

Force-Guiding Particle Chains for Shape-Shifting Displays

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Abstract—We present design and implementation of a chain of particles that can be programmed to fold the chain into a given curve. The particles guide an external force to fold, therefore the particles are simple and amenable for miniaturization. A chain can consist of a large number of such particles. Using multiple of these chains, a *shape-shifting display* can be constructed that folds its initially flat surface to approximate a given 3D shape that can be touched and modified by users, for example, enabling architects to interactively view, touch, and modify a 3D model of a building.

I. INTRODUCTION

The underlying goal of this work is the design and implementation of a “shape display” – a two-dimensional surface that can fold into the third dimension where the shape to be displayed can be freely programmed and dynamically changed. A user can not only view and touch the displayed 3D surface, but by means of touch gestures recognized through sensors built into the surface could interactively modify the displayed shape. Such a shape display would enable a wide range of interesting applications, for example, an architect could display and interactively modify a newly designed building; scientists could display complex 3D graphs and models to better understand them.

One approach to realize such a shape display is based on many tiny modular robots (i.e., *particles*) that can change their arrangement such that the aggregate surface formed by all the robots forms the displayed shape. However, approaches based on freely moving autonomous particles have the disadvantage that each particle needs complex actuators and latches to move into a desired configuration and latch into a mechanically stable shape. Also, power supply and communication among particles is challenging as they form dynamically changing connection topologies. Therefore, researchers have investigated approaches where neighboring particles are connected by joints and can change their relative orientation by means of actuators. The resulting fixed connection topology allows for wired networking and power supply, but actuators and latches are still needed in each particle. Specifically, these actuators and latches need to be strong enough to create large and mechanically stable particle aggregations. This represents a hurdle towards miniaturization of the particles, effectively limiting the “resolution” of the shape display to rather coarse structures.

Our contribution lies in removing the need for latches, only requiring mechanically weak and simple actuators in each particle, therefore enabling a better miniaturization of

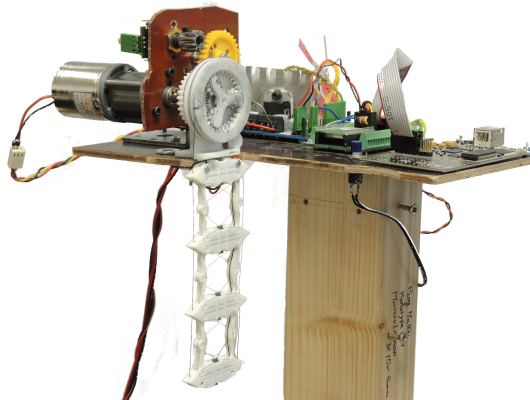


Fig. 1: Our prototype of a “force-guiding particle” chain.

the particles. We call our approach “force-guiding particles”, as a single strong actuator exerts a mechanical force on all particles in a chain and a weak and simple actuator inside each particle guides this force to fold the chain into a desired configuration. As the particles in the chain are connected by joints, two wires running through the chain provide power to and establish a communication network among all particles. Therefore, miniaturization of particles becomes feasible. By placing multiple such chains (all driven by a single external actuator) next to each other we obtain a “shape-shifting curtain” that can display 3D surfaces.

The remainder of the paper describes the electro-mechanical design of a force-guiding particle chain, as well as algorithms for control and planning of chain folding to approximate a desired shape. We describe a prototype (Fig.1) that has been produced with a 3D printer and characterize its performance. We also provide an analysis of the scaling properties of force-guiding particle chains.

II. RELATED WORK

“Shape-Shifting Material” and “Programmable Matter” are general terms for research on materials whose physical properties and in particular shape can be changed dynamically under program control or through direct interaction [1], [2], [3]. The approach is typically based on an extreme form of modular robotics, where small robotic “particles” can change their relative position by means of suitable actuators.

One can broadly identify two sub-classes of approaches. In the first the robots are detachable and can “climb” each other (e.g., [4], [5], [6], [7], [8], [9]). While arbitrary shapes can be formed, the individual robots are typically complex as they need complex actuators and latches as well as sophisticated power supply and networking as the robots are not permanently connected. That leads to costly robots which

cannot be easily miniaturized. Even when relocation relies on non-moving parts (e.g., magnets [5], [10]), miniaturization might still be difficult as each particle needs to exert strong forces on neighboring particles in order to form mechanically stable shapes consisting in a large number of particles.

In the second sub-class “robots” are connected in fixed topologies and the resulting substrate can be deformed by embedded actuators. These substrates can be two-dimensional and fold along crease patterns inspired by Origami (e.g., [11], [12], [13]), or one-dimensional chains that fold in 2D (e.g., [14]) or 3D (e.g., [15], [16]). While this reduces the complexity of the individual robots due to the fixed connection topology, miniaturization of the individual robots is still difficult as the force to fold and latch is still created by actuators inside each robot.

To overcome this intrinsic limitation, White et al. [17] devise an external manipulator to fold a chain of passive latching particles into 3D shapes. However, their external actuator is rather complex (i.e., essentially a robotic arm) and a separate actuator is needed for each chain, thus it does not scale well to larger systems with multiple chains. Also, the unfolding of the chain requires manual support. Hence, this approach is not suitable for a “shape display” as we envision it where displayed shapes can be dynamically changed.

III. REQUIREMENTS

Below we explicate the main requirements on force-guiding particle chains.

a) Formation of arbitrary shapes: It should be possible to display a wide range of connected 3D surfaces. As our shape display consists of multiple chains placed next to each other, each chain should be able to approximate a 2D slice (i.e., curve) of the 3D surface.

b) Miniaturization of particles: To enable a high-resolution display, it should be possible to miniaturize particles. This requires particles to have a simple structure. In particular, it should be possible to miniaturize the mechanical structures and actuators contained in a particle without compromising the mechanical stability of the particle chain.

c) Scalability: As particles are miniaturized, a growing number of particles are required to display surface shapes of realistic size. Therefore, it should be possible to include a large number of particles in each chain and to include many such chains into a shape display, i.e., it should be possible to scale up the number of particles. Again, mechanical structures and actuators play a key role here as they need to be strong enough to deal with the growing extension and weight of the chain as the number of particles increases.

IV. APPROACH: FORCE-GUIDING PARTICLES

Our shape display consists of many particle chains placed next to each other, thus forming a 2D surface. Each chain can fold within a plane that is perpendicular to this surface. Together, all folded chains approximate the 3D shape that is to be displayed.

Each chain resembles an articulated ladder-like skeleton (Fig. 2(a)). Here, a particle consists of the square formed

by two consecutive rungs and the two edges connecting these rungs. By folding one of the edges, the square can be reconfigured into a triangle, thus deflecting the chain. As each particle in the chain can be folded left or right, the chain can approximate any planar curve that does not self-intersect. Together, all chains can thus approximate any connected 3D surface, thus meeting the requirement of displaying arbitrary shapes. We outline a planning algorithm to approximate a given shape later in the paper.

The regular geometry of the structure allows a strong external actuator to exert a compressing force F_w on the whole chain by pulling two threads that run through the ladder structure on the left and on the right as shown in Fig. 2(a). By exerting a small force F_s on one of the edges it is slightly bent, such that F_w will compress the edge, thus transforming the rectangular particle into a triangle. Edges to which no force F_s is applied remain locked in straight configuration even if F_w is applied. If F_w is released, the chain returns into straight configuration due to gravity. Thus, particles “guide” the external force for folding.

The single external actuator (i.e., motor) is dimensioned such that it can pull the threads of *all* chains even if the number of particles in each chain becomes large. While that may imply a relatively big actuator, the dimensions of the particles (and thus the resolution of the shape display) are not affected. In that sense, we can scale up the number of particles in a chain without impact on the size and complexity of the particles, thus meeting the scalability requirement.

As we will explain in the following section, the force F_s in the particle can be generated by means of a Shape Memory Alloy wire that contracts when applying a current. By retaining the external force F_w , the chain remains folded without the need for latches inside each particle. Thus, the mechanics of a particle essentially consists of hinges, joints, and SMA wires – making it very simple and enabling miniaturization, thus meeting the miniaturization requirement.

V. HARDWARE AND SOFTWARE DESIGN

In this section we describe in more detail the design of force-guiding particle chains. We begin with a description of the mechanics and electronics, followed by discovery and localization of particles, and conclude with control and planning aspects.

A. Mechanical Design

Figure 2(a) shows the design of a particle chain, where a deformable particle correspond to the highlighted area. A particle chain consists of the following five main components (see also the number labels in the figure).

1. Rigid edge. Each particle consists of two rigid horizontal edges that are shared with the particles above and below. The particular stretched-hexagon shape ensures a precise triangular mesh when the particles fold. Its dimensions are tailored to encase the electronics that control the particle.

2. Flexible edge. Hinged to successive rigid edges via snap-fit joints, each of the two flexible edges of a particle embodies a monostable mechanism to lock/unlock the folding.

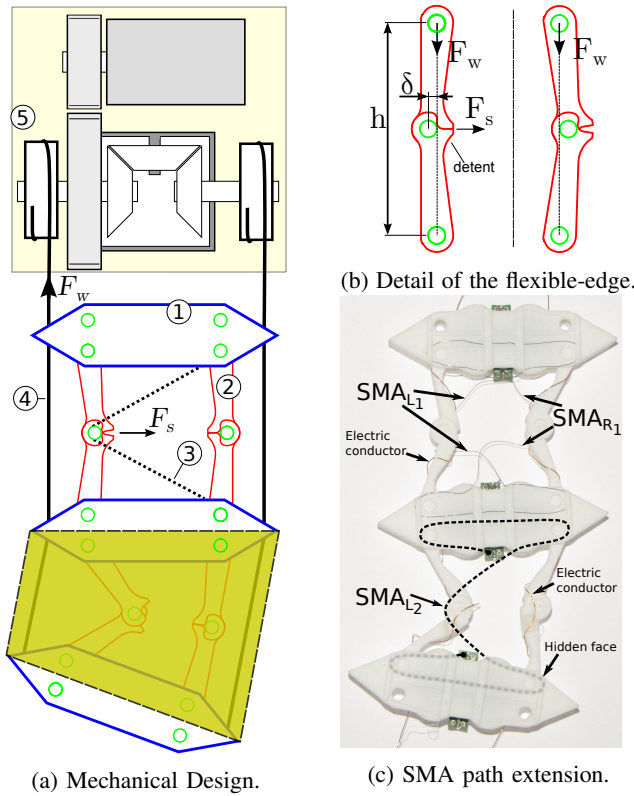


Fig. 2: Mechanical components of the system.

As shown in Fig. 2(b), the slight misalignment of the middle hinge w.r.t. the two hinges at the extremes, combined with a counteracting detent, lead to a “locked” straight configuration even when the external force F_w is applied. When, instead, a small force F_s pulls the middle hinge towards the centre of the particle, the link gets “unlocked”, and F_w deforms the particle into a triangular configuration.

3. Active strap. Made of Shape Memory Alloy (SMA), the contraction due to an electric current heating it up causes the lateral force F_s to unlock the flexible edge as described above. When the current is stopped, the SMA cools down and returns to the original length. Since the SMA features a relative contraction of about 4%, a longer SMA wire running through the opposite rigid edges amplifies the contraction effect by a factor of four as shown in Fig. 2(c).

4. Actuation thread. Next to the flexible edge, the actuation thread extends along the chain to propagate the force F_w the external actuator provides. The resulting effect on the chain is a compressing force on each side that, combined with the unlocking of a flexible edge, causes the deflection of the chain. By applying a constant tension, the chain retains the folded configuration without a need for latches.

5. External actuator. It provides the mechanical power to the system. An electric motor rolls up the two actuation threads, by means of spoolers connected via a differential gear. Because of the misalignment between flexible edges and threads, a particle deformation makes one thread roll up, while the other one slightly unrolls. To mechanically compensate this effect, the differential gearbox decouples the two spoolers. In this way, a single motor can even drive

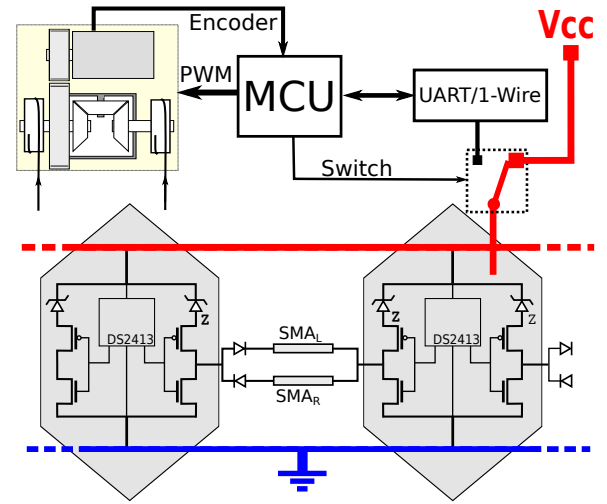


Fig. 3: Electrical block diagram.

multiple chains connected to the same shaft, provided an equivalent number of deforming particles among chains.

B. Electronics

Figure 3 shows the electronic architecture of the system, which consists of an MCU that controls the external actuator as well as the particles via a 1-wire bus that also supplies power to the particles.

MCU. Core of the system is an 8-bit MCU (ATmega128RFA1) that controls all the components: it remotely drives the local actuation of each particle and synchronizes the external actuation to fold the chain. As all the particles share the same bus to communicate with the MCU and power the SMA straps, the MCU controls the bus line switching it from communication mode (1-Wire[®]) to SMA power supply mode (Vcc) and vice-versa.

Actuation and feedback loop. The main actuator based on a 12 V DC motor (HN-34PGD-2416T) can rotate at variable speed (PWM controlled) in both directions to fold and unfold the chain. The planetary gear reduction embedded in the motor (gear ratio 410:1) prevents the backward rotation so the motor can be switched off once the chain has been folded. As no sensors are embedded in the particles, a quadrature encoder (48 cycle/rev.) connected through a gear (ratio 1:3) to the main shaft, allows feedback control to stop the motor when a desired configuration has been reached.

1-Wire[®] bus. Two conductors conveniently threaded through the chain form the data line and ground reference for a 1-Wire[®] bus that connects particles and MCU. 1-Wire[®] uses a master/slave protocol and supports data and power on the same bus. Power is supplied either when no communication is in progress or when a logic '1' is transmitted. In these two cases, a pull-up resistor (in the master) pulls the voltage on the bus to 5 V. While the logic '1' passively results from the pull-up resistor, the dominant '0' is obtained by short-cutting data line and ground. This limits the maximum current on the bus to 5.4 mA, as higher currents can introduce logic '0' in the communication. Each 1-Wire[®] slave exploits the period when the bus is powered

to harvest energy in a small capacitor. As interface between the MCU and the bus, we adopted a UART to 1-Wire[®] converter (DS2480B) that works as master.

Particle/Node. Each particle hosts a dual-channel-addressable-switch (DS2413) encased in the rigid edge of the particle which acts as a slave on the 1-Wire[®] bus. The MCU can control the output pins of these switches, with direct access to their memory. As depicted in Fig. 3, each output pin drives a half-H bridge connected to a pair of SMA straps: neighbouring nodes, connected to the same pair of SMA straps, form a full-H bridge. By controlling the polarity of the H-Bridge (i.e., one half-H bridge pulls up and the other one pulls down) and introducing two diodes in mutual exclusion in series with the straps, each strap (SMA_L for left edge and SMA_R for right edge) can be activated independently.

Bus commutator. As the maximum acceptable current on the 1-Wire[®] bus is 5.4 mA, while the SMA straps nominally draw 180 mA, a commutator decouples data line and power supply (Vcc). To obtain the same decoupling also within each particle, a zener diode (5.6 V) in series with the power circuit prevents the straps from being supplied when the bus is in data mode. Instead, when the external commutator switches the line to Vcc (15~20 V), all the straps previously enabled are powered. The local capacitor on each DS2413 switch preserves the settings during the voltage transition. Also, the device can withstand 28 V.

C. Node Discovery and Localization

Each DS2413 is addressed by a factory lasered 64-Bit ID. Since these addresses are unknown until the device is connected to the bus, a discovery algorithm is needed so that the MCU can find out the addresses of the particles. The DS2480B bus master provides this functionality. However, the discovery algorithm does not indicate the actual order of the particles along the bus. Since this is fundamental for controlling the straps, we devise a localization algorithm to infer the neighbor relationships among particles.

The algorithm exploits the fact that two neighboring particles can control their half-H bridges such that a current flows through the SMA wire connecting the two particles and the MCU can detect such a current by measuring the power draw over the 1-wire bus. The MCU hence picks one particle, configures its half H-bridge and then sequentially selects all other particles (their addresses are known after discovery) and configures their half-H bridge until power draw varies, in which case two neighboring particles are found. The procedure is repeated for the newly-found neighbor until no more particles are left. If no neighbor can be found, a node is at the end of the chain.

D. Control

Folding the chain requires the MCU to synchronize a set of N folding particles p_i and the main actuator. The process consists in four steps characterized by time constants:

- 1) The 1-Wire master initializes the communication synchronizing the slaves while discovering their addresses taking time t_{setup} ;

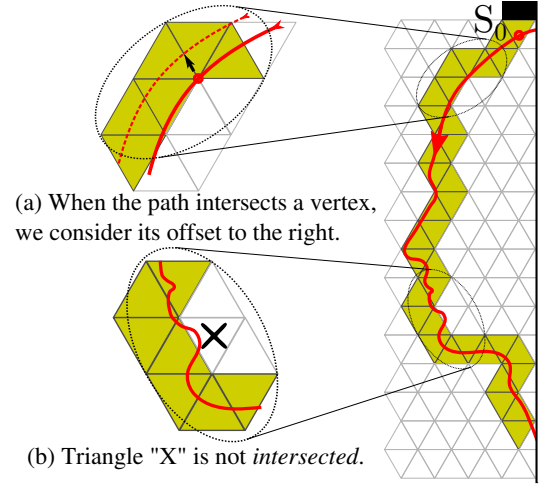


Fig. 4: For each possible folding configuration of a chain, the particles lie on a regular triangular grid.

- 2) The MCU sends the desired switch configuration to k particles taking time t_{config} for each;
- 3) The MCU switches the bus to power supply mode to activate the SMA taking a minimum time t_{SMA} to fully contract the SMA;
- 4) The MCU activates the external actuator until the chain is folded taking time t_{mot} .

If we assume that the folding operation involves N particles, for which the configuration of k addressable switches needs to be set, the total time to perform the folding is:

$$T(N, k) = t_{setup} + k \cdot t_{config} + t_{SMA} + N \cdot t_{mot} \quad (1)$$

As a result of the regular geometry of the chain, the number of revolutions the motor performs to deform each particle is constant. Then also t_{mot} is constant for a constant speed.

E. Planning

Planning consists in computing for each particle in a chain whether it should be folded left or right such that the folded chain approximates a given curve S that is to be displayed.

We observe that for each possible folding configuration of a chain, the folded triangular particles lie on a regular triangular grid as shown in Fig. 4, where the desired curve S resembles a head. We further assume that the chain hangs on a wall where the top particle is fixed to a wall bracket (black rectangle).

We choose the top most point S_0 of S , and from there start to walk along S to create the sequence of triangles intersecting S . A triangle t_i is said to intersect S if S enters t_i through edge e_i and eventually leaves t_i through a different edge $l_i \neq e_i$. For example, in zoomed Fig. 4(b), the path intersects an edge of triangle "X" and then leaves the triangle through the same edge, therefore, the triangle marked "X" does not intersect. In case the path S intersects a vertex on the grid, generating an undefined situation, we consider those triangles that the path would intersect, if it had a slight offset to the right (w.r.t the walking direction), as shown in Fig. 4(a). Each intersecting triangle t_i corresponds

to particle p_i in the chain. Likewise, each pair of edges e_i and l_i corresponds to the opposite rigid edges of particle p_i . Since we have chosen only adjacent intersecting triangles, the sequence of corresponding particles is also connected. To decide whether particle p_i should fold left or right, we can walk again along the path S and consider the side where the corner connecting e_i and l_i lies: if it lies on the left of S , the particle folds the left edge; if it lies on the right, the particles folds the right edge.

VI. PROTOTYPE

We have built a prototype of a force-guiding particle chain consisting of three particles. All the components of the chain are 3D-printed. The resistance and the flexibility of the used material PA22, along with the precision of the Selective-Laser-Sintering technology allowed us to realize the parts of a particle with snap-fit joints to simplify and speed up the assembly process. Specifically, each rigid edge is composed of two symmetrical parts that can be snapped into each other and encase the printed circuit board with the electronics. Each half-link of a flexible edge is printed separately and then snapped into the complementary part. The flexible edge is then snapped into the two rigid edges. The SMA wire is threaded through a thin tunnel (diameter 0.5 mm) to extend the SMA path as shown in Fig. 2(c). We also investigated the possibility of printing pre-assembled hinges, aiming to improve joint stability and minimize assembly overhead, but the printing process requires a minimum distance between surfaces which causes unacceptable backlash.

The differential gear and spoolers are also 3D printed. One specific requirement for that component is that it should be thin enough to allow placing multiple chains next to each other. For that reason, we designed a planetary differential gear integrated with the spoolers. The planet gears and the carrier are printed separately, then assembled into the spoolers. Similarly, the gear connecting the motor and the encoder is a custom 3D-printed component.

VII. EVALUATION

In this section we evaluate the design using the prototype described above. We first perform an experimental validation of the prototype and report on timing behavior and power consumption. We then analytically investigate the fundamental scaling properties of the design.

A. Experimental Validation

A PC connected via USB/UART to the MCU lets the operator control each particle individually and switches the bus between data and Vcc. We firstly checked the correct functioning of the 1-wire network, showing that the data line can be switched to Vcc for powering the SMA wires without issues for communication. We found that the 1-wire master loses synchronization with the slaves after this operation, requiring a reset to restore the communication. This introduces a delay after each operation equivalent to t_{setup} (Sec. V-D). We measure that t_{setup} takes 205 ms with 6 nodes for the initialization of the bus; later resets can be

performed within 78 ms, regardless of the number of nodes on the bus. The configuration of a node takes $t_{config}=22$ ms.

We installed a chain of three particles hanging from a custom support with the main actuator winding up the thread from the top. We empirically measured the activation time of the SMA to fully disengage the link (using video), and counted the actual revolutions of the spool to completely fold one particle. The SMA requires $t_{SMA} \geq 421$ ms to disengage the link when the system is supplied with 20 V and the SMA draws 280 mA (nominal value 180 mA). However, for safety reasons we increased this value to 500 ms.

Considering that to completely fold the chain the thread winds up by nominally 99.1 mm, the spooler (40 mm nominal diameter) requires 0.789 revolutions, which corresponds to approximately 113 cycles on the encoder (48 cycles/rev. connected to the shaft through a 1:3 gear). We reduce the motor speed using pulse width modulation. The time needed to actuate a single particle is then $t_{mot}=1050$ ms.

Inserting these values into Eq.1 and considering a single particle folding (i.e., two addressable switches have to be set), the time needed for this operation is $T(1, 2) = 78\text{ms} + 2 \cdot 22\text{ms} + 500\text{ms} + 1 \cdot 1050\text{ms} = 1672\text{ms}$, which matches the experimental result we obtain. Actuating the system with the motor at the nominal speed of 14 RPM, t_{mot} reduces to 56 ms, for which $T(1, 2)$ becomes 678 ms. The unfolding operation only requires 168 ms with three particles.

A complete actuation test, where all the particles are folded starting from the bottom and progressively proceeding to the top, demonstrated the correctness of the settings previously identified. Even though each deformation presents an error of ± 2 cycles on the encoder (mainly due to fabrication tolerance, but also due to the low accuracy of the measurements), the errors self-compensate when all the particles are folded.

The unfolding of the chain subject to gravity could not completely restore the initial configuration, although an additional rubber bands assist links to return to straight configuration. This happens because of the moderate weight of the components and especially due to the friction at the hinges. An additional weight of 16 g added to the tail of the chain improves the result, yet without completely unfolding the chain. Improvements in the manufacturing process to reduce the friction will address this issue.

B. Scalability

In this section we investigate the scaling limit of our approach, i.e., how many particles we can support in a single chain. For that we consider the worst-case situation depicted in Fig. 5, where N particles of a chain are already folded in a straight horizontal configuration (cantilever), while the $(N + 1)$ -th particle (top-right in the figure) is about to fold by applying F_s to the left flexible edge. We estimate the force F_w resulting on the pulling thread and compute the maximum number N of particles that can be supported such that the limited force F_s that can be exerted by the SMA is still sufficient to bend the flexible edge given F_w . Due to the leverage effect F_w is maximized in the shown configuration,

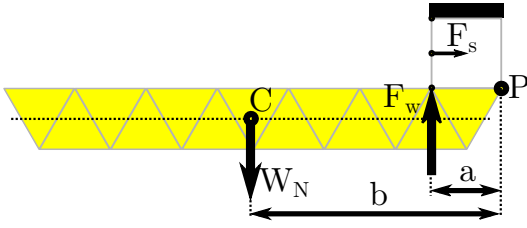


Fig. 5: Worst case configuration of a folded chain.

therefore it constitutes the worst case. For this we first have to compute the force F_s that is required to bend a flexible edge when a pulling force F_w is applied to the particle:

$$F_s = 2 \frac{\delta}{h} F_w \quad (2)$$

where δ is the offset between the three hinges of the monostable link and h the distance between the two hinges at the extremes as shown in Fig. 2(b).

Inserting the actual dimensions of our prototype into Eq. 2, we can estimate the mechanical advantage F_w/F_s . With $\delta=1.28$ mm and $h=32.11$ mm, the mechanical advantage is 12.5. A smaller δ would increase this value, but would also reduce the stability of the flexible edge.

Now we can establish a relationship between the number N of particles and the resulting force F_w . For this we observe that the configuration shown in Fig. 5 can be considered a lever of class 3, where P is the fulcrum, F_w the effort, and W_N the resistance. We define W_N as the total weight applied to its centre of mass, $W_N = NW_p$, where W_p is the weight of one particle (about 4.5 g in our prototype). The two distances a and b correspond to the edge length l of a particle and $\frac{1}{4}Nl$, respectively. Applying the law of the lever we obtain:

$$F_w \cdot l = \frac{1}{4}Nl \cdot (NW_p) \quad (3)$$

We can now substitute F_w with $12.5 \cdot F_s$ according to the mechanical advantage computed above based on Eq. 2 and solve for N :

$$N = 2\sqrt{\frac{F_w}{W_p}} = 2\sqrt{\frac{12.5 \cdot F_s}{W_p}} \quad (4)$$

The maximum F_s an SMA strap can exert, depends on its contraction force: with a diameter of $100 \mu\text{m}$ the nominal contraction force F_{SMA} is 14.7 N. As the arrangement of the strap around the hinge of the flexible edge (Fig. 2) doubles the effect of its contraction force, while the angle the strap forms with the vector $\overline{F_s}$ reduces its effect by $\cos(30^\circ)$, we can approximate $F_s = 2F_{SMA} \cdot \cos(30^\circ)$ which corresponds to 2.59 kg. Substituting this value and $W_p = 4.5$ g into Eq. 4, we obtain $N = 84$ particles in the worst case. We observe that N is independent of the size l of the particles and grows with the inverse of the square root of the weight W_p for a constant F_s . This means that if we can miniaturize particles and reduce their weight, we can increase the number of particles in the chain in the worst case. Hence, the miniaturization is only limited by the mechanical stability of the thread and the material from which the particles are made.

VIII. CONCLUSION AND FUTURE WORK

We have presented a force-guiding particle chain that can fold into arbitrary flat curves under program control as a building block to construct a shape-shifting display that can fold its surface into a 3D shape that can be viewed, touched, and modified by users. The key feature of force-guiding particle chains is that the mechanical force to fold the chain is generated by an actuator external to the chain, such that the particles of the chain are simple and require only a mechanically weak actuator to guide the external force. Thereby, particles are amenable to miniaturization and scale up to a large number of particles per chain. We presented the mechanical and electrical design, as well as algorithms for control and planning. We demonstrate and validate a working prototype.

Future work includes, among others, improvement of the mechanical design to reduce friction and the number of parts; inclusion of sensors into particles for recognizing touch gestures and folding state of particles; as well as advanced planning algorithm to minimize F_w .

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