Stateful Switch: Optimized Time Series Release with Local Differential Privacy

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Abstract—Time series data have numerous applications in big data analytics. However, they often cause privacy issues when collected from individuals. To address this problem, most existing works perturb the values in the time series while retaining their temporal order, which may lead to significant distortion of the values. Recently, we propose TLDP model [45] that perturbs temporal perturbation to ensure privacy guarantee while retaining original values. It has shown great promise to achieve significantly higher utility than value perturbation mechanisms in many time series analysis. However, its practicability is still undermined by two factors, namely, utility cost of extra missing or empty values, and inflexibility of privacy budget settings. To address them, in this paper we propose switch as a new two-way operation for temporal perturbation, as opposed to the one-way dispatch operation in [45]. The former inherently eliminates the cost of missing, empty or repeated values. Optimizing switch operation in a stateful manner, we then propose StaSwitch mechanism for time series release under TLDP. Through both analytical and empirical studies, we show that StaSwitch has significantly higher utility for the published time series than any state-of-theart temporal- or value-perturbation mechanism, while allowing any combination of privacy budget settings.

Index Terms—Local differential privacy; time series; temporal perturbation; switch operation

I. INTRODUCTION

In big data era, continual data, i.e., a sequence of values in the temporal order (a.k.a., *time series*), has numerous real-world applications [39]. Among them, many time series are collected from individuals, such as biosensors in telecare, IoT sensors in smart home, and trajectories for mobility tracking in COVID-19 pandemic. Directly releasing them to the public can cause privacy infringement [21], [28]. For example, the periodic heart rate readings from an Apple Watch may reveal the daily activity of its owner, e.g., sleeping, sitting, or walking.

To address this issue, many privacy-preserving time series publishing techniques have been proposed [32], [43], most of which are based on differential privacy [13], in either a centralized [14], [31] or a local setting [4], [9], [36], [42]. However, all these works are **value perturbation** mechanisms, i.e., they perturb the value at each timestamp so that no value at any timestamp can be inferred with high confidence. Unfortunately, in medical and financial applications, such mechanisms do not work because distorted values are useless or even harmful, for example ECG/blood pressure readings, and stock trading prices.

Our recent work [45] has proposed to mitigate this issue by perturbing the **temporal order** of a time series. The privacy model, namely local differential privacy in the temporal setting (TLDP), guarantees an adversary cannot infer the original timestamp of a value with high confidence. As temporal perturbation does not inject any noise to the value, the accuracy of most time series statistics (e.g., moving average, range count) and manipulations (e.g., window smoothing, resampling) can be significantly enhanced. The following is a concrete example beyond the medical or financial domain.

Example: Smart Meter. Utility (electricity, gas, and water) companies are deploying smart meters in households to collect real-time consumption data for usage prediction and resource scheduling. However, such data may disclose the activities in an individual household, such as away-from-home (low usage of all three utilities) and heavy washing (high usage of both electricity and water in a laundry room). To preserve privacy under differential privacy, unfortunately we cannot perturb these reading values as they must be accurately reflected in the utility bills. Therefore, temporal perturbation (independently on these three time series) becomes the natural way to achieve deniability and differential privacy.

Although TLDP is a promising privacy model in value-critical applications, there remain two issues in [45]. First, the proposed Threshold Mechanism (TM) is built on the operation of *dispatch*, which randomly moves the value of the current timestamp to a future timestamp within a sliding window of length k. But since this is a one-way operation (i.e., only from current to future, but not vice versa), it causes missing, empty or repeated values in a released time series. Second, TLDP has two privacy parameters, namely, the privacy budget ϵ and the sliding window length k. However, the Threshold Mechanism (TM) cannot effectively support all combinations of ϵ and k. A mismatch of ϵ and k could either cost TM extra missing or empty values to satisfy a small ϵ or a large k, or waste the large ϵ for a small k. The following two examples explain this mismatch issue of TM.

• Frequency counting, which counts the occurrences of a specific value in a time series, is sensitive to missing values. Using TM with $\epsilon=1.0$ and k=10 causes very large estimation error, 382 times higher than Randomized Response [38], a value-perturbation mechanism on time series.

• When k=4, the largest privacy budget TM can support is only 2.19. In other words, any ϵ larger than 2.19 has to be wasted [45].

In this paper, we present a switch-based mechanism for TLDP that addresses these two issues. As opposed to the dispatch operation in TM, switch is a two-way operation in a time series, which exchanges two values S_i and S_j of timestamps t_i and t_j . In essence, a switch is equivalent to two synchronous dispatch operations, and it is free of missing, empty or repeated values. Based on this operation, we propose StaSwitch (short for Stateful Switch) perturbation mechanism, which bounds each value's choice of switch by a stateful probability distribution. Furthermore, this new mechanism does not cause any mismatch on ϵ and k as in TM. As such, users have full flexibility on the choice privacy parameters without degrading data utility. To summarize, our contributions in this paper are three-fold.

- We propose a two-way atomic operation switch for temporal perturbation, which inherently eliminates missing, empty or repeated values in the released time series.
- We design two temporal perturbation mechanisms based on *switch* operation, namely the baseline mechanism *RanSwitch* and an optimized one *StaSwitch*. They are capable of offering full flexibility for users to set any privacy parameters without degrading data utility.
- We present detailed analysis on the privacy guarantee and utility cost of RanSwitch and StaSwitch. Through intensive analytical and empirical studies, we show that StaSwitch has significantly higher utility for the released time series than any state-of-the-art temporal-perturbation or value-perturbation mechanism.

The rest of the paper is organized as follows. Section II formulates the problem of time series release. Section III presents *switch* operation, together with our baseline mechanism RanSwitch. Section IV introduces StaSwitch mechanism with theoretical analysis on privacy guarantee and utility cost. Section V presents experimental results and case studies on both real and synthetic datasets. Finally, we review existing work in Section VI and conclude this paper in Section VII.

II. PROBLEM DEFINITION AND PRELIMINARIES

A. Problem Definition

In this paper, we define a time series as an infinite sequence of values $S = \{S_1, S_2, ..., S_n, ...\}$ in a discrete temporal domain $T = \{t_1, t_2, ..., t_n, ...\}$. Our task is to release a sanitized time series $R = \{R_1, R_2, ..., R_n, ...\}$ out of the original one S under local differential privacy, and as with [45], our goal is to minimize the collective cost arising from each value's missing, repetition, empty and misaligned between R and S. Specifically, a **missing cost**, whose unit is M, occurs when a value in S is missed in S; a **repetition cost**, whose unit is S, occurs when a value is duplicated once in S; an **empty cost**, whose unit is S, occurs when a timestamp in S has not been filled with any value, causing a default; and

finally a **misalignment cost** 1 occurs when a value is released at an earlier or delayed timestamp, and one timestamp of misalignment bears a unit cost of D.

B. Existing Value-Perturbation LDP Mechanisms for Time Series

A number of solutions have been proposed for time series release under LDP. Depending on the privacy requirements, canonical definitions of neighboring time series include *user-level* [31], *event-level* [14], and *w-event* [24] privacy. Given a definition of neighboring time series as above, a formal definition of (ϵ, δ) -LDP on time series is as below.

Definition 2.1: $((\epsilon, \delta)$ -LDP) Given privacy parameters ϵ and δ , a randomized algorithm \mathcal{A} satisfies (ϵ, δ) -LDP, iff for any two neighboring time series S and S', and any possible output R of \mathcal{A} , the following inequality holds:

$$\Pr(\mathcal{A}(S) = R) \le e^{\epsilon} \cdot \Pr(\mathcal{A}(S') = R) + \delta$$
 (1)

Since (ϵ, δ) -LDP is defined on difference in values, all existing works [4], [9], [17], [36] adopt *value-perturbation* mechanisms, such as Laplace mechanism [13], Gaussian mechanism [15], and Randomized Response [38], to inject noise to released value or statistics of the time series.

C. Local Differential Privacy in Temporal Setting

As opposed to the above *value-perturbation* LDP model, we follow the *temporal-perturbation* LDP model (TLDP) as in [45]. In TLDP, two (temporally) neighboring time series are defined as those can be turned into one another by exchanging the values of two timestamps.

Definition 2.2: (Neighboring Time Series) Two time series S and S' are neighbors if there exist two timestamps $t_i \neq t_j$ such that

- 1) |i j| < k, and
- 2) $S_i = S'_i$ and $S_j = S'_i$, and
- 3) for any other timestamp $t_l (l \neq i, j)$, $S_l = S'_l$.

In the above definition, k is the length of a **time sliding window**, which is an additional privacy parameter. The larger the k, the longer period the value remains sensitive to the user. For example, by setting k to 24 hours, a smart watch user can be assured that a released heart rate reading can be from anytime of that day; but if k is set to 1 hour, this period of "deniability" is shortened to 1 hour and might not be sufficient to preserve the user's privacy. Based on Definition 2.2, local differential privacy in the temporal setting, a.k.a. (ϵ, δ) -TLDP, is defined as follows.

Definition 2.3: $((\epsilon, \delta)$ -TLDP) Given privacy parameters ϵ and δ , a randomized algorithm \mathcal{A} satisfies (ϵ, δ) -TLDP, iff for any two neighboring (in a window of length k) time series S and S', and any possible output R of \mathcal{A} , the following inequality holds:

$$\Pr(\mathcal{A}(S) = R) \le e^{\epsilon} \cdot \Pr(\mathcal{A}(S') = R) + \delta$$
 (2)

The degree of privacy in TLDP is controlled by ϵ , δ , and k.

¹The delay cost in [45] is a special case of misalignment cost. The latter also considers the cost when a value is released in advance of the original timestamp.

III. SWITCH OPERATION AND RANSWITCH MECHANISM

In this section, we first define the *switch* operation for temporal perturbation, based on which we present a baseline mechanism RanSwitch to satisfy (ϵ, δ) -TLDP, together with its perturbation protocol, and privacy and utility analysis.

A. Switch Operation

To perturb a time series temporally, an intuitive operation is to probabilistically assign a temporal position for the incoming value at each timestamp. This is the rationale of the *dispatch* operation in [45]. However, since the dispatch position is independently selected at each timestamp, dispatch conflicts may occur. We argue that the root cause lies in the **one-way nature** of dispatch operation. For example, at timestamp t_i , dispatching its value S_i to timestamp t_j only decides the destination of S_i is t_j , but it is uncertain which value S_x should fill in t_i . In other words, the timestamp t_i 's "from" and "to" dispatches (i.e., $S_x \Rightarrow t_i$, and $S_i \Rightarrow t_j$) always happen asynchronously and independently, which leads to missing, empty and repeated values.² To address this issue, in this paper we propose *switch* as a **two-way atomic operation** for temporal perturbation, which is formally defined below.

Definition 3.1: (**Switch Operation**) Given a sliding window of length k, a switch operation $S_i \Leftrightarrow S_j$ exchanges two values S_i and S_j with each other in R, that is, $R_j = S_i$ and $R_i = S_j$, where $0 \le i - j < k$.

In essence, a switch operation $S_i \Leftrightarrow S_j$ is equivalent to two simultaneous and correlated dispatch operations $S_i \Rightarrow t_j$ (i.e., "to" dispatch at t_i) and $S_j \Rightarrow t_i$ (i.e., "from" dispatch at t_i) in [45]. Therefore, it inherently eliminates missing, empty or repeated values. In the sequel, to avoid confusion with "dispatch", we use term "**allocate**" for the one-way perturbation in a switch operation, i.e., S_i is allocated to t_j and S_j is allocated to t_i .

Another advantage of the switch operation is its inherit resemblance to neighboring time series in Definition 2.2. Recall that two temporally neighboring time series are those which can be turned into one another by exchanging the values of two timestamps. Therefore, to satisfy (ϵ, δ) -TLDP becomes intuitive — at each timestamp we just randomly switch the current value with another one within the sliding window of length k. This idea leads to our baseline perturbation mechanism RanSwitch for TLDP. In what follows, we will first present the perturbation protocol of RanSwitch in Sec. III-B, and then analyze its privacy guarantee in Sec. III-D and utility cost in Sec. III-E.

B. RanSwitch: A Baseline Mechanism

For a time series $S = \{S_1, S_2, ..., S_n, ...\}$, at each timestamp t_i , RanSwitch randomly selects a timestamp t_j in the slid-

ing window $\{t_i, t_{i+1}, ..., t_{i+k-1}\}$ according to the following **perturbation probability distribution**:

$$\Pr[j = i + l] = \begin{cases} p, & \text{if} \quad l = 0\\ q, & \text{if} \quad l \in \{1, 2, ..., k - 1\} \end{cases}$$
 (3)

and then applies the switch operation to values S_i and S_j . Here p denotes the probability of selecting the current timestamp (i.e., retaining S_i), while q is the probability of selecting one of the other k-1 timestamps, and p+(k-1)q=1. The pseudocode of RanSwitch mechanism is shown in Algorithm 1. S_i may be finally allocated to one of the k-1 backward timestamps $\{t_{i-k+1}, t_{i-k+2}, ..., t_{i-1}\}$, the current timestamp t_i , or one of the forward timestamps $\{t_{i+1}, t_{i+2}, ...\}$.

Algorithm 1 Perturbation protocol of RanSwitch

Input: Original time series $S = \{S_1, S_2, ..., S_n, ...\}$ Sliding window length k

Perturbation probabilities p and q

Output: Released time series $R = \{R_1, R_2, ..., R_n, ...\}$

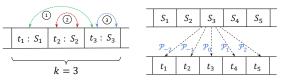
1: Initialize the released time series $R = \emptyset$

2: **for** each timestamp t_i ($i \in \{1, 2, ..., n, ...\}$) **do**

3: Randomly select an index j from $\mathbb{X} = \{l | i \le l \le i+k-1\}$ according to Eq. 3

4: Switch S_i and S_j

5: Set $R_i = S_i$ and release R_i



(a) Switch operation

(b) Probability distribution

Fig. 1. Dispatching probability of RanSwitch mechanism

We show an example of k=3 in Fig. 1(a). Since there are 3 timestamps, RanSwitch performs 3 switch operations 123, one at each timestamp. At timestamp t_1 , suppose S_1 switches with S_3 , then S_3 is allocated backward to t_1 and released. At timestamp t_2 , suppose t_2 itself is selected, then S_2 is released at the current timestamp. At timestamp t_3 , suppose t_3 itself is selected, then the original S_1 (denoted by S_3 now) is allocated to its forward timestamp t_3 and then released.

Since RanSwitch (or any TLDP perturbation mechanism) probabilistically allocates S_i to timestamps t_{i-k+1} , ..., t_i , ..., t_{i+k-1} , ... with probabilities irrespective of i, we can use $\{\mathcal{P}_{1-k},...,\mathcal{P}_{-1},\mathcal{P}_0,\mathcal{P}_1,...,\mathcal{P}_{k-1},...,\mathcal{P}_n\}$ to denote these probabilities, where the subscripts are the temporal deviation of the allocated timestamp from the original timestamp. Fig. 1(b) illustrates the allocating probabilities of S_3 to timestamps $\{t_1,t_2,t_3,t_4,t_5\}$. Note that $n \geq k-1$ and can be as large as infinity, since a value can be repeatedly allocated to its forward timestamps (although with rapidly decreasing probabilities). In this paper, we collectively call these probabilities the **allocating probability distribution** of

 $^{^2}$ According to [45], if the "from" dispatch fails with conflict, then t_i has to report an empty value; if the "to" dispatch fails, then the value S_i will be missed; if two or more values happen to be dispatched to the same timestamp, or if a value is repeatedly dispatched to more than one timestamp, some values will be overwritten and thus missed.

 $^{^3}$ As will be elaborated in Theorem 3.3, privacy analysis only needs the first 2k-1 allocating probabilities, i.e., $\{\mathcal{P}_{1-k},...,\mathcal{P}_{-1},\mathcal{P}_0,\mathcal{P}_1,...,\mathcal{P}_{k-1}\}$, which spans across two adjacent sliding windows.

RanSwitch, the derivation of which lays the foundation of privacy and utility analysis.

C. Allocating Probability Distribution in RanSwitch

We now derive the allocating probability distribution of RanSwitch. For each value S_i , RanSwitch can allocate it backward, stationarily, or forward. As such, we derive the three probabilities separately.

- Backward probability $\{\mathcal{P}_{1-k}, \mathcal{P}_{2-k}, ..., \mathcal{P}_{-1}\}$, when S_i is dispatched to a previous timestamp within a sliding window. To start with, \mathcal{P}_{1-k} denotes the probability when S_{i-k+1} switches with S_i so that S_i is released at timestamp t_{i-k+1} . Hence $\mathcal{P}_{1-k}=q$. The next probability \mathcal{P}_{2-k} is when S_i is released at t_{i-k+2} , which occurs when S_i has not been released at t_{i-k+1} and is then selected at timestamp t_{i-k+2} . Hence $\mathcal{P}_{2-k}=q(1-q)$. Similarly, we can derive other backward probabilities as $\mathcal{P}_j=q(1-q)^{k-1+j}$, where $j\in\{1-k,2-k,...,-1\}$.
- Current probability \mathcal{P}_0 , when the value S_i stays at the current timestamp. This occurs only when S_i has not been released until t_i (w.p. $(1-q)^{k-1}$) and is then released at t_i (w.p. p). As such, $\mathcal{P}_0 = p(1-q)^{k-1}$.
- Forward probability $\{\mathcal{P}_1, \mathcal{P}_2, ..., \mathcal{P}_{k-1}\}$, when the value S_i is released after t_i within a sliding window. Similar to the derivation of backward probability, for any $j \in \{1, 2, ..., k-1\}$, we have $\mathcal{P}_j = q(1-q)^{k-1+j}$.

Combining the above three cases, we reach the following Theorem 3.2 on allocating probability distribution in RanSwitch for all the 2k-1 timestamps.

Theorem 3.2: For $1 - k \le j \le k - 1$, the allocating probability distribution of RanSwitch mechanism is

$$\mathcal{P}_{j} = \begin{cases} p(1-q)^{k-1}, & \text{if } j = 0\\ q(1-q)^{k-1+j}, & \text{otherwise} \end{cases}$$
 (4)

D. Privacy Analysis of RanSwitch

Based on Theorem 3.2, the following theorem proves that RanSwitch satisfies (ϵ, δ) -TLDP.

Theorem 3.3: Given a sliding window of length k and the probabilities p and q, the mechanism RanSwitch satisfies (ϵ, δ) -TLDP, where $\epsilon = \ln \frac{p^2(1-q)^{2(k-1)}-q}{q^2(1-q)^{2(k-1)}}$ and $\delta = q$.

PROOF. For any two neighboring time series $S = \{S_1, S_2, ..., S_i, ..., S_j, ...\}$ and $S' = \{S_1, S_2, ..., S_i', ..., S_j', ...\}$, let t_i and t_j denote the two timestamps when S and S' differ, and |i-j| < k. As each value may be allocated to a forward/backward timestamp within a sliding window or any forward timestamp beyond this window, the difference of output time series occurs when the value S_i in S is allocated to a forward timestamp which S_i' in S' cannot reach. Thus,

$$\delta = \max\{\mathcal{P}_{1-k}, \mathcal{P}_{2-k}, ..., \mathcal{P}_{-2}, \mathcal{P}_{-1}\} = q$$

To derive ϵ , for any output time series R, ϵ must satisfy

$$\epsilon = \sup_{S, S', R} \ln \frac{\Pr[\mathcal{A}(S) = R] - \delta}{\Pr[\mathcal{A}(S') = R]}$$

For any output R by RanSwitch, suppose S_i (or S'_j) is allocated to timestamp t_{α} , and S_j (or S'_i) is allocated to timestamp t_{β} . Then, $\frac{\Pr[\mathcal{A}(S)=R]-\delta}{\Pr[\mathcal{A}(S')=R]} = \frac{\mathcal{P}_{\alpha-i}\cdot\mathcal{P}_{\beta-j}-\delta}{\mathcal{P}_{\alpha-j}\cdot\mathcal{P}_{\beta-i}}$. According to Eq. 4, except for the case of j=0, the allocated

According to Eq. 4, except for the case of j=0, the allocating probability decreases as j increases. So $\mathcal{P}_{1-k} > \mathcal{P}_{k-1}$. Depending on whether they are larger or smaller than P_0 , we derive ϵ separately:

• When $q \leq p(1-q)^{k-1}$, $\mathcal{P}_{k-1} < \mathcal{P}_{1-k} < \mathcal{P}_0$ holds. So

$$\epsilon = \ln \frac{\mathcal{P}_0^2 - \delta}{\mathcal{P}_{\alpha-\beta} \cdot \mathcal{P}_{\beta-\alpha}} = \frac{p^2 (1-q)^{2(k-1)} - q}{q^2 (1-q)^{2(k-1)}}$$
 (5)

• When $q>p(1-q)^{k-1}$, $\mathcal{P}_{1-k}>\mathcal{P}_0$ holds. Since p>q, $p(1-q)^{k-1}>q(1-q)^{2(k-1)}$ always holds. Therefore, $\mathcal{P}_{k-1}<\mathcal{P}_0<\mathcal{P}_{1-k}$. So

$$\epsilon = \max\{\ln \frac{\mathcal{P}_0 \cdot \mathcal{P}_{\beta-j} - \delta}{\mathcal{P}_{i-j} \cdot \mathcal{P}_{\beta-i}}, \ln \frac{\mathcal{P}_0^2 - \delta}{\mathcal{P}_{\alpha-\beta} \cdot \mathcal{P}_{\beta-\alpha}}\}$$
 (6)

As both $\frac{\mathcal{P}_0\cdot\mathcal{P}_{\beta-j}-\delta}{\mathcal{P}_{i-j}\cdot\mathcal{P}_{\beta-i}}=\frac{p(1-q)^{2(k-1)+\beta-j}-1}{q(1-q)^{2(k-1)+\beta-j}}<0$ and $\frac{\mathcal{P}_0^2-\delta}{\mathcal{P}_{\alpha-\beta}\cdot\mathcal{P}_{\beta-\alpha}}<\frac{p^2(1-q)^{k-1}-p}{q^2(1-q)^{k-1}}<0$ always hold when $q>p(1-q)^{k-1}$, any privacy budget $\epsilon>0$ is sufficient to satisfy Eq. 6.

Therefore, Eq. 5 becomes the only requirement of setting privacy budget, i.e., $\epsilon = \ln \frac{p^2(1-q)^{2(k-1)}-q}{q^2(1-q)^{2(k-1)}}$. \square

E. Utility Analysis of RanSwitch

In Sec. II-A, there are four costs that collectively determine the utility of a temporal perturbation approach. Fortunately, RanSwitch mechanism only involves misalignment cost, as there are no missing, empty or repeated values. As such, the expectation of the total cost $\mathbb{E}[C]$ is

$$\mathbb{E}[C] = -D\sum_{j=1-k}^{-1} j\mathcal{P}_j + D\sum_{j=1}^{\infty} j\mathcal{P}_j \qquad (7)$$

Note the second term means the count of (forward) misaligned timestamps j spans from 1 to ∞ . As the first two sliding window (i.e., $1-k \le j \le k-1$) dominates the total cost, we can derive a lower bound of it as

$$\mathbb{E}[C] < \sum_{j=1-k}^{-1} (-j) \cdot P_j + \sum_{j=1}^{k-1} j \cdot P_j$$

$$= \left(k\gamma - \frac{\gamma^2}{q} + (k-1)(\gamma - \gamma^2) \right) D$$
(8)

where $\gamma = 1 - (1 - q)^k$.

From Eq. 8, the lower bound of the total cost is approximately proportional to k. As such, if k is large enough, $\mathbb{E}[C]$ becomes very close to kD, which means the asymptotic utility of RanSwitch could be poor with respect to large k. To illustrate this, in Fig. 2 we plot its allocating probability distribution in green line for k=10 and the privacy budget $\epsilon=2$. We observe that the dispatching probabilities beyond the sliding window, i.e., \mathcal{P}_{10} , ..., \mathcal{P}_{19} , ..., are non-negligible. They form a long tail and dominate $\mathbb{E}[C]$, which is around 4.54^4 according to Eq. 7. Furthermore, for those \mathcal{P}_j within

 $^{^4\}mathrm{We}$ set the maximum misaligned timestamps to 2k in Fig. 2

the sliding window, the probabilities significantly decrease as jincreases, which pushes up the upper bound of the ratio of any two allocating probabilities and thus incurs heavy perturbation to guarantee privacy.

As a comparison, we also plot the allocating probability distribution of the ideal mechanism in blue line, which splits the probabilities equally among all timestamps except \mathcal{P}_0 , i.e., $\mathcal{P}_{-9} = \mathcal{P}_{-8} = \dots = \mathcal{P}_{-1} = \mathcal{P}_{1} = \dots = \mathcal{P}_{8} = \mathcal{P}_{9} = \mathcal{P}_{1} = \mathcal{P}_{1} = \mathcal{P}_{2} = \mathcal{P}_{3} = \mathcal{P}_{4} = \mathcal{P}_{5} = \mathcal{P}_{5}$ $(1-\mathcal{P}_0)/18$. As such, $\mathbb{E}[C]$ is only around 3.82 according to Eq. 8. Although this ideal mechanism cannot be designed in practice, because the perturbation probability distribution is accumulated throughout an infinite time series, it inspires us to design a close-to-ideal perturbation mechanism in the next section, namely StaSwitch, with a more balanced allocating probability distribution than RanSwitch.

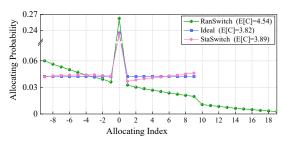


Fig. 2. Allocating probability distribution (k = 10 and $\epsilon = 2$)

IV. STASWITCH: A MECHANISM WITH STATEFUL SWITCH

The root cause of RanSwitch's high misalignment cost lies in the repeated deferment of a value through multiple switch operations. For example, a value at timestamp t_i is first forward switched to t_{i+j} , and then switched to t_{i+j+l} , so on and so forth. To reduce this cost, in this section we propose StaSwitch which keeps track of the switch state of each value and guarantees its final allocated timestamp is still within the initial sliding window. That is, if a value has never been delayed, its allocating space can be the whole sliding window; otherwise, for a value that has already been delayed for l(l < k) timestamps, its new allocation space is limited to the first k-l timestamps of the sliding window.

In what follows, we will first present the perturbation protocol of StaSwitch in Sec. IV-A, and then derive its allocating probability distribution in Sec. IV-B, followed by privacy analysis in Sec. IV-C and utility analysis in Sec. IV-D.

A. Perturbation Protocol of StaSwitch

Given a sliding window of length k, for each value S_i , StaSwitch mechanism allows at most k-1 timestamps delay. To guarantee this, at timestamp t_i , StaSwitchrandomly selects a timestamp t_j in the sliding window $\{t_i, t_{i+1}, ..., t_{i+k-1}\}$ according to the **perturbation probabil**ity distribution in Eq. 9, and then switches S_i and S_j .

$$\Pr[t_j] = \begin{cases} p + bq, & \text{if} \quad j = 0\\ q, & \text{if} \quad j \in \{1, 2, ..., k - 1 - b\} \\ 0, & \text{otherwise} \end{cases}$$
 (9)

TABLE I Perturbation probability distribution when k=4

	j = 0	j = 1	j=2	j = 3	
b = 0	p	q	q	q	
b=1	p+q	q	q	0	
b=2	p+2q	q	0	0	
b=3	p+3q	0	0	0	

Note that b denotes the timestamps S_i has been delayed. If the value S_i has never been delayed, then b=0 and Eq. 9 degrades to Eq. 3 of RanSwitch mechanism. Otherwise, S_i cannot be allocated to the last b timestamps $\{t_{k-b},...,t_{k-1}\}$ in the sliding window of t_i . And their perturbation probabilities, which is bq, are reclaimed by StaSwitch and allocated to t_i . As such, StaSwitch guarantees each value can be delayed up to k-1 timestamps. StaSwitch gets its name from **stateful** switch, as the perturbation probability distribution depends on the current state of delayed timestamps b. Table I shows an example under different delayed timestamps b when k = 4, where each cell shows the perturbation probability of S_i being allocated to t_{i+i} .

Algorithm 2 Perturbation protocol of StaSwitch

Input: Original time series $S = \{S_1, S_2, ..., S_n, ...\}$

Sliding window length k

Perturbation probabilities p and q

Output: Released time series $R = \{R_1, R_2, ..., R_n, ...\}$

1: Initialize a released time series $R = \emptyset$

2: Initialize a vector of delayed timestamps $b = \{0\}^{|S|}$

3: **for** each timestamp t_i ($i \in \{1, 2, ..., n, ...\}$) **do**

Randomly select an index j from $\mathbb{X} = \{l | i \leq l \leq i + k - 1 - b_i\}$ according to Eq. 9

Switch S_i and S_j

 $b_j = b_j + j - i$ $R_i = S_i$ and release R_i

Algorithm 2 shows the pseudo-code of StaSwitch mechanism. The procedure is similar to Algorithm 1, except for vector b, which record the current delayed timestamps of each value. At each timestamp t_i , an index j is randomly drawn from Eq. 9 (Line 4), and then the current value S_i and the selected one S_i are switched (Line 5). As such, S_i is delayed by j-i timestamps, so b_j , the delayed timestamps of S_j , is incremented by j-i (Line 6). Finally, the current value S_i (i.e., the original S_i) is released (Line 7).

B. Allocating Probability Distribution in StaSwitch

We now derive the allocating probability distribution $\mathcal{P} =$ $\{\mathcal{P}_{1-k},...,\mathcal{P}_{-1},\mathcal{P}_0,\mathcal{P}_1,...,\mathcal{P}_{k-1}\}$ of StaSwitch for privacy and utility analysis. To start with, we first derive the expected perturbation probability distribution over all b's from Eq. 9 as

$$\mathbb{E}[\Pr[t_j]] = \begin{cases} p_0 = \sum_{i=0}^{k-1} P_b^i(p+iq), & \text{if } j = 0\\ q_j = \sum_{i=0}^{k-j-1} P_b^i q, & \text{if } 1 \le j \le k-1 \end{cases}$$
(10)

where $P_b^i = \Pr[b=i]$. Note that, at timestamp t_{i+j} , P_b^i is the probability that the current value comes from S_i (i.e., with i timestamps delay) before switching value S_{i+j} . Since there are i such cases, we sum them up as below.

$$P_b^i = q \sum_{j=0}^{i-1} P_b^j \cdot \prod_{l=1}^{i-1-j} (1 - q_l), \tag{11}$$

where $P_b^0 = \prod_{j=1}^{k-1} (1-q_j)$ denotes the probability that the current value has never been delayed.

By substituting Eq. 11 for P_b^i in Eq. 10, we obtain the expected perturbation probability distribution over p_0 and all q_j . Then we derive the allocating probability distribution as follows. First, we know \mathcal{P}_{1-k} means S_{i-k+1} selects timestamp t_i , so S_i is released at t_{i-k+1} . Hence $\mathcal{P}_{1-k} = q_{k-1}$. Similarly, we can derive other allocating probabilities as follows. For any $1 \leq j \leq k-1$,

$$\mathcal{P}_{j-k} = q_{k-j} \prod_{i=1}^{j-1} (1 - q_{k-i})$$

$$\mathcal{P}_{0} = p \prod_{j=1}^{k-1} (1 - q_{k-j})$$

$$\mathcal{P}_{j} = q \sum_{l=0}^{j-1} P_{b}^{l} \left((p+jq) \prod_{i=1}^{j-l-1} (1-q_{i}) + \sum_{i=1}^{k-j-1} q_{i} \prod_{r=1}^{j-l-1} (1-q_{r+i}) \right)$$
(12)

To interpret the last equation, \mathcal{P}_j , the allocating probability of a value being deviated by j timestamps, is comprised of j joint probabilities, each first allocating a value with l ($0 \le l \le j-1$) timestamps deviation (i.e., P_b^l) and then allocating the same value with another j-l timestamps deviation. The latter probability is further comprised of two terms. The first term, $(p+jq)\prod_{i=1}^{j-l-1}(1-q_i)$, denotes the probability that a value with l timestamps deviation is directly switched to timestamp t_j , and the second term, $\sum_{i=1}^{k-j-1}q_i\prod_{r=1}^{j-l-1}(1-q_{r+i})$, is the probability that the value first selects a timestamp after t_j and then switches it with value S_j .

C. Privacy Analysis

This subsection establishes the privacy guarantee of StaSwitch mechanism. Lemmas 4.1 and 4.2 first show the monotonicity of the probability distribution $P_b^i (1 \le i \le k-1)$ and the allocating probability distribution $\mathcal{P}_i (1-k \le i \le k-1)$ respectively, based on which we prove StaSwitch satisfies (ϵ, δ) -TLDP in Theorem 4.3.

Lemma 4.1: For $j \in \{1, 2, ..., k-1\}$, the probability P_b^j monotonously increases with j, i.e., $P_b^1 < P_b^2 < ... < P_b^{k-1}$.

Lemma 4.2: The minimum of StaSwitch's allocating probabilities $\{\mathcal{P}_{1-k},...,\mathcal{P}_{-1},\mathcal{P}_0,\mathcal{P}_1,...,\mathcal{P}_{k-1}\}$ is $\mathcal{P}_{min}=\min\{\mathcal{P}_{1-k},\mathcal{P}_{-1},\mathcal{P}_1\}$.

Theorem 4.3: The StaSwitch satisfies (ϵ, δ) -TLDP, where $\epsilon = \ln \frac{\frac{p^2}{\sigma} - (p^2 - p + 2)}{q(1 + q - \frac{k(1 - p)q}{2(1 + q)} - \frac{q}{2 - p})}$, $\delta = \max\{\mathcal{P}_{1-k}, \mathcal{P}_{2-k}, ..., \mathcal{P}_{-1}\}$, $\sigma = \frac{(1 - p)(1 + p + q)(2 - p) - q}{2(k - 2)(1 + q)(2 - p)} + \frac{(k - 3)q^2(1 - q)^{k - 1}}{2}$, and p and q are parameters defined in Eq. 9.

PROOF. Let t_i and t_j denote the two timestamps that neighboring time series S and S' differ. That is, $S_i = S'_j$, and $S_j = S'_i$. In the output R, let t_α and t_β denote the timestamps

to which the values of S_i and S_j are allocated, respectively. Therefore,

$$\epsilon = \sup_{S,S',R} \ln \frac{\Pr[\mathcal{A}(S) = R] - \delta}{\Pr[\mathcal{A}(S') = R]} = \sup_{\alpha,\beta,i,j} \ln \frac{\mathcal{P}_{\alpha-i}\mathcal{P}_{\beta-j} - \delta}{\mathcal{P}_{\alpha-j}\mathcal{P}_{\beta-i}}$$
(13)

According to Lemma 4.2, the minimum allocating probability is $\mathcal{P}_{min} = \min\{\mathcal{P}_{1-k}, \mathcal{P}_{-1}, \mathcal{P}_1\}$, where $\mathcal{P}_{1-k} = q_{k-1}$, $\mathcal{P}_{-1} = q_1 \prod_{j=2}^{k-1} (1-q_j)$, and $\mathcal{P}_1 = \prod_{j=1}^{k-1} (1-q_j)q(p+q+1-p_0-q_{k-1})$.

Therefore, Eq. 13 is reduced to

$$\epsilon = \ln \frac{\mathcal{P}_{0}^{2} - \delta}{\mathcal{P}_{1} \cdot \mathcal{P}_{-1}} \le \ln \left(\frac{\mathcal{P}_{0}^{2}}{\mathcal{P}_{1} \cdot \mathcal{P}_{-1}} - \frac{1}{\mathcal{P}_{1}} \right)
< \ln \frac{p^{2} - q_{1}(p^{2} - p + 2)}{qq_{1}(p + q + 1 - p_{0} - \frac{q}{2 - p})}$$
(14)

From the above inequality, to derive an upper bound of ϵ , we need to have an upper bound of p_0 and a lower bound of q_1 respectively. First we know that

$$P_b^{k-1} = q \sum_{i=0}^{k-2} P_b^i \prod_{l=1}^{k-2-i} (1-q_l) < q \sum_{i=0}^{k-2} P_b^i = q - q P_b^{k-1}$$

Therefore, $P_b^{k-1} < \frac{q}{1+q}$, and hence,

$$p_0 = \sum_{i=0}^{k-1} P_b^i \cdot (p+iq) = p + q \sum_{i=1}^{k-1} P_b^i \cdot i$$

$$$$

Then we derive a lower bound of q_1 . For any $j \in \{1, 2, ..., k-2\}$, according to Eq. 10 and Lemma 4.1,

$$q_j - q_{j+1} = \sum\nolimits_{i = 0}^{k - j - 1} {{P_b^i}q} - \sum\nolimits_{i = 0}^{k - j - 2} {{P_b^i}q} = qP_b^{k - j - 1} > qP_b^1$$

Thus, $q_1 - (k-2)qP_b^1 > q_2 - (k-3)qP_b^1 > \cdots > q_{k-2} - qP_b^1 = q_{k-1}$. The first term $q_1 - (k-2)P_b^1$ must be greater than the average of the first k-2 terms, i.e.,

$$q_{1} - (k-2)qP_{b}^{1} > \frac{\sum_{i=1}^{k-2} q_{i} - qP_{b}^{1} \sum_{i=1}^{k-2} i}{k-2}$$
$$= \frac{1 - p_{0} - q_{k-1}}{k-2} - \frac{k-1}{2}qP_{b}^{1}$$

Therefore,

$$\begin{aligned} q_1 &> \frac{1 - p_0 - q_{k-1}}{k - 2} - \frac{k - 1}{2} q P_b^1 + (k - 2) q P_b^1 \\ &> \frac{(1 - p)(1 + p + q)(2 - p) - q}{2(k - 2)(1 + q)(2 - p)} + \frac{(k - 3)q^2(1 - q)^{k - 1}}{2} = \sigma \end{aligned}$$

Then following Eq. 14, the upper bound of ϵ becomes $\ln \frac{\frac{p^2}{\sigma} - (p^2 - p + 2)}{q\left(1 + q - \frac{k(1 - p)q}{2(1 + q)} - \frac{q}{2 - p}\right)}$. The proof for δ follows that of Theorem 3.3, where δ must cover the difference between the output space of any two neighboring time series S and S', that is, $\delta = \max\{\mathcal{P}_{1-k}, \mathcal{P}_{2-k}, ..., \mathcal{P}_{-2}, \mathcal{P}_{-1}\}$. \square

Theorem 4.3 provides a close-form of ϵ , but not δ . The following corollary gives a good estimation on it.

Corollary 4.4: The maximum of the first k-1 allocating probabilities can be approximated by \mathcal{P}_{max} , where $max = \lceil \frac{\sqrt{q^2-4p+8}-2}{2a} \rceil - (k+1)$.

PROOF. According to Eq. 11, $P_b^1 = \prod_{j=1}^{k-1} (1-p_j)q = qP_b^0$. For $j \in \{2,3,...,k-1\}$, by approximating P_b^j by P_b^1 , i.e., $P_b^j \approx P_b^1$, we have $\sum_{j=0}^{k-1} P_b^i \approx P_b^0 + (k-1)qP_b^0 = (2-p)P_b^0$. By solving $(2-p)P_b^0 \approx 1$, we know for $j \in \{1,...,k-1\}$,

$$P_b^0 \approx \frac{1}{2-p}, \ P_b^j \approx \frac{q}{2-p} \tag{15}$$

By substituting Eq. 15 in Eq. 10, we have

$$q_j \approx \frac{q + (k - j - 1)q^2}{2 - p} \tag{16}$$

Then according to Eq. 12,

$$\begin{split} \mathcal{P}_{j-k} - \mathcal{P}_{j-k-1} &= 0 \Leftrightarrow q_{k-j} - q_{k-j+1} - q_{k-j} \cdot q_{k-j+1} = 0 \\ &\Leftrightarrow \frac{(1 + (j-1)q)(1 + (j-2)q)}{2 - p} = 1 \\ &\Leftrightarrow j = \frac{\sqrt{q^2 - 4p + 8} + 3q - 2}{2q} \end{split}$$

Therefore, the maximum probability is \mathcal{P}_{max} , where $max = \lceil j - 0.5 \rceil - k = \lceil \frac{\sqrt{q^2 - 4p + 8} - 2}{2q} \rceil - (k + 1)$. \square

D. Utility Analysis

The only cost of StaSwitch is misalignment cost. So the expectation of the total cost is

$$\mathbb{E}[C] = D\left(\sum_{j=1}^{k-1} (-j) \mathcal{P}_{j-k} + \sum_{j=1}^{k-1} j \mathcal{P}_j\right)$$
 (17)

To derive $\mathbb{E}[C]$, we substitute Eqs. 15 and 16 in Eq. 12, then

$$\mathcal{P}_{j-k} \approx \frac{q + (j-1)q^2}{2-p} \prod_{i=1}^{j-1} \left(1 - \frac{q + (i-1)q^2}{2-p}\right)$$

$$\mathcal{P}_0 \approx p \prod_{j=1}^{k-1} \left(1 - \frac{q + (j-1)q^2}{2-p}\right) \tag{18}$$

$$\mathcal{P}_j \approx \frac{q \left(\Phi(j,0) + \Psi(j,0)\right)}{2-p} + \frac{q^2 \sum_{l=1}^{j-1} \left(\Phi(j,l) + \Psi(j,l)\right)}{2-p},$$

where

$$\begin{split} &\Phi(j,l) = (p+jq) \prod_{i=1}^{j-l-1} (1 - \frac{q + (k-i-1)q^2}{2-p}) \\ &\Psi(j,l) = \sum_{i=1}^{k-j-1} \frac{q + (k-i-1)q^2}{2-p} \prod_{r=1}^{j-l-1} (1 - \frac{q + (k-r-i-1)q^2}{2-p}) \end{split}$$

By substituting Eq. 18 in Eq. 17, we obtain an approximation of the expected total cost $\mathbb{E}[C]$ of StaSwitch mechanism. For example, we can obtain $\mathbb{E}[C]=3.89$ in Fig. 2. Compared with $\mathbb{E}[C]=3.82$ for the ideal mechanism, StaSwitch achieves very similar utility.

V. EXPERIMENTAL EVALUATION

In this section, we evaluate and compare RanSwtich and StaSwitch with state-of-the-art TLDP mechanism, namely (Extended) Threshold mechanism (TM/ETM) [45], and value-perturbation mechanisms for time series such as Randomized Response [38] and Piecewise mechanism [33].

A. Experimental Setting

Datasets. We conduct experiments on two real and one synthetic time series datasets.

- US stock [2] consists of historical daily prices of 14,058 trading days. We first extract all daily close price as a numerical time series Stock-N, and then derive another binary time series Stock-B by comparing each close price with its previous day, so each value indicates "up" or "down" of daily stock price.
- Trajectory [1] consists of 6,307 taxi trajectories, each of which has GPS coordinates in a 15-second interval and has at least 300 timestamps.
- SyntheticTS is a generated synthetic time series that consists of 10⁶ timestamps, whose values are integers randomly drawn from [0, 100].

Experiment Design. We design two sets of experiments. The first set evaluates the overall cost of the three TLDP mechanisms under various datasets and parameters, including the sliding window length k and privacy budget ϵ . The second set compares their utility in three real-world applications of popular time series manipulation, namely, simple moving average, frequency counting, and trajectory clustering.

We implement all mechanisms in Java and conduct experiments on a desktop computer with Intel Core i9-9900K 3.60 GHz CPU, 64G RAM running Windows 10 operating system.

B. Overall Cost Evaluation

This subsection evaluates the total cost of three TLDP mechanisms, i.e., RanSwitch, StaSwitch and TM/ETM. According to Sec. II-A, the total cost is

$$C = D \sum_{i} l_i + M \cdot n_1 + N \cdot n_2 + E \cdot n_3,$$

where l_i denotes each value S_i 's count of timestamps deviated after perturbation, and n_1 , n_2 and n_3 are the numbers of missing, empty and repeated values, respectively. In the experiment, we set the unit cost of misalignment D=1, and set unit cost of missing, repetition and empty M=N=E=k, which means these costs are as worse as allocating a value to the endpoint of the sliding window.

Fig. 3 plots the total cost of three mechanisms on the dataset SyntheticTS, by varying the length of sliding window from 10 to 80, and the privacy budget from 1 to 8.5 Overall, StaSwitch performs the best and consistently outperforms RanSwitch in all cases, thanks to its stateful switch operation. The gain of StaSwitch becomes more eminent with the increasing k, and

⁵The privacy parameter δ can be derived by Theorems 3.3 (for RanSwitch) and 4.3 (for StaSwitch) according to ϵ and k, so it is not shown in the figure.

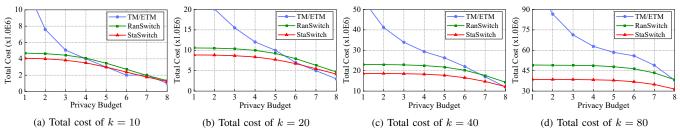


Fig. 3. Overall comparison of total cost of different TLDP mechanisms on SyntheticTS

 $\label{eq:table II}$ Total Cost of Larger Privacy Budgets (k=10)

ϵ	7	8	9	10	11	12	13	14
TM/ETM	2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
RanSwitch	1.96	1.34	0.89	0.56	0.35	0.22	0.13	0.08
StaSwitch	1.77	1.24	0.83	0.54	0.34	0.21	0.12	0.08

exceeds 20% when k=80. We observe that TM/ETM incurs higher cost than both RanSwitch and StaSwitch in most cases, especially for small privacy budgets. The reason is that TM/ETM has to introduce missing and empty values to fully satisfy the privacy guarantee. On the other hand, as we show the results in Table II, when the privacy budget becomes even larger (e.g., $\epsilon > 8$), TM/ETM cannot further benefit from it. This is because the utility gain of TM/ETM is capped at the upper limit of the threshold k-1, and is consistent with our analysis in Sec. I, which motivates this work.

C. Utility Evaluation in Real Applications

To compare the effectiveness of our proposed mechanisms RanSwitch and StaSwitch against TM/ETM, we measure their utilities in three real-world time series applications — simple moving average, frequency counting, and trajectory clustering. We also compare them with *value perturbation* mechanisms in each application. For frequency counting, as each value is binary, we use Randomized Response (RR) [38] which achieves the best performance for binary data [35]. For simple moving average and trajectory clustering, we use Piecewise mechanism (PM) [33], the state-of-the-art LDP solution for numerical value perturbation.

1) Simple Moving Average: We conduct simple moving average of the stock's daily close price on Stock-N and calculate the mean square error (MSE) of the estimated results as $\frac{1}{|S|-r+1}\sqrt{\sum_{i=1}^{|S|-r+1}(m_i-m_i')^2}, \text{ where } m_i=\frac{1}{r}\sum_{j=i}^{i+r-1}S_j \text{ and } m_i'=\frac{1}{r}\sum_{j=i}^{i+r-1}R_j \text{ are the moving averages from the original time series } S \text{ and the released one } R, \text{ with averaging range } r.$

Fig. 4 plots the results, where the privacy budget ϵ varies from 1 to 8, and the length of sliding window k and the averaging range r are set to 10 or 40, respectively. To best accommodate all the results, we re-scale the y-axis in Fig. 4. Under different k and r, PM always has the highest MSE, 1-3 orders of magnitude higher than that of TM/ETM and 3-5 orders of magnitude higher than that of RanSwitch and StaSwitch. This demonstrates the superiority of temporal perturbation over value perturbation for simple moving aver-

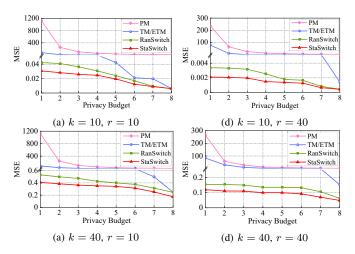


Fig. 4. Results of simple moving average on Stock-N

age. Both RanSwitch and StaSwitch outperform TM/ETM, and the gain is more eminent when privacy budget is small. This is because TM/ETM suffers from missing and empty values when the given privacy budget is relatively small, while RanSwitch and StaSwitch are free of missing or empty values thanks to the two-way nature of switch operation. Between RanSwitch and StaSwitch, StaSwitch consistently outperforms RanSwitch in all cases and the average gain exceeds 20%, as the stateful switch operation adopted by StaSwitch further reduces the misalignment cost in the released time series. In addition, when the averaging range rincreases from 10 to 40, the accuracy of all four mechanisms is improved. For PM, it is because more injected noise is canceled with each other according to law of large numbers; for TLDP mechanisms, it is because temporal perturbation is almost constrained within a sliding window of length k, but this disadvantage is mitigated when the averaging range becomes larger, e.g., k = 10 and r = 40.

2) Frequency Counting: To compare the accuracy of counting the frequency of value "up" on the dataset Stock-B, we adopt RR, TM/ETM, RanSwitch and StaSwitch respectively to perturb the time series, and measure their deviation from the ground-truth count by calculating the MSE as $\frac{1}{|S|}\sqrt{\sum_{i=1}^{|S|}(c_i-c_i')^2}, \text{ where } c_i=\sum_{j=1}^i\mathbb{1}(S_j=up) \text{ and } c_i'=\sum_{j=1}^i\mathbb{1}(R_j=up) \text{ are the "up" counts at timestamp } t_i \text{ in } S \text{ and } R, \text{ respectively.}$

Fig. 5 plots the MSE of these mechanisms, where the time window length k varies from 10 to 80 and the privacy budget ϵ varies from 1 to 8. Overall, StaSwitch performs the best

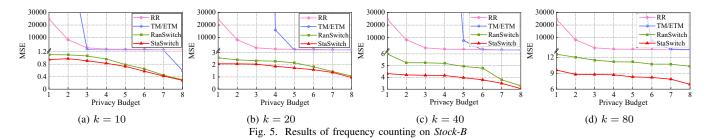


TABLE III NMI on Trajectory Clustering

ϵ	1	2	3	4	5	6	7	8
PM	0.003	0.005	0.006	0.007	0.008	0.008	0.007	0.008
TM/ETM	0.572	0.590	0.610	0.616	0.699	0.705	0.706	0.768
RanSwitch	0.593	0.613	0.609	0.623	0.703	0.713	0.726	0.845
StaSwitch	0.624	0.650	0.703	0.725	0.756	0.745	0.858	0.928

among the four mechanisms, followed by RanSwitch, and their gap becomes more eminent as the window length k gets larger (e.g., RanSwitch reduces 25% of MSE when k=80). On the other hand, TM/ETM no longer outperforms RR (a $value\ perturbation$ method) for small privacy budget (e.g., $\epsilon \leq 3$) or large sliding window (e.g., k=80). This is because frequency counting is very sensitive to missing values, as a missing of "up" decreases all subsequent counts by 1.

3) Trajectory Clustering: Same as [45], we adopt a classic clustering algorithm, namely the k-medoids algorithm [29], to cluster all 6,307 trajectories in the dataset Trajectory into 6 groups. The clustering result over the original dataset is regarded as the ground truth. Then we perturb all trajectories by three TLDP mechanisms (i.e., RanSwitch, StaSwitch and TM/ETM, k = 10) and one value perturbation mechanism, namely PM, and apply the same clustering again. To measure the similarity between the ground-truth clusters and those from perturbed trajectories, we adopt a classic metric, Normalized Mutual Information (NMI) [40], whose range is [0, 1] and a larger NMI means more similarity. Table III shows the results of four mechanisms with privacy budget from 1 to 8. We observe that PM has much smaller NMI than those TLDP mechanisms, because value perturbation heavily damages the utility of the released trajectories. On the other hand, StaSwitch always achieves the highest NMI, followed by RanSwitch, which indicates they have more similar clustering results to the ground truth than TM/ETM.

VI. RELATED WORK

In this section, we review existing works on differential privacy, with a focus on time series release.

Differential Privacy. Differential privacy was first proposed in the centralized setting [13]. To avoid relying on a trusted data collector, local differential privacy (LDP) was proposed to let each user perturb her data locally [12], [25]. In the literature, many LDP techniques have been proposed for various statistical collection tasks, such as frequency of categorical data [5], [17], [23], [35], and mean of numerical data [9], [26], [33]. Recently, the research focus in LDP has been shifted to more complex tasks, such as itemset mining [37], marginal release [8], [48], graph data mining [30], [44], key-value data

analysis [20], [46], [47], high-dimensional data analysis [10], [11], and learning problems [27], [49].

Centralized DP for Time Series. Existing works on centralized DP for time series focus on differentially private aggregate statistics, e.g., frequency estimation. Depending on the privacy requirement, a perturbation mechanism can satisfy *event-level* privacy [7], [14], *user-level* privacy [3], [31], or *w-event* privacy [24], [34]. There are also several strategies proposed to reduce the overall variance in the released statistics, such as Fourier transformation [31], sampling [18], clustering [3], and smoothing techniques [7], [19]. Another line of works also consider temporal correlation of continuously released time series data [6], [41].

LDP for Time Series. More recently, there are a number of studies on the problem of continual time series analysis under LDP. A technique based on *memoization* was first proposed in the local setting [9], [17]. Besides that, Joseph *et al.* [22] design an approach to track changing statistics by assuming that user data are sampled from several evolving distributions. Erlingsson *et al.* [16] further investigate a shuffle model for collecting correlated time series data. Wang *et al.* [36] develop a framework for estimating the sum of real values over a time interval, and Bao *et al.* [4] propose correlated Gaussian mechanism to reduce the noise injected to time series. Xue *et al.* [42] investigate continuous frequency estimation in the user population by exploring an optimal privacy budget allocation scheme to improve estimation accuracy.

The above works are all based on *value perturbation*. The most relevant work to this paper is [45], which is the first work on TLDP privacy model and adopts *temporal perturbation* to satisfy TLDP. However, this mechanism suffers from missing, repetition and empty cost, as well as limitations on settings of privacy parameters ϵ and k. These issues have been addressed in this paper.

VII. CONCLUSION

This paper studies the problem of time series release following TLDP privacy model. We first define *switch* as a two-way atomic operation for the time series perturbation, which inherently eliminates missing, empty or repeated values. Then we propose a baseline mechanism RanSwitch and an optimized mechanism StaSwitch, the latter of which adopts stateful switch to bound each value's timestamp deviation, and thus enhances the utility significantly. We compare RanSwitch and StaSwitch with the existing temporal-perturbation and value-perturbation mechanisms through extensive analytical and empirical analysis under various privacy budgets and

time window sizes, and show that the optimized mechanism StaSwitch always achieves the best performance in various tasks.

As for future work, we plan to extend this work to more complicated time series analysis tasks, such as temporally correlated time series release, time series forecasting, pattern recognition and curve fitting.

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