, etc. Specifically, Rezaabad et al. [38] studied the cell planning problem for the 5G network with both wired base stations and unwired base stations. They applied a genetic algorithm to minimize the cost while maximizing the coverage. Zhou et al. [39] proposed a LSTM (Long short-term memory)-based radio resource control algorithm to avoid/alleviate the congestion in dense 5G networks, improving the network throughput. Diez et al. [40] designed a Lagrange approximation Supple Radio controller to assign radio resources to users in an OFDMA-based multi-RAT (radio access technology) system, guaranteeing the stability of the network. Jacob et al. [41] proposed a spectrum sharing technique using 5G enabled bidirectional cognitive deep learning nodes along with dynamic spectrum sharing LSTM, improving the spectrum efficiency. Shahid et al. [42] presented a framework for sub-channel allocation and power allocation for ultra-dense networks, maximizing the energy efficiency of dual-access small cells. Yang et al. [43] maximized the energy efficiency in dense 5G networks. While these existing results could effectively enhance dense 5G networks, they are not applicable to our problem because of the fundamental differences shown in Table II.

III. Problem Formulation

In this section, we formulate the problem of joint AP placement and power-channel-RU assignment for IEEE 802.11ax-based dense WiFi networks. The objective is to minimize the number of APs while fulfilling two QoS requirements (i.e., two-tier throughput requirement and fault tolerance requirement). The quantities to be optimized include the following:

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TABLE II
MAIN DIFFERENCES BETWEEN OUR WORK AND THE EXISTING WORKS ON DENSE 5G NETWORKS.

Work	For Indoor?	Objective	QoS Requirement 1: Satisfy Two-tier Throughput Requirement?	QoS Requirement 2: Tolerate the failure of n APs?	Joint AP Placement and Power-Channel-RU Assignment?
Our work	Yes	Minimize cost	Yes	Yes	Yes
[38]	No	Minimize cost; Maximize coverage	No	No	No
[39]	No	Improve throughput	No	No	No
[40]	No	Guarantee system stability	No	No	No
[41]	No	Improve spectrum efficiency	No	No	No
[42] [43]	No	Maximize energy efficiency	No	No	No

number and positions of APs, power assignment, channel assignment, and RU assignment. These quantities are inter-dependent (e.g., if the AP placement is changed, the best power assignment, channel assignment and RU assignment are also changed), so it is necessary to jointly optimize these quantities. We present a network model in Section III-A and an interference model in Section III-B. Based on these models, we formulate the problem in Section III-C. We prove that this problem is NP-hard in Section III-D.

The symbols used in our model are shown in Table III.

TABLE III
SYMBOLS USED IN OUR MODEL

Symb	Meaning	Symb	Meaning
A	The set of APs	A_f	The set of fault APs
S	The set of stations (STA)	B	The set of channel widths
P	The set of power levels	C	The set of channels
K	Subcarrier number set	R_i	The data rate of STA i
δ_i	The throughput of STA i	$\delta_i^{(H)}$	1 if $\delta_i \geq \rho_H$, 0 otherwise
$I_{i,j}$	The interference range between APs i and j	$\delta_i^{(L)}$	1 if $\rho_H > \delta_i \geq \rho_L$, 0 otherwise
Ω	The set of AP candidate locations	$N(i)$	The set of neighboring APs of AP i
$A(i)$	The set of APs that cover STA i	$S(i)$	The set of STAs that associate with AP i

A. The Network Model

We make use of the following features of IEEE 802.11ax in our network model: OFDMA physical layer, resource unit, trigger frame, and dual bands etc. Below, we describe our network model in detail.

Our network consists of three kinds of devices: network controller [44], APs, and stations (STAs). The network controller is responsible for AP management and coordination [44]. It has global knowledge about the WiFi network. It can obtain the neighboring AP list of any AP and the channel being used by each AP. Thus, in our network, APs do not need to backoff before transmission under the coordination of the network controller.

We assume that the set of possible AP locations, denoted by Ω, is given beforehand. The target region is divided into multiple cells and APs can be placed only at the center of them. Within one cell location, 0, 1, or more APs can be placed according to the density of STAs. Hence, the elements in set Ω are the coordinates of the centers of all the cells.

Denote S and A the set of STAs and the set of APs, respectively. Any STA $i \in S$ can only associate with one AP $j \in A$ during a period of time. When a set of APs, A_f ($|A_f| < |A|$), fail, the STAs previously associated with the failed APs can re-associate with other APs $\in A \setminus A_f$ to obtain the WiFi services.

We adopt both the 2.4 and 5 GHz bands, in which a channel with b MHz ($b \in B$, where $B = \{20, 40, 80, 160\}$ MHz [5]) bandwidth can be assigned to an AP. The OFDMA physical layer is adopted. Instead of the whole channel, the RUs in the channel can be assigned to STAs for simultaneous transmissions. Each AP can only be assigned a channel in a given channel set, denoted by C. Each STA can only be assigned a j-tone RU [5] ($j \in K$, where K is the set of subcarrier numbers).

In addition, each AP is assigned a power level in a given power level set, denoted by P. Instead of setting lower power level for the STAs, we set the power level of the STAs to be the same as that of their AP [20]. This is equivalent to increasing the power spectral density of the STAs, which allows increasing modulation and coding scheme for uplink (UL) OFDMA transmissions. With this setting, the data rate of STAs for UL traffic is the same as that of downlink (DL) traffic.

The frame exchange procedure is shown in Fig. 1 [16], where TXOP, SIFS, M-BA, and OFDMA-BA represent the transmission opportunity, the short interframe space, the multi-station block ACK, and the OFDMA block ACK, respectively [5]. Under the OFDMA mechanism, STAs start to transmit uplink physical layer protocol data unit (UL PPDU) to their AP only after receiving a trigger frame (TF) that specifies which STAs are going to participate in subsequent frame exchange. The STAs reply an OFDMA-BA frame to the AP after receiving the DL PPDU.

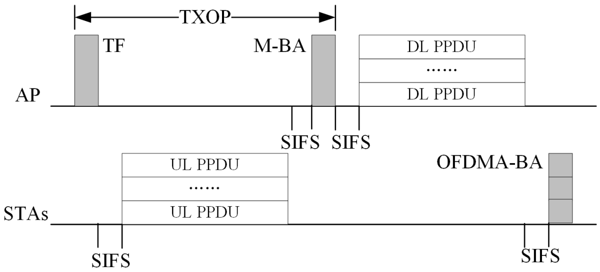

Fig. 1. Frame exchange procedure between an AP and its STAs [16].

B. The Interference Model

Let $l_{i,j}$ denote the link between nodes i and j (here, a node refers to an AP or a STA). To let node i receive frame properly from j over link $l_{i,j}$, the received signal strength (RSS) at node i must be no less than the frame decoding threshold θ_D [21]. In this case, we say that node i is within the transmission range of j and vice versa. In addition, node i is said to be interfered by j (here, nodes i and j are on different links whose channels overlap with each other) if the signal strength received by node i from j is greater than or equal to the interference signal strength threshold θ_I [21]. In this case, we say that node i is within the interference range of j and vice versa. Usually, $\theta_D > \theta_I$. To obtain the communication ranges and the interference ranges of APs, we resort to the following path loss model [45]: $RSS = P_j + G_{TX} - P_{lost} + G_{RX}$, where $P_{lost} = P_{ref} + 10\lg(d^{\eta}) + \chi$. Here, RSS is the received signal strength at the receiver; d is the distance between the sender and the receiver; P_j is the power level of sender node j; G_{TX} and G_{RX} are the antenna gains of the sender and the receiver respectively; P_{ref} is the path loss at a reference distance (which is usually 1 m); η is the path-loss exponent; and χ is the standard deviation associated with the degree of shadow fading. Thus, $d = \sqrt[\eta]{10^{(P_j+G_{TX}-P_{ref}-\chi+G_{RX}-RSS)/10}}$. Let r_j and γ_j denote the communication range and the interference range of node j, respectively. We have $d = r_j$ if $RSS = \theta_D$ and $d = \gamma_j$ if $RSS = \theta_I$. Next, we introduce the interference model [46]. Let $l_{i,x}$ and $l_{j,y}$ denote the links between AP i and STA x, and AP j and STA y, respectively. Let $d_{i,x}$ and $d_{j,y}$ denote the distance between AP i and STA x, and AP j and STA y, respectively. Let γ_x and γ_y denote the interference range of STA x and STA y, respectively. Fig. 2 depicts the interference ranges of links $l_{i,x}$ (the region enclosed by dotted line) and $l_{j,y}$ (the region enclosed by solid line).

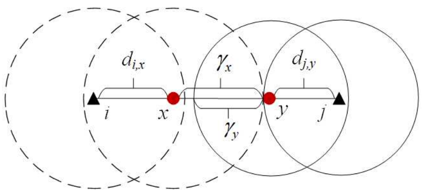

Fig. 2. Interference range between APs i and j [46].

Denote $S(i)$ and $S(j)$ the sets of STAs that are associated with APs i and j, respectively. According to Fig.2, we define the interference range between APs i and j, $I_{i,j}$, as

$$I_{i,j} \triangleq \max_{x \in S(i)} \{d_{i,x}\} + \max\{\gamma_x, \gamma_y\} + \max_{y \in S(j)} \{d_{j,y}\}. \quad (1)$$

If the distance between APs i and j is less than or equal to $I_{i,j} (i \neq j)$ and their channels overlap with each other, then links $l_{i,x}$ and $l_{j,y}$ interfere with each other. That is, they cannot be active simultaneously [21].

C. The Optimization Problem

The objective of our problem is to find out the minimum number of APs under the constraints that the fault tolerance and the two-tier throughput requirements can be fulfilled. Denote δ_i the throughput of STA i. Our problem can be formulated as

$$(\mathcal{P}_1) : \min |A|$$
$$s.t. \begin{cases} \text{C1:} & \sum_{j=1}^{|A \backslash A_f|} a_{i,j} = 1, i \in S; \\ \text{C2:} & \sum_{i=1}^{|S|} \delta_i^{(H)} \geq |S| \times \beta\%; \\ \text{C3:} & \sum_{i=1}^{|S|} (\delta_i^{(L)} + \delta_i^{(H)}) = |S|. \end{cases} \quad (2)$$

Here, $a_{i,j}$ is equal to 1 if STA i associates with AP j ($j \in A \backslash A_f$), 0 otherwise; $\delta_i^{(H)}$ is equal to 1 if $\delta_i \geq \rho_H$, 0 otherwise; $\delta_i^{(L)}$ is equal to 1 if $\rho_H > \delta_i \geq \rho_L$, 0 otherwise.

In (26), constraint C1 indicates that any STA $i \in S$ can associate with one AP $j \in A \setminus A_f$ when $|A_f| = n$ APs fail. Constraint C2 ensures that the throughput of at least β % of STAs is greater than or equal to ρ_H. Constraint C3 ensures that the throughput of $(100 - \beta)$ % of STAs is greater than or equal to ρ_L. We call C1 **fault tolerance requirement**, and C2 and C3 **two-tier throughput requirement**. To check whether C1 can be met, we need to deal with the AP placement problem, and to check whether C2 and C3 can be met, we need to obtain the throughput of STAs, which is based on solving some other problems, such as power, channel and RU assignment etc. In other words, our problem $\mathcal{P}_1$ contains several subproblems, which make it more complex.

D. Hardness Analysis of Our Problem

Theorem: Problem $\mathcal{P}_1$ is NP-hard.

Proof: We prove this theorem by reduction from the decision version of **set cover problem**, which is well-known to be NP-hard. The decision version of set cover problem can be described as follows. Given a finite set $\mathcal{X} = \{x_1, x_2, ..., x_{|\mathcal{X}|}\}$ and a set of subsets of $\mathcal{X}$, $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_{|\mathcal{F}|}\}$ where $\mathcal{F}_j \subseteq \mathcal{X}$ (i.e., set $\mathcal{F}_j$ covers one or more elements of set $\mathcal{X}$) and

$$\mathcal{X} = \bigcup_{\mathcal{F}_j \in \mathcal{F}} \mathcal{F}_j (j = 1, 2, ..., |\mathcal{F}|). \quad (3)$$

Does there exist a subset of $\mathcal{F}$, $\mathcal{F}' \subseteq \mathcal{F}$ with minimum size of $|\mathcal{F}'|$ that covers all elements of $\mathcal{X}$? The above decision version of set cover problem can be formulated as

$$(\mathcal{P}_2) : \text{whether exist}$$
$$\min |\mathcal{F}'|$$
$$s.t. \begin{cases} \mathcal{X} = \bigcup_{\mathcal{F}_j \in \mathcal{F}'} \mathcal{F}_j (j = 1, 2, ..., |\mathcal{F}'|); \\ \mathcal{F}' \subseteq \mathcal{F}. \end{cases} \quad (4)$$

Denote $\mathcal{P}_1^{(i)}$ the i-th subproblem of our problem $\mathcal{P}_1$, where $\mathcal{P}_1^{(1)}$ = "AP placement problem", $\mathcal{P}_1^{(2)}$ = "power assignment problem", $\mathcal{P}_1^{(3)}$ = "channel assignment problem", $\mathcal{P}_1^{(4)}$ = "RU assignment problem", etc. That is, our problem

$$\mathcal{P}_1 = \bigcup_{i=1,2,...} \mathcal{P}_1^{(i)}. \quad (5)$$

Obviously, the hardness of any subproblem $\mathcal{P}_1^{(i)}$ is smaller or equal to $\mathcal{P}_1$, denoted by $\mathcal{P}_1^{(i)} \leq \mathcal{P}_1$. If there is a NP-hard subproblem $\mathcal{P}_1^{(i)}$ in $\mathcal{P}_1$, then $\mathcal{P}_1$ is also NP-hard.

Here, let's study the decision version of $\mathcal{P}_1^{(1)}$ = "AP placement problem" in detail. We just consider the **basic AP placement problem**, in which the fault tolerance degree n is 0 and each AP candidate location in set Ω is limited to be placed zero or one AP. The decision version of this basic AP placement problem can be described as follows. Given a set of STAs $\mathcal{S} = \{s_1, s_2, ..., s_{|\mathcal{S}|}\}$ in the given region and a set of subsets of $\mathcal{S}$, $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_{|\Omega|}\}$ where set $\mathcal{A}_j \subseteq \mathcal{S}$ contains the STAs that are covered by the WiFi signal emitted from the j-th AP that is placed at the j-th AP candidate location in Ω (if the j-th AP candidate location is placed an AP) and

$$\mathcal{S} = \bigcup_{\mathcal{A}_j \in \mathcal{A}} \mathcal{A}_j (j = 1, 2, ..., |\Omega|). \tag{6}$$

Does there exist a subset of $\mathcal{A}$, $\mathcal{A}' \subseteq \mathcal{A}$ with minimum size of $|\mathcal{A}'|$ that covers all STAs of $\mathcal{S}$? The decision version of the basic AP placement problem can be formulated as

$$(\mathcal{P}_1^{(1)}) \text{ :whether exist} \\ \min |\mathcal{A}'| \\ s.t. \begin{cases} \mathcal{S} = \bigcup_{\mathcal{A}_j \in \mathcal{A}'} \mathcal{A}_j (j = 1, 2, ..., |\mathcal{A}'|); \\ \mathcal{A}' \subseteq \mathcal{A}. \end{cases} \tag{7}$$

In (7), the minimum $|\mathcal{A}'|$ is the minimum number of APs that are required to provide WiFi services and the subscripts of the elements in set $\mathcal{A}'$ are the locations (these subscripts are actually the number of the corresponding positions) where the APs are going to be placed.

Further, by comparing (4) and (7), we can easily reduce problem $\mathcal{P}_2$ to problem $\mathcal{P}_1^{(1)}$, which is actually more complicated than the basic AP placement problem, in polynomial time. Thus, we have

$$\mathcal{P}_2 \propto_p \mathcal{P}_1^{(1)} \leq \mathcal{P}_1. \tag{8}$$

Since $\mathcal{P}_2$ is NP-hard, our problem $\mathcal{P}_1$ is therefore NP-hard as well. ■

From (5), we can see that our problem $\mathcal{P}_1$ is much more complicated than set cover problem. As a result, the existing algorithms of the set cover problem cannot be used directly to our problem. Namely, we need to design a new efficient heuristic algorithm for $\mathcal{P}_1$.

IV. Heuristic Algorithm: Design and Analysis

A. Overall Framework

As mentioned earlier, in order to solve optimization problem (26), the throughput of STA i, δ_i ($i \in S$), should be obtained first. When the number and locations of the APs and STAs are given, we obtain the throughput of STAs via STAs-APs association in Section IV-B, power adjustment in Section IV-C, channel assignment and power re-adjustment in Section IV-D, and RU assignment in Section IV-E. Then we analyze the data rate of STAs in Section IV-F and the throughput of STAs in Section IV-G. That is, for a given set S and an AP placement scheme A that satisfies constraint C1 (if A cannot satisfy C1, we add APs to set A until it satisfies C1), we adopt Procedure 1 to obtain δ_i ($i \in S$).

Procedure 1: Resource assignment and obtaining the throughput of STAs
Input : A, S, P, C, etc.
Output: δ_i ($i \in S$).

Step 1. STAs-APs association.
Step 2. Power adjustment for the APs.
Step 3. Channel assignment and power re-adjustment for the APs.
Step 4. RU assignment for the STAs.
Step 5. Obtaining the data rate of STAs.
Step 6. Calculating the throughput of STAs.

In the steps of Procedure 1, we always try our best to improve the throughput of STAs as much as possible by strategically assigning resources (such as power, channel, and RU etc) to APs/STAs (see Section IV-B to IV-E). After δ_i ($i \in S$) is obtained, we can check whether set A fulfils constraints C2 and C3. If no, we add APs to set A until the requirement of C2 and C3 can be met. It's worth noting that, when the number of APs is large enough, it is not always true that C2 and C3 can be met. When there are more APs, there would be more signal interference between neighboring basic service sets. This may reduce the throughput of the users and consequently C2 and C3 may not be met. In general, it is necessary to optimize the number of APs, the locations of APs and resource allocation (where the resources include power levels, channels and resource units), so that C2 and C3 could possibly be met. We may try to reduce the number of APs in set A by removing the redundant APs and iteratively replacing two (or three) nearby APs by one (or two). Based on this idea, we design a heuristic algorithm to solve optimization problem (26) in Section IV-H and analyze its time complexity in Section IV-I.

B. STAs-APs Association

To do the STAs-APs association, we first obtain the set of APs from which their signals can cover STA i, denoted by $A(i)$. We initialize the power level of each AP in P to maximum level, which is going to be adjusted later, to cover as many STAs as possible. That is,

$$P_j = \max\{p_q\}, q \in \{1, 2, ..., |P|\}, j \in A, \tag{9}$$

where p_q denotes the q-th power level in P. If the distance between STA i and AP j, $d_{i,j}$, is less than or equal to the communication range of AP j, r_j, then the signal emitted from AP j can cover STA i. Thus,

$$A(i) = \{\text{AP } j | d_{i,j} \leq r_j\}, i \in S, j \in A. \tag{10}$$

Next, we associate STA i to the AP in $A(i)$ whose signal strength received by STA i is the strongest. If there are more than one AP in $A(i)$ with the strongest signal strength, we associate STA i to the AP with the fewest number of associated STAs. After the STAs-APs association, the set of STAs associated with AP j, $S(j)$, can also be obtained.

C. Power Adjustment

As mentioned earlier, the power level of each AP is initialized to the maximum in P. But in view of the higher the power level, the larger the interference range, we need to adjust the power levels of APs to reduce their interference among each other.

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Denote $r_j^{(q)}$ the communication range of AP j with power level p_q. We have $r_j^{(1)} < r_j^{(2)} <, ..., < r_j^{(|P|)}$ under the assumption of $p_1 < p_2 <, ..., < p_{|P|}$. After the STAs-APs association, we know the maximum distance between AP j and its STAs, $\max_{i \in S(j)}\{d_{i,j}\}$. Then the power level of AP j can be adjusted as

$$P_j = \begin{cases} p_q, & \text{if } \max_{i \in S(j)}\{d_{i,j}\} \in (r_j^{(q-1)}, r_j^{(q)}], q \in [2, |P|]; \\ p_1, & \text{if } \max_{i \in S(j)}\{d_{i,j}\} \le r_j^{(1)}. \end{cases} \tag{11}$$

With P_j ($j \in A$) in hands, we can further obtain the interference range between APs i and j, $I_{i,j} (i \neq j)$, which can help us to efficiently assign channels to the APs.

D. Channel Assignment and Power Re-adjustment

Specifically, we use the wireless spectrums of 5150 $\sim$ 5350 MHz [47] in the 5 GHz band and 2402 $\sim$ 2483 MHz in the 2.4 GHz band. The numbers and locations of the channels used in our model are shown in Fig. 3. Thus, the channel set $C = \{1, 2, ..., 19\}$.

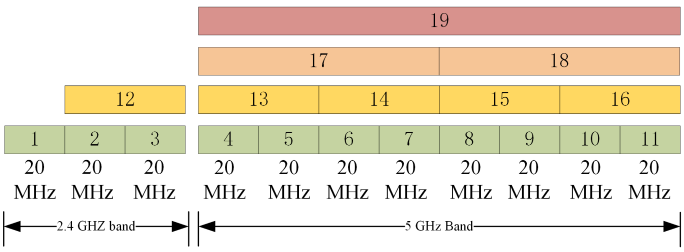


Fig. 3. The channels used in our model.

From Fig. 3 we observe that channel 12, which is composed by bonding channels 2 and 3, overlaps with channels 2 and 3 (actually, channel 12 can also be composed by bonding channels 1 and 2. Here, we use channels 2 and 3 to generate channel 12). Hence, we define two **overlapping channel sets (OCSs)** in the 2.4 GHz band, denoted by $\Gamma_1 = \{2, 12\}$ and $\Gamma_2 = \{3, 12\}$, respectively. Similarly, we define eight OCSs in the 5 GHz band, denoted by $\Gamma_3 = \{4, 13, 17, 19\}$, $\Gamma_4 = \{5, 13, 17, 19\}$, $\Gamma_5 = \{6, 14, 17, 19\}$, $\Gamma_6 = \{7, 14, 17, 19\}$, $\Gamma_7 = \{8, 15, 18, 19\}$, $\Gamma_8 = \{9, 15, 18, 19\}$, $\Gamma_9 = \{10, 16, 18, 19\}$, $\Gamma_{10} = \{11, 16, 18, 19\}$. We call channels 1 $\sim$ 11 **primary channels**.

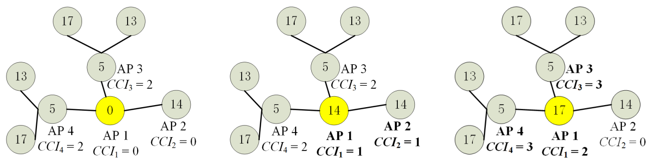
To reduce the interference from overlapping channels when performing channel assignment, we need to obtain the set of neighboring APs of AP i, denoted by $N(i)$, which is defined as

$$N(i) \triangleq \{\text{AP } j | D_{i,j} \le I_{i,j}\}, i, j \in A, i \neq j, \tag{12}$$

where $D_{i,j}$ denotes the distance between APs i and j.

The main policy for reducing the interference from overlapping channels is as follows. For any AP i, if AP $j \in N(i)$ and the non-overlapping channels are enough, then we assign a channel which does not overlap with AP j's channel to AP i. However, when the non-overlapping channels are not enough, the channel assigned to AP i may overlap with AP j's channel. In this case, we try to reduce the total interference degree between AP i and its neighbors as much as possible (see next paragraph). Obviously, if AP $j \notin N(i)$, then the channel assigned to AP i can be the same as that assigned to AP j since they do not interfere with each other.

We introduce a new metric, **channel conflict indicator (CCI)**, to measure the interference degree of APs. Let CCI_i denote the interference degree of AP i, which is defined as: the number of AP i's neighbors whose channels belong to the same OCS as the channel of AP i. To reduce the total interference degree between AP i and its neighbors in the case where the non-overlapping channels are not enough, we need to know the CCI values of AP i's neighbors first before assigning a channel to AP i. We introduce a CCI graph, which is shown in Fig. 4, to help us performing the channel assignment. In Fig. 4, the circle represents the AP whose channel number is presented at the center. The two APs connected by an edge are neighbors to each other. For example, in Fig. 4(a), we want to assign a channel to AP 1 whose channel number is initialized as 0. Suppose that the current channel assignment and the CCI values are shown in Fig. 4(a) and no other channels can be used. Thus, we can choose one only from {5, 13, 14, 17} for AP 1. Notice that channels 5, 13, 17 $\in \Gamma_4$, we assign channel 14 to AP 1 (see Fig. 4(b)). The reason is that any one from {5, 13, 17}, such as 17, assigning to AP 1 will increase the CCI values of AP 1 and its neighbors significantly (see Fig. 4(c)), which leads to higher degree of interference.



(a) Initial CCI value. (b) AP 1 $\leftarrow$ 14. (c) AP 1 $\leftarrow$ 17.

Fig. 4. CCI graph.

The channel assignment algorithm is shown in Algorithm 1. At the beginning, the channel numbers and CCI values of all APs are initialized as 0. First, we generate a channel assignment queue Q_a by sorting the APs in descending order according to the number of STAs served by each AP. Then we assign channels to APs one by one according to the AP order in Q_a (we construct corresponding CCI graph at the same time). That is, the APs with more STAs are given more priority to be assigned channels. In Algorithm 1, $c_{i,j}$ is equal to 1 if channel j ($j \in C$) is assigned to AP i ($i \in A$), 0 otherwise. Lines 8 $\sim$ 9 mean that we assign the first channel (i.e., the primary channel) in set C^* to AP i when the non-overlapping channels are enough (i.e., $C^* \neq \varnothing$). Lines 10 $\sim$ 13 mean that when the non-overlapping channels are not enough, AP i is always assigned the channel with the minimum increase of CCI values of AP i and its neighbors. Lines 14 $\sim$ 16 update the channels that have been assigned to the APs to channels with wider bandwidth.

After channel assignment, we re-adjust the power level of APs to increase the RSS of STAs (i.e., to increase their data rate). For each AP i whose power level p_q is lower than $p_{|P|}$,

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Algorithm 1: Channel assignment

```
Input : A, C, Γ_u (u = 1, 2, ..., 10), N(i) (i ∈ A).
Output: c_{i,j} (i ∈ A, j ∈ C).
j ← 0; c_{i,j} ← 1; CCI_i ← 0 (i ∈ A);
Generate queue Q_a;
for each AP i ∈ Q_a do
    C* ← C;
    for each AP i' ∈ N(i) do
        if j' ∈ Γ_u (u = 1, 2, ..., 10) ∧ c_{i',j'} = 1 then
            C* ← C* \ Γ_u;
    if C* ≠ ∅ then
        j ← C*(1); c_{i,j} ← 1;
    else
        Select a channel j in set C with the minimum increase of
        CCI values of AP i and its neighbors;
        c_{i,j} ← 1;
        Update CCI values of AP i and its neighbors;
for each AP i ∈ Q_a do
    if the bandwidth of channel j'(j' ∈ C, j' ≠ j) is wider than that
    of channel j ∧ assigning channel j' to AP i does not increase CCI
    values of AP i and its neighbors then
        j ← j'; c_{i,j} ← 1;
```

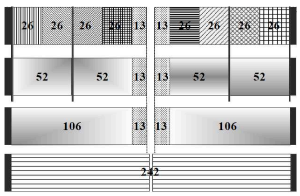
we increase its power level from p_q to p_{q+1} ($q \in [1, |P|-1]$), and then judge whether the signal emitted from AP i with power level p_{q+1} interferes with APs/STAs belonging to other basic service sets. If so, then we restore the power level to its original value (i.e., p_q) and stop; otherwise, we continue to try to upward adjust its power level from p_{q+1} to p_{q+2} until the power level is equal to $p_{|P|}$.

E. Resource Unit Assignment

There are seven types of RUs defined in IEEE 802.11ax [5], that is, the subcarrier (i.e., tone) number set $K = \{26, 52, 106, 242, 484, 996, 2 \times 996\}$. The maximum number of k-tone RUs ($k \in K$) for each channel width are shown in Fig. 5(a), where $b_1 = 20$, $b_2 = 40$, $b_3 = 80$, and $b_4 = 160$ (MHz) [5]. It implies that up to 9 STAs in 20 MHz, 18 STAs in 40 MHz, 37 STAs in 80 MHz, and 74 STAs in 160 MHz channel width are supported in an OFDMA transmission. Fig. 5(b) shows the RU locations in a 20 MHz PPDU [5]. The RU locations in a 40, 80, or 160 MHz PPDU can be found in [5]. As shown in Fig. 5, the maximum number of k-tone RUs

RU Type	b_1	b_2	b_3	b_4
26-tone	9	18	37	74
52-tone	4	8	16	32
…	…	…	…	…
2 × 996-tone	N/A	N/A	N/A	1

(a) Maximum number of RUs.



(b) RU locations (20 MHz).

Fig. 5. Maximum number of RUs and their locations [5].

($k \in K$) is determined by the channel width. Hence, when assigning RUs to AP i's STAs, we consider the number of STAs served by AP i, $|S(i)|$, as well as the channel width of AP i. We mainly focus on the following two aspects: 1) how to utilize the bandwidth of AP i's channel as much as possible; 2) how to balance the data rate of STAs in $S(i)$ as much as possible. **For aspect one**, we define m_b RU sets, $RU_{b,m}$ ($b \in B, m = 1, 2, ..., m_b$, where m_b is the maximum number of 26-tone RUs in b MHz channel width), for RU assignment and guarantee that the total bandwidth of the RUs in set $RU_{b,m}$ is as close to b as possible. For example, according to Fig. 5(b), we define $RU_{20,1}$ = {242} for 1 STA, $RU_{20,2}$ = {106, 106} for 2 STAs, $RU_{20,3}$ = {26, 106, 106} for 3 STAs, ..., $RU_{20,9}$ = {26, 26, 26, 26, 26, 26, 26, 26, 26} for 9 STAs. **For aspect two**, we assign larger-size RUs to STAs that are farther away from AP i and smaller-size RUs to STAs that are nearer to AP i. For any AP i with channel width b, we adopt the following steps to assign RUs to its STAs.

- Step 1. We divide $|S(i)|$ by m_b and the quotient of $\lfloor S(i)/m_b \rfloor$ with the remainder of rem are gotten.
- Step 2. If rem is not equal to zero, we divide AP i' $|S(i)|$ STAs into $\lfloor S(i)/m_b \rfloor + 1$ groups. The x-th group ($x = 1, 2, ..., \lfloor S(i)/m_b \rfloor$) contains m_b STAs and the ($\lfloor S(i)/m_b \rfloor + 1$)-th group contains rem STAs. If rem is equal to zero, we divide the above STAs into $S(i)/m_b$ groups with each containing m_b STAs.
- Step 3. If rem is not equal to zero, we assign the RUs in set RU_{b,m_b} to the STAs in the x-th group ($x = 1, 2, ..., \lfloor S(i)/m_b \rfloor$), and assign the RUs in set $RU_{b,rem}$ to the STAs in the ($\lfloor S(i)/m_b \rfloor + 1$)-th group. If rem is equal to zero, we assign 26-tone RU to each STA of AP i.

It's worth noting that the $\lfloor S(i)/m_b \rfloor + 1$ or $S(i)/m_b$ groups of AP i's STAs exchange frames with AP i in turn. In addition, we define the RU sets for 40, 80, and 160 MHz channel width, which can be found in [48].

F. Data Rate of STAs

We obtain the data rate of STAs according to the RSS and the RU of STAs. From [5], we can get the correspondence between the receiver minimum input level sensitivity and data rate, that is, ($MS_{b,1}, \sigma_{k,1}$), ($MS_{b,2}, \sigma_{k,2}$), ..., ($MS_{b,X}, \sigma_{k,X}$), where $MS_{b,x}$ ($b \in B, x = 1, 2, ..., X$) denotes the x-th minimum sensitivity (MS) in b MHz channel width; $\sigma_{k,x}$ denotes the x-th data rate for k-tone RU ($k \in K$). Here, $MS_{b,1} < MS_{b,2} < ... < MS_{b,X}$, and $\sigma_{k,1} < \sigma_{k,2} < ... < \sigma_{k,X}$. The data rate of an UL (or DL) from STA i to AP j (or from AP j to STA i) depends on the RSS at AP j (or STA i) [21]. Denote RSS_i the signal strength received by AP j from STA i, and R_i the data rate of STA i for UL data traffic. Thus, we have $R_i = \sigma_{k,q}$ if $MS_{b,q} \le RSS_i < MS_{b,q+1}, q \in [1, X-1]$; and $R_i = \sigma_{k,X}$ if $MS_{b,X} \le RSS_i$. Since we assume that the power level of STA i's AP is the same as that of STA i [20], the data rate of STA i for DL data traffic is the same as R_i.

G. Throughput of STAs

For any AP j with channel width b, there are $|S(j)|$ STAs associated with it. Thus, it needs

$$M_j = \begin{cases} |S(j)|/m_b, & \text{if } |S(j)| = zm_b, z \in Z^+; \\ \lfloor |S(j)|/m_b \rfloor + 1, & \text{otherwise,} \end{cases} \quad (13)$$

frame exchanges (including DL and UL traffics) to complete one communication round (i.e., each STA in $S(j)$ completes one UL transmission and one DL reception).

Let t_{TF}, t_{SIFS}, t_{UL_PPDU}, t_{M_BA}, t_{DL_PPDU}, and t_{OFDMA_BA} denote the duration of TF, SIFS, UL PPDU, M-BA, DL PPDU, and OFDMA-BA, respectively. Denote the duration of an UL transmission by T_{UL}, the duration of a DL reception by T_{DL}. According to Fig. 1, we have

$$T_{UL} = t_{TF} + 2t_{SIFS} + t_{UL\_PPDU} + t_{M\_BA}, \tag{14}$$

and

$$T_{DL} = 2t_{SIFS} + t_{DL\_PPDU} + t_{OFDMA\_BA}. \tag{15}$$

Denote the duration of one communication round of AP j by T_j, then

$$T_j = (T_{UL} + T_{DL})M_j. \tag{16}$$

Thus, the throughput of STA i associated with AP j is

$$\delta_i = \frac{R_i(t_{UL\_PPDU} + t_{DL\_PPDU})}{(CCI_j + 1)T_j} \tag{17}$$

where $CCI_j + 1$ means that AP j and its CCI_j neighbors interfere with each other. That is, they must be active in turn.

H. Overall Heuristic Algorithm

1) An Overview of Our Algorithm

To solve optimization problem (26) efficiently, we design a heuristic algorithm which consists of four stages. In stage one, we use the Greedy algorithm to iteratively place AP at the location around which the density of uncovered STAs is the highest until all STAs are covered and constraints C1 $\sim$ C3 in (26) can be met to get an initial set A_1. Since stage one may produce redundant APs, in stage two, we iteratively remove the redundant APs in A_1 one by one to get set A_2. In fact, in A_2, some APs may be close to each other and the number of STAs served by them may be small. Thus, in stage three, we iteratively replace two nearby APs in A_2 by one to get set A_3. In stage four, we try to further reduce the number of APs by iteratively replacing three nearby APs in A_3 by two to get the final solution A_4. Actually, after stage four, we may try to reduce the number of APs by iteratively replacing x nearby APs by $x - 1$ ($x = 4, 5, ...$), but there are two reasons that prevent us from doing that. The first one is about the time complexity for the next solution. The second one is about the improvement of the solution obtained as compared to our four-stage algorithm (see Section VI).

In our four-stage algorithm, the key operation in each stage is to check whether a given set A is feasible. For instance, in stage one, when adding one AP to the network, we should check whether constraints C1 $\sim$ C3 can be met. Thus, we first design Algorithm 2 for the feasibility test. The main step in Algorithm 2 is to check whether C2 $\sim$ C3 can be met, which is based on the throughput of STAs. Hence, Algorithm 2 calls Procedure 1 in line 6 to obtain the throughput of STAs.

2) The Detailed Design of Our Algorithm

In stage one, we use the Greedy algorithm as shown in Algorithm 3 to add APs one by one to the network until all STAs are covered. We call this process *one round Greedy placement*. After this process, each STA is covered by at least one AP. But according to C1, each STA must be covered by at least $n+1$ ($n = |A_f|$) APs. Therefore, to meet C1, the Greedy algorithm may need to run more than one round. After C1 is met, we continue to add APs to the network until C2 and C3 can also be met. The operations of stage one are shown in Algorithm 3, which returns an initial set A_1.

Algorithm 2: Feasibility test for a given set A

Input : A, S, Ω, P, C, n, etc.
Output: an indicator I (if A is feasible, then $I = TRUE$; otherwise $I = FALSE$).

1 $I \leftarrow TRUE$; $A^* \leftarrow A$;
2 **if** C1 can be met **then**
3   **for** each $A_f \subset A$ **do**
4         $A \leftarrow A^*$;
5         $A \leftarrow A \setminus A_f$;
6   Call **Procedure 1** to obtain the throughput of STAs;
7   **if** C2 and C3 cannot be met **then**
8             $I \leftarrow FALSE$;
9   Quit the loop;
10 **else**
11     $I \leftarrow FALSE$;
12 Return I;

Algorithm 3: Stage one-constructing an initial set of APs

Input : S, Ω, P, C, n, etc.
Output: A_1.

$A_1 \leftarrow \varnothing$; $i \leftarrow 0$;
repeat
    $i \leftarrow i + 1$;
    Place AP i at the location around which the density of uncovered STAs is the highest;
    $A_1 \leftarrow A_1 \cup \{\text{AP } i\}$;
    Call **Algorithm 2** to test the feasibility of A_1;
    if the output of Algorithm 2 is $FALSE$ **then**
        Return to the **repeat** statement;
until the output of Algorithm 2 is $TRUE$;
Return A_1;

In stage two, our goal is to remove the redundant APs in A_1. To achieve this goal, we try to iteratively remove APs one by one in a predefined order. The main idea is as follows. In each iteration, we generate an AP queue Q_b by sorting the APs in ascending order according to the number of STAs served by each AP. The order of APs in Q_b is the order in which the APs are going to be tried to remove. Then we try to remove the first AP in Q_b. If it cannot be removed (i.e., once it is removed, C1, C2 or C3 can not be met), then we hold it and try to remove the next one in Q_b; otherwise, we remove it and start the next iteration. The reason why we always try to remove the AP at the head of queue Q_b first in each iteration is that the AP with the least number of STAs is more likely to be removed. The operations of stage two are shown in Algorithm 4, which produces a better set A_2.

In stage three, we iteratively replace two nearby APs in set A_2 by one to reduce the number of APs. There are two main tasks in each iteration: 1) finding the pair of APs with the shortest distance; and 2) trying to replace the pair of APs with the shortest distance by a new one with a feasible location in Ω. The main steps in each iteration are as follows:

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Algorithm 4: Stage two-removing the redundant APs

Input : A_1.
Output: A_2.

$A_2 \leftarrow A_1$;
repeat
  Generate queue Q_b;
  for each AP $i \in Q_b$ **do**
    $A_2 \leftarrow A_2 \setminus \{\text{AP } i\}$;
    Call **Algorithm 2** to test the feasibility of A_2;
    if the output of Algorithm 2 is $FALSE$ **then**
      $A_2 \leftarrow A_2 \cup \{\text{AP } i\}$;
    else
      Return to the **repeat** statement;
until $i = |Q_b|$;
Return A_2;

- Step 1. We generate $\binom{|A_2|}{2}$ pairs of APs and calculate the distance of each pair of APs.
- Step 2. We generate a replacement queue $Q_c[i] = \{\text{AP } i_1, \text{AP } i_2\}$, where APs i_1 and i_2 represent the two APs of the i-th pair of APs ($i = 1, 2, ..., \binom{|A_2|}{2}$), by sorting the $\binom{|A_2|}{2}$ pairs of APs in ascending order according to the distance of each pair of APs.
- Step 3. We remove the pair of APs at the head of queue Q_c first and then try to search a feasible location in Ω for the new AP. If a feasible location can be found, then we place a new AP at that location and start the next iteration; otherwise, we have to put the original two APs back where they were. If a pair of APs in Q_c cannot be replaced by one, we try to replace the next pair until all pairs of APs in Q_c have been tried.

The operations of stage three are shown in Algorithm 5, which returns the third set A_3.

Algorithm 5: Stage three-replacing two nearby APs by one

Input : A_2.
Output: A_3.

$A_3 \leftarrow A_2$;
repeat
  Generate queue Q_c;
  for each $Q_c[i] \in Q_c$ **do**
    $A_3 \leftarrow A_3 \setminus Q_c[i]$;
    for each location $g \in \Omega$ **do**
      Place a new AP at location g;
      $A_3 \leftarrow A_3 \cup \{\text{the new AP}\}$;
      Call **Algorithm 2** to test the feasibility of A_3;
      if the output of Algorithm 2 is $FALSE$ **then**
        $A_3 \leftarrow A_3 \cup Q_c[i]$;
        $A_3 \leftarrow A_3 \setminus \{\text{the new AP}\}$;
      else
        Return to the **repeat** statement;
until $i = |Q_c|$;
Return A_3;

In stage four, we continue to try to reduce the number of APs by iteratively replacing three nearby APs in set A_3 by two. The main steps in each iteration are as follows:

- Step 1. We generate $\binom{|A_3|}{3}$ groups of APs with each containing 3 APs.
- Step 2. We calculate the distance between APs i_1 and i_2, D_{i_1,i_2}; APs i_2 and i_3, D_{i_2,i_3}; and APs i_1 and i_3, D_{i_1,i_3}, respectively, where APs i_1, i_2, and i_3 denote the three APs of the i-th group ($i = 1, 2, ..., \binom{|A_3|}{3}$).
- Step 3. We calculate the distance between APs i_1, i_2, and i_3, G_i, which is defined as $G_i \triangleq D_{i_1,i_2} + D_{i_2,i_3} + D_{i_1,i_3}$.
- Step 4. We generate a replacement queue $Q_d[i] = \{\text{AP } i_1, \text{AP } i_2, \text{AP } i_3\}$ by sorting the $\binom{|A_3|}{3}$ groups of APs in ascending order according to the values of G_i ($i = 1, 2, ..., \binom{|A_3|}{3}$).
- Step 5. We try to replace the group of APs at the head of queue Q_d by two new APs. If it can be replaced, then we start the next iteration; otherwise we try to replace the next group in Q_d until no group of APs can be replaced.

The operations of stage four are shown in Algorithm 6, which obtains the final solution A_4.

Algorithm 6: Stage four-replacing three nearby APs by two

Input : A_3.
Output: A_4.

$A_4 \leftarrow A_3$;
repeat
  Generate queue Q_d;
  for each $Q_d[i] \in Q_d$ **do**
    $A_4 \leftarrow A_4 \setminus Q_d[i]$;
    for each pair of locations $g_1, g_2 \in \Omega$ **do**
      Place two new APs at locations g_1 and g_2;
      $A_4 \leftarrow A_4 \cup \{\text{the two APs}\}$;
      Call **Algorithm 2** to test the feasibility of A_4;
      if the output of Algorithm 2 is $FALSE$ **then**
        $A_4 \leftarrow A_4 \cup Q_d[i]$;
        $A_4 \leftarrow A_4 \setminus \{\text{the two APs}\}$;
      else
        Return to the **repeat** statement;
until $i = |Q_d|$;
Return A_4;

I. Time Complexity Analysis

In stage one, we use the Greedy algorithm to add APs to the network one by one until constraints C1 $\sim$ C3 can be met. When adding an AP to the network, it needs to search $|\Omega|$ times for the current best location. For the network that contains x ($x = 1, 2, 3, ..., |A_1|$) APs, it needs to search a total of $x|\Omega|$ times. Thus, the time complexity of stage one is

$$O(\max_{x \in \{1,2,...,|A_1|\}} \{x\}|\Omega|) = O(|A_1||\Omega|). \tag{18}$$

In stage two, we remove the redundant APs in A_1. In the worst case, in each iteration, we need to do $|Q_b|$ ($|Q_b| = |A_1|, |A_1| - 1, |A_1| - 2, ..., n + 1$) removal attempts to try to remove one AP (here, we ignore the case where the number of APs in the network is less than $n + 1$). Thus, we have

$$O(\sum_{|Q_b|=n+1}^{|A_1|} |Q_b|) = O(|A_1|^2). \tag{19}$$

In stage three, we iteratively replace two nearby APs by one. In the worst case, we have to do $|Q_c|$ ($|Q_c| = \binom{x}{2}, x = |A_2|, |A_2| - 1, |A_2| - 2, ..., n + 1$) attempts to replace a pair of APs by one in each iteration. In each replacement attempt, we should search $|\Omega|$ times to find a feasible location for the new AP. Thus, we have

$$O(\sum_{x=n+1}^{|A_2|} \binom{x}{2}|\Omega|) = O(|A_2|^3|\Omega|). \tag{20}$$

In stage four, we iteratively replace three nearby APs by two. In the worst case, we have to do $|Q_d|$ ($|Q_d| = \binom{x}{3}, x = |A_3|, |A_3| - 1, |A_3| - 2, ..., n + 1$) attempts to replace three APs by two in each iteration. In each replacement attempt, we should search $(\binom{|\Omega|}{2} + |\Omega|)$ times to find a feasible pair of locations for two new APs (if two new APs do not overlap, there are $\binom{|\Omega|}{2}$ location combinations; if they overlap, there are $|\Omega|$ candidate locations). Thus, we have

$$O(\sum_{x=n+1}^{|A_3|} \binom{x}{3}(\binom{|\Omega|}{2} + |\Omega|)) = O(|A_3|^4|\Omega|^2). \tag{21}$$

It is not hard to see from (18) to (21) that the time complexity of our algorithm is determined by Algorithm 6 (i.e., stage four). Algorithm 6 calls Algorithm 2 to test the feasibility of the current AP placement scheme (see Algorithm 6 line 9). In addition, Algorithm 2 calls Procedure 1 to obtain the throughput of STAs (see Algorithm 2 line 6). Therefore, the time complexity of our algorithm is the product of the time complexity of Algorithm 6, Algorithm 2, and Procedure 1.

Suppose that the input of Algorithm 6 is A. According to (21), the time complexity of Algorithm 6 is $O(|A|^4|\Omega|^2)$. In Algorithm 2, for the network with fault tolerance degree n that contains $|A|$ APs, when n APs fail, there are $\binom{|A|}{n}$ AP fault combinations that need to be checked for testing the feasibility of set A. That is, the time complexity of Algorithm 2 is $O(\binom{|A|}{n}) = O(|A|^n)$. The time complexity of Procedure 1 is equal to the sum of the time complexity of its six steps, namely, $O(|S| + |A| + (|A|^2 + |A|) + |S| + |S| + |S|)$, where $(|A|^2 + |A|)$ is the time complexity of channel assignment and power re-adjustment. In fact, the time complexity of channel assignment is mainly determined by Algorithm 1 lines 3 $\sim$ 13, whose time complexity is equal to $O(\sum_{i=1}^{|A|} |N(i)|)$. In the worst case, $|N(i)| = |A| - 1$ ($i \in A$), which yields $O(\sum_{i=1}^{|A|} |N(i)|) = O(|A|^2)$. Based on the above analysis, the time complexity of our algorithm is $O(|A|^{4+n}|\Omega|^2(4|S| + |A|^2 + 2|A|)) = O(|A|^{4+n}|\Omega|^2(|S| + |A|^2))$, where the fault tolerance degree n is a given and typically small number (e.g., n = 2 or 3). When n is larger, the system could provide better fault tolerance at higher cost. In practice, n is small because the number of APs that fail at about the same time is typically small (e.g., n is equal to 2). According to our simulation experiment (where $n = 2$, $|S| = 400$, $\beta = 90$, $\rho_L = 0.5$ Mbps and $\rho_H = 1$ Mbps), the average execution time with a single CPU core on a personal computer is 15,367 s. Since the optimization is done in the planning phase (e.g., planning for a new WiFi for a stadium), this execution time should be acceptably fast.

V. Modified Problems and Solutions

In problem $\mathcal{P}_1$, we consider the application scenario where users are located at fixed locations. We now consider an application scenario where users move around.

A. Joint Optimization and Disjoint Optimization

For more comprehensive performance evaluation, we further consider the following factors.

- **Fixed users and mobile users:** We consider two scenarios. In the first scenario, users are located at fixed locations (e.g., they sit at fixed seats in a concert hall to enjoy music performance). In the second scenario, users move around (e.g., they move around in a multi-function hall for social activities).
- **Joint optimization and disjoint optimization:** The proposed algorithm adopts a joint optimization approach: it jointly optimizes the number of APs, the locations of APs and the resource allocation (where the resources include power levels, channels and resource units). For comparison, we consider a disjoint optimization approach: the proposed algorithm is modified such that optimization is done in two steps. Step I optimizes the number of APs and the locations of APs, so that all users are covered and the fault tolerance requirement is fulfilled. Step II optimizes the resource allocation, so that the two-tier throughput requirement is fulfilled.

B. Mobility Model

When users move around, we apply a well-known mobility model called Random Waypoint (RWP) [49] [50] to model their mobility. Consider $|S|$ users in a rectangular environment as shown in Fig. 6. Based on the RWP model, the moving process of user $i \in S$ is as follows:

Step 1. User i is assigned an initial source location (x_i, y_i), a destination location $(\hat{x}_i, \hat{y}_i)$, and a speed v_i. The locations (x_i, y_i) and $(\hat{x}_i, \hat{y}_j)$ are chosen independently and uniformly in the target region. The speed v_i is chosen uniformly on $[v_{min}, v_{max}]$.

Step 2. User i moves from (x_i, y_i) to $(\hat{x}_i, \hat{y}_j)$ along a straight line with speed v_i.

Step 3. After reaching $(\hat{x}_i, \hat{y}_j)$, user i pauses for a period of time, which is chosen uniformly on $[t_{min}, t_{max}]$.

Step 4. A new destination is chosen uniformly, and a new speed v_i is chosen uniformly on $[v_{min}, v_{max}]$, independently of all previous destinations and speeds. Then (x_i, y_i), $(\hat{x}_i, \hat{y}_j)$, and v_i are updated and go to Step 2.

To deploy the WiFi network, the serving environment is evenly divided into grid points (e.g., each grid point is a $1m \times 1m$ area) where each grid point serves as a reference point for the two QoS requirements. As a user moves, he may pass through multiple reference points.

C. Problems and Solutions

With the above factors (i.e., fixed versus mobile users and joint versus disjoint optimization), the resulting problems and solutions are listed as follows.

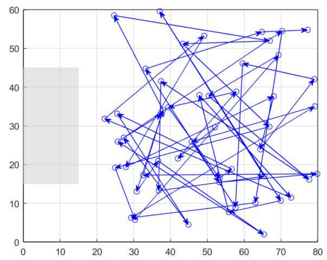

Fig. 6. The RWP model with moving steps of 50 (in a multi-function hall).

1) Users are at fixed locations

Joint optimization

Problem: our original problem.

Solution: our four-stage algorithm.

Disjoint optimization

Step I: AP placement.

Problem ($\mathcal{P}_3$):

$$\min |A| \quad s.t. \left\{ \text{C1:} \quad \sum_{j=1}^{|A \setminus A_f|} a_{i,j} = 1, i \in S. \right. \tag{22}$$

Solution: our four-stage algorithm without resource allocation (i.e., without Procedure 1) and the two-tier throughput requirement in Algorithm 2.

Step II: Resource allocation.

Problem ($\mathcal{P}_4$):

$$\begin{cases} \text{C2:} & \sum_{i=1}^{|S|} \delta_i^{(H)} \geq |S| \times \beta\%; \\ \text{C3:} & \sum_{i=1}^{|S|} (\delta_i^{(L)} + \delta_i^{(H)}) = |S|. \end{cases} \tag{23}$$

Solution: Procedure 1 of our four-stage algorithm.

2) Users move around

Joint optimization

Problem ($\mathcal{P}_5$):

$$\min |A| \quad s.t. \begin{cases} \text{C1:} & \sum_{j=1}^{|A \setminus A_f|} \tilde{a}_{i,j} = 1, i \in \tilde{S}; \\ \text{C2:} & \sum_{i=1}^{|\tilde{S}|} \tilde{\delta}_i^{(H)} \geq |\tilde{S}| \times \beta\%; \\ \text{C3:} & \sum_{i=1}^{|\tilde{S}|} (\tilde{\delta}_i^{(L)} + \tilde{\delta}_i^{(H)}) = |\tilde{S}|. \end{cases} \tag{24}$$

Here, $\tilde{a}_{i,j}$ is equal to 1 if reference point i "associates" with AP j ($j \in A \setminus A_f$), 0 otherwise. $\tilde{S}$ denotes the set of reference points. $\tilde{\delta}_i^{(H)}$ is equal to 1 if $\tilde{\delta}_i \geq \rho_H$, 0 otherwise. $\tilde{\delta}_i^{(L)}$ is equal to 1 if $\rho_H > \tilde{\delta}_i \geq \rho_L$, 0 otherwise. $\tilde{\delta}_i$ denotes the theoretical throughput of reference point i.

Disjoint optimization

Step I: AP placement.

Problem ($\mathcal{P}_6$):

$$\min |A| \quad s.t. \left\{ \text{C1:} \quad \sum_{j=1}^{|A \setminus A_f|} \tilde{a}_{i,j} = 1, i \in \tilde{S}. \right. \tag{25}$$

Solution: our four-stage algorithm without resource allocation (i.e., without Procedure 1) and the two-tier throughput requirement in Algorithm 2.

Step II: Resource allocation.

Problem ($\mathcal{P}_7$):

$$\begin{cases} \text{C2:} & \sum_{i=1}^{|\tilde{S}|} \tilde{\delta}_i^{(H)} \geq |\tilde{S}| \times \beta\%; \\ \text{C3:} & \sum_{i=1}^{|\tilde{S}|} (\tilde{\delta}_i^{(L)} + \tilde{\delta}_i^{(H)}) = |\tilde{S}|. \end{cases} \tag{26}$$

Solution: Procedure 1 of our four-stage algorithm.

VI. Performance Evaluation

A. Simulation Settings

Since ns-3 does not currently support some of the major functions (such as MU-OFDMA) of IEEE 802.11ax [51], we develop a simulator using MATLAB [48].

We adopt the propagation model investigated by Cisco in [45], in which the two bands at 2.4 GHz and 5 GHz have different path losses. The Cisco's model is an improved version of the widely used log-distance path loss model. It takes into account the antenna gains of both sender and receiver, so it is more accurate than the log-distance path loss model. Let $P_{ref}^{(2.4G)}$ and $P_{ref}^{(5G)}$ be the path losses at a reference distance (which is 1 m) in the 2.4 GHz band and the 5 GHz band, respectively. $P_{ref}^{(2.4G)}$ is typically smaller than $P_{ref}^{(5G)}$ [45]. Based on Cisco's study in [45], we set $P_{ref}^{(2.4G)} = 40$ dB and $P_{ref}^{(5G)} = 46$ dB. That is, the difference between the reference path loss of the two bands is $\tau = P_{ref}^{(5G)} - P_{ref}^{(2.4G)} = 6$ dB (which is the same as that in [45]).

To compensate for the larger path loss in the 5 GHz band, some major AP vendors (e.g., Cisco and Huawei) adopt higher power levels and larger antenna gain for the 5 GHz band [52] [53]. For example, for Huawei AirEngine 8760-X1-PRO AP [52], the maximum power level is 31 dBm for the 5 GHz band and 26 dBm for the 2.4 GHz band with an adjustment step of 1 dBm, while the antenna gains of the 5 GHz and 2.4 GHz bands are 5 dBi and 4 dBi, respectively. As a result, the actual maximum transmission power levels for the two bands are 31 + 5 = 36 dBm and 26 + 4 = 30 dBm respectively, where the difference is 6 dB. As another example, a Cisco's AP [53] also uses a similar power enhancement method for the 5 GHz band such that the actual maximum transmission power of the 5 GHz band is 5 dB higher than that of the 2.4 GHz band. With these settings, the resulting losses in the two bands are almost the same and hence the stations could access the dual-band channels fairly.

Based on the above observations, we adopt similar settings as the commercial APs to compensate for the larger path loss in the 5 GHz band. In particular, the power level set for the 2.4 GHz band is set to be $\overline{P} = \{\overline{p}_1, \overline{p}_2, ..., \overline{p}_{|\overline{P}|}\} = \{23, 24, 25, 26\}$ dBm, and the power level set for the 5 GHz band is set to be $\widetilde{P} = \{\widetilde{p}_1, \widetilde{p}_2, ..., \widetilde{p}_{|\widetilde{P}|}\} = \{28, 29, 30, 31\}$ dBm (i.e., same as that in [52]). In addition, the antenna gain for the 2.4 GHz band, denoted by $G^{(2.4G)}$, is set to be 4 dBi while the antenna gain for the 5 GHz band, denoted by $G^{(5G)}$, is set to be 5 dBi (i.e., same as that in [52]). Thus, we have $(\widetilde{p}_i + G^{(5G)}) - (\overline{p}_i + G^{(2.4G)}) = \tau$, where $i = 1, 2, ..., |\widetilde{P}|$. That is, the actual transmition power of each power level in the 5 GHz band is τ dB higher than that of the corresponding power level in the 2.4 GHz band. Besides, we

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set $G_{TX} = G_{RX} = G^{(2.4G)}$ or $G^{(5G)}$ when the APs/STAs work on the 2.4 or 5 GHz band, $\eta = 4$ and $\chi = 5$ dB [54].

In addition, the settings for the channel set C, θ_D, θ_I, the RU sets, the correspondence between the receiver minimum input level sensitivity and the data rate, and the throughput model (17) can be found in our previous paper [55].

We consider the following settings for the two-tier throughput requirement (unless otherwise specified): $\beta = 90$, $\rho_H = 1$, Mbps and $\rho_L = 0.5$ Mbps. The users' locations are the seats' locations, where the seats are arranged regularly (e.g., 20 rows × 15 columns) because this regular arrangement is common in the real world (such as recital halls). The specific seat arrangements will be provided when they are adopted. The following results are from the average of 30 simulation runs.

B. Effectiveness Evaluation

In this part, we set the area of the target region as 50 × 50 m^2, which is divided into 25 10×10 m^2 cells. Thus, $|\Omega| = 25$. We suppose that there are 100, 200, 300, 400, 500 seats in the target region, respectively. The arrangements of the seats are as follows: 100 seats: 10 rows × 10 columns; 200 seats: 20 rows × 10 columns; 300 seats: 20 rows × 15 columns; 400 seats: 20 rows × 20 columns; and 500 seats: 20 rows × 25 columns. Below, when we set $|S| = x, x \in \{100, 200, ..., 500\}$, it means that there are x seats in the target region and each seat is occupied by one STA.

1) Throughput Model Validation

Considering that the throughput model in (17) plays a key role in our algorithm, we verify it first via the comparison of the simulation and the analytical results. Setting $|S| = 500$ and $n = 0$, we obtain Fig. 7. The upper part of the figure compares the simulation results with the analytical ones derived directly from (17), while the lower part shows the errors. We can see that the simulation and analytical results match very well since the errors are less than 1.4%. Therefore, the throughput model in (17) is verified.

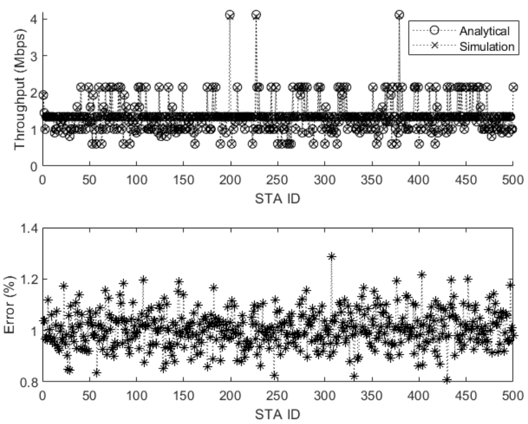


Fig. 7. Comparison of simulation and analytical results ($|S| = 500$).

2) Effectiveness Evaluation of Our Algorithm

We evaluate the effectiveness of the four algorithms by comparing them with the exhaustive search method that can obtain the optimal solution in an acceptable time only on the small-scaled case. The four algorithms are as follows: one-stage: stage 1; two-stage: stages 1 ∼ 2; three-stage: stages 1 ∼ 3; and four-stage: stages 1 ∼ 4. Setting $|S| = 100, 200, 300, 400, 500$, and $n = 0$, respectively, we obtain Tables IV and V, in which the data from row 2 to 6 represent the average number of APs and the average execution time of 30 runs, respectively. It can be seen from Table IV that compared with one-, two-, and three-stage, the four-stage obtains the least number of APs, which is equal to that obtained from the exhaustive search method, namely, our algorithm performs as good as the exhaustive search method under the small-scaled case. Table V shows that the four-stage saves 41 ∼ 82 % execution time as compared to exhaustive search method. In short, our algorithm is very effective in providing a high quality solution.

TABLE IV
COMPARISON OF THE NUMBER OF APs

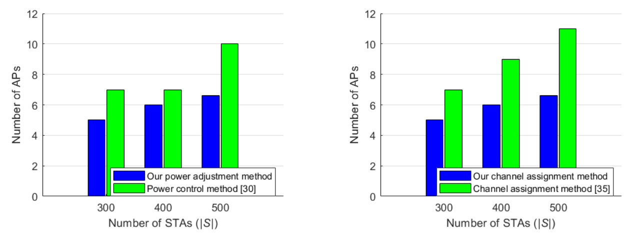
$\|S\|$	**100**	**200**	**300**	**400**	**500**
Exhaustive search	2.00	3.00	3.00	3.00	4.00
Four-stage	2.00	3.00	3.00	3.00	4.00
Three-stage	2.00	3.00	4.00	4.00	4.00
Two-stage	2.23	3.00	4.00	6.00	8.20
One-stage	3.00	3.20	4.00	6.00	8.20

TABLE V
COMPARISON OF THE EXECUTION TIME (SECOND)

$\|S\|$	**100**	**200**	**300**	**400**	**500**
Exhaustive search	0.18	1.09	3.01	19.09	56.20
Four-stage	0.08	0.25	0.85	2.78	4.16
Three-stage	0.08	0.10	0.65	1.76	2.61
Two-stage	0.04	0.04	0.19	0.45	1.13
One-stage	0.02	0.02	0.08	0.17	0.43

3) Sub-method Evaluation

We compare our algorithm for individual subproblems against solutions from the literature. That is, we replace our power adjustment method and channel assignment method in the four-stage algorithm by the method presented in [30] and [35], respectively, and compare them with the four-stage algorithm. Setting $|S| = 300, 400, 500$, and $n = 1$, respectively, we obtain Fig. 8, which illustrates that our power adjustment method and channel assignment method are better than existing ones.



(a) Power assignment methods. (b) Channel assignment methods.

Fig. 8. Comparison of power and channel assignment methods.

C. Parameter Sensitivity Analysis

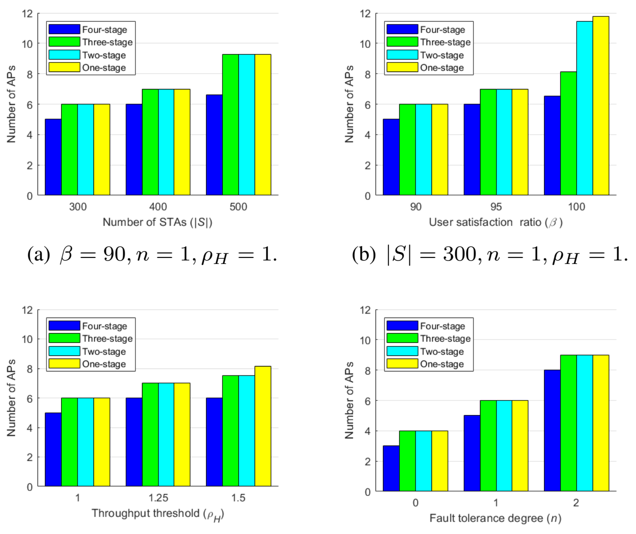
We analyze the sensitivity of various parameter settings by comparing the four algorithms mentioned above.

Firstly, we study the relationship between the number of APs and the value of $|S|$. Setting $|S| = 300, 400, 500$, and $n = 1$, respectively, we obtain Fig. 9(a). From Fig. 9(a) we observe that as the number of STAs increases, so does the number of APs. In fact, increasing the number of STAs increases the value of $|S| \times \beta\%$. According to constraint C2, the increase of the value of $|S| \times \beta\%$ means we should guarantee that more users' throughput is greater than or equal to ρ_H, which leads to an increase in the demand for total network throughput. As a result, it requires more APs to be placed.

Secondly, we study the impact of β on the number of APs. Setting $\beta = 90, 95, 100$, $|S| = 300$, $n = 1$, and $\rho_H = 1$ Mbps, respectively, we obtain Fig. 9(b). We observe that higher user satisfaction ratio β requires more APs. This is because the increase in β means that the WiFi network has to satisfy more users, which requires more APs in the network.

Thirdly, we investigate the change of the number of APs when ρ_H varies. Setting $\rho_H = 1, 1.25, 1.5$ (Mbps), $|S| = 300$, $\beta = 90$, and $n = 1$, respectively, we obtain Fig. 9(c). From the figure we find that the number of APs required in the network increases with the increase of ρ_H. This is because the increase of ρ_H leads to the increase of users' demand for the total network throughput, so more APs need to be arranged.

Finally, we study the number of APs when n varies. Setting $n = 0, 1, 2$, and $|S| = 300$, respectively, we obtain Fig. 9(d). From Fig. 9(d) we observe that increase in n increases the number of APs. In fact, when the fault tolerance degree is n, each STA must be covered by at least $n + 1$ APs. Obviously, the number of APs increases as n increase.



(a) $\beta = 90, n = 1, \rho_H = 1$. (b) $|S| = 300, n = 1, \rho_H = 1$.

(c) $|S| = 300, \beta = 90, n = 1$. (d) $|S| = 300, \beta = 90, \rho_H = 1$.

Fig. 9. Sensitivity analysis when parameters $|S|$, β, ρ_H, and n vary.

D. Performance Evaluation and Benefit Analysis

Because our problem is quite different from the others (see Tables I and II), it is difficult to find existing algorithms to compare with ours. Thus, we evaluate the performance of our algorithm by comparing it with the Greedy algorithm and the Random method, whose power-channel-RU assignment is performed in the same way as our algorithm, under the larger area where we cannot find the optimal solution in an acceptable time through an exhaustive search.

These baseline solutions have the following features.

- **Random Algorithm:** This algorithm is simple but valuable for performance comparison. As it uses the least effort approach, it could demonstrate the value of a well-designed algorithm.
- **Greedy Algorithm:** This algorithm is the first stage of our proposed algorithm and it uses sophisticated steps for reasonable optimization. Specifically, it determines the locally optimal choice in each step (e.g., when it adds an AP, it determines the best position for this new AP). For this purpose, it executes a series of algorithms: it repeatedly executes Algorithm 2, which in turn repeatedly executes Procedure 1, which in turn repeatedly executes the following: 1) Algorithm 1 for channel assignment, 2) the STAs-APs association steps given in Section IV-B, 3) the power adjustment steps given in Section IV-C, and 4) the RU assignment steps given in Section IV-E.

To fully evaluate the performance of our four-stage algorithm under different scenarios, we consider **the recital hall** and **the stadium** as the target region respectively. Due to the limited space of the paper, we present: (1) the results for the recital hall case in this paper, and (2) the results for all cases in the full version of this paper, which can be found in [56].

We consider the recital hall (80 × 60 m^2) whose layout is shown in Fig. 10(a). There is a stage on the left side for performances and there are many seats for audiences in the blank area. The locations of the audiences (i.e., STAs) are generated randomly. To compare the number of APs obtained from our algorithm and others intuitively, we show the solutions of an example with the settings of $|S| = 400$ and $n = 1$ in Figs. 10 (b) $\sim$ (d), in which the little red triangles, the little blue circles, and the big black circles represent the APs' locations, the users' locations, and the communication ranges of APs, respectively. The number next to the red triangle is the number of APs being placed at that location. We observe that some locations are placed more than one AP. This is because some areas have denser users than the others. In addition, we allow APs to automatically adjust their power level, so the communication ranges of them may be different from each other. From Figs. 10 (b) $\sim$ (d) we observe that our algorithm gets the best solution which needs only 8 APs, and the number of APs obtained from the Greedy and the Random solutions are 12 and 42, respectively.

Next, we compare the performance with various parameter settings. Setting $|S| = 300, 400, 500$, and $n = 1$, respectively, we obtain Fig. 11(a). Then setting $\beta = 90, 95, 100$, $|S| = 400$, $n = 1$, and $\rho_H = 1$ Mbps, respectively, we obtain Fig. 11(b). From these two figures we can see that the number of APs obtained form our algorithm is much smaller than that obtained form the Greedy algorithm and the Random method.

Suppose that it costs 1 unit to deploy 1 AP, the cost savings of our algorithm are shown in Table VI. The table shows that

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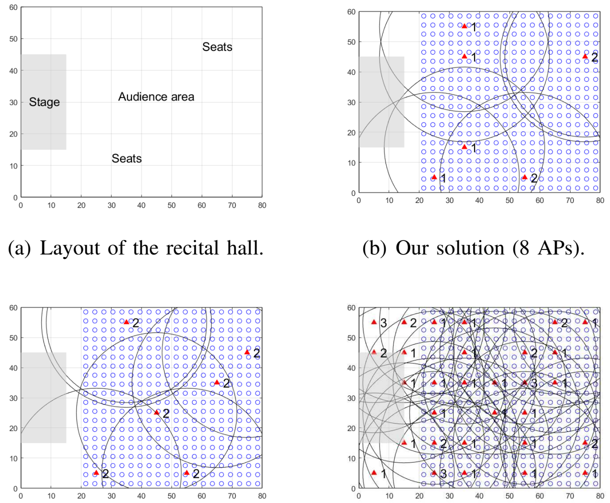
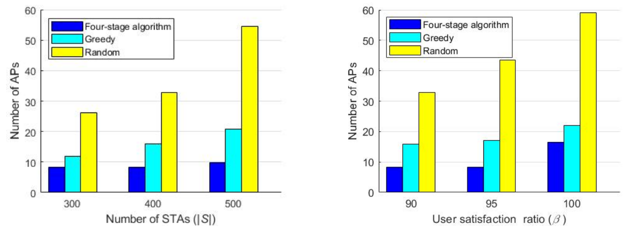


(a) Layout of the recital hall. (b) Our solution (8 APs).

(c) Greedy solution (12 APs). (d) Random solution (42 APs).

Fig. 10. Layout of the recital hall and the solutions of an example.



(a) $\beta = 90, n = 1, \rho_H = 1$. (b) $|S| = 400, n = 1, \rho_H = 1$.

Fig. 11. Performance comparison under the recital hall scenario.

our algorithm can save 34.9 $\sim$ 87.7 % of deployment cost under different parameter settings.

TABLE VI
COST SAVINGS COMPARISON FOR RANDOM SCENARIO(%)

Parameter	$\|S\|$			β		
Value	**300**	**400**	**500**	**90**	**95**	**100**
Compared with Greedy	**34.9**	43.3	62.0	43.3	60.6	60.5
Compared with Random	73.1	83.3	84.4	83.3	**87.7**	80.3

The above results show that our algorithm can save deployment cost significantly as compared to the Greedy algorithm and the Random method with various simulation settings.

Finally, we evaluate the value of the fault tolerance constraint in our optimization problem. Consider the two-tier throughput requirement and the indoor environment in the above simulation experiment (i.e., $\beta = 90$, $\rho_H = 1$ Mbps, $\rho_L = 0.5$ Mbps, and $|S| = 400$). Let δ denote the throughput of STAs. When there is no fault tolerance requirement (i.e., $n = 0$) but there is at least one AP failure, Table VII shows that the two-tier throughput requirement could not be fulfilled and consequently some stations suffer from significantly lower throughput. When there is fault tolerance requirement (i.e., $n > 0$), even when there are up to n AP failures, Table VIII and Table IX show that the two-tier throughput requirement could still be fulfilled such that all stations have good enough throughput. These results show that the fault tolerance requirement is indeed useful to ensure good enough performance even when there are AP failures.

TABLE VII
WITHOUT FAULT TOLERANCE REQUIREMENT
(I.E., $n = 0$)

Number of Failed APs	Percentage of Stations with the Following Throughput Range			Satisfy Two-tier Throughput Requirement?
	$\delta < \rho_L$	$\rho_L \leq \delta < \rho_H$	$\delta \geq \rho_H$	
0	0.00%	5.25%	94.75%	Yes
1	13.84%	19.69%	66.47%	No
2	36.92%	25.02%	38.06%	No
3	66.75%	19.06%	14.19%	No

TABLE VIII
WITH FAULT TOLERANCE REQUIREMENT
($n = 1$, I.E., TOLERATING AT MOST ONE AP FAILURE)

Number of Failed APs	Percentage of Stations with the Following Throughput Range			Satisfy Two-tier Throughput Requirement?
	$\delta < \rho_L$	$\rho_L \leq \delta < \rho_H$	$\delta \geq \rho_H$	
0	0.00%	0.33%	99.67%	Yes
1	0.00%	2.94%	97.06%	Yes
2	1.51%	5.73%	92.76%	No
3	4.93%	10.02%	85.05%	No

TABLE IX
WITH FAULT TOLERANCE REQUIREMENT
($n = 2$, I.E., TOLERATING AT MOST TWO AP FAILURES)

Number of Failed APs	Percentage of Stations with the Following Throughput Range			Satisfy Two-tier Throughput Requirement?
	$\delta < \rho_L$	$\rho_L \leq \delta < \rho_H$	$\delta \geq \rho_H$	
0	0.00%	1.75%	98.25%	Yes
1	0.00%	2.25%	97.75%	Yes
2	0.00%	1.54%	98.46%	Yes
3	0.09%	3.05%	96.86%	No

E. Effectiveness of Joint Optimization and Disjoint Optimization

We compare the effectiveness of joint optimization and disjoint optimization via two performance metrics:

1) The first performance metric is: percentage of cases in which feasible solutions (i.e., fulfilling both QoS requirements) could be found.

2) The second performance metric is: percentage difference between the objective function values (i.e., the number of APs) determined by joint optimization and disjoint optimization. This quantity is evaluated only for the cases in which both joint optimization and disjoint optimization could find feasible solutions (i.e., these solutions fulfill both QoS requirements).

We consider both fixed users (i.e., users are at fixed locations) and mobile users (i.e., users move around).

We first consider the fixed users in the recital hall scenario. Table X (where $|S|$ = 300, 350, 400, 450, 500; ρ_H = 1.25 Mbps) and Table XI (where ρ_H = 1, 1.125, 1.25, 1.375, 1.5

(Mbps); $|S|$ = 400) compare the performance of joint optimization and disjoint optimization. We observe the following from Tables X and XI:

- Joint optimization could always find feasible solutions (i.e., it could always fulfill the two QoS requirements). It is because joint optimization optimizes the number of APs, the locations of APs and the resource allocation as a whole. In the optimization process, it could adjust the number and the locations of APs in order to provide enough resources to fulfill both QoS requirements.
- Disjoint optimization may not find feasible solutions. As the number of users (stations) or the throughput requirement ρ_H increases, disjoint optimization could not find feasible solutions in more and more cases. It is because disjoint optimization separates AP placement (Step I) from resource allocation (Step II). Step I optimizes the number and the locations of APs to cover all users and fulfill the fault tolerance requirement. These APs may not provide enough resources (i.e., they may not support enough concurrent transmissions) to fulfill the two-tier throughput requirement in Step II. As a result, disjoint optimization may not find feasible solutions.
- When both joint optimization and disjoint optimization could find feasible solutions, they give the same solution quality (i.e., giving the same objective function value while fulfilling both QoS requirements). It is because the candidate solution space of disjoint optimization is included in that of joint optimization. When disjoint optimization can find a feasible solution, joint optimization can also find it (but the reverse is not necessarily true).

TABLE X
COMPARISON BETWEEN JOINT AND DISJOINT OPTIMIZATION WHEN USERS ARE AT FIXED LOCATIONS ($\beta = 90, n = 1, \rho_H = 1.25$)

Number of Stations	**Percentage of Cases in which Feasible Solutions could be Found**		**Difference Between the Objective Function Values***
	Joint	**Disjoint**	
300	100.00%	100.00%	0.00%
350	100.00%	76.67%	0.00%
400	100.00%	46.67%	0.00%
450	100.00%	20.00%	0.00%
500	100.00%	00.00%	Not applicable

TABLE XI
COMPARISON BETWEEN JOINT AND DISJOINT OPTIMIZATION WHEN USERS ARE AT FIXED LOCATIONS ($|S| = 400, \beta = 90, n = 1$)

Throughput Threshold (Mbps)	**Percentage of Cases in which Feasible Solutions could be Found**		**Difference Between the Objective Function Values***
	Joint	**Disjoint**	
1.000	100.00%	100.00%	0.00%
1.125	100.00%	90.00%	0.00%
1.250	100.00%	46.67%	0.00%
1.375	100.00%	10.00%	0.00%
1.500	100.00%	3.33%	0.00%

*Consider only the cases in which both joint optimization and disjoint optimization could find feasible solutions that fulfill both QoS requirements.

We now consider mobile users who move around in the multi-function hall scenario. We apply the RWP model to model the mobility of each user with the following parameter values: the moving speed interval is [0.5, 1.5] m/s, the pause time interval is [60, 600] s, and the target region is [20, 80] $\times$ [0, 60] m^2 in the multi-function hall. Each simulation instance is as follows. First, we obtain the feasible AP placement according to the $|\tilde{S}|$ = $|S|$ reference points by using joint optimization and disjoint optimization, respectively. Then we let $|S|$ users move around based on the RWP model. The statistical period is set to 160 hours and the sampling period is set to 3 minutes (i.e., the throughput of the STAs is sampled once every 3 minutes for a total of 160 hours, so the total number of samples is equal to 160 $\times$ 60 / 3 = 3200). Based on these samples, we obtain how well the two-tier throughput requirement is satisfied.

Table XII (where $|S|$ = 300, 350, 400, 450, 500, and ρ_H = 1.25 Mbps) and Table XIII (where ρ_H = 1, 1.125, 1.25, 1.375, 1.5 (Mbps), and $|S|$ = 400) compare the performance of joint optimization and disjoint optimization. Our observations from Table III are the same as that from Table II: (i) joint optimization could always find feasible solutions, (ii) disjoint optimization may not find feasible solutions, especially when the number of stations or the throughput requirement ρ_H is large, and (iii) when both joint optimization and disjoint optimization could find feasible solutions, they give the same solution quality.

TABLE XII
COMPARISON BETWEEN JOINT AND DISJOINT OPTIMIZATION WHEN USERS MOVE AROUND ($\beta = 90, n = 1, \rho_H = 1.25$)

Number of Stations	**Percentage of Cases in which Feasible Solutions could be Found**		**Difference Between the Objective Function Values***
	Joint	**Disjoint**	
300	100.00%	100.00%	0.00%
350	100.00%	90.00%	0.00%
400	100.00%	63.33%	0.00%
450	100.00%	30.00%	0.00%
500	100.00%	00.00%	Not applicable

TABLE XIII
COMPARISON BETWEEN JOINT AND DISJOINT OPTIMIZATION WHEN USERS MOVE AROUND ($|S| = 400, \beta = 90, n = 1$)

Throughput Threshold (Mbps)	**Percentage of Cases in which Feasible Solutions could be Found**		**Difference Between the Objective Function Values***
	Joint	**Disjoint**	
1.000	100.00%	100.00%	0.00%
1.125	100.00%	93.33%	0.00%
1.250	100.00%	63.33%	0.00%
1.375	100.00%	30.00%	0.00%
1.500	100.00%	3.33%	0.00%

VII. CONCLUSION

To set up an IEEE 802.11ax-based dense WiFi network, the following issues would be involved: access point placement, power assignment, channel assignment, and resource unit assignment. These issues are challenging because they are inter-dependent on each other while they would affect the resulting performance. In this paper, we formulated the problem of joint access point placement and power-channel-RU assignment for IEEE 802.11ax-based dense WiFi networks, in which the objective is to minimize the cost (in terms of the number of APs adopted) while fulfilling a given two-tier throughput requirement and a given fault tolerance requirement. We proved that this problem is NP-hard. To tackle this problem, we first derived the throughput of each station under the OFDMA mechanism and a widely used interference model and then designed a heuristic algorithm with polynomial time complexity. We presented extensive simulation results with comprehensive parameter settings to demonstrate that the proposed algorithm could efficiently find high-quality solutions.

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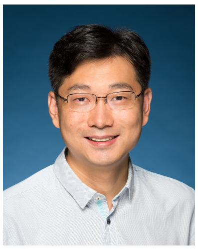

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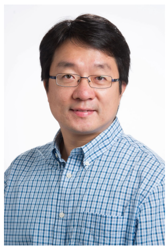

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