Route Swarm: Wireless Network Optimization through Mobility

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Abstract-In this paper, we demonstrate a novel hybrid architecture for coordinating networked robots in sensing and information routing applications. The proposed INformation and Sensing driven PhysIcally REconfigurable robotic network (INSPIRE), consists of a Physical Control Plane (PCP) which commands agent position, and an Information Control Plane (ICP) which regulates information flow towards communication/sensing objectives. We describe an instantiation where a mobile robotic network is dynamically reconfigured to ensure high quality routes between static wireless nodes, which act as source/destination pairs for information flow. The ICP commands the robots towards evenly distributed inter-flow allocations, with intra-flow configurations that maximize route quality. The PCP then guides the robots via potential-based control to reconfigure according to ICP commands. This formulation, deemed Route Swarm, decouples information flow and physical control, generating a feedback between routing and sensing needs and robotic configuration. We demonstrate our propositions through simulation under a realistic wireless network regime.

I. INTRODUCTION

RADITIONAL work on distributed cooperation in robotics has focused on position and motion configuration of collections of robots using localized algorithms, typically involving iterative exchange of state variables with single-hop neighbors, e.g., research on swarming, flocking, and formation control [1]–[3]. The current state of the art on distributed cooperation in robotics, focused on using only localized communication, can effectively solve problems in scenarios where there are relatively simple global application related objectives that do not change over time. However, due to the difficulties in translating multiple dynamically varying global objectives into local control actions, the problem of utilizing these algorithms in more complex sensing and communication networks remains an open question.

A recent advance in distributed cooperation techniques offers promise in utilizing simple swarm-like mobility in coordinating more complex tasks. A typical problem when considering only local communications is that global connectivity might be lost. Recent research has shown that this global property can be recovered even through local interactions [4]–[7]. We believe that this recent advance, enabling global connectivity to be maintained at all times while a collection of robots is moving, provides fundamental new opportunities as complex tasks can be decomposed into simple sub components, while maintaining overall network connectivity.

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Toward such goals, we first introduce at a high level a novel hybrid architecture for command, control, and coordination of networked robots for sensing and information routing applications, called INSPIRE (for INformation and Sensing driven PhysIcally REconfigurable robotic network). In the INSPIRE architecture, we propose two levels of control. At the low level there is a Physical Control Plane (PCP), and at the higher level is an Information Control Plane (ICP). At the PCP, iterative local communications between neighboring robots is used to shape the physical network topology by manipulating agent position through motion. At the ICP, more sophisticated multi-hop network algorithms enable efficient sensing and information routing (e.g., shortest cost routing computation, time slot allocation for sensor data collection, task allocation, clock synchronization, network localization, etc.). Unlike traditional approaches to distributed robotics, the introduction of the ICP provides the benefit of being able to scalably configure the sensing tasks and information flows in the network in a globally coherent manner even in a highly dynamical context by using multi-hop communications.

As a proof of concept of the INSPIRE architecture, we detail a simple instantiation, in which the robotic network is dynamically reconfigured in order to ensure high quality routes between a set of static wireless nodes (i.e. a *flow*) while preserving connectivity, where the number and composition of information flows in the network may change over time. In solving this problem, we propose ICP and PCP components that couple connectivity-preserving robot-to-flow allocations, with communication optimizing positioning through distributed mobility control; a heuristic we call *Route Swarm*. Finally, we demonstrate our propositions through simulation, illustrating the INSPIRE architecture and the Route Swarm heuristic in a realistic wireless network regime.

II. STATE OF THE ART

Distributed mobility control has been well investigated in the robotics community in recent years. In the context of multirobot systems, distributed coordination protocols endow agents with simple local interactions, yet yield fundamentally useful collective behaviors. Coordination algorithms can broadly be classified in three families, that is swarming, flocking and formation control. Swarming aims at achieving an aggregation of the team through local simple interaction [1], flocking is a form of collective behavior of a large number of agents with an agreement in the direction of motion and velocity [2], while formation control dictates the team reach a desired formation shape [3]. For all of these objectives, potential-based control techniques represent an effective solution [8],

combining provable performance with ease of control. As robots are usually required to either communicate or sense each other for all time, the connectivity maintenance of the network topology also needs to be addressed. Recent algorithms have been proposed to preserve the connectivity of the network topology over time, with approaches ranging from the control of addition and removal of edges [9], to the estimation and control of the algebraic connectivity [7].

The integration of mobile robotics and wireless networking is an emerging domain. Researchers have previously investigated deploying mobile nodes to provide sensor coverage in wireless sensor networks [10], [11]. In [4], the authors present a work to ensure connectivity of a wireless network of mobile robots while reconfiguring it towards generic secondary objectives. Going beyond connectivity, recently, research has also addressed how to control a team of robots to maintain certain desired end-to-end rates while moving robots to do other tasks, referred to as the problem of maintaining network integrity [12]. This is done by interleaving potential-field based motion control and at the higher level an iterative primal-dual algorithm for rate optimization. All of these works point to the need for a hybrid control framework where low-level motion control can be integrated with a higher-level network control plane such as the INSPIRE architecture illustrated in this work¹.

Closely related to our work is an early paper that advocated motion control as a network primitive in optimizing network information flows [13]. Although related in spirit, we provide in this work fundamental advances in flow-to-flow reallocations, dynamic and flexible connectivity maintenance allowing network reconfigurability, and refined potential-based control that requires only inter-agent distance in optimizing intra-flow positioning. Another, more recent work [14], focuses on a single-flow setting, but considers a more detailed fading model communication environment, and a slightly different path metric. In contrast to [14], we make novel contributions in multi-flow optimization which we have shown requires a more sophisticated network-layer information control plane. Moreover, the motion control presented in [14] can also be integrated with and adopted as a component of the PCP in the INSPIRE architecture presented here.

III. BACKGROUND MATERIAL

To begin, we give an overview of the background material and assumptions necessary for our contributions in this work.

A. Agent and Interaction Models

Consider a system of n=m+s agents consisting of m mobile robots indexed by $\mathcal{I}_M \triangleq \{1,\ldots,m\}$, and s static sensors indexed by $\mathcal{I}_S \triangleq \{m+1,\ldots,m+s\}$. The mobile robots are assumed to have single integrator dynamics

$$\dot{x}_i = u_i \tag{1}$$

where $x_i, u_i \in \mathbb{R}^2$ are the position and the velocity control input for an agent $i \in \mathcal{I}_M$, respectively. Assume that all agents

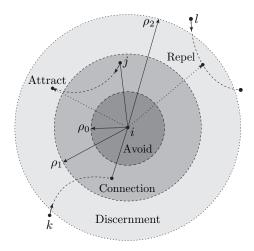


Fig. 1. Agent interaction model with radii determining sensing and communication $\|x_{ij}\| \leq \rho_2$, neighbor decisions relative to constraints $\rho_1 < \|x_{il}\| \leq \rho_2$, link establishment $\|x_{ik}\| \leq \rho_1$, and collision avoidance $\|x_{ij}\| \leq \rho_0$.

can intercommunicate in a proximity-limited way, inducing interactions (or topology) of a time varying nature. Specifically, letting $d_{ij} \triangleq \|x_{ij}\| \triangleq \|x_i - x_j\|$ denote the distance between agents i and j, and (i,j) a link between connected agents, the spatial neighborhood of each agent is partitioned by defining concentric radii $\rho_2 > \rho_1 > \rho_0$ as in Fig. 1, where we refer to ρ_2, ρ_1, ρ_0 as the *interaction*, *connection*, and *collision avoidance* radii, respectively. The radii introduce a *hysteresis* in interaction by assuming that links (i,j) are established only after $d_{ij} \leq \rho_1$, with link loss then occurring when $d_{ij} > \rho_2$, generating the annulus of $\rho_2 - \rho_1$ where *decisions* on link additions and deletions are made (c.f. Section V-A).

The above spatial interaction model is formalized by the undirected dynamic graph, $\mathbb{G} = (\mathcal{V}, \mathcal{E})$, with vertices (nodes) \mathcal{V} indexed by $\mathcal{I}_M \cup \mathcal{I}_S$ (the agents), and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ such that $(i,j) \in \mathcal{E} \Leftrightarrow (\|x_{ij}\| \leq \rho_2) \wedge \sigma_{ij}$, with switching signals [15]:

$$\sigma_{ij} = \begin{cases} 0, & (i,j) \notin \mathcal{E} \land ||x_{ij}|| > \rho_1 \\ 1, & \text{otherwise} \end{cases}$$
 (2)

where $(i,i) \notin \mathcal{E}$ (no self-loops) and $(i,j) \in \mathcal{E} \Leftrightarrow (j,i) \in \mathcal{E}$ (symmetry) hold for all $i,j \in \mathcal{V}$. Nodes with $(i,j) \in \mathcal{E}$ are called *neighbors* and the neighbor set for an agent i is denoted $\mathcal{N}_i = \{j \in \mathcal{V} \mid (i,j) \in \mathcal{E}\}.$

B. Assumptions and Problem Formulation

From the set of static sensors \mathcal{I}_S we construct f information flows indexed by $\mathcal{I}_F \triangleq \{n+1,\ldots,n+f\}$, each consisting of source-destination pairs defining a desired flow of network information. For a given flow $i \in \mathcal{I}_F$, we use the following notation: $\mathcal{F}_i \in \mathcal{I}_S \times \mathcal{I}_S$ represents the source and destination nodes for flow i, with $\mathcal{F}_i^s \in \mathcal{I}_S$ and $\mathcal{F}_i^d \in \mathcal{I}_S$ representing source and destination indices, respectively. Further, for convenience we use notation $x_i^s, x_i^d \in \mathbb{R}^2$ to represent the position of the source and destination for the flow $i \in \mathcal{I}_F$. The set of flow pairs is denoted $\mathcal{F} \triangleq \{\mathcal{F}_1,\ldots,\mathcal{F}_f\}$. At any time, a subset of these static pairs is active, forming the set of active

¹A complete characterization and general analysis of the INSPIRE architecture is the topic of our future work.

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flows \mathcal{I}_F , calling for dynamic configurability of the hybrid network, our contribution in this work. Thus, at a high level our system objective is to facilitate information flow for each source/destination pair by configuring the mobile robots such that each flow is *connected* and is at least approximately optimal in terms of data transmission, and that the entire network itself \mathbb{G} is connected to guarantee complete network collaboration.

Assumption 1 (Connectedness): It is assumed that the locations of the static nodes and the cardinality of mobile nodes is such that the set of connected graphs $\mathbb G$ that could be formed is non-empty, and that their communication graph is initiated to be in this set.

To measure link quality towards optimizing a given flow, we assume that each $(i,j) \in \mathcal{E}$ has a weight parameter w_{ij} that describes their cost with respect to transmitting information. A commonly used metric for link quality is ETX, i.e. the expected number of transmissions per successfully delivered packet. This can be modeled as the inverse of the successful packet reception rate λ_{ij} over the link. As the expected packet reception rate has been empirically observed and analytically shown to be a sigmoidal function of distance decaying from 1 to 0 as distance d_{ij} is increased [16], it can be modeled as follows:

$$\lambda_{ij} \approx 1 - \frac{1}{(1 + e^{-a(d_{ij} - b)})} \tag{3}$$

where $a, b \in \mathbb{R}_+$ are shape and center parameters depending on the communication range and the variance of environmental fading. Accordingly, the link weights w_{ij} , if chosen to represent ETX, can be modeled as a convex function of the inter-node distance d_{ij} :

$$w_{ij} = \frac{1}{\lambda_{ij}} = \frac{1}{1 - \frac{1}{1 + e^{-a(d_{ij} - b)}}} = 1 + e^{a(d_{ij} - b)}$$
(4)

The cost for flow $k \in \mathcal{I}_F$ is then taken to be the sum of ETX values on the path of the flow, i.e.:

$$W_k = \sum_{(i,j)\in\mathcal{E}_F^k} w_{ij} \tag{5}$$

where we apply notation $\mathbb{G}_F^k = (\mathcal{V}_F^k, \mathcal{E}_F^k)$ as the graph defining the interconnection over flow $k \in \mathcal{I}_F$ (we give a concrete definition of flow *membership* in Section IV-A). Our problem in this work is then formalized as follows:

Problem 1 (Multi-flow optimization): The network-wide goal then is to find an allocation and configuration of mobile agents so as to minimize the total cost function $\sum_{k=1}^f W_k$ while maintaining both intra-flow connectivity (to guarantee information delivery) and inter-flow connectivity (to guarantee flow-to-flow information passage/collaboration).

IV. INFORMATION CONTROL PLANE (ICP)

There are two key elements in solving Problem 1: on the one hand, within a given flow $k \in \mathcal{I}_F$, for a given allocation of a certain number of mobile nodes to that flow, node configuration should minimize the flow cost W_k . On the other hand, the number of mobile nodes allocated to each flow should minimize

the overall cost $\sum_k W_k$. We first consider these optimizations ideally, in the absence of connectivity constraints. The first, perflow element of the network optimization dictates the desired spatial configuration of allocated mobile nodes within a flow.

Theorem 4.1 (Equidistant optima): For a fixed number of mobile nodes $m_k \triangleq |\mathcal{V}_F^k|$ allocated to a given flow and arranged on the line between the source and destination of that flow, the arrangement which minimizes (5) is one where the nodes are equally spaced.

Proof: This follows from the following general result: to minimize a convex $y(\overrightarrow{z})$ s.t. $\sum z_i = c$, the first order condition (setting the partial derivative of the Lagrangian with respect to each element z_i to 0) yields that $\frac{\partial y}{z_i} = \mu$ where μ is the Lagrange multiplier. Now if $\frac{\partial y}{\partial z_i}$ is the same for all z_i , then the solution to this optimization is to set $z_i = \frac{c}{|z|}$, i.e. all variables are made to be the same.

The z_i in the above correspond to the inter-node distances d_{ij} , and the y corresponds to (5). Since w_{ij} is a convex function of distance between neighboring nodes, the path metric W_k is a convex function of the vector of inter-node distances. Further, since the sum of all inter-node distances is the total distance between the source and destination of the flow, which are static, it is constrained to be a constant d_k . Since the weight of each link is the same function of the inter-node distance, $\frac{\partial W_k}{d_{ij}}$ is the same for all pairs of neighboring nodes. Therefore the intra-flow optimization (i.e., choosing node positions to minimize W_k) is achieved by a equal spacing of the nodes.

The second, global cross-flow element of the network optimization dictates the number of mobile nodes allocated to each flow. The goal is to minimize the total network cost $\sum_k W_k$. Our approach to solving this optimization is motivated by the following observation: When the intra-flow locations of the robots are optimized to be equally spaced, W_k is a function of the number of nodes m_k allocated to flow k, and the total number of nodes allocated to all flows is constrained by the total number of mobile nodes. If we could show that W_k is a convex function, then to minimize the total network cost we need to identify the allocation at which the marginal costs for all flows are as close to equal as possible (ideally, if m_k was a continuous quantity, they would all be equal at the optimum point, but due to the discrete nature of m_k this is generally not possibly).

In the following, for ease of analysis, we consider the continuous relaxation of the problem, allowing m_k to be a real number, and hence W_k to be a continuous function.

Theorem 4.2 (Convexity): For any flow k that has been optimized to have the lowest possible cost W_k (i.e. all m_k mobile nodes are equally spaced), W_k is a convex function of m_k .

Proof:

$$W_k = (m_k + 1)w\left(\frac{d_k}{m_k + 1}\right) \tag{6}$$

$$\Rightarrow W_k' = w\left(\frac{d_k}{m_k + 1}\right) - \frac{d_k}{m_k + 1}w'\left(\frac{d_k}{m_k + 1}\right) \tag{7}$$

$$\Rightarrow W_k'' = w'' \left(\frac{d_k}{m_k + 1}\right) \frac{d^2}{(m_k + 1)^3}$$
 (8)

²The negative of the cost could be treated as a utility function. We therefore equivalently talk about cost minimization or utility maximization.

Since the link weight function $w(\cdot)$ is a convex function of the internode distance, we have that $w''(\frac{d_k}{m_k+1})$ is positive, and therefore we have that W''_k is positive, hence W_k is convex in m_k .

Note that in reality m_k is a discrete quantity, but the above argument suffices to show that W_k is a discrete convex function of m_k (since convexity over a single real variable implies discrete convexity over the integer discretization of the variable).

Theorem 4.3 (Optimality): The problem of minimizing $\sum_k W_k(m_k)$ subject to a constraint on $\sum_k m_k$ can be solved optimally in an iterative fashion by the following greedy algorithm: at each iteration, move one node from the flow where the removal induces the lowest increase in cost to the flow where its addition would yield the highest decrease in cost, so long as the latter's decrease in cost is strictly higher in absolute value than the former's increase in cost (i.e. so long as the move serves to reduce the overall cost).

We omit the detailed proof due to space constraint, but intuitively, this algorithm works by moving the system iteratively towards the optimum by following the steepest gradient in terms of cost reduction for the movement of each node. Since the overall optimization problem is convex, there is only a single optimum, to which this algorithm will converge. Moreover, since there is a strict improvement in each step and there are a finite number of nodes, the algorithm reaches the optimal arrangement in a finite number of steps.

Thus far, we have described both the intra-flow and inter-flow optimization problems in an ideal setting where both problems are convex optimization problems and as such can be solved exactly. However, in the robotic system we are considering there is one significant source of non-ideality/non-convexity, which is that the network must be maintained at all times in a connected configuration. This has two consequences. First, some of the mobile nodes may be needed as *bridge* nodes that do not participate in any flow and are instead used to maintain connectivity across flows. Second, the locations of some of the nodes even within each flow may be constrained in order to maintain the connectivity requirement. The solution for the constrained problem therefore may not correspond exactly to the solutions of the ideal optimization problems described above.

We therefore develop a heuristic solution that we refer to as Route Swarm, which is inspired by and approximates the ideal optimizations above but is adapted to maintain connectivity. Route Swarm has both an intra-flow and inter-flow component. As per the INSPIRE architecture, the intra-flow function is performed by the PCP, while the inter-flow function is performed by ICP. The ICP algorithm, shown as pseudocode as Algorithm 1, approximates the ideal iterative optimization described above by allocating mobile robots between flows greedily on the basis of greatest cost-reduction; to handle the inter-flow connectivity constraint, it incorporates a subroutine (Algorithm 2) to detect which nodes are not mobile because they must act as bridge nodes. Moreover, it also allocates any nodes that are no longer required to support an inactive flow to join active flows. And within each flow, the PCP algorithm (Algorithm 7) attempts to keep the robots as close to evenly

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Algorithm 1 ICP optimization algorithm.
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1: procedure InformationControlPlane

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2:
         Detect initial flow members based on shortest paths:
 3:
         for i \in \mathcal{I}_F do
              \mathcal{M} \leftarrow \text{SHORTESTPATH}(\mathbb{G}, \mathcal{F}_i)
4:
5:
         end for
         ▶ Detect flow-to-flow bridges for connectivity:
 6:
         b \leftarrow \text{DETECTBRIDGES}(\mathbb{G}, \mathcal{F}, \mathcal{M})
 7:
8:
         Detect best connectivity-preserving flow detachment:
 9:
         d \leftarrow \text{BestDetachment}(\mathbb{G}, \mathcal{F}, \mathcal{M}, b)
         ▷ Compute flow attachment with most utility:
10:
         a \leftarrow \text{BESTATTACHMENT}(\mathcal{F}, \mathcal{M})
11:
         ▶ Ensure optimizing reconfiguration exists (weighted
12:
    by \beta \in \mathbb{R}_{+}):
         if a > \beta d then
13:
14:
              ▷ Optimal command is best detach/attach pair:
              return C_d \leftarrow a
15:
         end if
16:
17: end procedure
```

Algorithm 2 ICP bridge detection.

```
1: procedure DETECTBRIDGES(\mathbb{G}, \mathcal{F}, \mathcal{M})
         ▷ Initialize supergraph with nodes for each flow:
         \mathbb{S} \leftarrow (\{i \in \mathcal{I}_F\}, \emptyset)
3:
         ▶ Append nodes/edges for non-flow members:
 4:
         \mathbb{S} \leftarrow \text{AddNonFlowMembers}(\mathbb{S}, \mathcal{F}, \mathcal{M})
5:
 6:
         ▶ Append edges for flow members:
 7:
         \mathbb{S} \leftarrow AddFlowMembers(\mathbb{S}, \mathcal{F}, \mathcal{M})
 8:
         ▶ Bridges lie on shortest path between all flow pairs:
         return b \leftarrow SHORTESTPATHS(\mathbb{S}, \mathcal{F})
10: end procedure
```

spaced as possible while taking into account the inflexibility of the bridge nodes. The details of the route swarm algorithm are given below.

A. The Route Swarm Heuristic

In solving the connectivity constrained version of the flow optimization of Theorems 4.1, 4.2, and 4.3, due to problem complexity we provide a heuristic algorithm to solve the interflow allocation problem, leaving the intra-flow optimization to the PCP (Section V). Algorithm 1 depicts the high-level components of the proposed heuristic, each of which is detailed in the sequel.

To begin, we define the flow membership for an agent as $i \in \mathcal{I}_M$ as $\mathcal{M}_i \in \mathcal{I}_F$, where the set of memberships is denoted as $\mathcal{M} \triangleq \{\mathcal{M}_1, \dots, \mathcal{M}_n\}$. We denote with $\mathbb{G}_F^i = (\mathcal{V}_F^i, \mathcal{E}_F^i)$ the graph defining the interconnection over flow $i \in \mathcal{I}_F$ of agents $j \in \mathcal{I}_S \cup \mathcal{I}_M$ with $i \in \mathcal{M}_j$. Thus we have $\mathcal{V}_F^i = \{j \in \mathcal{I}_S \cup \mathcal{I}_M \mid i \in \mathcal{M}_j\}$ and $(j,k) \in \mathcal{E}_F^i \Leftrightarrow (i \in \mathcal{M}_j \cap \mathcal{M}_k) \land (j \in \mathcal{N}_k)$, with $k \in \mathcal{I}_S \cup \mathcal{I}_M$. Notice that by definition $\mathbb{G}_F^i \subseteq \mathbb{G}$. We will also refer to the collection of flow graphs simply by \mathbb{G}_F . We also have flow neighbors defined as $\mathcal{N}_i^F = \{j \in \mathcal{N}_i \mid \mathcal{M}_j \cap \mathcal{M}_i \neq \emptyset\}$. Furthermore, we denote with $\mathcal{D}_i \in \mathcal{I}_M$ the detachable agents $j \in \mathcal{I}_M$ for flow $j \in \mathcal{I}_K$, i.e. those agents for which

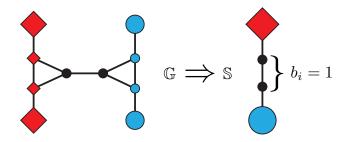


Fig. 2. Example supergraph construction for a network with f=2 flows, s=4 static nodes, and m=6 mobile robots. Flow membership is denoted by color and node shape. Notice that a multi-hop bridge is detected via the shortest path between flows.

reconfiguration does not impact network connectedness. The set of detachments is denoted by $\mathcal{D} \triangleq \{\mathcal{D}_1, \dots, \mathcal{D}_f\}$.

Due to the system connectivity constraint, the ICP is not free to select *any* mobile agent for flow reallocation, specifically as intra-flow connectivity or overall flow-to-flow connectivity may be lost. Thus, the ICP must detect *bridges*, i.e., mobile nodes whose reconfiguration might break flow-to-flow connectivity over the network, and also consider *safe detachments* (flow-to-flow motion), i.e., a node whose reconfiguration in the workspace does not impact the connectivity of the source flow.

Respecting our connectivity constraint, we first detect initial flow membership by computing the shortest path (in terms of link cost) for each source/destination pair per flow by using for example Dijkstra's algorithm (lines 3-5 of Algorithm 1). This defines the connected *backbone* for each flow implicitly identifying the mobile nodes required to maintain flow-connectivity. Additionally, we have optimality of the connected flow backbones as we maximize the link utility (or minimize path costs) between source and destination nodes.

In detecting bridge agents we follow the process outlined by Algorithms 2, 3, and 4, where we denote by $b_i \in \{0,1\}$ the status of agent $i \in \mathcal{I}_S \cup \mathcal{I}_M$ as a connectivity preserving bridge, with $b = \{b_1, \dots, b_n\}$. Briefly speaking, the primary problem of this process is the construction of a supergraph, denoted $\mathbb{S} = (\mathcal{V}_{\mathbb{S}}, \mathcal{E}_{\mathbb{S}})$, defining the interconnection of the flows. In this way, we can identify nodes which are critical in defining the flow-to-flow connectivity over the system. Figure 2 depicts an example of supergraph construction. We first add one node for each flow in the system, and then we add a node for each non-flow member in the system that represents potential bridge candidates. Edges between non-flow members are preserved, while there is only one edge between any given non-flow member and the node which represents a flow in the supergraph. Bridges can then be simply detected as the members of the shortest path between any pair of nodes representing flows in the supergraph (e.g. Fig. 2), where connectivity is guaranteed by construction:

Proposition 4.1 (Bridge detection): Consider the graph $\mathbb{G}_c \subseteq \mathbb{G}$ obtained by including all the flow-members for any flow and the non-flow members belonging to any shortest path of the supergraph \mathbb{S} . Then \mathbb{G}_c is a spanning-graph representing a connected component of the graph \mathbb{G} .

Proof: In order to prove this result we must show that both intra-flow and inter-flow connectedness is ensured. The

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Algorithm 3 ICP supergraph non-flow member nodes/edges.
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1: procedure ADDNONFLOWMEMBERS(\mathbb{S}, \mathcal{F}, \mathcal{M})
2: \triangleright Add nodes for each non-leaf, non-flow member:
3: \mathcal{V}_{+} \leftarrow \{i \in \mathcal{I}_{M} \mid (\mathcal{M}_{i} = \emptyset) \land (\mathcal{N}_{i} \geq 2)\}
4: \mathcal{V}_{\mathbb{S}} \leftarrow \mathcal{V}_{\mathbb{S}} \cup \mathcal{V}_{+}
5: \triangleright Add edges between non-flow members:
6: \mathcal{E}_{\mathbb{S}} \leftarrow \mathcal{E}_{\mathbb{S}} \cup \{(i,j) \mid (i,j \in \mathcal{V}_{+}) \land (j \in \mathcal{N}_{i})\}
7: \triangleright Add non-flow member to flow member edges:
8: \mathcal{E}_{\mathbb{S}} \leftarrow \mathcal{E}_{\mathbb{S}} \cup \{(i,j) \mid (i \in \mathcal{V}_{+}) \land (j \in \mathcal{N}_{i}) \land (\mathcal{M}_{j} \neq \emptyset)\}
9: return \mathbb{S}
10: end procedure
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Algorithm 4 ICP supergraph flow member edges.

```
1: procedure ADDFLOWMEMBERS(\mathbb{S}, \mathcal{F}, \mathcal{M})
2: \triangleright Add edges due to multiple flow memberships:
3: for i \in \mathcal{I}_M \mid |\mathcal{M}_i| \geq 2 do
4: \triangleright Add an edge for every membership pair:
5: \mathcal{E}_{\mathbb{S}} \leftarrow \mathcal{E}_{\mathbb{S}} \cup \{(i,j) \mid j \in \mathcal{I}_F \cap \mathcal{M}_i\}
6: end for
7: return \mathbb{S}
8: end procedure
```

former follows directly from the flow-membership definition while the latter follows from the connectedness between any pair of flows.

After identifying the bridge agents that maintain flowto-flow connectivity and the safely detachable agents perflow (those which are not on the backbone), the ICP issues reconfiguration commands $C_i \in \mathcal{I}_F$ to mobile agents $i \in \mathcal{I}_M$ indicating a desired flow membership towards optimizing interflow agent allocation. Specifically, as detailed in Algorithms 5 and 6, and as motivated by Theorem 4.3, we compute the safe detachment having the least in-flow utility, and couple it with the flow attachment (i.e. the addition of a contributing flow member) which improves most in terms of link utility and flow alignment (the primary contributions a mobile agent can have in information flow). This decision, if feasible (utility of attachment outweighs cost of detachment), is then passed to the PCP to execute the mobility necessary for reconfiguration, achieving our goal of dynamic utility improving and connectivity preserving network configurations.

V. PHYSICAL CONTROL PLANE (PCP)

The complementary component to the ICP in the INSPIRE architecture is the PCP which coordinates via state feedback, i.e. $\{\mathbf{x}, \mathbb{G}, \mathbb{G}_F\}$, to generate swarming behaviors that optimize the network dynamically in response to ICP commands. Our desire for generality in coordinating behaviors dictates that the PCP takes on a *switching* nature, associating a distinct *behavior controller* with each of a finite set of discrete agent states. Specifically, define

$$S_i \in \mathbb{B} \triangleq \{SWARMING, RECONFIGURE\}$$
 (9)

as the *behavior state* of a mobile robot $i \in \mathcal{I}_M$, where \mathbb{B} is the space of discrete agent behaviors. Here, when $\mathcal{S}_i = \mathit{SWARMING}$, agent i acts to optimize its assigned flow

Algorithm 5 ICP best flow detachment.

```
1: procedure BESTDETACHMENT(\mathbb{G}, \mathcal{F}, \mathcal{M}, b)
2: \triangleright Non-member, non-bridges are detachable:
3: for (i \in \mathcal{I}_M) \land (\mathcal{M}_i = \emptyset) \land (\neg b_i) do
4: j \leftarrow Flow most contributed to by agent i (utility)
5: \mathcal{M}_i \leftarrow j
6: \mathcal{D}_j \leftarrow \mathcal{D}_j \cup i
7: end for
8: \triangleright Best detachment has least in-flow utility:
9: return \operatorname{argmin}_{i \in \mathcal{D}}(\sum_{j \in \mathcal{N}_i^F} w_{ij})
10: end procedure
```

Algorithm 6 ICP best flow attachment.

```
1: procedure BESTATTACHMENT(\mathcal{F}, \mathcal{M})
2: 
ightharpoonup Determine utility of attachment per flow:
3: for <math>i \in \mathcal{I}_F do
4: 
ightharpoonup Weigh added node utility against flow path alignment:
5: <math>a_i \leftarrow (|\mathcal{V}_F^i| + 1) \sum_{j \in \mathcal{V}_F^i} 1/\|x_j - 1/2(x_i^s + x_i^d)\|
6: end for
7: return \underset{i \in \mathcal{I}_F}{\operatorname{end}} return \underset{i \in \mathcal{I}_F}{\operatorname{end}} end procedure
```

 \mathcal{M}_i . Otherwise, when $\mathcal{S}_i = RECONFIGURE$, agent i traverses the workspace fulfilling global allocation commands \mathcal{C}_i from the ICP. In this work, the state machine that drives the switching of the behavior controllers is depicted in Algorithm 7. Each component comprising the PCP switching is detailed in the sequel.

A. Constraining Agent Interaction

In order to control the properties of \mathbb{G} (i.e. connectivity) we exploit the *constrained interaction* framework proposed by Williams and Sukhatme in [9]. The constrained interaction framework acts through hysteresis (2) to regulate links spatially with simple application of attraction and repulsion to retain established links or reject new links with respect to topological constraints. Define the discernment region $||x_{ij}|| \in (\rho_1, \rho_2]$, where agent i decides relative to system constraints (here connectivity) whether agent j is a candidate for link addition $(j \notin \mathcal{N}_i)$ or deletion $(j \in \mathcal{N}_i)$, or if agent j should be attracted (retain $(i,j) \in \mathcal{E}$) or repelled (deny $(i,j) \notin \mathcal{E}$). Define predicates for link addition and deletion, $P_{ij}^a, P_{ij}^d : \mathcal{V} \times \mathcal{V} \to \{0, 1\},$ activated at ρ_2 and ρ_1 , respectively, that indicate constraint violations if the link (i, j) were allowed to be either created or destroyed, i.e. $||x_{ij}||$ transits ρ_1 or ρ_2 . The predicates designate for the ith agent the membership of nearby agents in link addition and deletion candidate sets C_i^a, C_i^d , and attraction and repulsion sets $\mathcal{D}_i^a, \mathcal{D}_i^r$. Link control is then achieved by choosing control u_i having attractive and repulsive potential *fields* between members of \mathcal{D}_i^a , \mathcal{D}_i^r , respectively.

In particular, to regulate network topology spatially, we design the agent controls as follows:

$$u_{i} = u_{i}^{e} + u_{i}^{o} - \nabla_{x_{i}} \left(\sum_{j \in \mathcal{D}_{i}^{a}} \psi_{ij}^{a} + \sum_{j \in \mathcal{D}_{i}^{r}} \psi_{ij}^{r} + \sum_{j \in \Pi_{i}} \psi_{ij}^{c} \right)$$
(10)

Algorithm 7 PCP switching logic.

```
1: procedure PhysicalControlPlane
 2:
        for i \in \mathcal{I}_M do

    Control mobile agents

 3:
             ▶ Reconfiguration Commanded:
             if C_i \neq \emptyset then
 4:
                 Set waypoint to target flow C_i \in \mathcal{I}_F
 5:
                 S_i \leftarrow RECONFIGURE
 6:
 7:
             end if
             ▶ Flow members, to-flow maneuver:
 8:
 9:
             if \mathcal{M}_i \neq \emptyset \land \mathcal{S}_i \neq RECONFIGURE then
                 if \neg(On Path Connecting Flow \mathcal{M}_i) then
10:
                     Set waypoint to flow \mathcal{M}_i \in \mathcal{I}_F
11:
                     S_i \leftarrow RECONFIGURE
12:
13:
                 else
                     S_i \leftarrow SWARMING
14:
15:
                 end if
             end if
16:
            ▶ Agent Behaviors:
17:
18:
             if S_i = SWARMING then
19:
                 Run dispersion controller optimizing flow \mathcal{M}_i
20:
             end if
             if S_i = RECONFIGURE then
21:
                 Run waypoint controller for reconfiguration
22:
                 if At Waypoint then
23:
                     S_i \leftarrow SWARMING
24:
                 end if
25:
26:
             end if
        end for
27:
28: end procedure
```

with potentials $\psi_{ij}^a, \psi_{ij}^r, \psi_{ij}^c \colon \mathbb{R}_+ \to \mathbb{R}_+$, serving the purposes of attraction, where $\Pi_i = \{j \in \mathcal{V} \mid \|x_{ij}\| \leq \rho_0\}$ is the collision avoidance, where for agent i. Further, each agent can also apply (based on \mathcal{S}_i) an exogenous objective (i.e. non-cooperative) controller $u_i^e \in \mathbb{R}^2$ and an inter-agent coordination objective $u_i^o \in \mathbb{R}^2$ (e.g. dispersion as will be seen in Section V-D). An appropriate attractive potential which we adopt for this work takes the following form:

$$\psi_{ij}^{a} = \frac{1}{\rho_{2}^{2} - \|x_{ij}\|^{2}} + \Psi_{a}, \quad \text{if} \quad \|x_{ij}\| \in [\rho_{1}, \rho_{2})$$
 (11)

where $\Psi_a(\|x_{ij}\|)$ is chosen such that (11) is smooth over the ρ_1 transitions. Similar to the attractive potential (11), the repulsive potential takes the form

$$\psi_{ij}^r = \frac{1}{\|x_{ij}\|^2 - \rho_1^2} + \Psi_r, \quad \text{if} \quad \|x_{ij}\| \in (\rho_1, \rho_2)$$
 (12)

where Ψ_r is chosen to guarantee ψ^r_{ij} is smooth over the ρ_2 transition. Finally, a basic collision avoidance is given by potential

$$\psi_{ij}^c = \frac{1}{\|x_{ij}\|^2} + \Psi_c, \quad \text{if} \quad \|x_{ij}\| \in (0, \rho_0)$$
 (13)

with Ψ_r chosen to guarantee ψ^c_{ij} is smooth over the ρ_0 transition.

The attractive and repulsive potentials are constructed such that $\psi^a_{ij} \to \infty$ as $d_{ij} \to \rho_2$ and $\psi^r_{ij} \to \infty$ as $d_{ij} \to \rho_1$, guaranteeing link retention and denial, respectively, and allowing us through predicates P^a_{ij} , P^d_{ij} to control desired properties of $\mathbb G$ (e.g. connectivity).

B. Connectivity Maintenance

Notice that in *maintaining* network connectivity, we require *only* link retention action, allowing us to immediately choose

$$P_{ij}^a \stackrel{\triangle}{=} 0, \quad \forall \ i \in \mathcal{I}_M, j \in \mathcal{I}_M \cup \mathcal{I}_S, (\mathcal{S}_i, \mathcal{S}_j) \in \mathbb{B} \times \mathbb{B}$$
 (14)

for the link addition predicates, effectively allowing link additions to occur between all interacting agents across all network states³.

Now, in accordance with Algorithm 7, the link deletion predicates are given by:

$$P_{ij}^{d} \triangleq \begin{cases} 1, & (\mathcal{S}_{i} \vee \mathcal{S}_{j} = SWARMING) \wedge \\ & ((\mathcal{M}_{i} \cap \mathcal{M}_{j} \neq \emptyset) \vee (b_{i} \vee b_{j} = 1)) \\ 0, & \text{otherwise} \end{cases}$$
 (15)

where by assumption $i,j \in \mathcal{I}_M$, i.e. only mobile agents apply controllers. We choose link retention (15) to guarantee that connectivity is maintained both within flows across \mathbb{G}_F , and from flow to flow across bridge agents over the supergraph \mathbb{S} , noting that idle agents with $\mathcal{M}_i = \emptyset$ and reconfiguring agents with $\mathcal{S}_i = RECONFIGURE$ are free to lose links as they have been deemed redundant by the ICP with respect to network connectivity.

C. Flow Reconfiguration Maneuvers

While maintaining connectivity as above, each agent further acts according to ICP reconfiguration commands towards optimizing inter-flow allocations. Specifically, in response to command C_i , agent i enters the reconfiguration state $S_i \leftarrow RECONFIGURE$, and begins to apply a waypoint controller as follows (c.f. lines 4-7 of Algorithm 7). When $S_i = RECONFIGURE$, agent i applies exogenous objective controller

$$u_i^e \triangleq \frac{x^w - x_i}{\|x^w - x_i\|} - \dot{x}_i \tag{16}$$

where $x^w \in \mathbb{R}^2$ is the target waypoint calculated as the midpoint of target flow C_i .

The input (16) is a velocity damped waypoint seeking controller, having unique critical point $x_i \to x^w$ (i.e. a point at which $u_i^e = 0$), guaranteeing that the target intra-flow positioning (and thus membership) for agent i is achieved. As the convergence of $x_i \to x^w$ is asymptotic in nature, to guarantee finite convergence and state switching, we apply a saturation $||x^w - x_i|| \le \epsilon^w$ with $0 < \epsilon^w << 1$ to detect waypoint convergence, initiating a switch to $S_i \leftarrow SWARMING$ as in lines 23-25 of Algorithm 7.

D. Intra-Flow Controllers

Once the ICP has assigned flow memberships $\mathcal{M}_i \ \forall \ i \in \mathcal{I}_M$ and all *commanded* reconfigurations \mathcal{C}_i have been completed, the mobile agents begin to seek to optimize the flow to which they are a member. First, we assume that flow members must configure along the line segment connecting flow source/destination pairs, yielding in the case of proximity-limited communication, a line-of-sight or beamforming style heuristic. The membership of an agent $i \in \mathcal{I}_M$ to a flow $j \in \mathcal{M}_i$ thus initiates a check to determine if x_i lies on the flow path $x_j^s + \tau x_j^d$, within a margin $0 < \epsilon_F < < 1$ (c.f. lines 9-16, Algorithm 7). To do so, the projection of x_i onto $x_j^s + \tau x_j^d$ is determined first by computing

$$\tau \triangleq \frac{(x_i - x_j^s) \cdot (x_j^d - x_j^s)}{\|x_i^d - x_j^s\|^2}$$
 (17)

defining whether the projection will lie within or outside of the flow path. Then we have the saturated projection

$$x_{i \to \mathcal{F}_j} = \begin{cases} x_j^s - \alpha \tau (x_j^d - x_j^s), & \tau < 0 \\ x_j^d - \alpha \tau (x_j^d - x_j^s), & \tau > 1 \\ x_j^s + \tau (x_j^d - x_j^s), & \tau \in (0, 1) \end{cases}$$
(18)

where $\alpha > 0$ is a biasing term such that the projection does not intersect x_i^s or x_i^d . We then have the state transition condition

$$||x_{i\to\mathcal{F}_i} - x_i|| \le \epsilon_F \tag{19}$$

which when satisfied gives $S_i \leftarrow SWARMING$ (line 14, Algorithm 7, and described below). If condition (19) is not satisfied, agent i transitions to state $S_i \leftarrow RECONFIGURE$, applying waypoint controller (16) with $x^w \triangleq x_{i \to \mathcal{F}_j}$, guaranteeing a reconfiguration, in a shortest path manner, to a point on the line segment defining its assigned flow \mathcal{M}_i .

Finally, when an agent i is in the swarming state $S_i = SWARMING$ (lines 18-20, Algorithm 7), after all necessary reconfigurations have been made (either by the ICP via C_i or internally by flow alignment), a *dispersive* interneighbor controller is applied in order to optimize the assigned flow. Specifically, each swarming agent $i \in \mathcal{I}_M$ applies a coordination controller (regardless of bridge status b_i):

$$u_i^o \triangleq -\nabla_{x_i} \sum_{j \in \mathcal{N}_i^A} \frac{1}{\|x_{ij}\|^2} - \sum_{j \in \mathcal{N}_i^S} \nabla_{x_j} \frac{1}{\|x_{ji}\|^2}$$
 (20)

where

$$\mathcal{N}_{i}^{A} \triangleq \{ j \in \mathcal{N}_{i} \mid (\mathcal{M}_{j} \cap \mathcal{M}_{i} \neq \emptyset) \land \\ [(\mathcal{S}_{i} = SWARMING) \lor (j \in \mathcal{I}_{S})] \}$$
 (21)

is the set of neighbors that share membership in flow \mathcal{M}_i , and who are either *in flow* and *actively* swarming (i.e. by condition (19)), or are a static source/destination node. Further, we define

$$\mathcal{N}_i^S \triangleq \{ j \in \mathcal{N}_i^A \,|\, j \in \mathcal{I}_S \} \tag{22}$$

as the set of static in flow neighbors for which compensation (Remark 5.1, below) must be applied. Controller (20) dictates that mobile flow members disperse equally only with fellow flow members and also with the source/destination nodes of their assigned flow \mathcal{M}_i .

³Although in this work we allow all link additions, link addition control could be useful for example in regulating neighborhood sizes to mitigate spatial interference, or to disallow interaction between certain agents.

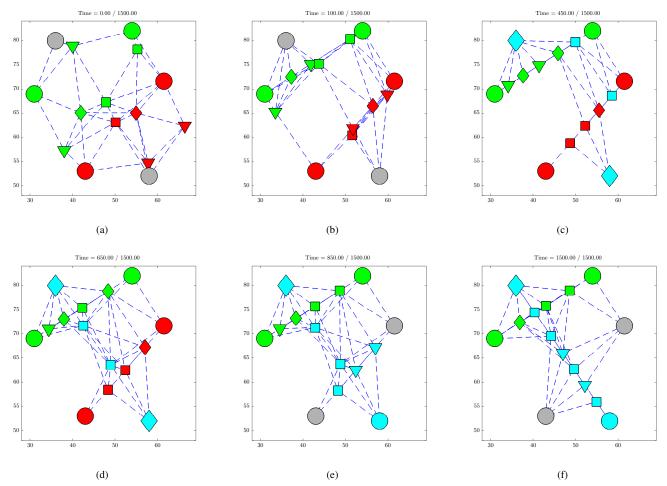


Fig. 3. Network progression for the simulated execution described in Section VI. Flow membership is indicated by color, where square nodes are the flow backbone, triangle nodes are redundant with respect to connectivity, and diamond nodes are bridges. Note that flow \mathcal{F}_3 is initially inactive and becomes active at t=450, while \mathcal{F}_2 is initially active and deactivates at t=850.

Remark 5.1 (Energy compensation): The inclusion of supplementary control terms for interactions with static neighbors $j \in \mathcal{N}_i \cap \mathcal{I}_S$ in (20) acts to retain the inter-agent symmetry required for the application of constrained interaction [9], specifically as static agents do not contribute to the system energy. We refer to this control action as energy compensation, an idea that will evolve in future work by Williams and Gasparri to treat systems with asymmetry in sensing, communication, or mobility.

While dispersive controllers generally yield equilibria in which inter-agent distant is maximized (up to ρ_2) [17], as each flow is constrained by static source/destination nodes, the dispersion (20) generates our desired equidistant intra-flow configuration as formalized below:

Proposition 5.1 (Equidistant dispersion): Consider the application of coordination objective (20) to a set of mobile agents $i \in \mathcal{I}_M$ within the context of interaction controller (10), each sharing membership to a flow $k \in \mathcal{I}_F$, i.e. $\mathcal{M}_i = k, \ \forall i$. It follows that at equilibrium the agents are configured such that the equidistant spacing condition

$$||x_{ij}|| \to \frac{||x_k^d - x_k^s||}{|\mathcal{V}_F^k| - 2}, \quad \forall \ i \mid j \in \mathcal{N}_i^F$$
 (23)

holds asymptotically over flow k.

A formal proof is beyond the scope of this work⁴, however note that our controllers operate using *only* inter-agent distance, an advancement beyond related works such as [13].

VI. SIMULATION RESULTS

In this section, we present a simulated execution of our described INSPIRE proof-of-concept, Route Swarm. Consider a system operating over a workspace in \mathbb{R}^2 , having n=15 total agents, m=9 of which are mobile and s=6 of which are static information source/destinations. Assume we have f=3 flows (green, red, and blue indicate flow membership), with the initial system configuration depicted as in Fig. 3a (notice that \mathbb{G} is initially connected), with the system dynamics shown in Fig. 3b through 3f. We simulate a scenario in which \mathcal{F}_3 is initially *inactive* (gray), allowing the ICP to optimize agent allocation over only f=2 flows, as in Fig. 3b to 3c. By Fig. 3c, flows \mathcal{F}_1 and \mathcal{F}_2 have been assigned an evenly distributed allocation of mobile agents, where the PCP has provided equidistant agent spacing for each flow. At this same time (650 time steps), the flow \mathcal{F}_3 activates, initiating a reconfiguration by

⁴Informally, an energy balancing argument establishes the result.

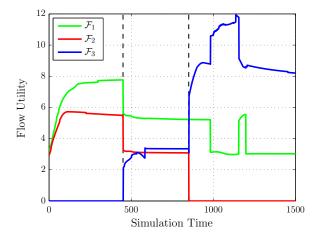


Fig. 4. Flow utilities for the simulated execution described in Section VI.

the ICP to optimize the newly added flow, as in Fig. 3d, noting that initially in Fig. 3c, \mathcal{F}_3 is poorly served by the network configuration. Finally, in Fig. 3e, flow \mathcal{F}_2 is deactivated, forcing another reconfiguration yielding the equilibrium shown in Fig. 3f. The per-flow utility over the simulation, given for a flow $i \in \mathcal{I}_F$ by $\sum_{(j,k)\in\mathcal{E}} |_{i\in\mathcal{M}_j\cap\mathcal{M}_k} w_{ij}$ (the sum of the link utilities associated with each flow), is depicted in Fig. 4. Finally, to better illustrate the dynamics of our proposed algorithms, we direct the reader to http://anrg.usc.edu/www/Downloads for the associated simulation video.

Remark 6.1 (Dynamic vs. static): The optimizations proposed in this work are advantageous in terms of dynamic information flow needs and changing system objectives, when compared to static solutions. On flow switches, static placements fail to fulfill the information flow needs of the altered system configuration. Additionally, our methods allow for dynamics in \mathcal{I}_M itself, as the ICP can adaptively reconfigure the system to utilize the available agents across the network flows.

VII. CONCLUSION

In this paper, we illustrated a novel hybrid architecture for command, control, and coordination of networked robots for sensing and information routing applications, called INSPIRE (for INformation and Sensing driven PhysIcally REconfigurable robotic network). INSPIRE provides of two control levels, namely Information Control Plane and Physical Control Plane, so that a feedback between information and sensing needs and robotic configuration is established. An instantiation was provided as a proof of concept where a mobile robotic network is dynamically reconfigured to ensure high quality routes between static wireless nodes, which act as source/destination pairs for information flow. Future work will be focused on the validation of the proposed architecture in a real-world scenario having mobile robotic interaction with a sensor network testbed.

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