Divided Difference Kalman Filter for Indoor Mobile Localization

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***Abstract*— Knowledge of the positions of sensor nodes is crucial for numerous applications in wireless sensors network. In this paper, we propose to use the Divided Difference Kalman Filter (DDKF) as a solution for locating and tracking a mobile node. This approach is an alternative variant of the nonlinear Kalman filtering already used in this type of applications. The advantage of this approach is that it does not require calculation of the Jacobian as for the Extended Kalman Filter (EKF) and it does not need to use several parameters, as for the Unscented Kalman Filter (UKF) which accuracy is closely dependent on the good choice of such parameters. In this work, a comparative performance study of four localization methods was conducted, namely the DDKF, the EKF, the UKF and the Least Squares Kalman Filter (LS-KF) which is a method based on multilateration in the least squares sense, followed by a smoothing step, using Kalman filtering. This study reveals many advantages in favor of the DDKF which, when applied for indoor localization, provides up to 40% gain in terms of Root Mean Squares Errors (RMSE) in position estimation, as compared to the other considered methods and which has a location error that is less than 2 meters in 95% of the considered cases.**

Keywords- Indoor Localization, Wireless Sensor Network, Divided Difference Kalman Filter

I. INTRODUCTION

Wireless Sensor networks (WSN) are often composed of a large number of nodes, called sensors. These nodes are entities that can operate autonomously to collect and send data relative to their environment. In recent years, WSN open up a multitude of applications: military, civil security (control of fire risks, natural disasters), medical field (remote monitoring of physiological information, tracking doctors and patients), etc.

Several applications in WSN depend heavily on the ability of nodes to know their position. This problem can be solved by the use of GPS system in networks installed outdoors, but cannot be adapted to those deployed inside buildings.

In this latter case sensor network localization requires the knowledge of the positions of some sensor nodes called anchors and uses inter-sensors measurements such as received signal strength indicator (RSSI) measurements, angle of arrival (AOA) measurements, and propagation time based measurements (time of arrival/ time difference of arrival TOA/TDOA) [1]. Generally TOA, TDOA and AOA based methods show good location estimation accuracy, however, the three methods require precise synchronization among the local oscillators of wireless nodes, several types of signals with different velocities and multiple antennas at nodes, respectively. Therefore, they are disadvantageous in terms of cost and energy consumption of sensor communication nodes. On the other hand, RSSI-based location estimation method is advantageous in terms of cost and energy consumption, because most of the current wireless communication standards have a function of measuring RSSI in their protocols [2].

Several approaches use RSSI metric for estimating the position, such as multilateration which is the most basic and intuitive method. This method computes a node's position via the intersection of three circles or more. To estimate its position using multilateration, a node needs to know its distance from anchors using RSSI measurement [3]. Another widely used approach is the fingerprinting based positioning approach, which consists of two phases: a training phase and an online phase. In the training phase, a fingerprint database is built. In the online phase, a sensor node measures the fingerprint vector of RSSIs from different anchor nodes. The fingerprint vector is then compared with fingerprints stored in the fingerprint database for determining the location of the sensor node. The drawback of this approach is that it is expensive for building fingerprint database in the training phase and the database needs to be re-established when the application scenario changes [4-5]. Whatever the used method, since the measurements are noisy, the accuracy of the estimated positions is not very good. The use of the Kalman filter may improve this accuracy.

The purpose of the research presented in this paper is to introduce and evaluate an algorithm, based on the Divided Difference Kalman filter (DDKF) for indoors localization and tracking. This algorithm is an alternative to other Kalman variants already used, such as the extended Kalman filter (EKF) [6-7] and the unscented Kalman filter (UKF) [8-9], It is shown by means of Monte Carlo simulations that the proposed algorithm achieves better location accuracy.

The rest of this paper is organized as follows:

The measurement model and the localization and tracking algorithms are presented in section II. In section III, the

computer simulation results are given and the performance is analyzed and compared to that of other algorithms. Finally, conclusions are given in section IV.

II. LOCALIZATION ALGORITHMS

In order to evaluate the performance of the DDKF, we propose to compare it to other methods such as the LS-KF, the EKF and the UKF. Before describing these tracking algorithms, we will first present the metric chosen in our work.

A. Received Signal Strength Indicator (RSSI)

RSSI is an indicator of the power of the signal at the time a message is received. A very common model exists to represent the RSSI according to the distance: the log-normal shadowing path loss model [6]. According to this model, the received signal strength is given by:

$$P(d) = P_0 - 10\eta \log_{10}(d/d_0) + X_\sigma \quad (1)$$

where P_0 is the measured path loss in decibels at a distance d_0 from the transmitter, η is a path loss exponent dependant on the surroundings and building type, d is the distance between the transmitter and receiver in meters, d_0 is typically one meter, and X_σ is a normal (Gaussian) random variable in decibels that has zero mean and standard deviation σ. This random variable is introduced to account for fading and shadowing effects.

B. Location Tracking

1) Least Square-Kalman Filter (LS-KF)

This method consists of two steps; first a static location of the node is performed using the multilateration approach, then linear Kalman filtering is applied for smoothing.

a) Multilateration

Multilateration is the algorithm used by the GPS. Several other positioning systems also use variants of multilateration. This method allows finding the position of a blind node from the positions of a number of anchors and their distances to this blind node. Assuming that the anchors coordinates are $(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n)$, the blind node coordinates are (x,y) and the distances between the blind node and the anchors are $d_1,d_2,\ldots,d_n$, as estimated from RSSI measurements, then:

$$\begin{cases} (x-x_1)^2 + (y-y_1)^2 = d_1^2 \\ (x-x_2)^2 + (y-y_2)^2 = d_2^2 \\ \vdots \\ (x-x_n)^2 + (y-y_n)^2 = d_n^2 \end{cases} \quad (2)$$

Using a linearization (i.e. by subtracting the first equation from the others) and least squares estimation [10], we can obtain the coordinates of the blind node as follows:

$$S = \begin{bmatrix} x \\ y \end{bmatrix} \quad (3)$$

$$\hat{S} = (A^T A)^{-1} A^T B \quad (4)$$

Where

$$A = \begin{bmatrix} x_1 - x_2 & y_1 - y_2 \\ \vdots & \vdots \\ x_1 - x_n & y_1 - y_n \end{bmatrix} \quad (5)$$

and

$$B = \begin{bmatrix} x_1^2 - x_2^2 + y_1^2 - y_2^2 + d_1^2 - d_2^2 \\ \vdots \\ x_1^2 - x_n^2 + y_1^2 - y_n^2 + d_1^2 - d_n^2 \end{bmatrix} \quad (6)$$

b) Kalman Filtering

In order to improve the accuracy of the tracking, the Kalman filter is used to estimate the trajectory of the blind node. It is an iterative estimator based on a recurrence relation, which means that only the state previously estimated and actual measurements are needed to calculate the estimate of the current state. The system of equations used in the Kalman filter corresponds to two models: one representing the process and the second modeling the measures. These equations are given below:

$$X_k = F X_{k-1} + V_k \quad (7)$$

$$Z_k = H X_k + W_k \quad (8)$$

Here, the state vectors $X_k = [x,y,v_x,v_y]^T$ consists of the positions x and y and the velocities v_x and v_y at time k , the matrix F is the state transition matrix which relates the state at time k to the state at time k-1 and V_k is the vector representing the process noise, which is assumed to have a zero mean normal distribution with covariance Q. Z_k is the vector of observations (measurements) at time k, H is the observation matrix, which links the state vector to the measurements and W_k is the observation noise, which is assumed to be zero mean white Gaussian with covariance R.

A cycle of the Kalman filter consists of two steps, a prediction step and a correction step.

Prediction step:

The estimated state at time k-1 is propagated to obtain the a priori state estimate (predicted) at time k, as follows:

$$X_k^- = F X_{k-1} \quad (9)$$

Assuming a constant velocity movement model, the state transition matrix is given by:

$$F = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

Where T is the sampling period.

The covariance matrix of the predicted errors P_k^- may be expressed as a function of the previous covariance matrix of the estimated errors P_{k-1} and process noise covariance matrix Q as follows:

$$P_k^- = F\, P_{k-1}\, F^T + Q \tag{11}$$

If the fluctuations of the acceleration around zero are assumed to be constant during each update time interval and if they are modeled by a white noise with variance σ_Q^2, then Q is given by [11]:

$$Q=\begin{bmatrix} \frac{T^4}{4} & 0 & \frac{T^3}{2} & 0 \\ 0 & \frac{T^4}{4} & 0 & \frac{T^3}{2} \\ \frac{T^3}{2} & 0 & T^2 & 0 \\ 0 & \frac{T^3}{2} & 0 & T^2 \end{bmatrix} \sigma_Q^2 \tag{12}$$

Update step:

This step requires in the first the calculation of the Kalman gain defined by:

$$K_k = P_k^-\, H^T\, [H\ P_k^- H^T + R\ ]^{-1} \tag{13}$$

Where H is the observation matrix and R is the measurement covariance matrix.

Since the measurement is the position obtained from the multilateration step, the matrix H is given by:

$$H=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{14}$$

Assuming that the components of the measurement noise along the x and y axis are zero mean and uncorrelated, with variances σ_x^2 and σ_y^2, R is given by:

$$R=\begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \tag{15}$$

Finally, the *a posteriori* state estimate X_k and corresponding covariance matrix P_k are updated as follows:

$$X_k = X_k^- + K_k\, [Z_k - H\, X_k^-] \tag{16}$$

$$P_k = (I - K_k\, H)\, P_k^- \tag{17}$$

2) Extended Kalman Filter (EKF)

In the LS-KF method, described previously, the multilateration is used to calculate the positions from RSSI measurements then the positions are filtered using the Kalman filter; another alternative is to use the RSSI measurements, provided by the sensors, directly in a nonlinear filter to estimate the positions. Several nonlinear filters exist [12], among which the EKF is probably the most common and most popular. The EKF solves the problem of non-linearity by calculating the Jacobian of the nonlinear measurement equation around the estimated state. The equation measurement is expressed as:

$$Z_k = h\,(X_k) + W_k \tag{18}$$

Since in our case the process equation is linear, the prediction step in the EKF is the same as in the Kalman filter (see equation (9)). However, since the equation measurement is nonlinear the measurement matrix H in the correction step must be replaced by the Jacobien. Without loss of generality, the expression of this Jacobien will be given below in the case of four anchors. In this case the observation vector Z_k consists of RSSI measurements collected from the four anchors, assumed to be located at (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4). The observation function $h(X_k^-)$ and the corresponding Jacobian H_k are derived from the log-normal shadowing path loss model [6]:

$$h\,(X_k^-) = \begin{bmatrix} P_0 - 10\,\eta \log_{10}(d_1/d_0) \\ P_0 - 10\,\eta \log_{10}(d_2/d_0) \\ P_0 - 10\,\eta \log_{10}(d_3/d_0) \\ P_0 - 10\,\eta \log_{10}(d_4/d_0) \end{bmatrix} \tag{19}$$

$$H_k = \frac{-10\eta}{\ln(10)} \begin{bmatrix} \frac{(x-x_1)}{d_1^2} & \frac{(y-y_1)}{d_1^2} & 0 & 0 \\ \frac{(x-x_2)}{d_2^2} & \frac{(y-y_2)}{d_2^2} & 0 & 0 \\ \frac{(x-x_3)}{d_3^2} & \frac{(y-y_3)}{d_3^2} & 0 & 0 \\ \frac{(x-x_4)}{d_4^2} & \frac{(y-y_4)}{d_4^2} & 0 & 0 \end{bmatrix} \tag{20}$$

Where d_n is the Euclidean distance function expressed as:

$$d_n = \sqrt{(x - x_n)^2 + (y - y_n)^2} \tag{21}$$

In the above equation x, y are the first two elements of the predicted state vector X_k^-.

If the measurement noises of the anchors are assumed to be independent with each other, the measurement covariance matrix is given by:

$$R = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \tag{22}$$

By applying the Kalman filter to the linearised system, we obtain the following equations for the update step:

$$K_k = P_k^-\, H_k^T\, [H_k\ P_k^- H_k^T + R\ ]^{-1} \tag{23}$$

$$X_k = X_k^- + K_k\, [Z_k - h(X_k^-)] \tag{24}$$

$$P_k = (I - K_k H_k) P_k^- \quad (25)$$

3) Unscented Kalman Filter

The EKF works well for systems with moderate nonlinearities; but when the nonlinearity becomes too strong the EKF is not efficient [12]. The UKF has recently become an alternative to the EKF, It uses the unscented transformation which allows calculating the mean and covariance of a transformed variable from a set of sample points, called sigma points that are propagated using the non-linear transformation.
The different steps of the UKF are described in the following. Given the predicted state vector, the sigma points are calculated and stored in the columns of the matrix χ_k of size L × (2L +1), where L is the dimension of the state vector. These points are calculated using a deterministic algorithm:

$$(\chi_k)_0 = X_k^- \quad (26)$$

$$(\chi_k)_i = X_k^- + \left(\sqrt{(L+\lambda) P_k^-}\right)_i, \; i = 1 \ldots\ldots\ldots L \quad (27)$$

$$(\chi_k)_i = X_k^- - \left(\sqrt{(L+\lambda) P_k^-}\right)_i, \; i = L+1 \ldots\ldots\ldots 2L \quad (28)$$

where $(\chi_k)_i$ denotes the i^{th} column of matrix χ_k and λ is defined by:

$$\lambda = \alpha^2 (L+\kappa) - L. \quad (29)$$

In (29), α and κ control the spread of the sigma points. α is usually set to $0 \leq \alpha \leq 1$ and κ is a secondary scaling parameter which is usually set to zero.
First, the Kalman Filter state prediction is applied (equations (9) and (11)) due to the assumed linear process dynamics.
Then in the correction step, the sigma points $(\chi_k)_i$ are transformed by the measurement function,

$$(Z_k)_i = h((\chi_{k-1})_i), \; i = 0 \ldots\ldots\ldots 2L \quad (30)$$

and their weighted mean is computed:

$$\hat{Z}_k = \sum_{i=0}^{2L} w_i^{(m)} (Z_k)_i \quad (31)$$

Where $w_i^{(m)}$ is the weight associated with the sigma point i, defined by:

$$w_0^{(m)} = \lambda / (L+\lambda) \quad (32)$$

$$w_i^{(m)} = 1 / \{2(L+\lambda)\}, \; i = 1 \ldots\ldots 2L \quad (33)$$

The vector $\hat{Z}_k$, plays the role of the predicted measurement vector that may be used to calculate the a posteriori state estimate:

$$X_k = X_k^- + K_k (Z_k - \hat{Z}_k) \quad (34)$$

Where Z_k is the vector of real measurements and K_k is the Kalman gain, given by:

$$K_k = P_{xz} P_{zz}^{-1} \quad (35)$$

Where

$$P_{zz} = \sum_{i=0}^{2L} w_i^{(c)} [(Z_k)_i - \hat{Z}_k] [(Z_k)_i - \hat{Z}_k]^T + R \quad (36)$$

$$P_{xz} = \sum_{i=0}^{2L} w_i^{(c)} [(\chi_k)_i - X_k^-] [(Z_k)_i - \hat{Z}_k]^T \quad (37)$$

In (36) R represents the covariance matrix of the measurement noise, and the weight $w_i^{(c)}$ is defined by:

$$w_0^{(c)} = \frac{\lambda}{(L+\lambda)} + (1 - \alpha^2 + \beta) \quad (38)$$

$$w_i^{(c)} = \frac{1}{2(L+\lambda)}, \; i = 1 \ldots\ldots 2L \quad (39)$$

β is a parameter used to incorporate any prior knowledge about the error distribution (for Gaussian distribution, $\beta = 2$ is optimal).
Finally the estimated covariance matrix is updated as follows:

$$P_k = P_k^- - K_k P_{zz} K_k^T \quad (40)$$

4) Divided Difference Kalman Filter (DDKF)

Based on Stirling's interpolation, the DDKF is proposed to solve nonlinearity problem by approximating the mean and the covariance of stochastic variables generated by nonlinear transformation of stochastic variables with known mean and covariance [13]. Stirling polynomial interpolation formula, limited to the second order for a non linear function h is:

$$h(x) = h(\bar{x}) + \tilde{D}_{\Delta x} h + \frac{1}{2} \tilde{D}^2_{\Delta x} h \quad (41)$$

Where $\tilde{D}_{\Delta x}$ and $\tilde{D}^2_{\Delta x}$ are the first and the second order operators of the central divided difference of h(x). In the scalar case, these operators are given by:

$$\tilde{D}_{\Delta x} = (x - \bar{x}) \frac{h(\bar{x}+\xi) - h(\bar{x}-\xi)}{2\xi} \quad (42)$$

$$\tilde{D}^2_{\Delta x} = (x - \bar{x})^2 \frac{h(\bar{x}+\xi) - h(\bar{x}-\xi) - 2h(\bar{x})}{\xi^2} \quad (43)$$

ξ is the interval step-size for the approximation, $\sqrt{3}$ is the optimal value for a Gaussian distribution [11].

Two filters were proposed in [14], the DDKF1, based on first-order approximation, and the DDKF2, based on second-order approximation. These filters are described below.
In both filters, the prediction step is the same as in the linear Kalman filter, since in our case the process equation is linear.

a) The First Order Divided Difference Kalman Filter (DDKF1)

The DDKF1 compute the mean and covariance in the first order polynomial approximation.

Let S_k denote the Cholesky decomposition of the predicted covariance matrix P_k^-:

$$S_k = \text{Chol}(P_k^-), \quad (44)$$

and $\hat{Z}_k$ the predicted measurement

$$\hat{Z}_k = h(X_k^-) \quad (45)$$

The Kalman gain can be calculated according to:

$$K_k = P_{xz} P_{zz}^{-1} \quad (46)$$

Where P_{zz} and P_{xz} are given by:

$$P_{zz} = H(X_k^-, S_k, \xi) H^T(X_k^-, S_k, \xi) + R \quad (47)$$

$$P_{xz} = S_k H^T(X_k^-, S_k, \xi) \quad (48)$$

In the two previous equations the (i,j) element of H is defined as:

$$H_{j,i}(X_k^-, S_k, \xi) = (h_j(X_k^- + \xi (S_k)_i) - h_j(X_k^- - \xi (S_k)_i))/2\xi \quad (49)$$

The a posteriori state vector X_k and the associated covariance matrix P_k are updated according to:

$$X_k = X_k^- + K_k(Z_k - \hat{Z}_k) \quad (50)$$

$$P_k = P_k^- - K_k P_{zz} K_k^T \quad (51)$$

b) The Second Order Divided Difference Kalman Filter (DDKF2)

The DDKF2 is obtained by calculating the mean and covariance using the second order polynomial approximation. The implementation of the DDKF2 filter follows the same steps as the DDKF1. The only difference is that equations (45) and (47) should be replaced by the following equations:

$$\hat{Z}_k = \frac{\xi^2 - L}{\xi^2} h(X_k^-) + \frac{1}{2\xi^2} \sum_{i=1}^{L} (h(X_k^- + \xi (S_k)_i) + h(X_k^- - \xi (S_k)_i)) \quad (52)$$

$$P_{zz} = H(X_k^-, S_k, \xi) H^T(X_k^-, S_k, \xi) + H^{(2)}(X_k^-, S_k, \xi) H^{(2)T}(X_k^-, S_k, \xi) + R \quad (53)$$

Where

$$H_{j,i}^{(2)}(X_k^-, S_k, \xi) = \frac{\sqrt{\xi^2 - 1}}{2\xi^2} (h_j(X_k^- + \xi (S_k)_i) + h_j(X_k^- - \xi (S_k)_i) - 2 h_j(X_k^-) \quad (54)$$

III. RESULTS

A number of Monte Carlo simulations were carried out to assess the algorithms performance, in terms of both accuracy and numerical cost, of the localization and tracking methods, described in the previous section. In the first simulation, one mobile node moves at a speed of 1 *m/s* in a 10m*10m rectangular area, with four anchors located at the four corners of this area, as shown in figure 1.

A. Accuracy

The localization accuracy is an important criterion for assessing localization algorithms; it is evaluated using two metrics, which are the Root Mean Square Error (RMSE) in position and the cumulative distribution function (CDF) of the estimation errors. The RMSE is calculated as follows:

$$RMSE(k) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_0(k) - x_{i,est}(k))^2 + (y_0(k) - y_{i,est}(k))^2} \quad (55)$$

Where N is the total number of runs, $x_{i,est}(k)$ and $y_{i,est}(k)$ is the estimated position at run i and time k, and $x_0(k)$ $y_0(k)$ is the true position of the mobile node, at time k.

The results obtained and averaged over 500 runs are presented in figures 1 to 3. In figure 1 the true trajectory is plotted together with the trajectories estimated by different filters, for one run, while in figure 2, the RMSE in position obtained with different filters is plotted versus time. Finally, in figure 3 the cumulative distribution function is plotted, for different filters.

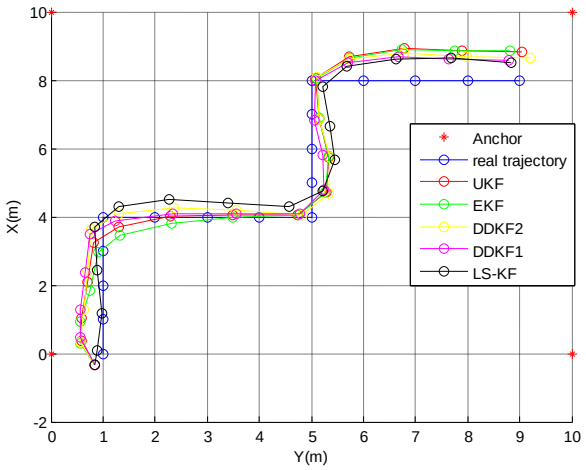

Figure 1. True and estimated trajectories for a particular run.

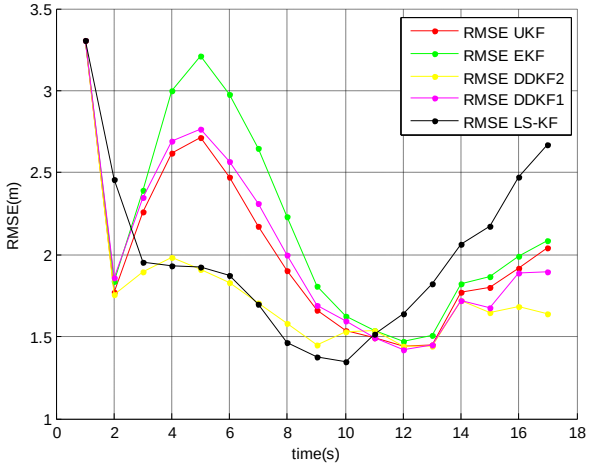

Figure 2. RMSE in position obtained with different filters.

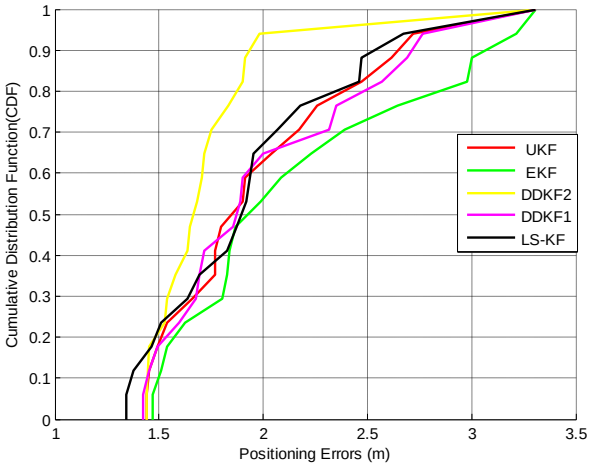

Figure3. Cumulative distribution function of the location errors

From figures 2 and 3 it can be observed that the DDKF2 is the most effective approach with an error less than 2 meters in 95% of cases, whereas for the other approaches the same accuracy is achieved in only between 55 % and 68 % of cases. The LS-KF provides positions with high accuracy when the mobile node is at the center of the area, but this accuracy decreases when it moves away from the center. This is due to accuracy of the used multilateration method, which is position dependent.

For a further assessment of the performances of the studied tracking algorithms, two factors that affect the localization accuracy are examined: anchors' density, and anchors' placement.

B. Density of the Anchor Nodes

Increasing the density of the anchor nodes may minimize localization errors. This is confirmed by the results presented in Figures 5 to 9, where the CDF of the different filters are plotted for different anchors' densities. For 4 anchors, the placement is the same as in figure 1. The placement of 8 anchors is obtained from that of 4 anchors by inserting an additional anchor midway between every two neighboring anchors. The placement of 12 anchors corresponds to that of 8 anchors with 4 additional anchors placed at the center of the rectangular area. Finally, the placement of 16 anchors is obtained by inserting 4 new anchors midway between the 4 previous anchors, as shown in figure 4.

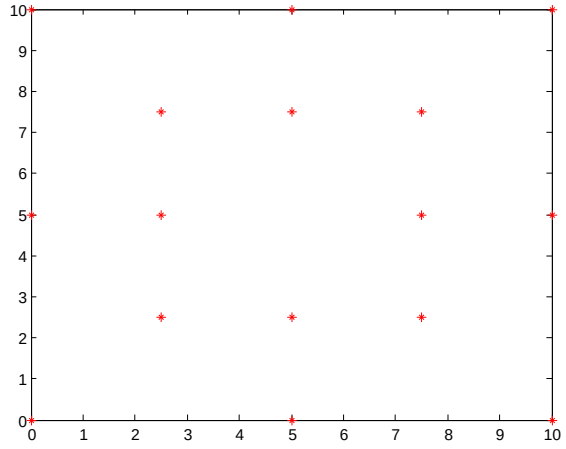

Figure 4. Anchors Geometry

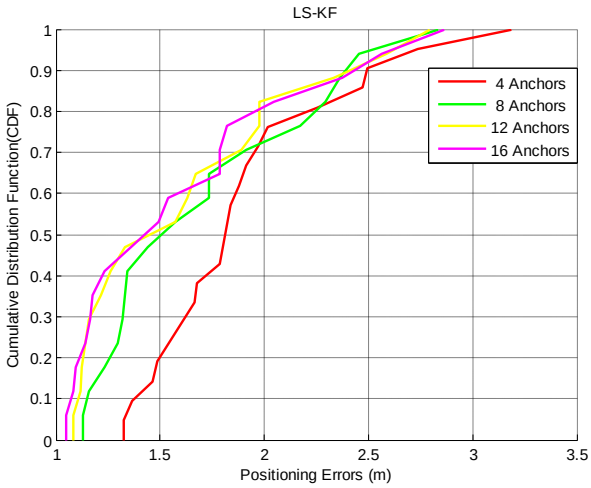

Figure 5. Cumulative distribution function of the location error for LS-KF

Figure 5 shows that the number of anchors does not greatly affect the accuracy obtained by LS-KF method.

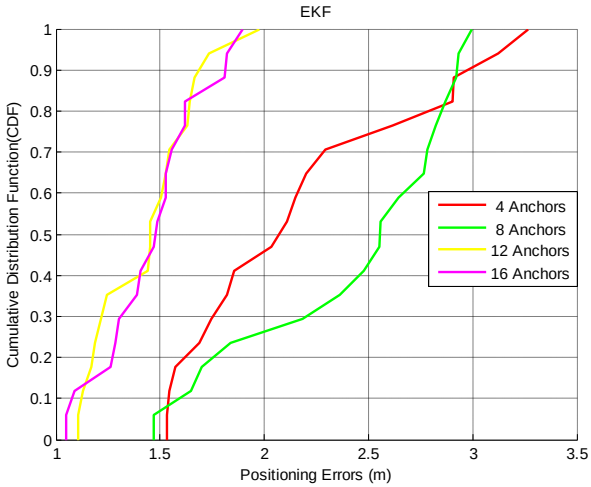

Figure 6. Cumulative distribution function of the location error for EKF

Figure 6 shows that the precision obtained using 12 or 16 anchors with the EKF estimator is approximately the same and is significantly higher than the accuracy obtained by using 4 and 8 anchor nodes. We note that the EKF is very sensitive to the geometry of anchors, which explains the divergence of the filter when it uses 8 anchors. By using 12 or 16 anchor nodes, the positioning errors are less than 1.5 m in 50% of the time.

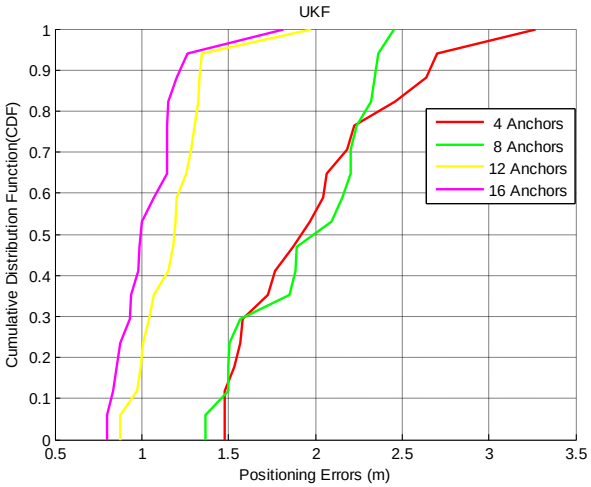

Figure 7. Cumulative distribution function of the location error for UKF

Figure 7 shows that, for the UKF, approximately 95% of positionning errors are less then 1.3m using 12 or 16 anchors, while 50% are less then 2m using 4 or 8 anchors.

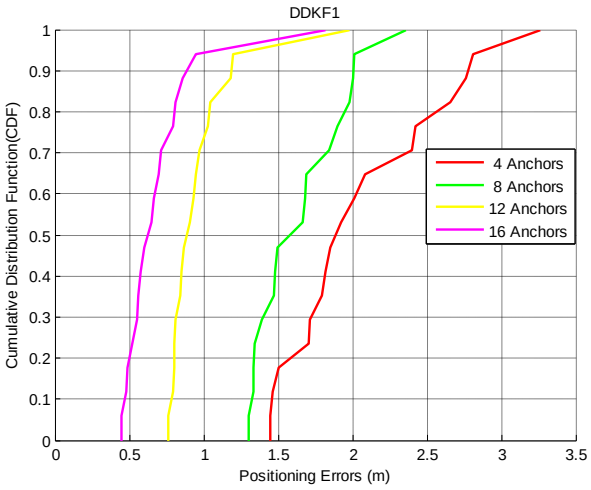

Figure 8. Cumulative distribution function of the location error for DDKF1

Figures 8 and 9 shows that the DDKF1 and DDKF2 can achieve a localization error of a few tens of centimeters when using more than 8 anchor nodes.

For the DDKF1 estimator, in 90% of the time, the positioning errors are approximately less than: 0.8m, 1.2m, 2m and 2.75m using, respectively, 16, 12, 8 and 4 anchor nodes.

For the DDKF2 estimator, in 90% of the time, the positioning errors are approximately less than: 0.6m, 0.8m, 1.2m and 1.8 m using, respectively, 16, 12, 8 and 4 anchor nodes.

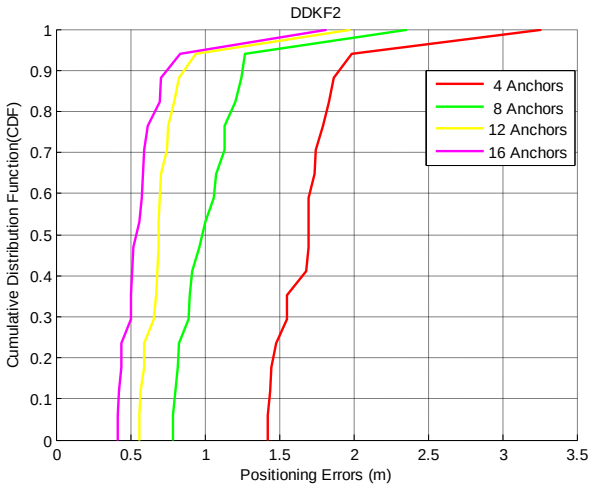

Figure 9. Cumulative distribution function of the location error for DDKF2

To summarize, we can say that increasing the number of anchors reduces the localization errors mainly when using methods based on nonlinear Kalman filtering. However, no significant gain is brought about by increasing the number of anchor nodes beyond a certain value. This has to be avoided, since increasing this number increases mainly the complexity.

C. Geometry of the Anchor Nodes

The geometry of the anchor nodes also affects localization accuracy. To confirm this fact, three scenarios have been considered. As illustrated in figure 10, in the first scenario the anchor nodes are placed on the perimeter of the area, whereas in the second scenario they are placed at the center, and in the third scenario they are placed in a corner of the area. The mobile follows the same trajectory as in figure 1.

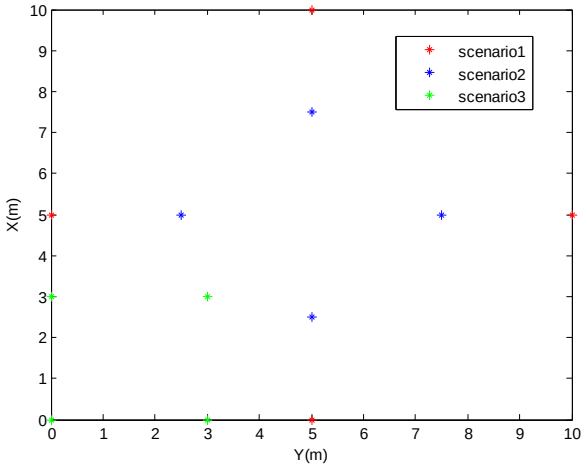

Figure 10. Geometry scenarios

The following table summarizes the time average RMSE obtained with the different filters, for the three scenarios.

	Scenario1	Scenario2	Scenario3
LS-KF	2.83	3.76	3.65
EKF	2.58	2.08	2.09
UKF	2.19	2.33	1.88
DDKF1	2.53	2.24	6.41
DDKF2	1.67	1.98	3.29

Table1 Impact of anchor nodes placement

From this table, it can be observed that the geometry of the anchor nodes plays a major role in the accuracy of the estimated positions. It can also be observed that although the most suitable scenario is filter dependent, globally the best one corresponds to anchor nodes placed at the perimeter of the area. Notice also that the DDKFs are the most sensitive to the geometry of the anchor nodes, which means that the approximations that they are based on are not valid for certain anchor nodes geometries.

D. Computation requirement

Another metric to compare the different methods is the numerical cost. This has been evaluated by counting the number of additions, subtractions, multiplications and divisions, necessary for the implementation of one cycle of each filter. The results are given in table 2.

Method	Number of operations per cycle
LS-EKF	766
EKF	1370
UKF	2341
DDKF1	1831
DDKF2	2122

Table 2. Computation requirement

As it can be seen from this table, the LS-KF is the least computationally intensive; it is followed by the EKF and the DDKF1 then by the DDKF2 and the UKF.

IV. CONCLUSION

In this paper a divided difference Kalman Filter has been proposed to solve the indoor tracking problem in sensor networks. The performance of this filter has been compared to other variants of the Kalman filter, using Monte Carlo simulations. The results of simulations show that a significant improvement in the accuracy localization is brought about by using the DDKF2. Another advantage of this filter is that it does not require the calculation of the Jacobian, which is not often obvious, as is the case in the EKF, and does not depend on the clinching choice of some parameters that affect the estimation accuracy, as it is the case in the UKF.

In this paper the impact of the density of anchors on the location accuracy has also been examined. Indeed, as expected, in general the localization accuracy increases when the number of anchors increases, but only up to certain given value. However the price to pay for this is an increase in the computational load. Also, the location of anchors has a significant impact on the localization accuracy. The choice of the best anchors' geometry is an open subject that deserves a further study.

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