Proceedings of the 2003 IEEE/RSJ
Intl. Conference on Intelligent Robots and Systems
Las Vegas, Nevada · October 2003

Closed Loop Navigation for Mobile Agents in Dynamic Environments

Savvas G. Loizou* Herbert G. Tanner† Vijay Kumar‡ Kostas J. Kyriakopoulos*

* Control Systems Lab., National Technical University of Athens, Athens, Greece
† Mechanical Engineering Dept., University of New Mexico, Albuquerque NM, USA
‡ GRASP Lab, CIS, University of Pennsylvania, Philadelphia PA, U.S.A
sloizou@mail.ntua.gr

Abstract— **We apply a novel motion planning and control methodology, which is based on a non-smooth navigation function, to a point mobile robot moving amongst moving obstacles. The chattering introduced by the discontinuous potential field is suppressed using nonsmooth backstepping. The combined controller guarantees global asymptotic convergence and collision avoidance. This controller is particularly suitable for real time implementation on systems with limited computational resources. The effectiveness of the proposed scheme is verified through computer simulations.**

I. Introduction

Navigation function methodologies [5], [7], [11] have a long history of application in mobile robot navigation with encouraging results in terms of guaranteed collision avoidance and convergence. However the use of this method has been limited to cases where the environment is stationary.

The basic motivation for this work comes from the need for simple and fast real time navigation algorithms for autonomous agents navigating in dynamic environments, with limited sensing and computational resources. Such algorithms need to be robust enough to deal with uncertainty. Our approach is to develop a controller based on a non-smooth navigation function and implement it using a nonsmooth integrator backstepping technique [9]. The moving obstacles are assumed to be disk shaped, an assumption that is not too restrictive since arbitrary star shaped obstacles can be diffeomorphically mapped to disks in R^2 [5]. Methods for constructing diffeomorphisms are discussed in [7] for point robots and in [10] for rigid body robots. The obstacles move in the workspace under the assumptions that an upper bound of the obstacle speeds is known a' priori, the minimum approach distance between the obstacles is nonzero and the obstacles eventually will not sit forever on the robot's target configuration.

The rest of this paper is organized as follows: Section II discusses the mathematical preliminaries necessary for the analysis. In section III we present the construction of non-smooth navigation functions while section IV discusses the controller synthesis. In section V we present our simulation results and the paper concludes with section VI.

II. Preliminary Definitions

Our approach relies on the notion of the generalized gradient which is a fundamental concept in nonsmooth analysis:

Definition 1 ([1]): For a locally Lipschitz function $V : R^2 \to R$ define the generalized gradient of V at $x \in R^2$ by:

$$\partial V(x) = \overline{co}\left\{\lim \nabla V(x) \,|x_i \to x, x_i \notin \Omega_V\right\}$$

where Ω_V is the set of measure zero where the gradient of V is not defined and $\overline{co}$ denotes the convex closure.

Definition 2: We call generalized critical point of a locally Lipschitz function $\varphi : R^2 \to R$, a point $x \in R^2$ for which:

$$0 \in \partial V(x)$$

Without loss of generality we assume that the origin is the desired configuration. Let $\mathcal{W} \subset R^2$ be the robot workspace with $\mathbf{0} \in \mathcal{W}$.

Definition 3: A function $\mathcal{F} : \mathcal{W} \to R$ is called a non-smooth navigation function (**NNF**) if it has the following properties:

i. $\mathcal{F}$ is absolutely continuous in $\mathcal{W}$,
ii. $\mathcal{F}$ has exactly one minimum at the origin,
iii. $\mathcal{F}$ has a countable number of isolated generalized critical points,
iv. $F(x_b) > F(x_i), \forall x_b \in \mathcal{B}_{\mathcal{W}}, \forall x_i \in \mathcal{W} \backslash \mathcal{B}_{\mathcal{W}}$

where $\mathcal{B}_{\mathcal{W}}$ denotes the boundary of $\mathcal{W}$

Definition 4: Let $f : R^n \to R$ be an absolutely continuous function and consider the differential equation $\dot{x} = -\nabla f$. Then [2] a vector function $x(\cdot)$ is called a solution of the differential equation (in the Filippov sense) if $x(\cdot)$ is absolutely continuous and $\dot{x} \in -\partial f(x)$.

Proposition 1: If $\mathcal{F} : \mathcal{W} \to R$ is an NNF, then for the Filippov solutions of $\dot{x} = -\nabla \mathcal{F}$, the following statements are true:

i. The set $\mathcal{W}$ is a positive invariant set.
ii. The positive limit set of $\mathcal{W}$ consists of the generalized critical points of $\mathcal{F}$.
iii. There is a dense open set $\tilde{J} \subset \mathcal{W}$ whose limit set consists of the unique minimum of $\mathcal{F}$.

Proof: From property (iv) of definition 3 it follows that the negated gradient over the boundary of $\mathcal{W}$ is directed in the interior of $\mathcal{W}$. Hence $\mathcal{W}$ is positively invariant. A limit set of the Filippov solution $x(\cdot)$ by application of LaSalle's theorem [8] for $\mathcal{F}$ in $\mathcal{W}$ is a generalized critical point since for $0 \in \dot{\mathcal{F}}$ to be true, it must hold that $0 \in \partial F(x)$. To prove the last property take the minimum of $\mathcal{F}$ and define a circle of radius ε around it. Since the critical points are isolated, we can always have an $\varepsilon > 0$ for which the set $\tilde{J} = \{x : \|x\| < \varepsilon\}$ contains only one critical point, which is the unique minimum of $\mathcal{F}$. ■

III. Non-Smooth Navigation Functions

Let $\sigma : [0, \infty) \to [0, \infty)$ be a smooth function having the following properties:

i. $\sigma(0) = 0$
ii. $\sigma'(0) = 0$,
iii. $0 < \sigma'(x) < \mu$, $x \in \mathcal{R}^+$, with $\mu \in \mathcal{R}^+$

Let $\gamma : (0, \delta] \to [0, \infty)$ be a smooth function having the following properties:

i. $\gamma(\delta) = 0$,
ii. $\lim_{x \to 0} \gamma(x) = \infty$,
iii. $-\infty < \gamma'(x) < -\mu - \theta, \forall x \in (0, \delta]$

where $\theta > 0$ is a parameter. Define the function $V(q) = \sigma(\|q\|)$ where $q \in \mathcal{R}^2$ is the robot's position, and $b_i(q) = \gamma(\|q - q_i\| - r_i)$, where q_i the center of the circle representing the obstacle i. We assume that each obstacle is assigned an "external" radius, r_{t_i}, where the robot can sense the obstacle and begins the avoidance maneuver. The robot may continue approaching the obstacle until it reaches the "internal" radius r_i beyond which there is collision. We have that $r_{t_i} = r_i + \delta$ with $\delta > 0$. We assume that $\|q_i - q_j\| > (r_{t_i} + r_{t_j})$ for $i \neq j$ or equivalently the minimum distance between the obstacle volumes is greater than 2δ.

Let us define the following function:

$$F(q) = \begin{cases} V(q), & d > 0 \\ V(q) + b_j(q) & d \leq 0 \end{cases} \tag{1}$$

where $d = \|q - q_j\| - r_{t_j}$ and $j = \arg\min_{k \in \mathcal{O}} \|q - q_k\|$, where $\mathcal{O}$ is the set of obstacle indices. By construction $F(q)$ is absolutely continuous, since $b_j(q) = 0$ when $d = 0$. We can now define the discontinuous vector field:

$$f(q) = -\nabla F(q) \tag{2}$$

Consider the following differential equation:

$$\dot{x} = f(x) \tag{3}$$

Then, according to Def. 4, the Filippov solutions: $x(\cdot)$ of (3), are absolutely continuous and $\dot{x} \in K[f](x)$, where

$$K[f](x) = \overline{co}\{\lim f(x_i) \,|\, x_i \to x, x_i \notin N\}$$

and N is a set of measure zero. Let $f^+(q) = f(q)$ when $d > 0$ and $f^-(q) = f(q)$ when $d \leq 0$. We call f^+ and f^- the branches of f. Across the surface of discontinuity, the set $K[f](x)$ is a linear segment [2] joining the endpoints of the vectors f^+ and f^-.

When the system reaches the surface of discontinuity, we can distinguish the following cases [2] (see figure 1):

- Region b: $f^+ \cdot f^- > 0$ and the solutions pass from G^- to G^+.
- Region c: $f^+ \cdot f^- \leq 0$ and function $x(t)$ satisfies [2] the equation :

$$\dot{x} = f^0(x) \tag{4}$$

  which describes a sliding motion . Plane P is tangent to the surface of discontinuity at point x. The segment $K[f](x)$ intersects P and the intersection is the endpoint of vector $f^0(x)$. Then

$$f^0 = af^+ + (1 - a)f^- \tag{5}$$

  where

$$a = \frac{f_N^-}{f_N^- - f_N^+}$$

  and f_N^+, f_N^- are the projections of the vectors f^+ and f^- to the normal to the plane P.
- Points "a": $f^+ \cdot f^- = 0$ and the solutions depart from the surface of discontinuity.

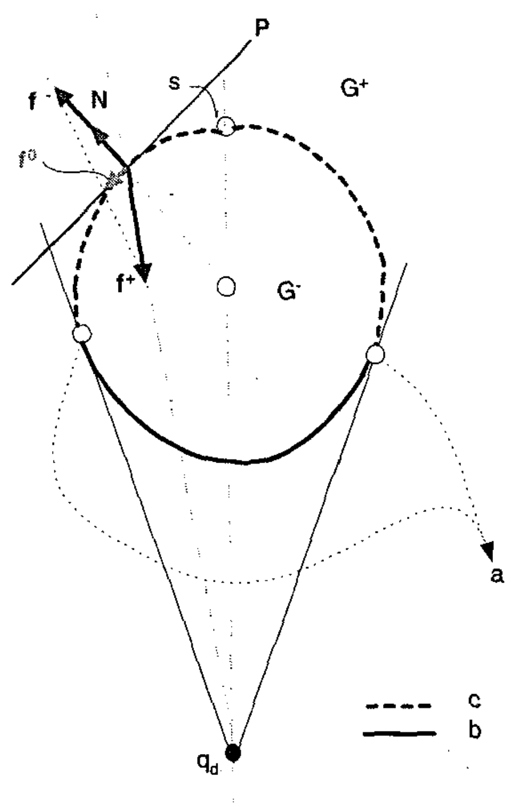

Fig. 1. Surfaces of discontinuity

- Point "s": $f^+ = -\lambda f^-$, with $\lambda > 0$ hence from eq. (5) $f^0 = 0$ and from eq. (4) we have that $\dot{\mathbf{x}} = 0$.

Lemma 1: Point "s" is unique.

Proof: Setting $f^+ = -\lambda f^-$, $\lambda > 0$ we get $\sigma' \frac{x}{\|x\|} = -\lambda \cdot \gamma' \frac{x - x_o}{\|x - x_o\|}$, where x and x_o the robot and obstacle center positions. For this to be true since $\gamma' < 0$ we need $\frac{x}{\|x\|} = \frac{x - x_o}{\|x - x_o\|}$ which for a disk obstacle this holds for $x = \hat{x}_o (\|x_o\| + \lambda_1 \cdot r_{t_i})$, with $\lambda_1 = \pm 1$. Since the origin is not contained in the set defined by the obstacle's external radius, then $\lambda_1 = +1$ and the proof is complete. ■

Lemma 2: Point "s" is a saddle point

Proof: A normal to the tangential plane P at "s" is $N = \mathbf{x} - \mathbf{x_0}$ where $\mathbf{x}$ is the position vector of point "s" and $\mathbf{x_0}$ is the position vector of the center of the encountered circle. At that point $f^- = -\nabla\gamma(\|\mathbf{x} - \mathbf{x_0}\| - r_o) - \nabla\sigma(\|\mathbf{x}\|)$ and $f^+ = -\nabla\sigma(\|\mathbf{x}\|)$. We have that $f_N^- = \hat{N} \cdot f^-$ and $f_N^+ = \hat{N} \cdot f^+$. Substituting a in eq. (5) we get : $f^0 = \begin{bmatrix} f_x^0 & f_y^0 \end{bmatrix}^T = h \cdot [y - y_0 \quad -(x - x_0)]^T$, with $h = \sigma' \frac{x y_0 - y x_0}{\left((x - x_0)^2 + (y - y_0)^2\right)\sqrt{x^2 + y^2}}$. The matrix ∇f^0 is the hessian of the potential function that produces f^0. Note also that f^0 is the negated gradient of a potential field. Hence it is sufficient to show that ∇f^0 has a positive eigenvalue and that the corresponding eigenvector is in the tangential plane P. Point "s" has coordinates: $x = x_0 + r \frac{x_0}{\sqrt{x_0^2 + y_0^2}}$ and $y = y_0 + r \frac{y_0}{\sqrt{x_0^2 + y_0^2}}$ since the target is not contained in the circle (see proof of lemma 1). Substituting the coordinates in ∇f^0 and calculating the eigenvalues, we get: $\lambda_1 = 0$ and $\lambda_2 = \sigma' \frac{\sqrt{x_0^2 + y_0^2}}{r \cdot \left(\sqrt{x_0^2 + y_0^2} + r\right)}$ with corresponding eigenvectors: $v_1 = \begin{bmatrix} \frac{x_0}{y_0} & 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -\frac{y_0}{x_0} & 1 \end{bmatrix}$ respectively. We can see that $\lambda_2 > 0$, since by definition $\sigma' > 0$. Moreover $N \cdot v_2 = 0$ which means that v_2 is in the tangent space of P, which denotes the existence of a feasible direction of movement along the surface of discontinuity.

The value of λ_1 (which is actually a set of values due to the discontinuous function) does not affect the behavior of our system, since the point "s" belongs to the discontinuity surface, which is a set of measure zero. Hence point "s" will be an isolated critical point as long as $\lambda_2 \neq 0$, regardless of the value of λ_1. ■

Proposition 2: Function F has one local minimum at the origin and a countable number of isolated saddle points.

Proof: At a critical point we have that $\nabla F = 0$. Expanding: $\nabla F(q) = \nabla\sigma(\|q\|) + \xi \nabla\gamma(\|(q - q_i)\| - r_i) = \frac{q}{\|q\|} \cdot \sigma' + \xi \cdot \frac{q - q_i}{\|q - q_i\|} \gamma'$, where $\xi = 0$ when $d > 0$ or $\xi = 1$ when $d \leq 0$. For $\xi = 0$ the only critical point is the origin, where $\lim_{q \to 0} \nabla F(q) = [\cos(\theta) \quad \sin(\theta)]^T \cdot \sigma'(\|q\|) = 0$ where θ is the angle with which q approaches zero. This is the unique minimum of $F(q)$ for $\xi = 0$, since $\sigma'(\|q\|) > 0, \forall \|q\| > 0$. For $\xi = 1$ at a critical point it must be: $\frac{q}{\|q\|} \cdot \sigma' + \xi \cdot \frac{q - q_i}{\|q - q_i\|} \gamma' = 0$. This condition can never be satisfied since $\sigma' \neq -\gamma'$ by definition. Hence the only critical points that are not local minima are generalized critical points that lie on the surface of discontinuity. But by Lemmas 1 and 2 there is only one isolated generalized critical point per obstacle which is a saddle point. ■

We can now state the main result of this section:

Proposition 3: Function $F : \mathcal{W} \to R$ defined in (1) is an NNF.

Proof: Property (i) of definition 3 is satisfied since $V(q)$ and $b_j(q)$ are smooth and over the switching surface $b_j(q) = 0$. Properties (ii) and (iii) are satisfied by proposition 2. Property (iv) is satisfied since for any $\|q\| < \infty$ it holds that $\lim_{q \to \mathcal{B}_{\mathcal{W}}} F(q) = \infty$ and only for $q \to \mathcal{B}_{\mathcal{W}}$. This is due to the properties (ii) and (iii) of γ and the fact that $\sigma(\|q\|)$ is finite for finite $\|q\|$ because of property (iii) of it's definition. Hence $F(q) < \infty$, $\forall q \in \mathcal{W} \backslash \mathcal{B}_{\mathcal{W}}$ and property (iv) of definition 3 is satisfied. ■

IV. Controller Synthesis

Assume that the robot kinematics are described by:

$$\dot{\mathbf{x}} = u \tag{6}$$

and let the following equation describe the motion of the obstacles in the workspace:

$$\dot{q}_i = h_i(t), \quad i = 1 \dots n_O \tag{7}$$

where n_O represents the number of obstacles and $h_i(t)$ are unknown functions with the following properties:

i. $\|q_i - q_j\| > r_{t_i} + r_{t_j}, \;\; i \neq j$
ii. The obstacles will not stop at a configuration where they cover the origin:
$$\lim_{t \to \infty} (\|q_i(t)\|) \geq r_{t_i} + \varepsilon$$
with $\varepsilon > 0$, and
iii. the obstacle speed is bounded: $\|\dot{q}_i\| \leq M$

Let $x^{\perp}$ be a vector across the eigen-direction of the positive eigenvalue at the saddle point calculated in the proof of Proposition 2 with $\left\|x^{\perp}\right\| = \|x\|$. As discussed in the proof of Proposition 2, this is perpendicular to x. Let the switch $\zeta(d) = 1$ for $d \leq 0$ and $\zeta(d) = 0$ for $d > 0$ with d as defined in (1) and the switch $\eta(f(x)) = 1$ for $f(x) \neq [0 \;\; 0]$ and $\eta(f(x)) = 0$ for $f(x) = [0 \;\; 0]$, with f as defined in (2) .

Proposition 4: The system (6) under the control law:

$$u = \zeta \cdot \left(\dot{q}_i + \lambda \frac{x - q_i}{\|x - q_i\|}\right) + v$$

where $v = \eta \cdot f(x) + (1 - \eta) \cdot x^{\perp}$, is globally asymptotically stable.

Proof: Using the navigation function F defined in (1) as a Lyapunov function candidate, we have that: $\dot{F} = \frac{\partial F}{\partial t} + u \cdot \nabla F$ with $\frac{\partial F}{\partial t} = -\zeta \cdot \gamma' \frac{x - q_i}{\|x - q_i\|} \cdot \dot{q}_i$ and $\nabla F = \zeta \cdot \gamma' \frac{x - q_i}{\|x - q_i\|} + \sigma' \frac{x}{\|x\|}$. For ζ and η at non-switching positions, $\dot{F}$ attains unique values. Over the switching surfaces, $\dot{F}$ is a multi-valued function and it's variations are considered in the generalized sense, i.e. $\nabla F \in \partial F(x)$, $\frac{\partial F}{\partial t} \in \partial F(t)$ and $u \in \overline{co}\{\lim u(x_i, t_i) | (x_i, t_i) \to (x, t), (x_i, t_i) \notin \Omega_u\}$ with Ω_u the set where u is not defined. For the case $\zeta = 0$ we have that: $\dot{F} = v \cdot \nabla F$. But $\eta = 0$ only at a saddle point or at the origin. By construction the saddle point is on the boundary of the discontinuous surface where $\zeta = 1$ which contradicts the fact that $\zeta = 0$. At the target point $v \in \overline{co}\left\{\lim \eta \cdot f(x) + (1 - \eta) \cdot x^{\perp} | x \to 0, \eta \in [0, 1]\right\}$ but $f(0) = x^{\perp}(0) = 0$ hence $v = 0$ and $\dot{F} = 0$. For all other points we have $\dot{F} = -f \cdot f < 0$. So for $\zeta = 0$ we have that $\dot{F} \leq 0$ with the equality holding at the origin.

For the case $\zeta = 1$ we have that: $\dot{F} = -\gamma' \frac{x - q_i}{\|x - q_i\|} \cdot \dot{q}_i + \left(\dot{q}_i + \lambda \frac{x - q_i}{\|x - q_i\|} + v\right) \cdot \left(\gamma' \frac{x - q_i}{\|x - q_i\|} + \sigma' \frac{x}{\|x\|}\right)$ which after algebraic manipulation simplifies to:

$$\dot{F} = \lambda \cdot \gamma' + \sigma' \cdot \hat{x} \cdot \left(\dot{q}_i + \lambda \frac{x - q_i}{\|x - q_i\|}\right) + v \cdot \nabla F \tag{8}$$

For the term $v \cdot \nabla F = \left(\eta \cdot f(x) + (1 - \eta) \cdot x^{\perp}\right) \cdot (-f) \leq 0$ since when $\eta = 1$ then $-f \cdot f \leq 0$, for $\eta = 0$ or η switching (because $f = [0 \;\; 0]$ and $x \cdot x^{\perp} = 0$), $v \cdot \nabla F = 0$. Hence $v \cdot \nabla F \leq 0$ and from eq. (8) we have: $\dot{F} \leq \lambda \cdot \gamma' + \lambda \cdot \sigma' + \sigma' \cdot \|\dot{q}_i\|$. But $\gamma' < -\sigma' - \theta$ and substituting we get: $\dot{F} \leq -\lambda \cdot \theta + \sigma' \cdot \|\dot{q}_i\|$. Now since $\|\dot{q}_i\| \leq M$ and $\sigma' < \mu$ we have: $\dot{F} \leq -\lambda \cdot \theta + \mu \cdot M$ and $\dot{F} < 0$ as long as $\theta > \frac{\mu \cdot M}{\lambda}$.

For the case of ζ switching, ζ assumes all the values in the range $[0, 1]$. We have $\dot{F} \in \overline{co}\{\lim \varphi | x \to x_P, x \notin N\}$ with $\varphi = \zeta \cdot \left(\lambda \cdot \gamma' + \sigma' \cdot \hat{x} \cdot \left(\dot{q}_i + \lambda \frac{x - q_i}{\|x - q_i\|}\right)\right) + v \cdot \nabla F$ and x_P a point at the surface of discontinuity. The first term was examined in the case $\zeta = 1$ and if the conditions defined there are satisfied, then the term is made negative semidefinite (zero at the zero value of ζ). The last term can be written as $v \cdot \nabla F = \left(\eta f + (1 - \eta) x^{\perp}\right) \cdot (-f) = -\eta f \cdot f - (1 - \eta) x^{\perp} \cdot f$ with $-f \cdot f \leq 0$ and the term $x^{\perp} \cdot f = x^{\perp} \cdot \left(\zeta \cdot \gamma' \frac{x - q_i}{\|x - q_i\|} + \sigma' \frac{x}{\|x\|}\right) = 0$ since $x^{\perp} \cdot x = 0$ and at the point "s" where $x = q_i + \hat{x} \cdot r_{t_i}$, $x^{\perp} \cdot \frac{x - q_i}{\|x - q_i\|} = 0$. At the origin $x^{\perp}(0) = 0$. Hence $v \cdot \nabla F \leq 0$, with the equality holding at the origin and at the saddle points. So for all $\omega \in K[f](x, t)$, we have that

$\omega \overset{a.e}{<} 0$ [1]. Since the positive limit set of the obstacle configurations is away from the origin, the largest invariant set eventually (i.e. as $t \to \infty$) contains only the origin. (The system cannot identically stay on the saddle points since when there $u \neq 0$). Hence by applying the nonsmooth version of LaSalle's invariance principle [8], we have that the origin of the system is (eventually) globally asymptotically stable. ∎

The discontinuous control law u defined in proposition 4 when applied to the system (6) results in chattering in the neighborhood of the surfaces of discontinuity. To reduce chattering we use an integrator backstepping technique for nonsmooth systems [9], [6]. To this extend consider the augmented system:

$$\begin{aligned} \dot{x} &= k \cdot z + a \\ \dot{z} &= -c \cdot z - f \end{aligned} \tag{9}$$

where a is a stabilizing controller of system (6), z a virtual state, k, c are positive constants and f as defined in eq. (2). Obviously system (6) can perform the trajectories of system (9) if the input to system (6) is set to be $u = k \cdot z + a$. For the constructive procedure the interested reader is referred to [6], [9].

Proposition 5: The origin of the system (9) is globally asymptotically stable.

Proof: Let us construct a control Lyapunov function for our system: Let $V_a = F + \frac{k}{2}z^2$ with F defined in (1). We have $\dot{V}_a = \dot{F} + k \cdot z \cdot \dot{z} = \frac{\partial F}{\partial t} + \dot{x} \cdot f + k \cdot z \cdot \dot{z}$. Then $\dot{V}_a = \frac{\partial F}{\partial t} + (k \cdot z + a) \cdot f + k \cdot z \cdot (-c \cdot z - f) = \frac{\partial F}{\partial t} + a \cdot \nabla F + k \cdot z(-c \cdot z - f + f) = \dot{F}_0 - k \cdot c \cdot z^2 \leq 0$, where $\dot{F}_0$ is the time derivative of F along the trajectories of (6) and is negative semidefinite as was proved in proposition 4, with the equality holding at the origin. Hence $\dot{V}_a \leq 0$ with the equality holding at the origin. Using the results from the proof of proposition 4 and applying the nonsmooth version of LaSalle's invariance principle [8], we have that the origin of the system is (eventually) globally asymptotically stable. ∎

V. Simulation Results

To verify the properties of the proposed scheme, we run simulation examples using the parameters of Table 1, which are in accordance with the specifications prescribed in our analysis.

[1] a.e.: almost everywhere i.e. everywhere except a set of measure zero

Case Study 1:

The robot was originally at $x(0) = [3\ 4]^T$, functions h_i were chosen as $h_1(t) = M[cos(t)\ sin(t)]^T$ and $h_2(t) = \frac{2M}{5}[sin(t)\ \ -cos(t)]^T$ and their initial conditions were $q_1(0) = [4\ 0]^T$ and $q_2(0) = [-1\ 6]^T$. Figure 2 shows the trajectories of the robot and the obstacles. The trajectory of the robot is represented with black line. The red spots over the robot's trajectory represent successive robot positions over constant time intervals. The trajectory of obstacle O_1 is represented with a magenta line and of obstacle O_2 with a green line. Over the trajectories of O_1 and O_2 the cyan and magenta spots respectively represent successive obstacle positions over the same constant time interval with the red spots over the robot's trajectory. At the initial positions of the obstacles O_1 and O_2 two concentric circles are drawn representing the obstacle's internal and external radius. The system avoids the obstacles and navigates to the origin.

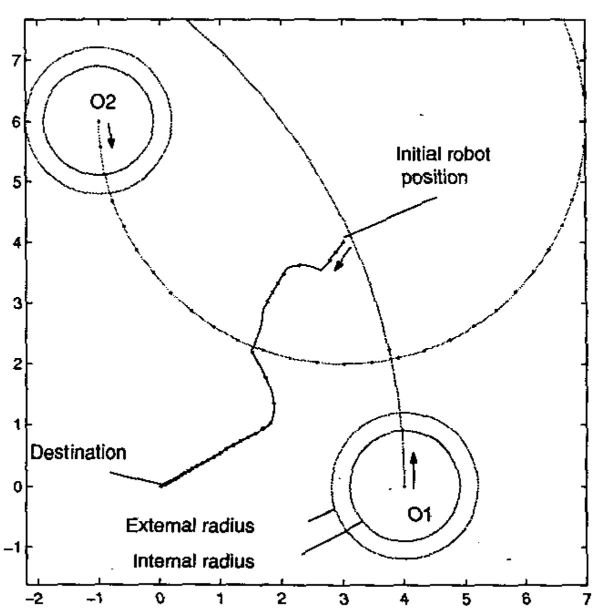

Fig. 2. Case Study 1 simulation results.

Functions		Variables			
$\sigma(x)$	$\mu(x - ln(1+x))$	μ	1.0	M	10
$\gamma(x)$	$a \cdot x^{-1} - b$	a	17.28	λ	1.0
		b	14.4	θ	11
		r_i	0.9	r_{t_i}	1.2

TABLE I

Simulation Parameters

Case Study 2:

The robot was originally at the origin $x(0) = [0\ 0]^T$, functions h_i were chosen as $h_1(t) = M[8 - 6 \cdot cos(10 \cdot t)]^T$ and $h_2(t) = [8\ 6 \cdot cos(10 \cdot t)]^T$ and their initial conditions were $q_1(0) = [-2\ 0.3]^T$ and $q_2(0) = [-5\ -0.3]^T$. Figure 3 depicts the trajectories of the system. The colored spots over the system's trajectories are in the same context with case study 1. The robot successfully avoids collisions with the moving obstacles and returns to the origin.

It must be noted that in both case studies the backstepping integrator is successful in suppressing the chattering effects of the underlying discontinuous controller.

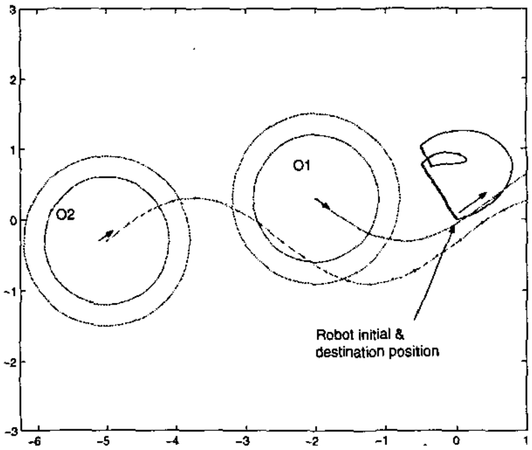

Fig. 3. Case Study 2 simulation results.

VI. Conclusions-Issues for further research

In this paper we developed a methodology for navigating point robots in dynamic environments. A new class of nonsmooth navigation functions (NNFs) was introduced. The methodology provides a simple and computationally inexpensive closed form feedback solution, enabling fast feedback and rendering the algorithm particularly suitable for real time motion planning and control of autonomous agents with limited computational resources and limited sensing range. The proposed scheme guarantees both convergence and collision avoidance. Application of nonsmooth integrator backstepping significantly reduces chattering, resulting in smooth motion paths for the system.

Our future plans include the study of robot navigation in dynamic environments (2D and 3D) with arbitrarily shaped obstacles, in multiple robot scenarios with kinematic constraints.

Acknowledgements: The authors want to acknowledge the contribution of the European Commission through contract IST – 2001 – 33567 – MICRON and contract IST – 2001 – 32460 – HYBRIDGE .

VII. References

[1] F. Clarke. *Optimization and Nonsmooth Analysis.* Addison - Wesley, 1983.

[2] A. Filippov. *Differential equations with discontinuous right-hand sides.* Kluwer Academic Publishers, 1988.

[3] K.J.Kyriakopoulos. *A Supervisory Control Strategy for Real-Time Navigation of Mobile Robots in Dynamic Environments.* PhD thesis, Rensselaer Polytechnic Institute, 1991.

[4] K.J.Kyriakopoulos and G.N.Saridis. Optimal and sub optimal motion planning for collision avoidance of mobile robots in non-stationary environments. *Journal of Intelligent and Robotic Systems*, 12:1–45, 1995.

[5] D. E. Koditschek and E. Rimon. Robot navigation functions on manifolds with boundary. *Advances Appl. Math.*, 11:412–442, 1990.

[6] M. Krstić, I. Kanellakopoulos, and P. Kokotović. *Nonlinear and Adaptive Control Design.* Wiley-Interscience, 1995.

[7] E. Rimon and D. E. Koditschek. Exact robot navigation using artificial potential functions. *IEEE Trans. on Robotics and Automation*, 8(5):501–518, 1992.

[8] D. Shevitz and B. Paden. Lyapunov stability theory of nonsmooth systems. *IEEE Trans. on Automatic Control*, 49(9):1910–1914, 1994.

[9] H. Tanner and K.J. Kyriakopoulos. Backstepping for nonsmooth systems. *Automatica*, 39:1259–1265, 2003.

[10] H. G. Tanner, S. G. Loizou, and K. J. Kyriakopoulos. Nonholonomic stabilization with collision avoidance for mobile robots. *Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pages 1220–1225, 2001.

[11] H. G. Tanner, S. G. Loizou, and K. J. Kyriakopoulos. Nonholonomic navigation and control of cooperating mobile manipulators. *IEEE Trans. on Robotics and Automation*, 19(1):53–64, 2003.