arXiv:2309.16712v1 [cs.NI] 17 Aug 2023

Joint Participation Incentive and Network Pricing Design for Federated Learning

Ningning Ding, Lin Gao, and Jianwei Huang

***Abstract*—Federated learning protects users' data privacy through sharing users' local model parameters (instead of raw data) with a server. However, when massive users train a large machine learning model through federated learning, the dynamically varying and often heavy communication overhead can put significant pressure on the network operator. The operator may choose to dynamically change the network prices in response, which will eventually affect the payoffs of the server and users. This paper considers the under-explored yet important issue of the joint design of participation incentives (for encouraging users' contribution to federated learning) and network pricing (for managing network resources). Due to heterogeneous users' private information and multi-dimensional decisions, the optimization problems in Stage I of multi-stage games are non-convex. Nevertheless, we are able to analytically derive the corresponding optimal contract and pricing mechanism through proper transformations of constraints, variables, and functions, under both vertical and horizontal interaction structures of the participants. We show that the vertical structure is better than the horizontal one, as it avoids the interests misalignment between the server and the network operator. Numerical results based on real-world datasets show that our proposed mechanisms decrease server's cost by up to 24.87% comparing with the state-of-the-art benchmarks.**

***Index Terms*—Federated learning, incentive mechanism, dynamic network pricing, interaction structure comparison**

I. Introduction

A. Background and Motivations

With the fast development of machine-type communications to support the Internet of Things (IoT), user devices are generating unprecedented amount of data[1] to power intelligent machine learning models [2]. However, users' privacy concerns often make it risky (or even illegal) to centrally collect and store all users' data for model training. This motivates the deployment of federated learning, which enables effective collaborative learning while protecting users' data privacy. During the model training process, distributed users keep their private data on their own devices and only share intermediary model parameters with the central server [3].

Although promising, federated learning still has several under-explored performance bottlenecks, including lack of incentives for participation and heavy communication overhead [4]. Most existing studies made the optimistic assumption that users are willing to participate in the federated learning training process (e.g., [5]). This is not always possible if the users do not receive enough incentives (rewards) to compensate their computation and communication costs [6]. Although some important earlier works explored the incentive mechanism design for federated learning, they did not consider the large and dynamically changing communication overhead and the network operator's resource pricing (e.g., [7]–[11]).

Two aspects lead to the heavy and dynamically varying communication overhead. First, federated learning applications involving a large number of edge devices[2] increasingly involve complex deep neural networks (DNNs) (e.g., Gboard [13] and federated automatic driving [14]). The model parameter update uploaded by each user consists of a large size of gradient vector[3], leading to heavy communication overhead [16]. Second, as mobile users experience different and dynamically changing network connectivities, they may choose to upload their model updates in different time slots (i.e., asynchronously) [3], [17]. This makes the total communication overhead of the federated learning system dynamically changing over time [18]. As a result, such heavy and dynamic communication demand can significantly influence the network operator's resource usage and pricing strategy, which in turn affect users' incentives to join the federated learning system [19].

To overcome the above bottlenecks, we focus on the joint design of incentive mechanism and network pricing in federated learning, with several challenges to tackle. First, the dynamic resource demand motivates the network operator to set dynamic prices to manage the network quality and maximize its profit, which in turn will change the resource demand distribution over time (e.g., a high price discourages users'

This work is supported in part by the National Key Research and Development Program of China (Grant No. 2021YFB2900300), the National Natural Science Foundation of China (Grant 62271434 and 61972113), Shenzhen Science and Technology Program (Projects JCYJ20210324120011032, JCYJ20190806112215116, KQTD20190929172545139, and ZDSYS20210623091808025), Guangdong Basic and Applied Basic Research Foundation (Project 2021B1515120008), Guangdong Provincial Key Laboratory of Big Data Computing, and the Shenzhen Research Institute of Big Data.

Ningning Ding is with the Department of Electrical and Computer Engineering, Northwestern University, Evanston, IL 60208, USA and Shenzhen Research Institute of Big Data, Shenzhen 518172, China.

Lin Gao is with the School of Electronics and Information Engineering and the Guangdong Provincial Key Laboratory of Aerospace Communication and Networking Technology, Harbin Institute of Technology, Shenzhen, China.

Jianwei Huang is with School of Science and Engineering, Shenzhen Research Institute of Big Data, The Chinese University of Hong Kong, Shenzhen, Shenzhen 518172, China (corresponding author, e-mail: jianwei-huang@cuhk.edu.cn).

[1]IoT big data statistics show that the amount of data generated by IoT devices is expected to reach 73.1 ZB (zettabytes) by 2025, corresponding to more than 300% growth over the 2019 output [1].

[2]Business Insider Intelligence expects vehicles in the Internet of Vehicles systems to rise from 33 million in 2017 to over 77 million by 2025 [12].

[3]As the state-of-the-art image classification model, Google's NASNet achieves over 80% accuracy on ImageNet but has a 355MB size [15].

usage) [20]. Moreover, each user's network usage affects other users' payoffs through network congestion (i.e., network externality). It is difficult for the network operator to design the optimal prices considering all heterogeneous users' different usage choices and their network externality. Second, users' private information (e.g., training costs) increases the difficulty for server's incentive design and network operator's pricing. Selfish users can misreport their information for a more favorable outcome [10]. Third, the complex interaction among users, server, and network operator also significantly affects how they can derive their optimal strategies. Specifically, there are two widely-considered interaction structures in the market depending on participants' relative market powers [21]–[23]:

- *Horizontal interaction structure:* the network operator and federated learning server announce their pricing and incentive mechanisms simultaneously, based on which users make participation decisions (to be introduced in Section II-D2).
- *Vertical interaction structure:* the network operator, server, and users make their decisions sequentially (to be introduced in Section II-D1)[4].

Different structures require different incentive and pricing considerations. When it is feasible for both structures to exist in a market, it is also important to compare the performance of these structures and provide policy guidelines regarding which one is more beneficial to the society.

These challenges motivate us to answer the following interesting questions in a federated learning system:

Key Question 1: *What is the server's optimal incentive mechanism for heterogeneous users with private information, considering the heavy communication overhead?*

Key Question 2: *How should the network operator set the prices to maximize its profit, considering the dynamically changing network resource demand?*

Key Question 3: *Which interaction structure is better in terms of the payoffs of the server, the network operator, and users?*

B. Contributions

We summarize our key novelty and contributions below.

- *Incentive mechanism design considering dynamically changing network resource demand*. To the best of our knowledge, this is the first analytical study on incentive mechanism design for federated learning considering dynamic network resource demand. Such a study is practically important for the sustainable development of federated learning systems.
- *Joint design of optimal contract and network pricing under different interaction structures*. We propose multi-stage games to analyze the server's optimal contract and the network operator's optimal dynamic pricing, under both horizontal and vertical interaction structures. With heterogeneous users' private information, the optimization problems of the network operator and the server are non-convex and of a high complexity (e.g., a large number of constraints). We solve these problems by converting the constraints into simpler but equivalent ones and properly transforming variables and functions to obtain convex or analyzable problems.
- *Comparison of interaction structures*. We show that the vertical interaction structure is better than the horizontal structure for users, server, and network operator. This is because the sequential decision process under the vertical structure avoids the scenarios where users incentivized by the server cannot afford the network payment.
- *Insights about network pricing and demand distribution*. When users are congestion-tolerant, we show that it is optimal for the network operator to achieve a water-filling network demand distribution and set the same price for the time slots chosen by at least one user. However, when users are congestion-sensitive, time slots with less background network demands encourage the federated learning users' selection but still have less total network demands. This is because the network operator needs to consider users' total congestion cost in each selected time slot.
- *Performance evaluation*. Numerical results based on real-world datasets show that our proposed mechanisms decrease the server's cost by up to 24.87% and increase the network operator's profit by up to 1245.25%, compared with the state-of-the-art benchmarks.

[4]For example, Cosmo (selling smartwatches) and Things Mobile (selling smartwatch SIM cards) have similar market power, and they usually forms a horizontal interaction structure; China Mobile has larger market power than BYD Auto, and they usually forms a vertical structure [22].

C. Related Work

Most studies on federated learning focused on improving training efficiency (e.g., [24], [25]), enhancing security (e.g., [26], [27]), and preserving privacy (e.g., [28], [29]). Most of the results were derived under an optimistic assumption that users are willing to participate in federated learning, which may not be realistic without proper incentives.

A carefully designed incentive mechanism can elicit users' truthful information, promote cooperation, and enhance system efficiency in federated learning [4]. Although federated learning has seen increasingly more applications in practice, there are only a few important earlier works on the incentive mechanism design (e.g., [5], [7]–[11], [30]), with a few limitations. For example, Sarikaya et al. in [7] studied a complete information scenario where the server knows the private information of users. Kang et al. in [5] and Jiao et al. in [30] focused on the incentive mechanism design under incomplete information yet without closed-form solutions. Feng et al. in [8] and Zhan et al. in [9] modeled users' independent communication costs, without considering users' mutual influence of network usage (e.g., network congestion). Building upon these earlier work, we propose a more general and practical model with private information and users' network externality.

More importantly, our work has two key novelties compared with prior studies on incentive design for federated learning. First, to the best of our knowledge, prior related literature did not consider the impact of dynamic network resource demand, which is challenging to analyze yet practically significant. Instead of only focusing on the interaction between the server and users, our work will perform the joint optimization of network operator's resource pricing and server's incentive design for users. Second, no prior work studied incentive mechanism for federated learning under different interaction structures. However, this is important for both the system participants and policy makers.

The rest of the paper is organized as follows. We first introduce the system model in Section II. We then study the optimal incentive mechanism and network pricing design under the vertical structure and the horizontal structure in Sections III and IV, respectively. We present some interesting insights about congestion-tolerant users in Section V. We perform simulations based on real-world datasets in Section VI and conclude in Section VII.

II. System Model

We consider a typical federated learning platform (e.g., federated automatic driving in the Internet of Vehicles (IoV)), where the model training is distributed over I users and coordinated by a central server. The communication (i.e., model updates transmission) between users and the server during the model training is supported by a mobile network operator (e.g., AT&T). In the following, we will first introduce the federated learning process, then specify the strategies and payoffs of users, server, and network operator, and finally formulate the games among these participants.

A. Federated Learning Process

Federated learning is a distributed machine learning paradigm, in which many users collaboratively train a shared learning model under a server's coordination. Consider an example of data (x_a, y_a), where x_a is the input (e.g., an image) and y_a is the label (e.g., the object in the image). The objective of learning is to find the proper model parameter w that can predict the label y_a based on the input x_a. Let us denote the prediction value as $\tilde{y}(x_a; w)$. The gap between the prediction $\tilde{y}(x_a; w)$ and the ground truth label y_a is characterized by the prediction loss function $f_a(w)$. If user i uses a set $\mathcal{S}_i$ of local data with data size s_i to train the model, the loss function of user i is the average prediction loss on all his data $a \in \mathcal{S}_i$:

$$F_i(w) = \frac{1}{s_i} \sum_{a \in \mathcal{S}_i} f_a(w). \tag{1}$$

The purpose of federated learning is to compute the model parameter w by using all users' local data. The optimal model parameter w^* minimizes global loss function, which is a weighted average of all users' loss functions:

$$w^* = \arg\min_{w} f(w) = \arg\min_{w} \sum_{i=1}^{I} \frac{s_i}{s} F_i(w), \tag{2}$$

where s is the total data size of all users [3].

We consider the widely adopted synchronous federated learning that proceeds in rounds of communication. Each global training round starts when the server broadcasts the current global model parameter to all users and ends after all users upload their local parameter updates to the server for aggregation (so that the server can produce a new global model parameter). The key advantage of the synchronous algorithms is that they have provable convergence (e.g., [31], [32]).

Next, we model the strategies of the network operator, server, and users in each training round.

B. Time Frame, User Type, and Strategies

1) Time Frame: We refer to one training round as one time frame, which is divided into $\mathcal{T} \triangleq \{1, 2, ..., T\}$ time slots. For example, the time frame can be one day, which can be further divided into 24 time slots (each with one hour). In our model, we focus on the optimization in one typical time frame.

2) User Type: We consider a set $\mathcal{I} \triangleq \{1, 2, ..., I\}$ of users in the federated learning system. Users are distinguished by their marginal data-usage costs θ. We refer to a user with θ_j as a type j user. Without loss of generality, I users belong to a set $\mathcal{J} \triangleq \{1, 2, ..., J\}$ of J types. Each type j has I_j users, with $\sum_{j\in\mathcal{J}} I_j = I$. We assume that $\theta_1 < \theta_2 < ... < \theta_J$, and the maximum data size that a user can generate is $d^{\max}$. The total number of users I and the specific number of each type I_j are public information, but each user's type is private information[5].

3) Network Operator's Pricing: The network operator has the flexibility of setting different network prices $\boldsymbol{p} \triangleq \{p(t)\}_{t\in\mathcal{T}}$ in different time slots $t \in \mathcal{T}$. Due to regulatory concerns, the maximum price for any time slot will be p_0.

4) Server's Contract: The server wants users to upload their local training results by the end of the time frame and contribute as much data for local training as possible. The server will design a contract[6] $\boldsymbol{\phi} \triangleq \{\phi_j\}_{j\in\mathcal{J}}$, which contains J contract items (one for each user type). Each contract item $\phi_j \triangleq (d_j, r_j)$ specifies the relationship between each type-j user's data size (for local training) and reward. Here d_j is the required training data size in each training round, and r_j is the corresponding reward if a type-j user completes his training task by the end of the current data frame (i.e., within the current training round) with required data size.

5) Users' Choices: Each user decides whether to participate in the training, (if yes) which contract item to choose, and which time slot to upload the training results. The choice of different time slots may lead to different network congestion costs and network prices for users. A user will not participate if his payoff (defined in Section II-C1) is negative.

C. Payoffs and Profits

1) Users: Since a user's type is private information, he can choose a contract item not designed for his type. When a user $i \in \mathcal{I}$ chooses the contract item ϕ_j and the time slot t_i, his payoff is the difference between the reward from the server and the costs (on his data usage, network payment, and network congestion):

$$W_U^i(\phi_j, t_i) = r_j - \theta_i d_j - p(t_i) - \beta \left(\sum_{k\in\mathcal{I}} \mathbb{1}_{t_k = t_i} + h(t_i) \right)^2, \tag{3}$$

where $\sum_{k\in\mathcal{I}} \mathbb{1}_{t_k=t_i}$ is the normalized network usage in this system (i.e., the number of federated learning users who

[5]It is easy for the server and the network operator to have the knowledge about statistics of type information through market research and past experiences, but it is hard to know each user's private type [33].

[6]Both contract theory and auction theory are promising and widely adopted theoretic tools for dealing with incentive problems with private information. Contract is more applicable to the case where the server knows user type distribution but does not know each user's type, while auction is more suitable when the server does not even know the user type distribution [34], [35].

choose to upload model parameters in the same time slot t_i as user i), $h(t_i)$ is the network usage from other systems at time slot t_i (i.e., background network usage at time slot t_i), and $\beta\left(\sum_{k\in\mathcal{I}}\mathbb{1}_{t_k=t_i}+h(t_i)\right)^2$ is the congestion cost. The quadratic congestion cost captures the increasing marginal cost feature of congestion-sensitive users[7] [36], [37].

2) Server: The server's cost is determined by the accuracy loss of the global model and the total rewards for users[8].

First, we characterize the expected accuracy loss of the global model. The model accuracy loss after D training rounds is measured by the difference between the prediction loss with parameter w_D and that with the optimal parameter w^*, i.e., $f(w_D)-f(w^*)$ (defined in Section II-A). The expected difference is bounded by $O(1/\sqrt{BD}+1/D)$ [38], [39], where B is all users' total training data size in each round, i.e., $B=\sum_{j\in\mathcal{J}}I_jd_j$. Given a fixed D (as we optimize the each-round accuracy), minimizing the accuracy loss bound is mathematically equivalent to minimizing $1/\sqrt{\sum_{j\in\mathcal{J}}I_jd_j}$.[9] It is clear that the model accuracy loss decreases in users' total training data size.

Then, we consider the server's total rewards for all users. If all users choose to participate in the contract and choose their corresponding contract items[10], the total rewards is $\sum_{j\in\mathcal{J}}I_jr_j$.

To summarize, the server's cost is:

$$W_S=\frac{1}{\sqrt{\sum_{j\in\mathcal{J}}I_jd_j}}+\xi\sum_{j\in\mathcal{J}}I_jr_j, \tag{4}$$

where ξ is server's weight on the rewards. A larger ξ means that the server is more concerned about minimizing the reward and less concerned about minimizing the accuracy loss.

3) Network Operator: Network operator's profit is the difference between the revenue from users and the total cost for providing network service in all time slots of a time frame:

$$W_O=\sum_{i\in\mathcal{I}}p(t_i)-\gamma\sum_{t\in\mathcal{T}}\left(\sum_{i\in\mathcal{I}}\mathbb{1}_{t_i=t}+h(t)\right)^2. \tag{5}$$

Here the quadratic network cost captures the widely considered increasing marginal cost feature (e.g., [40], [41]). Intuitively, network operator's cost monotonically increases in the network usage amount (which includes the network usage in this system and the background usage from other systems). However, due to the network operator' limited network resource, when there is already a huge amount of network usage, further increasing the usage will lead to even more significant costs. The γ indicates network operator's weight on the network cost. A larger γ means that the server is more concerned about minimizing the network cost and less concerned about maximizing the revenue.

[7] For example, due to the high requirements on network quality, users in an IoV system can be very sensitive to the network congestion, especially when the congestion is serious. We will study the congestion-tolerant users in Section V.

[8] We consider that the server's network cost for broadcasting the model at the beginning of the training round is a constant. Mathematically, it will not affect the optimization and analysis in this paper, so we do not model it here.

[9] Note that optimizing the single-round accuracy will also guarantee the performance of the entire training process, as it is equivalent to minimizing the expected accuracy loss given any total training round D, i.e., $1/\sqrt{D\sum_{j\in\mathcal{J}}I_jd_j}+1/D$. Mathematically, the constant D here will not affect the optimization. Moreover, we will use experimental accuracy loss in the simulations in Section VI to validate our analytical results.

[10] As we shall see in Section III, without loss of generality, we will design the contract to ensure that each user will choose the contract item designed for his type (i.e., incentive compatibility).

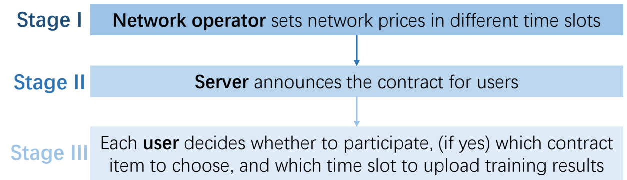

Fig. 1. Vertical interaction structure.

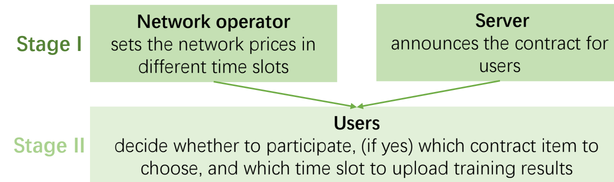

Fig. 2. Horizontal interaction structure.

D. Game and Interaction Structure

We focus on two widely-considered practical interaction structures: vertical and horizontal structures [21], [22]. Different structures correspond to different game formulations.

1) Three-Stage Game (Vertical Structure): As shown in Fig. 1, we model the interaction among the participants as a three-stage Stackelberg game under the vertical structure. The network operator sets the network prices in different time slots in Stage I. After observing the network prices, the server announces the contract for users in Stage II. Given the network operator's prices and the server's contract, each user decides whether to participate and (if yes) chooses the contract item and the time slot for uploading training results in Stage III.

2) Two-Stage Game (Horizontal Structure): As shown in Fig. 2, we model their interaction as a two-stage Stackelberg game. In Stage I, the network operator sets the prices in different time slots, and meanwhile the server announces the contract for users. In Stage II, users then make their decisions, which also leads to a non-cooperative game.

Next, we will use backward induction to analyze these two interaction structures in the next two sections.

III. Incentive Mechanism and Network Pricing Design in Three-Stage Game (Vertical Structure)

In this section, we study the optimal incentive mechanism design and network resource pricing under the vertical interaction structure. Specifically, we first study users' optimal strategies in Section III-A, then calculate the server's optimal contract in Section III-B, and finally derive the network operator's optimal pricing in Section III-C.

A. Users' Optimal Strategies in Stage III

We first formally define users' non-cooperative game in Stage III and the corresponding equilibrium as follows.

Game 1 (Stage III: Users' Game under the Vertical Structure)**.** *The game among users in Stage III is*

- *Players: I users in set $\mathcal{I}$.*
- *Strategy space: each user $i \in \mathcal{I}$ decides whether to participate, which contract item $\phi_i \in \boldsymbol{\phi}$ to choose, and which time slot $t_i \in \mathcal{T}$ to upload his model parameters.*
- *Payoff function: each user $i \in \mathcal{I}$ maximizes his payoff* $W_U^i(\phi_i, t_i; \boldsymbol{\phi_{-i}}, \boldsymbol{t_{-i}}) =$

$$r_i - \theta_i d_i - p(t_i) - \beta\left(\sum_{k\in\mathcal{I}} \mathbb{1}_{t_k=t_i} + h(t_i)\right)^2, \quad (6)$$

where $\boldsymbol{\phi_{-i}} \triangleq \{\phi_{i'}\}_{i'\in\mathcal{I}\setminus\{i\}}$ and $\boldsymbol{t_{-i}} \triangleq \{t_{i'}\}_{i'\in\mathcal{I}\setminus\{i\}}$.

Definition 1 (Equilibrium of Game 1)**.** *The equilibrium of Game 1 is a choice profile $\{(\phi_i^*, t_i^*)\}_{i\in\mathcal{I}}$, such that each user achieves his maximum payoff assuming other users are following the equilibrium strategies, i.e., $\forall \phi_i \in \boldsymbol{\phi}$, $\forall t_i \in \mathcal{T}$,*

$$W_U^i(\phi_i^*, t_i^*; \boldsymbol{\phi_{-i}^*}, \boldsymbol{t_{-i}^*}) \geq W_U^i(\phi_i, t_i; \boldsymbol{\phi_{-i}^*}, \boldsymbol{t_{-i}^*}). \quad (7)$$

By solving Game 1, we have the following lemma:

Lemma 1. *At the equilibrium of Game 1, all chosen time slots $\{t_i^*\}_{i\in\mathcal{I}}$ have identical lowest user network cost:*

$$c(\boldsymbol{p}) \triangleq \min_{t\in\mathcal{T}}\left(p(t) + \beta\Big(\sum_{k\in\mathcal{I}} \mathbb{1}_{t_k^*=t} + h(t)\Big)^2\right), \quad (8)$$

and all unselected time slots have network costs larger than $c(\boldsymbol{p})$. Each user i will choose the contract item ϕ_i^* that maximizes his payoff and gives him a non-negative payoff.

As shown in Lemma 1, users will choose the time slots with the lowest network cost (i.e., sum of network price and congestion cost) at the equilibrium. Each user can have multiple optimal choices in time slots, each corresponding to a different equilibrium but the same lowest network cost. We denote the lowest network cost as $c(\boldsymbol{p})$, which depends on the network pricing in Stage I. Moreover, users' contract item choices depend on the server's contract design in Stage II.

B. Server's Optimal Contract in Stage II

Given the network operator's prices in Stage I, the server needs to design the contract $\boldsymbol{\phi}$ in Stage II, considering the optimal strategies of users in Stage III. In this case, the server's optimization problem under the vertical structure is as follows.

Problem 1 (Server's Contract Design in Stage II)**.**

$$\begin{aligned}
\min\ & \frac{1}{\sqrt{\sum_{j\in\mathcal{J}} I_j d_j}} + \xi \sum_{j\in\mathcal{J}} I_j r_j \\
\text{s.t. } & W_U^i(\phi_i(\boldsymbol{\phi}), t_i(\boldsymbol{\phi})) \geq 0, \forall i \in \mathcal{I} \textit{ (IR)} \\
& W_U^i(\phi_i(\boldsymbol{\phi}), t_i(\boldsymbol{\phi})) \geq W_U^i(\phi_k(\boldsymbol{\phi}), t_i(\boldsymbol{\phi})), \forall i, k \in \mathcal{I} \textit{ (IC)} \\
& 0 \leq d_j \leq d^{\max}, \forall j \in \mathcal{J} \\
\text{var. } & \boldsymbol{\phi} = \{(d_j, r_j)\}_{j\in\mathcal{J}}
\end{aligned}$$

The server needs to design the contract under the *Individual Rationality* (IR) and *Incentive Compatibility* (IC) constraints. Specifically, individual rationality means that a user will participate if and only if he can obtain a positive payoff, and incentive compatibility means that a user maximizes his payoff by choosing the contract item intended for him.

Solving Problem 1 involves two challenges. First, users' contract item choices ϕ_i (related to their marginal training costs θ_i) and upload time choices t_i will both affect users' payoffs and thus the server's optimal strategies, leading to a challenging multi-dimensional contract design. However, based on Lemma 1, we can simplify the analysis into a one-dimensional contract design only about θ_i, as all participating users' different time slot choices lead to the same network cost $c(\boldsymbol{p})$ (i.e., same impact on their payoffs). Second, as the total number of IR and IC constraints is large (i.e., I^2), it is challenging to obtain the optimal contract directly. To overcome such a complexity issue, we first transform the constraints into a smaller number of equivalent ones. Then, we derive the server's optimal reward $r_j^*(\boldsymbol{d})$ for any given data size $\boldsymbol{d}$ in the contract (Lemma 2). Finally, we calculate the optimal contract $\boldsymbol{\phi}^*$ (Theorem 1).

We denote the set of user types incentivized by the sever as $\mathcal{J}' \triangleq \{1_{\mathcal{J}'}, 2_{\mathcal{J}'}, ..., J'_{\mathcal{J}'}\}$ (to be derived in Stage II), in which we reindex the types according to the ascending order of marginal cost θ.[11] The following Lemma 2 characterizes the optimal rewards for any feasible data size:

Lemma 2. *For any given data size $\boldsymbol{d} = \{d_j\}_{j\in\mathcal{J}}$ (even if it is not optimal), the optimal rewards satisfy:*

- *for any user type $j \in \mathcal{J}'$,*

$$r_j^*(\boldsymbol{d}, \boldsymbol{p}) = \begin{cases} \theta_j d_j + c(\boldsymbol{p}), & \text{if } j = J'_{\mathcal{J}'}, \\ \theta_j d_j + \sum_{k=(j+1)_{\mathcal{J}'}}^{J'_{\mathcal{J}'}} (\theta_k - \theta_{k-1}) d_k + c(\boldsymbol{p}), & \text{if } j = 1'_{\mathcal{J}'}, ..., (J'-1)_{\mathcal{J}'}, \end{cases}$$

- *for any user type $j \notin \mathcal{J}'$, $r_j^* = 0$.*

Based on Lemma 2, the following theorem characterizes the server's optimal contract given any network price:

Theorem 1. *Given the network operator's price $\mathbf{p}$, there exists a unique threshold type $x^*(\boldsymbol{p})$,*

$$x^*(\boldsymbol{p}) = \arg\min_{x\in\mathcal{J}} W_S(x, \boldsymbol{p}), \quad (9)$$

where $W_S(x, \boldsymbol{p})$ is given in (10) on the next page, such that the server's optimal incentivized type set is $\mathcal{J}'^* \triangleq \{1, 2, ..., x^*(\boldsymbol{p})\}$ and the optimal contract item for type-j users is

$$\phi_j^*(\boldsymbol{p}) = (d_j^*(\boldsymbol{p}), r_j^*(\boldsymbol{p})) =$$

$$\begin{cases} \left(d^{\max}, \theta_{x^*(\boldsymbol{p})-1} d^{\max} + (\theta_{x^*(\boldsymbol{p})} - \theta_{x^*(\boldsymbol{p})-1}) d^*_{x^*(\boldsymbol{p})} + c(\boldsymbol{p})\right), \forall j < x^*(\boldsymbol{p}), \\ \left(d^*_{x^*(\boldsymbol{p})}, \theta_{x^*(\boldsymbol{p})} d^*_{x^*(\boldsymbol{p})} + c(\boldsymbol{p})\right), \quad j = x^*(\boldsymbol{p}), \\ \mathbf{0}, \quad \forall j > x^*(\boldsymbol{p}), \end{cases}$$

where $d^*_{x^*(\boldsymbol{p})}$ is

[11] For example, if types 1, 3, and 5 are in this set, then we reindex them by $1_{\mathcal{J}'} \triangleq 1$, $2_{\mathcal{J}'} \triangleq 3$, and $3_{\mathcal{J}'} \triangleq 5$.

$$
W_S(x,\boldsymbol{p})=\begin{cases}\frac{1}{\sqrt{\sum_{j=1}^{x} I_j d^{\max}}}+\xi\left(\sum_{j=1}^{x} I_j\theta_x d^{\max}+\sum_{j=1}^{x} I_j c(\boldsymbol{p})\right), & \text{if } d_j^*=d^{\max},\forall j<x \text{ and } d^{\max}<d^{th1},\\ \left(\frac{2\xi}{I_x}\right)^{\frac{1}{3}}\left[\left(\sum_{j=1}^{x} I_j\right)\theta_x-\left(\sum_{j=1}^{x-1} I_j\right)\theta_{x-1}\right]^{\frac{1}{3}}+\xi\left(\frac{1}{I_x^{\frac{1}{3}}(2\xi)^{\frac{2}{3}}}-\frac{\left(\sum_{j=1}^{x} I_j\right)\left(\sum_{j=1}^{x-1} I_j\right)(\theta_x-\theta_{x-1})d^{\max}}{I_x}+\sum_{j=1}^{x} I_j c(\boldsymbol{p})\right), & \\ & \text{if } d_j^*=d^{\max},\forall j<x \text{ and } d^{th1}\le d^{\max}<d^{th2},\\ \infty, & \text{if } \exists j<x, d_j^*<d^{\max} \text{ or } d^{\max}\ge d^{th2}.\end{cases} \tag{10}
$$

$$
\begin{cases}\frac{1}{I_{x^*(\boldsymbol{p})}^{\frac{1}{3}}(2\xi)^{\frac{2}{3}}\left[\left(\sum_{j=1}^{x^*(\boldsymbol{p})} I_j\right)\theta_{x^*(\boldsymbol{p})}-\left(\sum_{j=1}^{x^*(\boldsymbol{p})-1} I_j\right)\theta_{x^*(\boldsymbol{p})-1}\right]^{\frac{2}{3}}} & \\ \quad -\frac{\left(\sum_{j=1}^{x^*(\boldsymbol{p})-1} I_j\right)d^{\max}}{I_{x^*(\boldsymbol{p})}}, & \text{if } d^{\max}\ge d^{opt},\\ d^{\max}, & \text{if } d^{\max}<d^{opt}.\end{cases} \tag{11}
$$

Due to space limit, we do not show the complex expressions of thresholds d^{th1}, d^{th2}, and d^{opt} in (10) and (11) here.

Theorem 1 shows that it is optimal for the server to incentivize the users with relatively small marginal costs. The server sets positive contract items for the incentivized user types and zero for the not incentivized ones. Under such a contract, the threshold type users (i.e., type-$x^*(\boldsymbol{p})$ users) only obtain a zero payoff, as the server's optimal rewards just cover their training costs and network costs. Users with marginal costs smaller than type-$x^*(\boldsymbol{p})$ users will obtain positive payoffs. The specific values of rewards and data sizes depend on the network operator's pricing $\boldsymbol{p}$ in Stage I.

C. Network Operator's Optimal Pricing in Stage I

Considering the server's optimal contract in Stage II and users' optimal strategies in Stage III, the network operator needs to design prices $\boldsymbol{p}\triangleq\{p(t)\}_{t\in\mathcal{T}}$ to maximize its profit:

Problem 2 (Network Operator's Pricing in Stage I)**.**

$$
\begin{aligned}
\max \quad & \sum_{i\in\mathcal{I}} p(t_i^*(\boldsymbol{p}))-\gamma\sum_{t\in\mathcal{T}}\left(\sum_{i\in\mathcal{I}}\mathbb{1}_{t_i^*(\boldsymbol{p})=t}+h(t)\right)^2\\
\text{s.t.} \quad & 0\le p(t)\le p_0,\ t\in\mathcal{T}\\
\text{var.} \quad & \{p(t)\}_{t\in\mathcal{T}}
\end{aligned}
$$

It is challenging to solve Problem 2 for several reasons. First, pricing not only directly affects the network operator's revenue but also indirectly determines the network cost by influencing users' decisions. Second, the network operator needs to consider all users' optimal time choices, the complex form of which makes the optimization problem non-convex.

We tackle the above challenges by decomposing the analysis of Problem 2 into two steps. First, we compute the network operator's optimal network demand distribution (i.e., the number of users in each time slot) under a given set of participating users (Lemma 3), by leveraging proper transformations of variables and functions. Given this demand distribution, we then compute the network operator's optimal prices in Theorem 2, by decomposing the multi-variable optimization into sequential single-variable optimizations.

For the convenience of presentation, we first introduce several notations. We denote the set of users selected by the network operator (i.e., those can afford the network prices of the chosen time slots and participate in the federated learning) by $\mathcal{X}_O$ (to be derived in Theorem 2). We denote n_t as the number of users in this federated learning system who upload results at time slot t. We define the set of time slots that will be chosen by at least one user as $\mathcal{Q}$, and the set of time slots that will not be chosen by any user as $\bar{\mathcal{Q}}$, i.e., $\mathcal{Q}\cup\bar{\mathcal{Q}}=\mathcal{T}$. In other words, for each time slot $t\in\mathcal{Q}$, we have $n_t>0$; and for each time slot $t\in\bar{\mathcal{Q}}$, we have $n_t=0$.

Lemma 3. *Given a set of selected users $\mathcal{X}_O$, the network operator's optimal time slot sets $\mathcal{Q}^*(\mathcal{X}_O)$ and $\bar{\mathcal{Q}}^*(\mathcal{X}_O)$ are*

$$
\begin{aligned}
\mathcal{Q}^*(\mathcal{X}_O)&=\{t: h(t)(\beta h(t)+2\gamma)\le h(\tilde{t})(\beta h(\tilde{t})+2\gamma)\},\\
\bar{\mathcal{Q}}^*(\mathcal{X}_O)&=\{t: h(t)(\beta h(t)+2\gamma)> h(\tilde{t})(\beta h(\tilde{t})+2\gamma)\},
\end{aligned} \tag{12}
$$

where the threshold time slot $\tilde{t}$ is the unique value that makes $\mathcal{Q}^*(\mathcal{X}_O)$ and $\bar{\mathcal{Q}}^*(\mathcal{X}_O)$ satisfy

$$
\begin{cases}\max\limits_{t\in\mathcal{Q}^*(\mathcal{X}_O)} h(t)(\beta h(t)+2\gamma)\le-\lambda\le\min\limits_{t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O)} h(t)(\beta h(t)+2\gamma), & \text{(13a)}\\ \sum\limits_{t\in\mathcal{Q}^*(\mathcal{X}_O)}\sqrt{(\beta h(t)-\gamma)^2-3\beta\lambda}=3\beta|\mathcal{X}_O|+\sum\limits_{t\in\mathcal{Q}^*(\mathcal{X}_O)}(2\beta h(t)+\gamma). & \text{(13b)}\end{cases}
$$

The network operator's optimal demand distribution is

$$
n_t^*(\mathcal{X}_O)=
\begin{cases}\frac{\sqrt{(\beta h(t)-\gamma)^2-3\beta\lambda}-(2\beta h(t)+\gamma)}{3\beta}, \forall t\in\mathcal{Q}^*(\mathcal{X}_O), & \text{(14a)}\\ 0, \quad \forall t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O). & \text{(14b)}\end{cases}
$$

Lemma 3 indicates that the network operator wants users to choose the time slots with small values of $h(t)(\beta h(t)+2\gamma)$, which can be manipulated by the network operator though proper prices (to be shown in Theorem 2). The criterion $h(t)(\beta h(t)+2\gamma)$ indicates that the network operator considers the network costs of both itself (indicated by term $2\gamma h(t)$) and users (indicated by term $\beta h(t)^2$).

Moreover, Lemma 3 shows that the time slots with less background network demands encourage the federated learning users' selection but still have less total network demands ($n_t^*+h(t)$). This is because the network operator needs to consider users' total congestion cost in each selected time slot, which cubically increases in the number of users who choose that slot. We will illustrate this by simulations in Section VI.

Based on the optimal demand distribution in Lemma 3, we present the network operator's optimal pricing in Theorem 2.

Theorem 2. *The network operator's optimal selected user set $\mathcal{X}_O^*$ contains users of types $\{1,2,...,x_O^*\}$ with*

$$
\begin{aligned}
x_O^*=\arg\max_{x_O\in\mathcal{J}}\Big(&|\mathcal{X}_O|\tilde{C}^*(x_O)-\\
&\beta\sum_{i\in\mathcal{X}_O}\left(n_{t_i}^*(\mathcal{X}_O)+h(t_i)\right)^2-\gamma\sum_{t\in\mathcal{T}}\left(n_t^*(\mathcal{X}_O)+h(t)\right)^2\Big),
\end{aligned} \tag{15}
$$

where

$$\tilde{C}^*(x_O)=\max\Big\{c\!:c<p_0+\min_{t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O)}\beta h(t)^2, W_S(x_O,\boldsymbol{p})\le\min_{j\in\mathcal{J}}W_S(j,\boldsymbol{p}),$$
$$\max_{t\in\mathcal{Q}^*(\mathcal{X}_O)}\Big(\beta\left(n_t^*(\mathcal{X}_O)+h(t)\right)^2\Big)\le c\le p_0+\min_{t\in\mathcal{Q}^*(\mathcal{X}_O)}\Big(\beta(n_t^*(\mathcal{X}_O)+h(t))^2\Big)\Big\}.$$

The network operator's optimal prices in Stage I are

$$p(t)^*=\begin{cases}\tilde{C}^*(x_O^*)-\beta\left(n_t^*(\mathcal{X}_O^*)+h(t)\right)^2, & t\in\mathcal{Q}^*(\mathcal{X}_O^*),\\ \textit{any value}\in\Big(\max_{t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O^*)}\{\tilde{C}(x_O^*)-\beta h(t)^2\},p_0\Big], & t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O^*).\end{cases}$$

Theorem 2 indicates that the network operator needs to consider a trade-off among the prices, the number of participating users, and its network resource cost. If the network operator sets larger prices, the number of participating users will decrease but the network resource cost will also decrease. The optimal prices in Theorem 2 maximize the network operator's profit under such a trade-off.

Next, we focus on the analysis of the two-stage game under the horizontal structure.

IV. Incentive Mechanism and Network Pricing Design in Two-Stage Game (Horizontal Structure)

For the horizontal structure, the analysis for users in Stage II is the same as that of Stage III under the vertical structure in Section III-A. Next, we present the analysis of the equilibrium strategies of the server and the network operator in Stage I.

We first define the server and the network operator's non-cooperative game and its equilibrium as follows:

Game 2 (Stage I: Game of Server and Network Operator under Horizontal Structure)**.** *The game between the server and the network operator in Stage I is*

- *Players: server and network operator.*
- *Strategy space: the sever designs the contract items $\boldsymbol{\phi}=\{(d_j,r_j)\}_{j\in\mathcal{J}}$, where $d_j\in[0,d^{\max}]$ and $r_j\in[0,+\infty)$, for each $j\in\mathcal{J}$. The network operator sets its prices $\boldsymbol{p}=\{p(t)\}_{t\in\mathcal{T}}$ at each time slot t, where $p(t)\in[0,p_0]$, for each $t\in\mathcal{T}$.*
- *Payoff function: the server minimizes its cost*

$$W_S(\boldsymbol{\phi};\boldsymbol{p})=\frac{1}{\sqrt{\sum_{j\in\mathcal{J}}I_jd_j}}+\xi\sum_{j\in\mathcal{J}}I_jr_j, \tag{16}$$

and the network operator maximizes its profit

$$W_O(\boldsymbol{p};\boldsymbol{\phi})=\sum_{i\in\mathcal{I}}p(t_i^*(\boldsymbol{p}))-\gamma\sum_{t\in\mathcal{T}}\Big(\sum_{i\in\mathcal{I}}\mathbb{1}_{t_i^*(\boldsymbol{p})=t}+h(t)\Big)^2.$$

Definition 2 (Equilibrium of Game 2)**.** *The equilibrium of Game 2 is a profile $(\boldsymbol{\phi}^*,\boldsymbol{p}^*)$, such that the server and the network operator achieve their minimum cost or maximum profit assuming each other is following the equilibrium strategy:*

$$W_S(\boldsymbol{\phi}^*;\boldsymbol{p}^*)\le W_S(\boldsymbol{\phi};\boldsymbol{p}^*),\forall d_j\in[0,d^{\max}],r_j\in[0,+\infty),j\in\mathcal{J},$$
$$W_O(\boldsymbol{p}^*;\boldsymbol{\phi}^*)\ge W_O(\boldsymbol{p};\boldsymbol{\phi}^*),\forall p(t)\in[0,p_0],t\in\mathcal{T}. \tag{17}$$

According to the definitions, we will first analyze the best responses of the server and network operator in Section IV-A, and then find out the fixed point of the best responses (which is the equilibrium) in Section IV-B.

A. Best Responses of Sever and Network Operator in Game 2

First, the server's optimal contract $\boldsymbol{\phi}^*(\boldsymbol{p})$ given the network operator's prices $\boldsymbol{p}$ (i.e., best response of the server) is the same as the analysis in Section III-B.

Next, we present the network operator's optimal pricing $\boldsymbol{p}^*(\boldsymbol{\phi})$ given any server's contract $\boldsymbol{\phi}$ (i.e., best response of the network operator) in Lemma 4. The main analysis difference from that under the vertical structure in Section III-C is that, the network operator does not know the server's optimal strategies under the horizontal structure here.

Lemma 4. *Given the server's contract $\boldsymbol{\phi}$, the network operator's optimal selected user set $\mathcal{X}_O^*(\boldsymbol{\phi})$ is*

$$\mathcal{X}_O^*(\boldsymbol{\phi})=\arg\max_{\mathcal{X}_O\subseteq\mathcal{I}}\Big(|\mathcal{X}_O|\tilde{C}^*(\mathcal{X}_O,\boldsymbol{\phi})-\beta\sum_{i\in\mathcal{X}_O}\left(n_{t_i}^*(\mathcal{X}_O)+h(t_i)\right)^2-\gamma\sum_{t\in\mathcal{T}}\left(n_t^*(\mathcal{X}_O)+h(t)\right)^2\Big), \tag{18}$$

where users' maximum acceptable network cost $\tilde{C}^*(\mathcal{X},\boldsymbol{\phi})$ is

$$\tilde{C}^*(\mathcal{X}_O,\boldsymbol{\phi})=\max\Big\{\tilde{C}\!:\max_{t\in\mathcal{Q}^*(\mathcal{X}_O)}\left(\beta\left(n_t^*(\mathcal{X}_O)+h(t)\right)^2\right)\le\tilde{C}\le p_0+\min_{t\in\mathcal{Q}^*(\mathcal{X}_O)}\left(\beta\left(n_t^*(\mathcal{X}_O)+h(t)\right)^2\right),\tilde{C}<p_0+\min_{t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O)}\beta h(t)^2,\ \max\{r_i-\theta_id_i\}_{i\notin\mathcal{X}_O}<\tilde{C}\le\min\{r_i-\theta_id_i\}_{i\in\mathcal{X}_O}\Big\}. \tag{19}$$

The network operator's optimal prices are

$$p(t)^*(\boldsymbol{\phi})=\begin{cases}\tilde{C}^*(\mathcal{X}_O^*,\boldsymbol{\phi})-\beta\left(n_t^*(\mathcal{X}_O^*)+h(t)\right)^2, & t\in\mathcal{Q}^*(\mathcal{X}_O^*),\\ \textit{any value}\in\Big(\max_{t\in\bar{\mathcal{Q}}^*}\{\tilde{C}^*(\mathcal{X}_O^*,\boldsymbol{\phi})-\beta h(t)^2\},p_0\Big], & t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O^*).\end{cases}$$

Next, we combine the best responses of the server and the network operator to obtain the equilibrium under the horizontal structure.

B. Equilibrium in Stage I and Structure Comparison

For the convenience of presentation, we first introduce the following definition:

$$H\triangleq\max\Big\{c(\boldsymbol{p}):W_S(x^*,\boldsymbol{p})\le\min_{j\in\mathcal{J}}W_S(j,\boldsymbol{p})\Big\}-\max\Big\{p_0+\min_{t\in\bar{\mathcal{Q}}^*(\mathcal{X}^*)}\beta h(t)^2, p_0+\min_{t\in\mathcal{Q}^*(\mathcal{X}^*)}\beta\left(n_t^*(\mathcal{X}^*)+h(t)\right)^2\Big\}, \tag{20}$$

where x^* equals x_O^* in (15) if $\tilde{C}^*(x_O)$ in (15) equals

$$\max\Big\{c\!:c<p_0+\min_{t\in\bar{\mathcal{Q}}^*(\mathcal{X}_O)}\beta h(t)^2, \max_{t\in\mathcal{Q}^*(\mathcal{X}_O)}\Big(\beta\left(n_t^*(\mathcal{X}_O)+h(t)\right)^2\Big)\le c\le p_0+\min_{t\in\mathcal{Q}^*(\mathcal{X}_O)}\Big(\beta(n_t^*(\mathcal{X}_O)+h(t))^2\Big)\Big\}, \tag{21}$$

and $\mathcal{X}^*$ contains users of types $\{1,2,...,x^*\}$.

Given the definition of H, we present the equilibrium existence and equilibrium strategies under the horizontal structure in Theorem 3:

Theorem 3. *If $H\ge0$, the equilibrium exists under the horizontal structure and is the same as that under the vertical structure. If $H<0$, the equilibrium does not exist under the horizontal structure .*

The condition $H\ge0$ in Theorem 3 means that under the horizontal structure, both the network operator and the server obtain the maximum profit (or minimum cost) by incentivizing the same group of users. The equilibrium does not exist if the

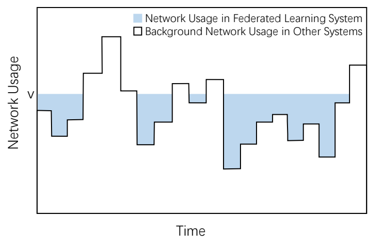

Fig. 3. Illustration example of water-filling network usage distribution.

network operator and the server are interested in incentivizing different groups of users (i.e., $H < 0$).

Moreover, Theorem 3 shows that the equilibrium under the horizontal structure (if it exists) is the same as the vertical one. Two aspects contribute to such a phenomenon. First, under the vertical structure, it is the users who choose the time slots and pay the network operator, which is the same as the interaction under the horizontal structure. Second, the equilibrium under horizontal structure only exists when the network operator and the sever coincidentally incentivize the same group of users, same as the decision alignment under the vertical structure.

To conclude, Theorem 3 shows that the vertical structure is better than the horizontal one, as the vertical structure always ensures the existence of an equilibrium which is no worse than that of the horizontal structure (if it exists).

The analysis so far focused on federated learning applications with congestion-sensitive users. Next, we study the special scenario with congestion-tolerant users.

V. Congestion-Tolerant Users

In this section, we study the special case where users are tolerant of congestion, i.e., users do not have congestion costs ($\beta = 0$). This is motivated by some practical scenarios where users do not have high requirements on network qualities[12].

The analysis of the two structures in Sections III and IV is still applicable to this special case. Meanwhile, we will be able to reveal some additional new insights, as presented in Propositions 1 and 2.

Proposition 1. *When $\beta = 0$, at the equilibrium, each user i chooses a time slot with the lowest price, i.e.,*

$$t_i^* = \arg\min_{t\in\mathcal{T}} p(t), \ \forall i \in \mathcal{I}. \tag{22}$$

As users are network congestion tolerant, they only care about the network price when selecting the time slots. However, if too many users choose the same time slot, the network operator's resource cost at this time slot will be very large, which may increase the network price for this slot. Therefore, users' network usage at the equilibrium depends on the network operator's pricing.

Proposition 2. *When $\beta = 0$, it is optimal for the network operator to have a water-filling network usage distribution and the same price for users' chosen time slots. The total network usage of the chosen time slots (i.e., water level) v satisfies*

$$\sum_{t\in\mathcal{T}} [v - h(t)]^+ = \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{I}} \mathbb{1}_{t_i^*=t} \right), \tag{23}$$

where

$$[v - h(t)]^+ \triangleq \begin{cases} v - h(t), & \text{if } v \geq h(t), \\ 0, & \text{if } v < h(t). \end{cases} \tag{24}$$

The water-filling network usage distribution is illustrated by the example in Fig. 3, where participating users will choose the time slots with small background network usage such that the total network usage in all chosen time slots will be the same (i.e., v). When users are not concerned about their congestion costs, the network operator only needs to consider its own network cost in different time slots when designing the prices.

Next, we use real-world datasets to validate the performance of our proposed mechanisms.

VI. Simulation

In this section, we perform numerical experiments to validate our analytical results and evaluate the performance of the proposed mechanisms. We first introduce the experiment setting in Section VI-A, then show the experiment results of the optimal contract and pricing in Section VI-B, and finally compare the performance of our mechanism with two state-of-the-art benchmarks in Section VI-C.

A. Experiment Setting

We use the hourly mobile phone data usage obtained from a real-world dataset as the background network usage distribution (as shown in Fig. 4). The dataset covers all base stations of the Elisa Oyj network operator in the Uusimaa region in Southern Finland, from late October 2017 till early January 2018 [42].

Regarding the system parameters, we consider the time frame of one day that consists of 24 time slots, i.e., $T = 24$. There are five types of users with marginal costs $\theta_1 = 2$, $\theta_2 = 4$, $\theta_3 = 6$, $\theta_4 = 8$, and $\theta_5 = 10$. Each type has $I_j = 1000$ users, and the maximum data size that each user can contribute in one round is $d^{\max} = 10$ MB. The maximum network price that the network operator can set is $p_0 = 2000$ cents. The normalized total background network usage is $\sum_{t\in\mathcal{T}} h(t) = 10^5$ users in other systems. We choose the congestion sensitive coefficients of users and the network operator as $\beta = \gamma = 10^{-4}$, and set the server's cost coefficient as $\xi = 5 \times 10^{-10}$ to balance different parameters' units and orders of magnitude.

To obtain the experimental model accuracy loss, we consider that users with non-IID data train a federated learning model on the CIFAR-10 dataset. Specifically, each user is randomly assigned 2 labels and each label has 50 data points. We assume that users' data distribution is independent of their marginal cost distribution. Our convolutional neural network (CNN) model consists of six 3×3 convolution layers (with 64, 64, 128, 128, 256, 256 channels, respectively, and every two followed with 2×2 max pooling), a Drop-out layer (0.5), a fully-connected layer with 10 units and ReLU activation, and a final softmax output layer.

[12]For example, compared with the users in automatic driving, mobile phone users participating in the next-word-prediction learning task usually care less about time delay. Mobile users themselves may even intentionally delay the parameter uploading due to considerations such as battery conditions [3].

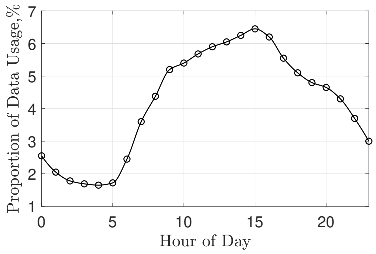

Fig. 4. Hourly Mobile Phone Data Usage.

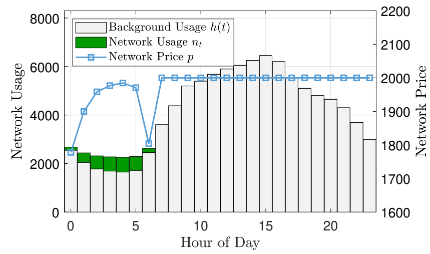

Fig. 5. Network usage distribution and network operator's optimal prices in different time slots.

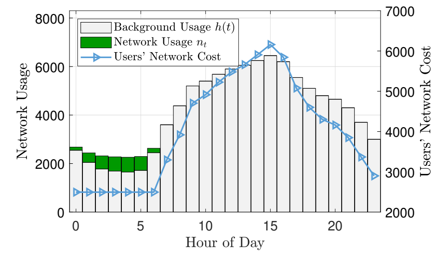

Fig. 6. Network usage distribution and users' network costs in different time slots.

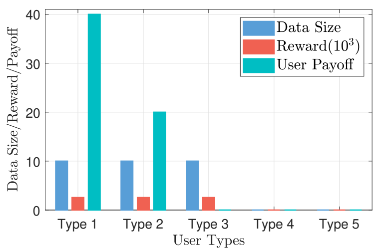

Fig. 7. Server's optimal contract and users' payoffs for different types of users.

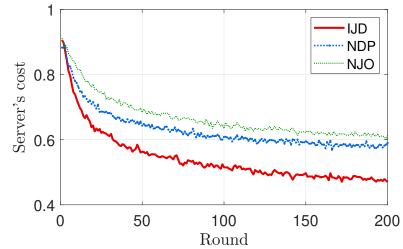

Fig. 8. The comparison of server's costs under three mechanisms.

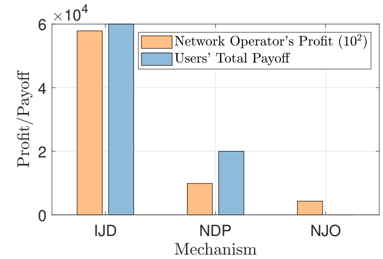

Fig. 9. The comparison of network operator's profits and users' total payoffs under three mechanisms.

B. Experiment Results: Optimal Pricing and Contract

As the horizontal structure has no equilibrium or the same equilibrium with the vertical structure, next we only present the numerical results under the vertical structure[13].

1) Network Operator's Optimal Demand Distribution: Fig. 5 shows that when users are congestion-sensitive, the optimal network demand distribution is not a water-filling solution, i.e., the total network usages (green plus gray in Fig. 5) in users' chosen time slots are not the same. Users will choose the time slots with small background network usages $h(t)$ (i.e., 0:00-6:00). Interestingly, among these chosen time slots, the slots with smaller background usages (gray) still have smaller total network usages (green plus gray). This is consistent with the results in Lemma 3, because the network operator does not want too many users choose the same time slot, as the total congestion cost of users in a time slot cubically increases in the number of users who choose this time slot.

2) Network Operator's Optimal Prices: As shown in Fig. 5, the optimal prices for time slots chosen by at least one user are different, i.e., there is a larger price for a smaller total network usage. As shown in Fig. 6, under the optimal prices, users at different chosen time slots (i.e., 0:00-6:00) have the same minimum network cost (the sum of the price and the congestion cost), which is consistent with Lemma 1.

3) Server's Optimal Contract: As shown in Fig. 7, the server sets positive contract items for type-1, type-2, and type-3 users and zero contract item for other users, which validate Theorem 1. The threshold type users (i.e., type-3 users) only obtain a zero payoff, as the server's designed optimal rewards just cover their training costs and network costs. Type-1 and type-2 users (with marginal costs smaller than type-3 users) will obtain positive payoffs.

[13]We can validate that under the same experiment setting in Section VI-A, the equilibrium under the horizontal structure does not exist. If we change the experiment setting, the equilibrium under the horizontal structure can exist and will be the same as that under the vertical structure. Due to space limit, we will not show the detailed simulation results under the horizontal structure.

C. Performance Comparison with Benchmarks

To evaluate the performance, we list two benchmarks and our proposed mechanism as follows.

- *No Joint Optimization (NJO) [30]*: the server designs the incentive mechanism without considering the network operator's strategies.
- *No Dynamic Pricing (NDP)*: the server designs the incentive mechanism by assuming that the network operator sets a same price in all time slots[14].
- *Our proposed pricing mechanism (IJD)*: the server designs the Incentive mechanism with the Joint consideration of network operator's optimal Dynamic pricing.

As shown in Fig. 8 and Fig. 9, our proposed mechanism outperforms the NJO and NDP benchmarks in terms of the server's cost, the network operator's profit, and users' total payoff. Compared with the NJO benchmark, the server's cost reduction and network operator's profit growth of our IJD mechanism reach 24.87% and 1245.25%, respectively.

VII. Conclusion

To the best of our knowledge, this is the first study on the important issue of joint participation incentive and network pricing design in federated learning. We compared two typical interaction structures of the server, network operator, and users in the system. We showed that the vertical interaction structure is better than the horizontal one for all participants. Moreover, we demonstrated that when users are congestion-sensitive, time slots with less background network demands encourage the federated learning users' selection but still have less total network demands. The simulations showed that our proposed mechanisms decrease the server's cost by up to 24.87% and increase network operator's profit by up to 1245.25%, compared with the state-of-the-art benchmarks.

[14]Due to space limit, we will not present the closed-form optimal incentives and prices in the benchmark cases.

REFERENCES

[1] "Internet of Things statistics for 2022 - Taking Things Apart," https://dataprot.net/statistics/iot-statistics/.

[2] S. K. Goudos, P. I. Dallas, S. Chatziefthymiou, and S. Kyriazakos, "A survey of IoT key enabling and future technologies: 5G, mobile IoT, sematic web and applications," *Wireless Personal Communications*, vol. 97, no. 2, pp. 1645–1675, 2017.

[3] H. B. McMahan, E. Moore, D. Ramage, S. Hampson, and B. A. y Arcas, "Communication-efficient learning of deep networks from decentralized data," in *Artificial Intelligence and Statistics*, 2017.

[4] P. Kairouz, H. B. McMahan, B. Avent, A. Bellet, M. Bennis, A. N. Bhagoji, K. Bonawitz, Z. Charles, G. Cormode, R. Cummings *et al.*, "Advances and open problems in federated learning," *Foundations and Trends® in Machine Learning*, vol. 14, no. 1–2, pp. 1–210, 2021.

[5] J. Kang, Z. Xiong, D. Niyato, S. Xie, and J. Zhang, "Incentive mechanism for reliable federated learning: A joint optimization approach to combining reputation and contract theory," *IEEE Internet of Things Journal*, vol. 6, no. 6, pp. 10 700–10 714, 2019.

[6] N. Tran, W. Bao, A. Zomaya, and C. Hong, "Federated learning over wireless networks: Optimization model design and analysis," in *IEEE INFOCOM*, 2019.

[7] Y. Sarikaya and O. Ercetin, "Motivating workers in federated learning: A stackelberg game perspective," *IEEE Networking Letters*, 2019.

[8] S. Feng, D. Niyato, P. Wang, D. I. Kim, and Y. Liang, "Joint service pricing and cooperative relay communication for federated learning," in *International Conference on Internet of Things (iThings)*, 2019.

[9] Y. Zhan, P. Li, Z. Qu, D. Zeng, and S. Guo, "A learning-based incentive mechanism for federated learning," *IEEE Internet of Things Journal*, 2020.

[10] N. Ding, Z. Fang, and J. Huang, "Optimal contract design for efficient federated learning with multi-dimensional private information," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 1, pp. 186–200, 2020.

[11] Y. Deng, F. Lyu, J. Ren, Y.-C. Chen, P. Yang, Y. Zhou, and Y. Zhang, "FAIR: Quality-aware federated learning with precise user incentive and model aggregation," in *IEEE INFOCOM*, 2021.

[12] "How 5G & IoT technologies are driving the connected smart vehicle industry," https://www.businessinsider.com/iot-connected-smart-cars.

[13] "Gboard," https://apps.apple.com/us/app/gboard-the-google-keyboard/id1091700242.

[14] A. Nguyen, T. Do, M. Tran, B. X. Nguyen, C. Duong, T. Phan, E. Tjiputra, and Q. D. Tran, "Deep federated learning for autonomous driving," *arXiv preprint arXiv:2110.05754*, 2021.

[15] B. Zoph, V. Vasudevan, J. Shlens, and Q. V. Le, "Learning transferable architectures for scalable image recognition," in *IEEE conference on computer vision and pattern recognition*, 2018.

[16] L. Wand, W. Wang, and B. Li, "CMFL: Mitigating communication overhead for federated learning," in *IEEE 39th International Conference on Distributed Computing Systems (ICDCS)*, 2019.

[17] X. Zhang, X. Zhu, J. Wang, H. Yan, H. Chen, and W. Bao, "Federated learning with adaptive communication compression under dynamic bandwidth and unreliable networks," *Information Sciences*, vol. 540, pp. 242–262, nov 2020.

[18] A. H. Estiri and M. Maheswaran, "Attentive federated learning for concept drift in distributed 5g edge networks," *arXiv preprint arXiv:2111.07457*, 2021.

[19] H. T. Nguyen, V. Sehwag, S. Hosseinalipour, C. G. Brinton, M. Chiang, and H. V. Poor, "Fast-convergent federated learning," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 1, pp. 201–218, 2021.

[20] X. Wang and L. Duan, "Dynamic pricing and capacity allocation of UAV-provided mobile services," in *IEEE INFOCOM*, 2019.

[21] A. Ghosh and S. Sarkar, "Pricing for profit in internet of things," *IEEE Transactions on Network Science and Engineering*, vol. 6, no. 2, pp. 130–144, 2018.

[22] N. Ding, L. Gao, J. Huang, X. Li, and X. Chen, "Optimal pricing under vertical and horizontal interaction structures for IoT networks," in *IEEE INFOCOM*, 2022.

[23] C. Syverson, "Macroeconomics and market power: Context, implications, and open questions," *Journal of Economic Perspectives*, vol. 33, no. 3, pp. 23–43, 2019.

[24] J. Ren, G. Yu, and G. Ding, "Accelerating DNN training in wireless federated edge learning systems," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 1, pp. 219–232, 2020.

[25] B. Luo, X. Li, S. Wang, J. Huang, and L. Tassiulas, "Cost-effective federated learning in mobile edge networks," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 12, pp. 3606–3621, 2021.

[26] C. Fung, C. Yoon, and I. Beschastnikh, "Mitigating sybils in federated learning poisoning," *arXiv:1808.04866*, 2018.

[27] J. Tan, Y.-C. Liang, N. C. Luong, and D. Niyato, "Toward smart security enhancement of federated learning networks," *IEEE Network*, vol. 35, no. 1, pp. 340–347, 2020.

[28] M. Hao, H. Li, G. Xu, S. Liu, and H. Yang, "Towards efficient and privacy-preserving federated deep learning," in *IEEE International Conference on Communications (ICC)*, 2019.

[29] P. Sun, X. Chen, G. Liao, and J. Huang, "A profit-maximizing model marketplace with differentially private federated learning," in *IEEE INFOCOM*, 2022.

[30] Y. Jiao, P. Wang, D. Niyato, B. Lin, and D. I. Kim, "Toward an automated auction framework for wireless federated learning services market," *IEEE Transactions on Mobile Computing*, vol. 20, no. 10, pp. 3034–3048, 2020.

[31] W. Lim, N. Luong, D. Hoang, Y. Jiao, Y. Liang, Q. Yang, D. Niyato, and C. Miao, "Federated learning in mobile edge networks: A comprehensive survey," *arXiv:1909.11875*, 2019.

[32] P. Kairouz, H. B. McMahan, B. Avent, A. Bellet *et al.*, "Advances and open problems in federated learning," *arXiv:1912.04977*, 2019.

[33] Z. Wang, L. Gao, and J. Huang, "Multi-dimensional contract design for mobile data plan with time flexibility," in *ACM International Symposium on Mobile Ad Hoc Networking and Computing*, 2018.

[34] P. Bolton and M. Dewatripont, *Contract theory*. MIT press, 2004.

[35] V. Krishna, *Auction theory*. Academic press, 2009.

[36] R. Burra, C. Singh, and J. Kuri, "Service scheduling for bernoulli requests and quadratic cost," in *IEEE INFOCOM*, 2019.

[37] V. Kalashnikov, N. Kalashnykova, and J. G. Flores-Muniz, "Toll optimization problems with quadratic costs," in *International Conference on Unconventional Modelling, Simulation and Optimization*, 2018.

[38] M. Li, T. Zhang, Y. Chen, and A. Smola, "Efficient mini-batch training for stochastic optimization," in *ACM SIGKDD international conference on Knowledge discovery and data mining*, 2014.

[39] O. Dekel, R. Gilad-Bachrach, O. Shamir, and L. Xiao, "Optimal distributed online prediction using mini-batches," *Journal of Machine Learning Research*, vol. 13, no. January, pp. 165–202, 2012.

[40] A. Tahiri, D. Ladeveze, P. Chiron, B. Archimede, and L. Lhuissier, "Reservoir management using a network flow optimization model considering quadratic convex cost functions on arcs," *Water Resources Management*, vol. 32, no. 10, pp. 3505–3518, 2018.

[41] T. T. Doan and C. L. Beck, "Distributed lagrangian methods for network resource allocation," in *IEEE Conference on Control Technology and Applications (CCTA)*, 2017.

[42] C. Bergroth, O. Järv, H. Tenkanen, M. Manninen, and T. Toivonen, "A 24-hour population distribution dataset based on mobile phone data from Helsinki Metropolitan Area, Finland," *Scientific data*, vol. 9, no. 1, pp. 1–19, 2022.