Proceedings of the 2003 IEEE/RSJ
Intl. Conference on Intelligent Robots and Systems
Las Vegas, Nevada · October 2003

Dynamics and Generation of Gaits for a Planar Rollerblader

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Abstract— **We develop the dynamic model for a planar ROLLERBLADER. The robot consists of a rigid platform and two planar, two degree-of-freedom legs with in-line skates at the foot. The dynamic model consists of two unicycles coupled through the rigid body dynamics of the planar platform. We derive the Lagrangian reduction for the ROLLERBLADING robot. We show the generation of some simple gaits that allow the platform to move forward and rotate by using cyclic motions of the two legs.**

I. INTRODUCTION

The last decade has seen a great deal of interest in undulatory robotic locomotion systems, including the Eel [11], the Snakeboard [12], the Variable Geometric truss [9], the Roller Racer [10] and various snake like robots [6]. The configuration space for such robots consists of two types of variables, *group* variables which represent the aggregate motion of the robot and *shape* variables. Propulsion or net motion of these robots is a result of cyclic *shape* variations. Such cyclic shape variations are often referred to as *gaits*. Unlike more conventional locomotion systems, the dynamics of such systems are quite complicated and it is often difficult to determine how shape motions can be synthesized and controlled to generate desired group motions.

Robots like the snakeboard are affected by the presence of *nonholonomic* or non-integrable constraints. Such constraints are usually expressed as linear functions of group and shape velocities. Exploring the interplay between the constraints and the dynamics is necessary to understand the build up and decay of momentum in the system [12].

In this paper, we analyze and present simulation results for a planar rollerblading robot called the ROLLERBLADER. The ROLLERBLADER consists of a central platform with two extensible links attached to it, as shown in Fig. 1. Each link has a rotary actuator mounted at the joint joining the link to the central platform and a linear actuator that controls the extension of the link. The links make contact with the ground using an inline skate at the end of each link. The skate is fixed perpendicular to the link. Thus the system considered in this paper has a total of four inputs that control the movement of the legs, and seven degrees of freedom. Note that there are many variations on this geometry. It is possible to include additional legs, and to add additional degrees of freedom to each leg. For example, articulating the in-line skate with respect to the leg provides an additional input in the system. However, we consider here the simplest geometry needed to perform the rollerblading motion. Thus, we do not consider issues of balance that introduce an additional level of complexity in terms of analysis and development.

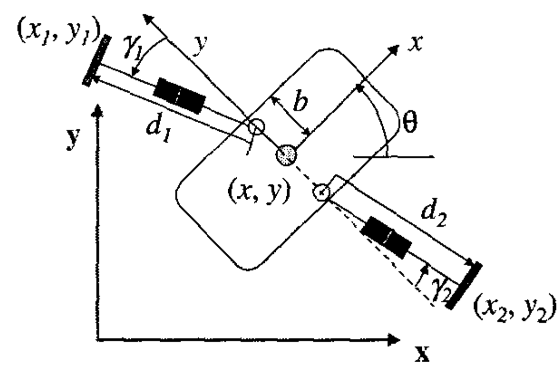


Fig. 1. Schematic of the ROLLERBLADER. (x, y, θ) are the group variables and $(\gamma_1, d_1, \gamma_2, d_2)$ are the shape variables.

This work builds on the recent body of literature on nonholonomic mechanics of locomotion systems ([10], [12], [11], [1]). The systems that are most closely related to this paper are the Roller Racer [10] and the Roller Walker [8]. In fact, the ROLLERBLADER is based on the design of the Roller Walker, a four legged machine with in-line skates at the feet. An experimental prototype was shown to be able to generate skating motions in the forward and rotary directions and also follow a figure-eight path [5]. Our goal is to better understand the mechanics of the skating motion and the process of generation of gaits.

The organization of the paper is as follows. We first provide some general background information in the next section (see [2], [12] and [9] for more details). In Section III, we derive the dynamics for the system in Figure 1. In Section IV, we use the BKMM Lagrangian reduction technique [2] to develop simplied equations of motion. The process of generating momentum in undulatory locomotion systems and the generation of gaits are discussed in Section V. Numerical simulation results provided in Section V illustrate gaits for forward motion and rotation. We conclude with a brief discussion of problems that remain to be solved and our ongoing work in this direction.

II. BACKGROUND

The configuration manifold for the robot is given by $Q = SE(2) \times S \times \mathbb{R} \times S \times \mathbb{R}$. S denotes the group of rotations on $\mathbb{R}^2$. $SE(2)$, the special Euclidean group, represents rotation and translation of a rigid body in a plane. A point $h = (x_1, y_1, \alpha) \in SE(2)$ can be represented using homogeneous coordinates:

$$h = \begin{pmatrix} \cos\alpha & -\sin\alpha & x_1 \\ \sin\alpha & \cos\alpha & y_1 \\ 0 & 0 & 1 \end{pmatrix}.$$

We can associate with $SE(2)$ a left action,

$$L_h : SE(2) \to SE(2) : g \to hg.$$

We also denote by $T_h L_h$ the differential map of the left action evaluated at $h \in G$. We choose $q = (x, y, \theta, \gamma_1, d_1, \gamma_2, d_2)^T$ to represent the configuration of the system where (x, y) is the position of the central platform in a inertial reference frame, θ is the orientation of the robot in the inertial reference frame, (γ_1, γ_2) denotes the angular position of links 1 and 2 with respect to the central platform, (d_1, d_2) are the extensions of links 1 and 2.

The configuration space can be naturally put into a *trivial principal fiber bundle* denoted by $Q(\mathcal{M}, G)$. Q is called the *total space* while $\mathcal{M} = S \times \mathbb{R} \times S \times \mathbb{R}$ represents the *shape* or *base* space. We choose $r = (\gamma_1, d_1, \gamma_2, d_2)$ as local coordinates for the shape variables. $G = SE(2)$ represents the fiber space. $g = (x, y, \theta) \in SE(2)$ are often referred to as the fiber variables or *fiber directions*.

We denote by $T_q Q$ the tangent space of Q at $q = (g, r) \in Q$ and by $v_q \in T_q Q$ a tangent vector at q. For our system, $v_q = (\dot{x}, \dot{y}, \dot{\theta}, \dot{\gamma}_1, \dot{d}_1, \dot{\gamma}_2, \dot{d}_2)$. We will use $\xi \in T_e G$ to denote the body velocity where $T_e G$ is the tangent space of G at the identity e.

We are interested in the left action of $SE(2)$ on the configuration space, $\Phi : SE(2) \times Q \to Q$ given by $\Phi_h(q) = \Phi_h(g, r) = (L_h g, r)$ for $h \in G$ and $q = (g, r) \in G \times \mathcal{M} = Q$. We will also require the tangent map of $\Phi_h(q)$ given by $T_h \Phi_h v_q : T_q Q \to T_q Q$:

$$T_h \Phi_h v_q = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & | \\ \sin\alpha & \cos\alpha & 0 & 0_{3\times4} \\ 0 & 0 & 1 & | \\ - & 0_{4\times3} & - & I_{4\times4} \end{pmatrix} v_q. \quad (1)$$

where $h = (x_1, y_1, \alpha) \in SE(2)$. As seen later, $SE(2)$ is the symmetry group for the ROLLERBLADER. The tangent space at $q \in Q$ to the orbit of $SE(2)$ is given by:

$$\mathcal{V}_q = T_q Orb(q) = \mathrm{sp}\left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \theta}\right\}.$$

III. DYNAMIC MODEL

In this section we derive the full dynamics for the ROLLERBLADER. Let the mass and rotational inertia of the central platform of the robot be M and I_c respectively. Let each link have rotational inertial I_p. The mass of the link is assumed to be negligible. Each skate has mass m, but is assumed to have no rotational inertia. Then the Lagrangian for the robot is given by

$$L = \frac{1}{2} I_c \dot{\theta}^2 + \frac{1}{2} I_p (\dot{\theta} + \dot{\gamma}_1)^2 + \frac{1}{2} I_p (\dot{\theta} + \dot{\gamma}_2)^2 + \frac{1}{2} M (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} m (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m (\dot{x}_2^2 + \dot{y}_2^2). \quad (2)$$

where (x_1, y_1) and (x_2, y_2) represent the position of the skates in the inertial reference frame:

$$\begin{aligned} x_1 &= x + b\sin\theta + d_1 \sin(\theta + \gamma_1), \\ y_1 &= y - b\cos\theta - d_1 \cos(\theta + \gamma_1). \\ x_2 &= x - b\sin\theta - d_2 \sin(\theta + \gamma_2), \\ y_2 &= y + b\cos\theta + d_2 \cos(\theta + \gamma_2). \end{aligned}$$

Differentiating, we get

$$\begin{aligned} \dot{x}_1 &= \dot{x} + b\cos\theta\dot{\theta} + d_1 \cos(\theta+\gamma_1)(\dot{\theta}+\dot{\gamma}_1) + \dot{d}_1 \sin(\theta+\gamma_1), \\ \dot{y}_1 &= \dot{y} + b\sin\theta\dot{\theta} + d_1 \sin(\theta+\gamma_1)(\dot{\theta}+\dot{\gamma}_1) - \dot{d}_1 \cos(\theta+\gamma_1). \\ \dot{x}_2 &= \dot{x} - b\cos\theta\dot{\theta} - d_2 \cos(\theta+\gamma_2)(\dot{\theta}+\dot{\gamma}_2) - \dot{d}_2 \sin(\theta+\gamma_2), \\ \dot{y}_2 &= \dot{y} - b\sin\theta\dot{\theta} - d_2 \sin(\theta+\gamma_2)(\dot{\theta}+\dot{\gamma}_2) + \dot{d}_2 \cos(\theta+\gamma_2). \quad (3) \end{aligned}$$

Using the body velocity $\xi = (\xi_x, \xi_y, \xi_\theta)$ of the system, the Lagrangian can be seen to simplify to

$$\begin{aligned} L = \frac{1}{2}[&(M+2m)\xi_x^2 + (M+2m)\xi_y^2 + I_p(\xi_\theta + \dot{\gamma}_1)^2 + \\ &I_p(\xi_\theta + \dot{\gamma}_2)^2 + (I_c + 2mb^2){\xi_\theta}^2 + m(\dot{d}_1^2 + \dot{d}_2^2 \\ &+ d_1^2(\xi_\theta + \dot{\gamma}_1)^2 + d_2^2(\xi_\theta + \dot{\gamma}_2)^2 + (2d_1 \cos\gamma_1(\xi_\theta + \dot{\gamma}_1) \\ &- 2d_2 \cos\gamma_2(\xi_\theta + \dot{\gamma}_2) + 2\dot{d}_1 \sin\gamma_1 + 2\dot{d}_2 \sin\gamma_2)\xi_x + \\ &2bd_1\xi_\theta \cos\gamma_1(\xi_\theta + \dot{\gamma}_1) + 2bd_2\xi_\theta \cos\gamma_2(\xi_\theta + \dot{\gamma}_2) \\ &+ (2b\dot{d}_1 \sin\gamma_1 + 2b\dot{d}_2 \sin\gamma_2)\xi_\theta + (2d_1 \sin\gamma_1(\xi_\theta + \dot{\gamma}_1) \\ &- 2\dot{d}_1 \cos\gamma_1 - 2d_2 \sin\gamma_2(\xi_\theta + \dot{\gamma}_2) + 2\dot{d}_2 \cos\gamma_2)\xi_y)] \quad (4) \end{aligned}$$

where

$$\begin{aligned} \xi_x &= \dot{x}\cos\theta + \dot{y}\sin\theta, \\ \xi_y &= -\dot{x}\sin\theta + \dot{x}\cos\theta, \\ \xi_\theta &= \dot{\theta}. \end{aligned} \quad (5)$$

The Lagrangian defined here is invariant to the left group action, i.e. $L(\Phi_h q, T_h \Phi_h v_q) = L(q, v_q)$. The invariance of the Lagrangian to the left action is also evident from the fact that the group $g = (x, y, \theta)$ does not appear in the expression for the Lagrangian written in terms of the body velocity (Equation (4)).

The formulation of the dynamics of the system must handle the *non-holonomic constraints* acting on the system. It is necessary then to specify the *constraint distribution* on Q which contains the allowable directions of motion of the system. In order to formulate the constraint distribution, the constraints are expressed as linear functionals of the velocities or *one-forms*. The nonholonomic constraints are specified as

$$\dot{y}_1\cos(\theta+\gamma_1)-\dot{x}_1\sin(\theta+\gamma_1)=0,$$
$$\dot{y}_2\cos(\theta+\gamma_2)-\dot{x}_2\sin(\theta+\gamma_2)=0.$$

Using Equation (3), we have

$$-\sin(\theta+\gamma_1)\dot{x}+\cos(\theta+\gamma_1)\dot{y}-b\sin(\gamma_1)\dot{\theta}-\dot{d}_1=0, \quad (6)$$
$$-\sin(\theta+\gamma_2)\dot{x}+\cos(\theta+\gamma_2)\dot{y}+b\sin(\gamma_2)\dot{\theta}+\dot{d}_2=0. \quad (7)$$

We can now define the constraint one-forms as

$$\omega_q^1=-\sin(\theta+\gamma_1)dx+\cos(\theta+\gamma_1)dy-b\sin(\gamma_1)d\theta-dd_1,$$
$$\omega_q^2=-\sin(\theta+\gamma_2)dx+\cos(\theta+\gamma_2)dy-b\sin(\gamma_2)d\theta+dd_2.$$

The constraint distribution $\mathcal{D}_q$ is given by the intersection of the kernels of the two one-forms given above. A basis for the distribution can be written as

$$\xi_Q^1=\begin{bmatrix}\frac{-\cos(\gamma_1+\theta)}{\sin(\gamma_1-\gamma_2)} & \frac{-\sin(\gamma_1+\theta)}{\sin(\gamma_1-\gamma_2)} & 0 & 0 & 0 & 0 & 1\end{bmatrix}^T,$$
$$\xi_Q^2=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}^T,$$
$$\xi_Q^3=\begin{bmatrix}\frac{-\cos(\gamma_2+\theta)}{\sin(\gamma_1-\gamma_2)} & \frac{-\sin(\gamma_2+\theta)}{\sin(\gamma_1-\gamma_2)} & 0 & 0 & 1 & 0 & 0\end{bmatrix}^T,$$
$$\xi_Q^4=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}^T,$$
$$\xi_Q^5=\begin{bmatrix}a_1 & a_2 & a_3 & 0 & 0 & 0 & 0\end{bmatrix}^T.$$

where,

$$a_1=(\xi_Q^5)_1=-b(\cos(\gamma_2+\theta)\sin(\gamma_1)+\cos(\gamma_1+\theta)\sin(\gamma_2)),$$
$$a_2=(\xi_Q^5)_2=-b(\sin(\gamma_2)\sin(\gamma_1+\theta)+\sin(\gamma_1)\sin(\gamma_2+\theta)),$$
$$a_3=\sin(\gamma_1-\gamma_2). \quad (8)$$

The constraint distribution is given by

$$\mathcal{D}_q=\mathrm{sp}\left\{\xi_Q^1,\xi_Q^2,\xi_Q^3,\xi_Q^4,\xi_Q^5\right\}.$$

The Lagrange's equations of motion with the constraints contain all the relevant information on the dynamics. In the next section, we show how the BKMM reduction process can be used to rewrite the equations by projecting them along the unconstrained directions.

IV. Reduction

We now present the process of **Reduction** due to Bloch, et. al. [2] which leads to simplified equations of motion, allowing us to write them in a lower-dimensional space. It also provides insight into the geometry of the system. In the course of reduction of a system, the externally applied constraints and certain internal constraints which often represent momentum conservation laws are used to define a *connection* on the principal fiber bundle. The *connection* relates the *fiber velocities* to the *shape motions*. In the presence of nonholonomic constraints there may exist one or more momenta along the unconstrained directions. The evolution of this momentum vector, referred to as the *generalized momentum*, is governed by a *generalized momentum equation* (first derived in [2]). The connection and the generalized momentum equation can then be used to reduce the dynamics of the *base* space. A fairly detailed approach to carrying out such a reduction can be found in [2] and [12].

For the ROLLERBLADER, we first derive the unconstrained directions and then the generalized momentum equation. In addition, we also derive the *connection* for the system and the reduced equations for the evolution of the *base* variables. This set of equations then defines the complete dynamics of the system. A process of **Reconstruction** can then be used to recover any variables which were removed in the **Reduction** process.

A. Constrained Fiber Distribution

To derive the generalized momentum equations, we need to calculate the unconstrained directions of the system in the presence of the nonholonomic constraints. These directions are represented by the *constrained fiber distribution*($\mathcal{S}_q$) which is defined as the intersection of the constraint distribution $\mathcal{D}_q$ and the *fiber distribution* $\mathcal{V}_q$. The fiber distribution contains all the infinitesimal motions of the system that do not alter the shape of the system. The fiber distribution can be written as

$$\mathcal{V}_q=\mathrm{sp}\left\{\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial \theta}\right\}.$$

Every vector $\xi_Q^q\in\mathcal{S}_q$ must be in both the fiber distribution and the constraint distribution. Thus, we can write ξ_Q^q in terms of the basis elements for $\mathcal{D}_q$ and $\mathcal{V}_q$.

$$\xi_Q^q=\nu_1\frac{\partial}{\partial x}+\nu_2\frac{\partial}{\partial y}+\nu_3\frac{\partial}{\partial \theta}, \quad (9)$$
$$\xi_Q^q=u_1\xi_Q^1+u_2\xi_Q^2+u_3\xi_Q^3+u_4\xi_Q^4+u_5\xi_Q^5. \quad (10)$$

Using Equations (9) and (10) and the basis for the constraint distribution, we can write

$$u_1=0,u_2=0,u_3=0,u_4=0,$$
$$\nu_1=a_1u_5,\nu_2=a_2u_5,\nu_3=a_3u_5,$$

where (a_1, a_2, a_3) are given by Equation (8). Thus, $\mathcal{S}_q$ is one-dimensional except at the singular point ($\gamma_1 = \gamma_2 = 0$) and we can write

$$\mathcal{S}_q = \mathrm{sp}\{\xi_Q^q\}.$$

where,

$$\xi_Q^q = a_1 \frac{\partial}{\partial x} + a_2 \frac{\partial}{\partial y} + a_3 \frac{\partial}{\partial \theta}. \tag{11}$$

Let $(\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$ represent a basis for the *Lie Algebra* $se(2)$ corresponding to $SE(2)$. The infinitesimal generator corresponding to $\xi^q = \xi_1\mathcal{A}_1 + \xi_2\mathcal{A}_2 + \xi_3\mathcal{A}_3 \in se(2)$ is

$$\xi_Q^q = (\xi_2 - y\xi_1)\frac{\partial}{\partial x} + (\xi_3 + x\xi_1)\frac{\partial}{\partial y} + \xi_1 \frac{\partial}{\partial \theta}.$$

A given vector field $\xi_Q^q = \nu_1 \frac{\partial}{\partial x} + \nu_2 \frac{\partial}{\partial y} + \nu_3 \frac{\partial}{\partial \theta}$ can be considered as the infinitesimal generator of an element $\xi^q \in se(2)$, under the group action Φ. Then,

$$\xi^q = \nu_3\mathcal{A}_1 + (\nu_1 + y\nu_3)\mathcal{A}_2 + (\nu_2 - x\nu_3)\mathcal{A}_3.$$

Thus, we can write

$$\left(\frac{d\xi^q}{dt}\right)_Q = (\dot{a}_1 + a_3\dot{y})\frac{\partial}{\partial x} + (\dot{a}_2 - a_3\dot{x})\frac{\partial}{\partial y} + \dot{a}_3\frac{\partial}{\partial \theta}. \tag{12}$$

B. Generalized Momentum

Given the constrained fiber distribution and a Lagrangian, L, for the given system the generalized momentum, p, is given by

$$p = \frac{\partial L}{\partial \dot{q}_i}\left(\xi_Q^q\right)^i. \tag{13}$$

where summation over the index i is implied. ξ_Q^q is given by Equation (11).

The generalized momentum of the system can be physically interpreted (after a scaling factor) as the angular momentum of the system around the center of rotation defined by the two skate constraints. The center of rotation has coordinates $(a_2/a_3, a_1/a_3)$ in an inertial reference frame fixed at the center of the robot. The angular momentum of the robot about the point of rotation is then given as

$$p' = I_c\dot{\theta} + M\dot{x}\frac{a_1}{a_3} + M\dot{y}\frac{a_2}{a_3} + I_p(\dot{\theta} + \dot{\gamma}_1) + I_p(\dot{\theta} + \dot{\gamma}_2).$$

Evaluating Equation (13), we find p is given by p' multiplied by a scaling factor(a_3).

Using Noether's theorem [12], the generalized momentum equation specifying the evolution of the momentum can be written as

$$\frac{dp}{dt} = \frac{\partial L}{\partial \dot{q}_i}\left(\frac{d\xi^q}{dt}\right)_Q^i + \tau_i(\xi_Q^q)^i. \tag{14}$$

Here, τ is the one-form of the input torques and forces. For the Rollerblader, this is given as $\tau = (0, 0, 0, \tau_{\gamma_1}, f_{d_1}, \tau_{\gamma_2}, f_{d_2})^T$ where $\tau_{\gamma_1}, f_{d_1}, \tau_{\gamma_2}$ and f_{d_2} are the input torques/forces corresponding to the γ_1, d_1, γ_2 and d_2 degrees of freedom respectively. The expression for $\left(\frac{d\xi^q}{dt}\right)_Q$ is given by Equation (12).

C. The Reduced Equations

To carry out the required reduction, we use the invariance of the momentum under the group action. Evaluating Equations (6), (7) and (13) at $(x, y, \theta) = (0, 0, 0)$ gives us a set of three equations which can be solved for the body velocities in terms of the base velocities and the momentum. (Thus, the momentum equation gives us an extra constraint.) We can write the resultant reduced equations in the form [12]:

$$\xi = -A(r)\dot{r} + \tilde{I}^{-1}p, \tag{15}$$

$$\dot{p} = \frac{1}{2}\dot{r}^T\sigma_{\dot{r}\dot{r}}(r)\dot{r} + p^T\sigma_{p\dot{r}}(r)\dot{r} + p^T\sigma_{pp}(r)p + \tilde{\tau}, \tag{16}$$

$$\tilde{M}(r)\ddot{r} + \dot{r}^T\tilde{C}(r)\dot{r} + \tilde{N}(r, \dot{r}, p) = \tau. \tag{17}$$

Very briefly, $A(r)$ is the nonholonomic connection, $\tilde{I}$ and $\tilde{M}$ can be derived from the inertia tensor for the system, $\tilde{C}(r)$ represents Coriolis and centrifugal terms, $\tilde{N}$ includes the influence of the momentum on the dynamics of the shape variables. Detailed expressions are provided in [4]. Using Equations (15) and (16), we can numerically solve for the evolution of the body velocities and momentum. We can then solve for $(\dot{x}, \dot{y}, \dot{\theta})$ using Equation (5).

V. Generation of Gaits

The generation of gaits for the ROLLERBLADER is a complex problem. Periodic/vibratory motion of the shape variables has often been used to generate motion. In [3], Brockett presented a mathematical formulation to understanding such actuation. Periodic inputs have since been used to drive a variety of robots including the RollerRacer [9] and the Snakeboard [12]. In [11], McIsaac and Ostrowski used a periodic input traveling down the length of an articulated Eel-like robot to generate motion. Periodic inputs were also used to generate motion for snake-like robots [7] and the RollerWalker [5]. Motivated by these examples, we examined the use of periodic inputs to generate motion for the ROLLERBLADER.

For simplicity, we assume that we have direct control over the shape inputs and are able to drive them directly. This is equivalent to assuming the motors are controlled by a feedback controller that cancels the dynamics in Equation (17) allowing the direct control of $r(t)$. Now, the simplest possible gait that can be used is the one where the motions of the two legs are symmetric with respect to the longitudinal axis of symmetry (x axis of the body fixed frame - see Figure 1). We call such a gait, where

$\gamma_2 = -\gamma_1$ and $d_1 = d_2$, a *symmetric gait*. Thus,

$$\gamma_1 = -\gamma_2, \dot{\gamma}_1 = -\dot{\gamma}_2, d_1 = d_2, \dot{d}_1 = \dot{d}_2. \quad (18)$$

$$\sigma_{pp} = 0, \tilde{\tau} = 0. \quad (19)$$

Now, substituting the conditions in Equation (18), we find:

$$\sigma_{\dot{r}\dot{r}} = 0. \quad (20)$$

Thus, Equation (16) simplifies to

$$\dot{p} = p\sigma_{p\dot{r}}(r)\dot{r}. \quad (21)$$

We note that the above equation is *linear* in p. This implies that, for a system starting from rest, the generalized momentum stays at 0 for all t, i.e. $p(t=0) = 0 \Rightarrow p(t) = 0$. Using this fact, we can simplify Equation (15) to

$$\xi = g^{-1}\dot{g} = -A(r)\dot{r}.$$

Using the conditions in Equation (18), the above equation simplifies to

$$\xi = \left(-\dot{d}_1/(\sin\gamma_1), 0, 0\right)^T.$$

Thus, for a system starting from rest, the symmetric gait generates motion only in the forward direction.

A similar analysis can be conducted for an *anti-symmetric gait*, i.e. a gait for which $(\gamma_1 = \gamma_2, d_1 = d_2)$. For an anti-symmetric gait we have

$$\gamma_1 = \gamma_2, \dot{\gamma}_1 = \dot{\gamma}_2, d_1 = d_2, \dot{d}_1 = \dot{d}_2. \quad (22)$$

Again, we find that the generalized momentum for the system starting from rest is conserved. Now, Equation (15) *simplifies to*

$$\xi = \left(0, 0, -\dot{d}_1/(b\sin\gamma_1)\right)^T.$$

Thus, an anti-symmetric gait gives rise to pure rotational motion of the robot. Note that both these gaits are characterized by zero generalized momentum. This of course does not mean that the momentum of the ROLLERBLADER is zero and that no motion is possible.

We now present simulation results for forward and rotary gaits of the robot. The physical parameters used for the simulations are $(m = 2, M = 25, I_c = 20, I_p = 10, b = 0.05)$.

Forward motion gait A symmetric gait with $\gamma_1 = -\gamma_2$ is used to generate forward motion. Since the base variables are considered to be directly controllable, they are used directly as inputs. The inputs are specified as sinusoids:

$$\begin{aligned} d_1 &= d_{1o} + d_{1c}\sin(2\pi t/T_{d_1} + \phi_{d_1}), \\ \gamma_1 &= \gamma_{1o} + \gamma_{1c}\sin(2\pi t/T_{\gamma_1} + \phi_{\gamma_1}), \\ d_2 &= d_{2o} + d_{2c}\sin(2\pi t/T_{d_2} + \phi_{d_2}), \\ \gamma_2 &= \gamma_{2o} + \gamma_{2c}\sin(2\pi t/T_{\gamma_2} + \phi_{\gamma_2}). \end{aligned} \quad (23)$$

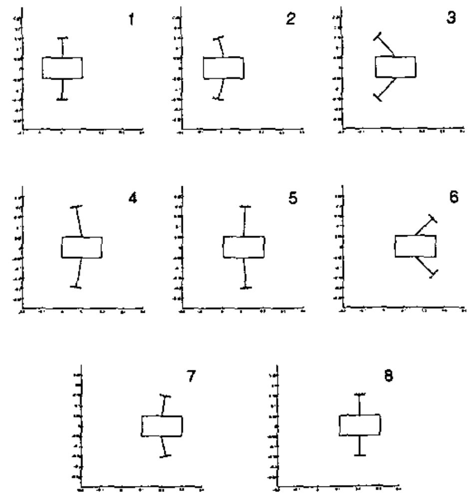
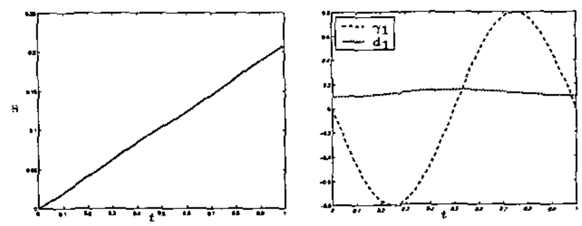


Fig. 2. Snapshots of the gait for forward motion at $t = (0, 0.05, 0.31, 0.46, 0.51, 0.76, 0.96, 1)$ seconds.



(a) Forward motion of the ROLLERBLADER.

(b) Shape variables.

Fig. 3. Forward motion gait for the ROLLERBLADER.

where $(d_{1o} = d_{2o} = 0.125, \gamma_{2o} = 0)$, $(d_{1c} = d_{2c} = 0.025, \gamma_{2c} = -\gamma_{1c} = 0.8)$ are the amplitudes of the sinusoidal inputs, $(\phi_{d_1} = \frac{3\pi}{2}, \phi_{d_2} = \frac{3\pi}{2}, \phi_{\gamma_1} = 0, \phi_{\gamma_2} = 0)$ and $(T_{d_1} = T_{d_2} = T_{\gamma_1} = T_{\gamma_2} = 1)$ are phase offsets and time periods respectively for the inputs.

Fig. 2 shows snapshots of the forward motion starting from an initial position $(x, y, \theta, \gamma_1, d_1, \gamma_2, d_2, p) = (0, 0, 0, 0, 0.1, 0, 0.1, 0)$. Figure 3 shows the straight line motion and inputs for the gait. Note that a closer investigation of Figure (3(a)) (not shown) reveals that the forward velocity, ξ_x, is not constant.

Rotary gait By using an anti-symmetric gait with inphase movements of the legs, we can get the robot to turn in place. The inputs for the gait are still given by Equation (23) but now $\gamma_{1c} = \gamma_{2c} = 0.8$. All the other parameters have the same values as in the forward motion gait. Thus, the gait is similar to the one for the forward motion except that $\gamma_1 = \gamma_2$. Figure 4 shows snapshots for the rotary motion for initial condition: $(x, y, \theta, \gamma_1, d_1, \gamma_2, d_2, p) = (0, 0, 0, 0, 0.1, 0, 0.1, 0)$. Figure 5 shows the evolution of θ and the inputs for the gait.

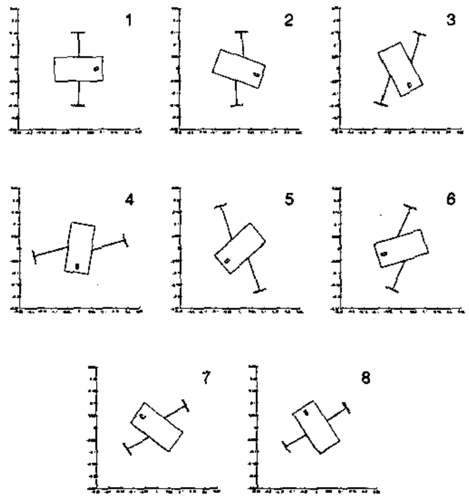


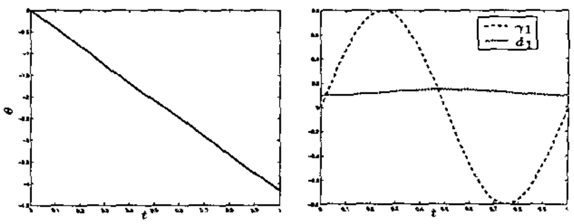
Fig. 4. Snapshots of the gait for rotary motion at $t = (0, 0.085, 0.26, 0.41, 0.58, 0.68, 0.76, 0.91, 0.99)$ seconds.

VI. Discussion and Conclusion

In this paper, we have formulated the dynamics for the ROLLERBLADER. The process of *reduction* helps to formulate the dynamics in terms of a smaller set of variables, providing insight into the process of momentum generation for the robot. We showed how two basic gaits can be developed for the robot enabling it to move forward and rotate.

There are several directions to our ongoing research. In order to systematically design gaits and motion plans for such a robot, we are investigating the controllability of the system. Since the Rollerblader is not a drift-free system, a logical course of action is to use Sussman's condition [14] to determine whether it is *Small Time Locally Controllable (STLC)*. If the system is *STLC*, optimal control techniques can be used to generate gaits for the system. In [13], an optimal control method was used to generate gaits for the Snakeboard and is potentially applicable to our system as well. A second direction of ongoing research is motion planning for the ROLLERBLADER using a combination of the two gaits demonstrated in this paper. Finally, we are in the process of building a prototype of the ROLLERBLADER using which we hope to demonstrate both these gaits and their composition. The long term goal of our research effort is to be able to automatically generate dynamic models, controllers and motion planning algorithms for locomotion systems consisting of a planar rigid body with an arbitrary combination of wheels, legs and skates. This will lead to a general framework for modular *hybrid* locomotion systems.

Acknowledgement The authors would like to acknowledge Dr. Jim Ostrowski, Evolution Robotics, for his help. The support of NSF grants FRG DMS01-39747 and IIS02-22927 is gratefully acknowledged.



(a) Orientation of the robot. (b) Shape variables.

Fig. 5. Rotary motion gait for the ROLLERBLADER

VII. References

[1] A. M. Bloch. *Nonholonomic Mechanics and Control.* Springer, 2003.

[2] A. M. Bloch, P. S. Krishnaprasad, J. E. Marsden, and R. M. Murray. Nonholonomic mechanical systems with symmetry. *Archive for Rational Mechanics and Analysis,* 136(1):21–99, 1996.

[3] R. W. Brockett. On the rectification of vibratory motion. *Sensors and Actuators,* 20, no. 1-2:91–96, 1989.

[4] S. Chitta and V. Kumar. Dynamics and generation of gaits for a planar rollerblader. preprint available at http://www.seas.upenn.edu/~sachinc/roller.pdf.

[5] G. Endo and S. Hirose. Study on Roller-Walker : Multi-mode steering control and self-contained locomotion. In *Proc. IEEE Int. Conf. Robotics and Automation,* pages 2808–2814, San Francisco, April 2000.

[6] S. Hirose and A. Morishima. Design and control of a mobile robot with an articulated body. *The International Journal of Robotics Research,* 9(2):99–114, 1990.

[7] S. Hirose and Peter Cave (Translated by) and Charles Goulden (Translated by). *Biologically Inspired Robots : Serpentile Locomotors and Manipulators.* Oxford University Press, July 1993.

[8] S. Hirose and H. Takeuchi. Study on Roller-Walker (Basic Characteristics and its Control). In *Proc. IEEE Int. Conf. Robotics and Automation,* pages 3265–3270, Minneapolis, 1996.

[9] P. S. Krishnaprasad and D. P. Tsakiris. G-snakes: Nonholonomic kinematic chains on lie groups. In *Proc.* 33^{rd} *IEEE Conf. on Decision and Control,* Buena Vista, FL, 1994.

[10] P. S. Krishnaprasad and D. P. Tsakiris. Oscillations, SE(2)-Snakes and Motion Control: A Study of the Roller Racer. Technical report, Center for Dynamics and Control of Smart Structures (CDCSS), University of Maryland, College Park, 1998.

[11] K. A. McIsaac and J. P. Ostrowski. Motion planning for dynamic eel-like robots. In *Proc. IEEE Int. Conf. on Robotics and Automation,* San Francisco, April 2000.

[12] J. P. Ostrowski. *The Mechanics and Control of Undulatory Robotic Locomotion.* Ph.D. thesis, California Institute of Technology, Pasadena, CA, 1995.

[13] J. P. Ostrowski, J. P. Desai, and V. Kumar. Optimal gait selection for nonholonomic locomotion systems. *The International Journal of Robotics Research,* 19(3):1–13, March 2000.

[14] H. J. Sussman. A general theorem on local controllability. *SIAM Journal on Control and Optimization,* 25(1):158–194, January 1987.