

Transparency Compensation for Bilateral Teleoperators with Time-Varying Communication Delays

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Abstract—Passivity-based bilateral teleoperation control systems can offer robust stability against arbitrarily large communication delays at the expense of poor transparency. In fact, most passive control frameworks are designed for a particular task and do not adjust transparency when transitioning between different environments. This paper presents a bilateral control strategy that passively compensates transparency when transitioning between free motion and hard contact motion scenarios. The proposed control framework exploits the effect that the wave impedance (a design parameter of the passivity-based scattering transformation) has on transparency without compromising closed-loop stability regardless of time-varying communication delays. To adjust transparency, the control scheme smoothly switches the wave impedance between a low value, ideal for free motion, and a sufficiently large value, suited for hard contact scenarios. We show, by rigorous mathematical treatment and simulations, that the proposed control strategy can effectively adjust the transparency of the system without compromising stability.

I. INTRODUCTION

The two foremost goals of any bilateral teleoperation control architecture are to guarantee transparency and stability of the closed-loop system. According to [1], transparency is achieved when the transmitted impedance to the operator equals the environmental impedance. An alternate definition is given in [2], where a teleoperator is said to be transparent if the positions and forces at the master and slave sides are equal. Unfortunately, it is well known that perfect transparency in a bilateral teleoperation system cannot be achieved without compromising stability due to the presence of inherent time delays in the communication channel [1].

Motivated by the need of transparent yet stable time-delayed teleoperators, several control frameworks have been proposed in the literature (refer to [3], [4] for a review of control schemes and a comparison among different methods). A major breakthrough in the field came in the late 1980s when passivity-based control and scattering theory were combined to guarantee the stability of force feedback teleoperators independently of arbitrarily large constant delays [5]. Ever since, the scattering and passivity formulation, refined later with the notion of wave variables [6], has arguably become one of the most predominant control approaches for stabilizing bilateral teleoperators.

Despite the popularity of passivity-based control schemes, most of them (including wave-based approaches) tend to suffer from poor transparency [1], [7]. Two of the main reasons for the lack of transparency compensation are 1)

the conservatism incurred by passivity-based control techniques and 2) the tuning of the control gains based on the expected environmental dynamics which, more than often, are uncertain. For instance, in wave-based architectures, transparency highly depends on a control parameter namely the wave impedance. For free motion, the ideal wave impedance should be infinitesimal such that the increase of inertia induced by the delay is barely perceived by the operator. In contrast, for stiff or hard contact environments, the desired wave impedance should be infinitely large such that a stiff environment is felt by the operator [7]. Tailoring or averaging the wave impedance to best satisfy both scenarios leads to a teleoperation system that feels sluggish in free motion with substantial position errors when interacting with stiff environments. A similar effect is experienced when tuning traditional proportional-derivative (PD) architectures where control gains are limited by stability constraints and, consequently, position errors arise while in contact motion [8].

Online compensation of position errors during contact tasks, aimed to improve static transparency, has been previously addressed in [9] and [10] via a wave-based scheme that introduces the notion of a variable rest length. The role of the variable rest length is to modify or shift the master's and slave's desired targets such that their position difference converges to zero. In both methods, the communication delays must be known and constant in order to perform the position compensation.

Recently, in [11], a wave-based control framework was proposed where the wave impedance passively switches between an arbitrary small value, ideal for free motion, and an arbitrary large value, suited for stiff environments. The idea of modifying the wave impedance according to the remote environment was previously explored in [12]. However, the switching strategy of [12] requires the mechanical and control systems to dissipate a minimum amount of energy to perform stable transitions. Other recent methods designed for transparency compensation include [13] and [14]. In [13], a two-layer approach is proposed where the first layer is designed to improve the transparency of the system based on the knowledge of the task and environment, while the second layer oversees the preservation of passivity. In [14], a switching two-channel control architecture is developed for linear systems with constant delays.

Herein, we extend the time-varying wave impedance control architecture of [11] to bilateral teleoperators with time-varying communication delays. We present a stable update strategy that smoothly adjusts the wave impedance value

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according to the current environmental dynamics without compromising the stability of the closed-loop system. In addition, we remove the need of transmitting remote contact information to the master robot by updating the wave impedance at the slave side rather than at the master side. Simulation results validate the proposed control architecture.

II. MODELING THE TELEOPERATION SYSTEM

We address the task of remotely controlling an n -degree-of-freedom (DOF) slave robot coupled bilaterally to an n -DOF master robot through a time delayed communication channel. The master and slave teleoperator have nonlinear Euler-Lagrangian dynamics given by¹

$$M_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \mathbf{f}_i + \bar{\boldsymbol{\tau}}_i \quad (1)$$

where $\mathbf{q}_i = \mathbf{q}_i(t) \in \mathbb{R}^n$ are the generalized coordinates, $M_i(\mathbf{q}_i) \in \mathbb{R}^{n \times n}$ are the bounded, positive definite inertia matrices, $C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n \times n}$ are the centrifugal and Coriolis matrices, $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^n$ are the gravitational forces, $\mathbf{f}_i = \mathbf{f}_i(t) \in \mathbb{R}^n$ are the human and environmental forces, and $\bar{\boldsymbol{\tau}}_i = \bar{\boldsymbol{\tau}}_i(t) \in \mathbb{R}^n$ are the control inputs for the master ($i = m$) and slave ($i = s$) robots. Due to its Euler-Lagrangian dynamic structure, we assume that (1) satisfies the well known passivity property $\dot{M}_i(\mathbf{q}_i) = C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + C_i^T(\mathbf{q}_i, \dot{\mathbf{q}}_i)$. In addition, we consider robotic systems where $M_i(\mathbf{q}_i)$ is upper and lower bounded, which in turns implies that $\|C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\| \leq \kappa_{C_i} \|\dot{\mathbf{q}}_i\|$ for some $\kappa_{C_i} \geq 0$.

In the following analysis we make the assumption that delays on the transmission lines from master to slave, $T_m(t)$, and from slave to master, $T_s(t)$, are finite (i.e., $\exists T > 0$ such that $T_m(t) - T_s(t) \leq T$) but not necessarily equal.

III. BILATERAL CONTROL FRAMEWORK

Our control goal is to design the inputs $\bar{\boldsymbol{\tau}}_i$ such that stability and transparency of the close-loop system (1) are achieved. Explicitly, we would like $\bar{\boldsymbol{\tau}}_i$ to enforce position coordination for finite time-varying delays, i.e.,

$$\mathbf{q}_m(t) - \mathbf{q}_s(t) \rightarrow 0 \quad (2)$$

and static force reflection, i.e.,

$$\mathbf{f}_m(t) \rightarrow -\mathbf{f}_s(t) \quad (3)$$

as $\dot{\mathbf{q}}_i \rightarrow 0$; regardless of the structure of the remote environment. Furthermore, we would like the operator to perceive low and high impedance values when interacting with free and stiff environments, respectively. Accordingly, we propose the design of the control inputs as

$$\bar{\boldsymbol{\tau}}_i = \boldsymbol{\tau}_i - M_i(\mathbf{q}_i)\Lambda\dot{\mathbf{q}}_i - C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\Lambda\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) - K_i\dot{\mathbf{q}}_i \quad (4)$$

where $\Lambda \in \mathbb{R}^{n \times n}$ and $K_i \in \mathbb{R}^{n \times n}$ are, without loss of generality, diagonal positive-definite constant matrices and $\boldsymbol{\tau}_i = \boldsymbol{\tau}_i(t) \in \mathbb{R}^n$ are the coordination control inputs to be designed. Then, the dynamics of the system (1) reduces to

$$M_i(\mathbf{q}_i)\ddot{\mathbf{r}}_i + C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\mathbf{r}_i = \mathbf{f}_i - K_i\dot{\mathbf{q}}_i + \boldsymbol{\tau}_i \quad (5)$$

where $\mathbf{r}_i(t) = \dot{\mathbf{q}}_i(t) + \Lambda\mathbf{q}_i(t)$. It can be shown that (4) is a passivity-based control method, which means that the master and slave robots are passive with respect to input $\mathbf{f}_i + \boldsymbol{\tau}_i$ and output \mathbf{r}_i . For completeness, we now define passivity.

Definition 3.1: [15] A system with input \mathbf{x} and output \mathbf{y} is said to be passive if

$$\int_0^t \mathbf{x}^T \mathbf{y} d\theta \geq -\kappa^2 + \nu^2 \int_0^t \mathbf{x}^T \mathbf{x} d\theta + \rho^2 \int_0^t \mathbf{y}^T \mathbf{y} d\theta \quad (6)$$

for some $\kappa, \nu, \rho \in \mathbb{R}$. Moreover, it is said to be lossless if equality persists and $\nu = \rho = 0$, input strictly passive if $\nu \neq 0$, and output strictly passive if $\rho \neq 0$.

A fundamental property of passive systems is that the two-port connection of two passive systems is also passive. This implies that we only need to enforce the passivity of the communication channel in order to guarantee the passivity of the entire system.

A. Passivity of Communication Channel

To passivize the communication channel, we propose the use of the scattering transformation along with the wave variables \mathbf{u}_i and \mathbf{v}_i [5], [6]. For the slave side, the outputs of the scattering transformation are computed as

$$\mathbf{v}_s(t) = (2B_s(t))^{-\frac{1}{2}} (B_s(t)\mathbf{r}_{sd}(t) - \boldsymbol{\tau}_s(t)) \quad (7)$$

$$\mathbf{r}_{sd}(t) = (2B_s^{-1}(t))^{\frac{1}{2}} \mathbf{u}_s(t) - B_s^{-1}(t)\boldsymbol{\tau}_s(t) \quad (8)$$

where $B_s(t) \in \mathbb{R}^{n \times n}$, namely the wave impedance, is a bounded, diagonal, time-varying, positive definite matrix that will be designed under transparency concerns; and $\mathbf{u}_s(t) = \gamma_s(t)\mathbf{u}_m(t - T_m(t))$ is the incoming wave variable from the master's scattering transformation. The scale factor $\gamma_s(t)$ is a positive semi-definite scalar function that will be designed under passivity constraints and $\mathbf{r}_{sd}(t) \in \mathbb{R}^n$ is a new variable. Then, the coordination control input can be computed as

$$\boldsymbol{\tau}_s(t) = B_s(t)(\mathbf{r}_{sd}(t) - \mathbf{r}_s(t)). \quad (9)$$

Likewise, for the master side, the outputs of the scattering transformation are computed as

$$\mathbf{u}_m(t) = (2B_m(t))^{-\frac{1}{2}} (B_m(t)\mathbf{r}_{md}(t) - \boldsymbol{\tau}_m(t)) \quad (10)$$

$$\mathbf{r}_{md}(t) = (2B_m^{-1}(t))^{\frac{1}{2}} \mathbf{v}_m(t) - B_m^{-1}(t)\boldsymbol{\tau}_m(t) \quad (11)$$

where $\mathbf{r}_{md}(t)$ is a new variable, $B_m(t) = B_s(t - T_s(t))$, and $\mathbf{v}_m(t) = \gamma_m(t)\mathbf{v}_s(t - T_s(t))$ for some positive semi-definite function $\gamma_m(t)$. Similar to the slave case,

$$\boldsymbol{\tau}_m(t) = B_m(t)(\mathbf{r}_{md}(t) - \mathbf{r}_m(t)). \quad (12)$$

The reader can verify that using the scattering transformation and the control inputs (9) and (12), (8) and (11) reduce to

$$\mathbf{r}_{md}(t) = \frac{1}{2} (\gamma_m(t)\mathbf{r}_s(t - T_s(t)) + \mathbf{r}_m(t)) \quad (13)$$

$$\mathbf{r}_{sd}(t) = \frac{1}{2} (\gamma_s(t)\Gamma(t)\mathbf{r}_m(t - T_m(t)) + \mathbf{r}_s(t)) \quad (14)$$

where $\Gamma(t) = B_s(t - T_m(t) - T_s(t - T_m(t)))^{\frac{1}{2}} B_s(t)^{-\frac{1}{2}}$.

Now, manipulating (7) to (12), we obtain that the power equation for the communication channel is given by

$$-(\boldsymbol{\tau}_m^T \mathbf{r}_{md} + \boldsymbol{\tau}_s^T \mathbf{r}_{sd}) = \frac{1}{2} (\mathbf{u}_m^2 - \mathbf{v}_m^2 + \mathbf{v}_s^2 - \mathbf{u}_s^2) \quad (15)$$

¹For simplicity, we will omit time dependence of signals except when considered necessary.

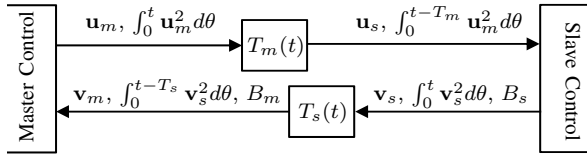


Fig. 1. Flow of Information between Master and Slave Robots.

where the negative sign at the left side of the equation is owed to the power inflow. Integrating (15) with respect to time yields the total energy in the communication channel

$$\begin{aligned} & - \int_0^t (\tau_m^T(\theta) \mathbf{r}_{md}(\theta) + \tau_s^T(\theta) \mathbf{r}_{sd}(\theta)) d\theta \\ & = \frac{1}{2} \int_0^t (\mathbf{u}_m^2(\theta) - \gamma_s(\theta) \mathbf{u}_m^2(\theta - T_m(\theta))) d\theta \\ & \quad + \frac{1}{2} \int_0^t (\mathbf{v}_s^2(\theta) - \gamma_m(\theta) \mathbf{v}_s^2(\theta - T_s(\theta))) d\theta. \end{aligned} \quad (16)$$

Note that the energy is independent of B_i . It, however, depends on the delay and the choice of γ_i .

From (16) we see that a sufficient condition to guarantee the passivity of the communication channel is

$$\int_0^t \mathbf{u}_m^2(\theta) d\theta \geq \int_0^t \gamma_s^2(\theta) \mathbf{u}_m^2(\theta - T_m(\theta)) d\theta \quad (17)$$

$$\int_0^t \mathbf{v}_s^2(\theta) d\theta \geq \int_0^t \gamma_m^2(\theta) \mathbf{v}_s^2(\theta - T_s(\theta)) d\theta. \quad (18)$$

Therefore, we will design γ_s and γ_m such that (17) and (18) are satisfied. To this end, we propose to transmit the total energy of the incoming wave variables along with the wave variables (see Fig. 1 for a pictorial representation). The equations governing γ_s and γ_m are given by

$$\gamma_i = \begin{cases} 1, & \text{if } E_i \geq \beta_i \\ \frac{2\beta_i^2 E_i^2}{E_i^4 + \beta_i^4}, & \text{otherwise} \end{cases} \quad (19)$$

where $\beta_i > 0$ are constant design parameters and

$$E_m = \int_0^{t-T_s(t)} \mathbf{v}_s^2(\theta) d\theta - \int_0^t \mathbf{v}_m^2(\theta) d\theta \quad (20)$$

$$E_s = \int_0^{t-T_m(t)} \mathbf{u}_m^2(\theta) d\theta - \int_0^t \mathbf{u}_s^2(\theta) d\theta. \quad (21)$$

are the energy stored (also called energy reservoirs) in the communication channels. Note that the energy is never negative, since $E_i \rightarrow 0$ implies that $\gamma_i \rightarrow 0$ and, consequently, the outgoing wave variables also converge to zero. Therefore, (17) and (18) are satisfied for all t .

Next, we will adapt the wave impedance according to the remote environmental dynamics to improve the transparency of the teleoperation system.

B. Tuning of the Wave Impedance

Transparency in wave-based control frameworks, as previously discussed in Section I, highly depends on the wave impedance, $B_i(t)$. Ideally, $B_i(t)$ should alternate between a small value, B_{min} , when the slave is free to move, and a large value, B_{max} , when the slave makes contact with a stiff environment [7]. Therefore, we propose the update law for the diagonal jj^{th} entry of B_s to be given as

$$\dot{B}_s^{jj}(t) = \begin{cases} \bar{\beta}^j(t), & \text{if } \|f_s^j(t)\| > 0 \\ -\underline{\beta}^j(t), & \text{otherwise} \end{cases} \quad (22)$$

where f_s^j is the j^{th} component of \mathbf{f}_s and $\bar{\beta}^j$ and $\underline{\beta}^j$ are nonnegative, bounded scalar functions that drive B_s^{jj} to B_{max}^{jj} and B_{min}^{jj} , respectively. The motivation behind (22) is to smoothly drive the wave impedance to its ideal value.

IV. STABILITY ANALYSIS

We evaluate now the stability and transparency (in the sense of (2) and (3)) of the bilateral control system under different scenarios. First, we prove passivity and, therefore, closed-loop stability when the environment and the human operator are assumed to be passive. Then, we relax the assumption on the human operator by considering the interaction of the slave robot with a stiff environment.

Proposition 4.1: Consider the teleoperation system in (1) with control law governed by (4), (9), (12), (19), and (22). Suppose that $T_m(t)$ and $T_s(t)$ are time-varying and finite. Furthermore, assume that the human and remote environment are passive with respect to \mathbf{r}_i , i.e., $\exists \kappa_i \in \mathbb{R}$ such that

$$- \int_0^t \mathbf{f}_i^T \mathbf{r}_i d\theta \geq -\kappa_i^2, \quad \text{for } i = \{m, s\}. \quad (23)$$

Then, the closed-loop teleoperator is stable, the coordination error is bounded, and the velocities converge to zero.

Proof: Let $\mathbf{x} = [\mathbf{q}_m^T, \mathbf{q}_s^T, \mathbf{r}_m^T, \mathbf{r}_s^T, \mathbf{f}_m^T, \mathbf{f}_s^T]^T$ and define $\mathbf{x}_t = \mathbf{x}(t + \theta) \in \mathcal{C}, \theta \in [-T_m - T_s, 0]$, where $\mathcal{C} = \mathcal{C}([-r, 0], \mathbb{R}^{6n})$ denotes the space of continuous functions taking the interval $[-r, 0]$ into \mathbb{R}^{6n} for $r \geq 0$ [16]. Consider the following positive definite function

$$\begin{aligned} V(t, \mathbf{x}_t) = & \mathcal{H}_m + \mathcal{H}_s + \kappa_m^2 + \kappa_s^2 - \int_0^t (\mathbf{f}_m^T \mathbf{r}_m + \mathbf{f}_s^T \mathbf{r}_s) d\theta \\ & - \int_0^t (\tau_m^T \mathbf{r}_{md} + \tau_s^T \mathbf{r}_{sd}) d\theta \end{aligned} \quad (24)$$

where $\mathcal{H}_i = \frac{1}{2} \mathbf{r}_i^T M_i(\mathbf{q}_i) \mathbf{r}_i + \frac{1}{2} \mathbf{q}_i^T \Lambda K_i \mathbf{q}_i \geq 0$ for $i \in \{m, s\}$. Note that \mathbf{r}_{md} , τ_m , \mathbf{r}_{sd} , and τ_s are all functions of \mathbf{r}_m and \mathbf{r}_s . Taking the time derivative of (24) we obtain

$$\begin{aligned} \dot{V}(t, \mathbf{x}_t) = & \mathbf{f}_m^T \mathbf{r}_m + \tau_m^T \mathbf{r}_m - \dot{\mathbf{q}}_m^T K_m \dot{\mathbf{q}}_m + \mathbf{f}_s^T \mathbf{r}_s + \tau_s^T \mathbf{r}_s \\ & - \dot{\mathbf{q}}_s^T K_s \dot{\mathbf{q}}_s - \mathbf{f}_m^T \mathbf{r}_m - \mathbf{f}_s^T \mathbf{r}_s - \tau_m^T \mathbf{r}_{md} - \tau_s^T \mathbf{r}_{sd} \\ = & -(\mathbf{r}_{md} - \mathbf{r}_m)^T B_m (\mathbf{r}_{md} - \mathbf{r}_m) - \dot{\mathbf{q}}_m^T K_m \dot{\mathbf{q}}_m \\ & - (\mathbf{r}_{sd} - \mathbf{r}_s)^T B_s (\mathbf{r}_{sd} - \mathbf{r}_s) - \dot{\mathbf{q}}_s^T K_s \dot{\mathbf{q}}_s \leq 0. \end{aligned} \quad (25)$$

Since $\dot{V}(t, \mathbf{x}_t) \leq 0 \forall \mathbf{x}_t$, we can conclude that the closed-loop teleoperation system is stable and that $V(t, \mathbf{x}_t) \leq V(0, \mathbf{x}_t) < \infty$, which implies that \mathbf{r}_m , \mathbf{r}_s , \mathbf{q}_s , \mathbf{q}_m , and therefore, $\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_s$ are bounded. The latter also implies that $\dot{\mathbf{q}}_m$ and $\dot{\mathbf{q}}_s$ are bounded. Moreover, from the fact that $\int_0^t \dot{V}(\phi, \mathbf{x}_\phi) d\phi \leq \infty$, we conclude that $\dot{\mathbf{q}}_m, \dot{\mathbf{q}}_s \in \mathcal{L}_2$. The last result, along with the boundedness of B_i , implies that $\tau_i \in \mathcal{L}_\infty$ (where we applied the fact that M_i, C_i , and \mathbf{g}_i are all bounded). Consequently, we obtain that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Then, using Barbalat's Lemma [17] we can finally conclude that $\dot{\mathbf{q}}_i(t) \rightarrow 0$ as $t \rightarrow \infty$. ■

The above proposition shows boundedness of the coordination error and asymptotic convergence to zero by the velocities. The next statement will provide a bound to the coordination error along with sufficient conditions for its asymptotic convergence to zero.

Proposition 4.2: Assume that $\bar{\beta}^j(t), \underline{\beta}^j(t) \leq c$, $T_m(t) + T_s(t) \leq T$, and $|\dot{T}_i| \leq d$ for some $c, T, d \geq 0$. Then, the coordination error converges to a ball of radius $(cT/B_{min}) \|\mathbf{q}_m\|$.

Moreover, if $\exists t_*$ such that $\gamma_m(t) = \gamma_s(t) = 1 \forall t \geq t_*$, then, the system achieves position coordination and static force reflection in the sense of (2) and (3).

Proof: Consider (24) and its time-derivative (25). Since $\int_0^t \dot{V}(\phi, \mathbf{x}_\phi) d\phi \leq \infty$, we have that $2(\mathbf{r}_{md}(t) - \mathbf{r}_m(t)) = \gamma_m(t)r_s(t - T_s(t)) - \mathbf{r}_m(t)$ and $2(\mathbf{r}_{sd}(t) - \mathbf{r}_s(t)) = \gamma_s(t)\Gamma(t)r_m(t - T_m(t)) - \mathbf{r}_s(t)$ belong to \mathcal{L}_2 , where we used (13) and (14). From Proposition 4.1, we also have that \mathbf{q}_i , $\dot{\mathbf{q}}_i$ and $\ddot{\mathbf{q}}_i$ are bounded, which together with $|\dot{T}_i| \leq d$, implies that $\dot{\gamma}_i$, \dot{r}_{id} , and \dot{r}_i are all bounded. Using then Barbalat's Lemma, we can conclude that $\mathbf{r}_m - \mathbf{r}_{md}$ and $\mathbf{r}_s - \mathbf{r}_{sd}$ converge to zero. Since, $\dot{\mathbf{q}}_i \rightarrow \mathbf{0}$, the latter result also implies that $\gamma_m(t)\mathbf{q}_s(t - T_s(t)) - \mathbf{q}_m(t) \rightarrow \mathbf{0}$ and $\gamma_s(t)\Gamma(t)\mathbf{q}_m(t - T_m(t)) - \mathbf{q}_s(t) \rightarrow \mathbf{0}$. Since T_i are finite, we have that $\gamma_m\mathbf{q}_s \rightarrow \mathbf{q}_m$ and $\gamma_s\Gamma\mathbf{q}_m \rightarrow \mathbf{q}_s$.

Now, let us assume that the energy filters (20) and (21) are initialized at a nonzero value. This assumption does not violate the passivity of the communication channel according to Definition 3.1. Then, we have that $\gamma_i \in (0, 1]$. Similarly, note that $0 \leq \Gamma(t) \leq 1 + cT/B_{min}$ for all t , since both \dot{B}_s and T_i are bounded. Using the latter two statements yields that either $\mathbf{q}_m(t) \rightarrow \mathbf{q}_s(t) \rightarrow \mathbf{0}$, which implies (2), or that $\gamma_m(t)\gamma_s(t)\Gamma(t) \rightarrow 1$, which implies that $\mathbf{q}_m(t) - \mathbf{q}_s(t)$ converges to a ball of radius $(cT/B_{min})\|\mathbf{q}_m\|$. Hence, the proof for the first statement is complete. To prove the second statement, let us assume that $\gamma_m(t) = \gamma_s(t) = 1$ for all $t \geq t_*$. Then, from the previous conclusion we obtain that $\mathbf{q}_s(t) = \gamma_m(t)\mathbf{q}_s(t) \rightarrow \mathbf{q}_m(t)$ as $t \rightarrow \infty$, which implies position coordination in the sense of (2). Similarly, since steady-state conditions imply that $\ddot{\mathbf{q}}_i(t), \dot{\mathbf{q}}_i(t) \rightarrow \mathbf{0}$ and $B_m(t) \rightarrow B_s(t)$, we can show that (5) reduces to

$$2\mathbf{f}_m = -B_m\Lambda(\mathbf{q}_s - \mathbf{q}_m), \quad 2\mathbf{f}_s = -B_s\Lambda(\mathbf{q}_m - \mathbf{q}_s)$$

which yields that $\mathbf{f}_m = -\mathbf{f}_s$ and the proof is complete. ■

Remark 4.1: We showed the stability of the teleoperator when the human and environment are modeled as passive systems. We also showed that the coordination error converges to zero if $\exists t_*$ such that $\gamma_i(t) = 1 \forall t \geq t_*$. The latter implies that $E_i(t) \geq \beta_i \forall t \geq t_*$. Therefore, let us examine the behavior of the energy reservoirs in (20) and (21). We have that the delayed wave variables are compressed during periods of decreasing delay and stretched during periods for which the delay increases. Therefore, since $0 \geq \gamma_i \geq 1$, the energy reservoirs (20) and (21) will increase/decrease when the delay decreases/increases. However, as soon as an energy reservoir decreases below its threshold value β_i , the scale factor γ_i goes below unity attenuating the energy extracted from the energy reservoir. This means that the proposed energy control strategy tends to favor the build up of stored energy in the communication channel. In practice, this strategy leads to $E_i \geq \beta_i$ after some time.

We now relax the previous assumption of passivity on the human operator by considering the interaction of the slave robot with a stiff environment. In the following, we will model the human operator's and environmental forces as

$$\mathbf{f}_m(t) = \boldsymbol{\eta}_m(t) - \rho_m^2 \mathbf{r}_m(t), \quad \mathbf{f}_s(t) = -\rho_s^2 \mathbf{r}_s(t) \quad (26)$$

where ρ_i are positive constants and $\boldsymbol{\eta}_m$ is a bounded force, i.e., $\exists \eta \in (0, \infty)$ such that $\|\boldsymbol{\eta}_m(t)\| \leq \eta \forall t \geq 0$. The human operator's model simulates a non-passive system where the human exerts a bounded force on the master robot but his/her action is resisted by his/her own passive dynamic component $-\rho_m^2 \mathbf{r}_m$ (see [18]). The environment is modeled as a strictly passive system (e.g., a hard surface) with $\rho = \rho_s$.

Proposition 4.3: Consider the teleoperation system in (1) with control law governed by (4), (9), (12), (19), and (22). Let the human operator and environment be modeled as in (26). Assume that $T_m(t)$ and $T_s(t)$ are time-varying and finite. Then, the system is input-to-state stable [17] and the positions and velocities are uniformly ultimately bounded.

Proof: Let $\mathbf{x} = [\dot{\mathbf{q}}_m^T, \dot{\mathbf{q}}_s^T, \mathbf{r}_m^T, \mathbf{r}_s^T]^T$ and $\mathbf{y} = [\mathbf{q}_m^T, \mathbf{q}_s^T, \mathbf{r}_m^T, \mathbf{r}_s^T]^T$. Both vectors are related through a linear diffeomorphism, i.e., $\mathbf{x} = \mathcal{T}\mathbf{y}$, where $\mathcal{T} \in \mathbb{R}^{4n \times 4n}$ is a nonsingular, bounded matrix [19]. Therefore, $\mathbf{x} = \mathbf{0} \iff \mathbf{y} = \mathbf{0}$ and $\mathbf{x} \in \mathcal{L}_\infty \iff \mathbf{y} \in \mathcal{L}_\infty$. Note also that \mathbf{r}_{md} , $\boldsymbol{\tau}_m$, \mathbf{r}_{sd} , and $\boldsymbol{\tau}_s$ are all functions of \mathbf{r}_m and \mathbf{r}_s .

Now, define $\mathbf{x}_t = \mathbf{x}(t + \theta) \in \mathcal{C}$, $\theta \in [-T, 0]$ and consider $V(t, \mathbf{x}_t) = \mathcal{H}_m + \mathcal{H}_s - \int_0^t (\boldsymbol{\tau}_m^T \mathbf{r}_{md} + \boldsymbol{\tau}_s^T \mathbf{r}_{sd}) d\theta$ as Lyapunov candidate function. Taking its time derivative yields

$$\begin{aligned} \dot{V}(t, \mathbf{x}_t) &= \boldsymbol{\eta}_m^T \mathbf{r}_m - \rho_m^2 \mathbf{r}_m^T \mathbf{r}_m - \rho_s^2 \mathbf{r}_s^T \mathbf{r}_s \\ &\quad - \dot{\mathbf{q}}_s^T K_s \dot{\mathbf{q}}_s - (\mathbf{r}_{md} - \mathbf{r}_m)^T B_m (\mathbf{r}_{md} - \mathbf{r}_m) \\ &\quad - \dot{\mathbf{q}}_m^T K_m \dot{\mathbf{q}}_m - (\mathbf{r}_{sd} - \mathbf{r}_s)^T B_s (\mathbf{r}_{sd} - \mathbf{r}_s) \\ &\leq \eta \|\mathbf{r}_m\| - \sum_{i \in \{m, s\}} \left(\rho_i^2 \|\mathbf{r}_i\|^2 + \underline{\sigma}(K_i) \|\dot{\mathbf{q}}_i\|^2 \right) \end{aligned}$$

where $\underline{\sigma}(\cdot)$ denotes the smallest eigenvalue of a square matrix. Next, define $\epsilon = \min\{\rho_m^2, \rho_s^2, \underline{\sigma}(K_m), \underline{\sigma}(K_s)\}$ and let $\epsilon_0 \in (0, \epsilon)$ be an arbitrarily small positive constant. We can then upper bound $\dot{V}(t, \mathbf{x}_t)$ as

$$\dot{V}(t, \mathbf{x}_t) \leq \eta \|\mathbf{x}_t\| - (\epsilon - \epsilon_0) \|\mathbf{x}_t\|^2 - \epsilon_0 \|\mathbf{x}_t\|^2 \quad (27)$$

and obtain that $\dot{V}(t, \mathbf{x}_t) \leq -\epsilon_0 \|\mathbf{x}_t\|^2 \forall \|\mathbf{x}_t\| \geq \frac{\eta}{\epsilon - \epsilon_0}$. Therefore, we can conclude that the system, with state variable \mathbf{x}_t , is input-to-state stable with ultimate bound given by $\eta/(\epsilon - \epsilon_0)$. Since boundedness of \mathbf{x} implies boundedness of \mathbf{y} , we also conclude that the positions and velocities are uniformly ultimately bounded. ■

Next, we will evaluate the case where the human operator exerts a bounded force (26) without a passive component.

Proposition 4.4: Consider the teleoperation system in (1) with control law governed by (4), (9), (12), (19), and (22). Let the human operator and environment be modeled as in (26) for $\rho_m = 0$. Assume that $T_m(t)$ and $T_s(t)$ are time-varying and finite and that $\dot{T}_s < 1$. Then, the system is input-to-state stable and the positions and velocities are uniformly ultimately bounded.

Proof: Suppose that $\exists \bar{\delta} \leq \bar{\delta} < 1$ such that $\bar{\delta} \leq \dot{T}_s(t) \leq \bar{\delta} \forall t \geq 0$ and consider the following positive definite, radially unbounded function

$$\begin{aligned} V(t, \mathbf{x}_t) &= \mathcal{H}_m + \mathcal{H}_s + \rho_0^2 \int_{t-T_s(t)}^t \mathbf{r}_s^T \mathbf{r}_s d\theta \\ &\quad - \int_0^t (\boldsymbol{\tau}_m^T \mathbf{r}_{md} + \boldsymbol{\tau}_s^T \mathbf{r}_{sd}) d\theta. \end{aligned} \quad (28)$$

where $\rho_0^2 = \rho_s^2/(1 - \bar{\delta}) - \rho_1^2 > 0$ for some small enough $\rho_1 > 0$ (note that such ρ_0 and ρ_1 exist as long as $\dot{T}_s < 1$). The time-derivative of (28) can be computed as

$$\begin{aligned} \dot{V}(t, \mathbf{x}_t) = & \mathbf{f}_m^T \mathbf{r}_m + \boldsymbol{\tau}_m^T \mathbf{r}_m - \dot{\mathbf{q}}_m^T K_m \dot{\mathbf{q}}_m + \mathbf{f}_s^T \mathbf{r}_s + \boldsymbol{\tau}_s^T \mathbf{r}_s \\ & \rho_0^2 (1 - \dot{T}_s) (\mathbf{r}_s^T \mathbf{r}_s - \mathbf{r}_s^T(t - T_s) \mathbf{r}_s(t - T_s)) \\ & - \dot{\mathbf{q}}_s^T K_s \dot{\mathbf{q}}_s - \boldsymbol{\tau}_m^T \mathbf{r}_{md} - \boldsymbol{\tau}_s^T \mathbf{r}_{sd} \end{aligned}$$

which after some manipulation can be bounded as

$$\begin{aligned} \dot{V}(t, \mathbf{x}_t) \leq & \eta \|\mathbf{r}_m\| - \underline{\sigma}(K_m) \|\dot{\mathbf{q}}_m\|^2 - \rho_1^2 \|\mathbf{r}_s^T\|^2 \\ & - \rho_2^2 \|\mathbf{r}_s^T(t - T_s)\|^2 - \underline{\sigma}(K_s) \|\dot{\mathbf{q}}_s\|^2 \\ & - \frac{\underline{\sigma}(B_m)}{4} \|\gamma_m(t) \mathbf{r}_s(t - T_s) - \mathbf{r}_m\|^2 \end{aligned}$$

where $\rho_2^2 = \rho_0^2(1 - \bar{\delta}) \leq \rho_0^2(1 - \dot{T}_s)$ is a positive constant. Now, using (13) and combining terms we obtain that

$$\begin{aligned} \dot{V}(t, \mathbf{x}_t) \leq & \eta \|\mathbf{r}_m\| - \underline{\sigma}(K_m) \|\dot{\mathbf{q}}_m\|^2 - \rho_1^2 \|\mathbf{r}_s^T\|^2 \\ & - \underline{\sigma}(K_s) \|\dot{\mathbf{q}}_s\|^2 - \left(\frac{1}{4} \underline{\sigma}(B_m) - \mu^2\right) \|\mathbf{r}_m\|^2 \end{aligned}$$

where $\mu = \frac{1}{4} \underline{\sigma}(B_m) (\rho_2^2 + \frac{1}{4} \underline{\sigma}(B_m))^{-1/2}$. If we define $\varepsilon = \min\{\rho_1^2, \frac{1}{4} \underline{\sigma}(B_m) - \mu^2, \underline{\sigma}(K_m), \underline{\sigma}(K_s)\} > 0$ and let $\varepsilon_0 \in (0, \varepsilon)$, we obtain (27) and conclude that $\dot{V}(t, \mathbf{x}_t) \leq -\varepsilon_0 \|\mathbf{x}_t\|^2, \forall \|\mathbf{x}_t\| \geq \frac{\eta}{\varepsilon - \varepsilon_0}$. The latter implies that the system is input-to-state stable with ultimate bound given by $\eta/(\varepsilon - \varepsilon_0)$. Since boundedness of \mathbf{x} implies boundedness of \mathbf{y} , we also conclude that the positions and velocities are uniformly ultimately bounded. ■

V. NUMERICAL EXAMPLE

We simulate the response of two identical 1-DOF linear robots with asymmetric, time-varying communication delays. The dynamics for the master and slave robots are given according to (1) for $M_i = 1$ [kg], $C_i = 0$ [kg/s], and $g_i = 0$ [N]. The communication delays are taken as $T_m(t) = 0.5 + 0.2 \cos(5t)$ [s] and $T_s(t) = 0.5 + 0.2 \sin(5t)$ [s] for a maximum round-trip delay of $T = 1 + \frac{\sqrt{2}}{5}$ [s]. The environment is modeled as a stiff wall located at $\mathbf{q}_w = 6$ [m] with a spring-damper reaction force given by $\mathbf{f}_s = 10^4(\mathbf{q}_w - \mathbf{q}_s) - 10^2 \dot{\mathbf{q}}_s$ [N] if $\mathbf{q}_s \geq \mathbf{q}_w$ and 0 [N] otherwise. Likewise, the human operator is modeled as a saturated PD-type controller with $\mathbf{f}_m(t) = \text{sat}_{-15,15}\{20(\mathbf{q}_d(t) - \mathbf{q}_m(t)) - \dot{\mathbf{q}}_m\}$, where \mathbf{q}_d is the desired trajectory (traced in Fig. 2) and $\text{sat}_{\underline{x}, \bar{x}}\{\cdot\}$ is the saturation function with \underline{x} and \bar{x} as lower and upper bounds.

The gains for the controller are chosen as $K_i = 0.2$ [N/m] and $\Lambda = 0.5$ [1/s], while the update laws for B_s are

$$\begin{aligned} \bar{\beta}(t) &= 0.05(B_{max} - B_s(t)) + 0.1(B_{max} - B_s(t))^{\frac{3}{2}} \\ \underline{\beta}(t) &= 0.05(B_s(t) - B_{min}) + 0.1(B_s(t) - B_{min})^{\frac{3}{2}} \end{aligned} \quad (29)$$

where $B_{min} = 2$ [N · s/m] and $B_{max} = 50$ [N · s/m] are assumed to be the ideal values.

Fig. 2 to 5 contrast the motion and force response of the teleoperator under a time-constant and a time-varying wave impedance value. For the constant wave impedance case we chose a trade-off (average) between the two ideal values, i.e., $B_i(t) = B_{ave} = (B_{min} + B_{max})/2$. Observe that for both

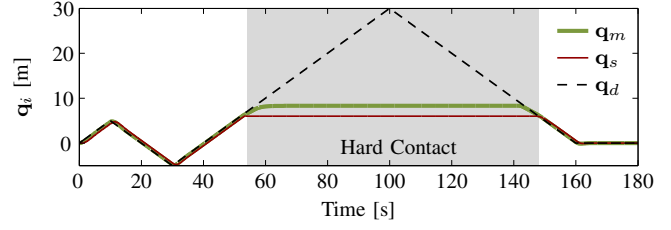


Fig. 2. Position of Master and Slave Robots when $B_{m,s} = B_{ave}$.

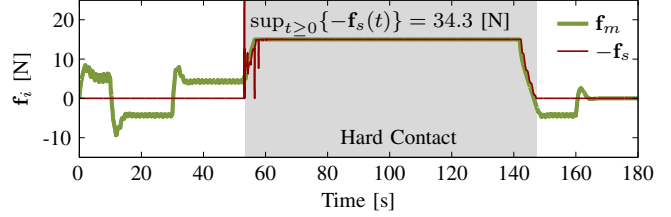


Fig. 3. Human and Environmental Forces when $B_{m,s} = B_{ave}$.

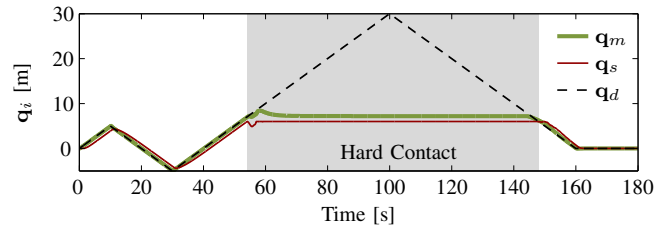


Fig. 4. Position of Master and Slave Robots when $B_i(t)$ are Time-Varying.

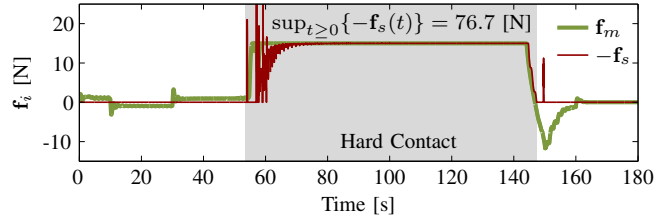


Fig. 5. Human and Environmental Forces when $B_i(t)$ are Time-Varying.

cases, the master and slave robots track the desired trajectory \mathbf{q}_d with relatively small error during the free motion segment of the simulation, that is, for $t \in [0, 54]$ [s] and $t \geq 148$ [s]. However, note from the force plots (see Fig. 3 and 5) that the operator has to apply a larger force during free motion when using the constant wave impedance value. This is an adverse effect on transparency due to the use of a trade-off value that is larger than the ideal wave impedance B_{min} .

The hard contact scenario takes place at $t \in [54, 148]$ [s] and is highlighted in the plots by the gray-shadowed rectangular area. Observe that static force reflection is achieved more rapidly (refer to Fig. 5) when a constant wave impedance is used. Note, however, that both controllers eventually achieve a satisfactory force tracking behavior. In terms of the master-slave coordination error (see Fig. 2 and 4), the time-varying wave impedance case yields a smaller error.

For comparison purposes, Fig. 6 plots the master-slave

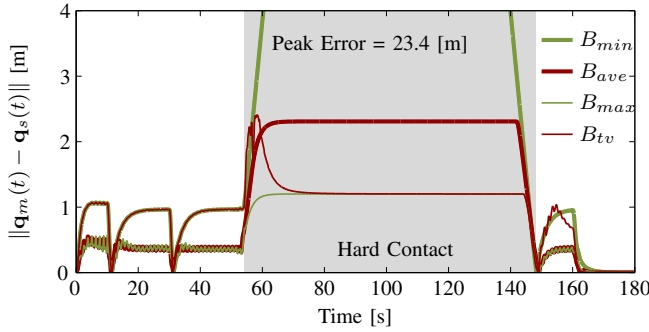


Fig. 6. Master-Slave Coordination Error for Constant Wave Impedance Values $B_{min} = 2$, $B_{max} = 50$, and $B_{ave} = (B_{min} + B_{max})/2$, and Time-Varying Wave Impedance $B_{tv} \in [B_{min}, B_{max}]$.

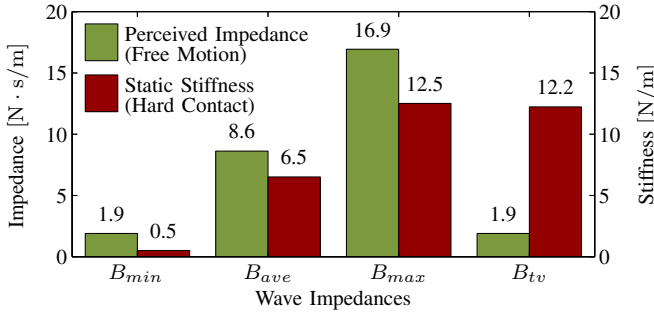


Fig. 7. Impedance and Static Stiffness Perceived by the Human Operator During Free Motion and Hard Contact, respectively.

coordination error using different impedance values. Note that the proposed time-varying wave impedance approach satisfactorily assimilates the ideal behavior of using a constant lower impedance while the slave is in free motion and the behavior of using a larger impedance once the slave robot makes contact with the wall. Likewise, the average transmitted impedance and static stiffness to the human operator while in free and hard contact motion scenarios, respectively, are plotted in Fig. 7 for different values of B_i . The average impedance Z_{ave} and static stiffness K_{ave} where computed as

$$Z_{ave} = \frac{1}{t_b - t_a} \int_{t_a}^{t_b} \frac{\mathbf{f}_m}{\dot{\mathbf{q}}_m} dt, \quad K_{ave} = \frac{1}{t_d - t_c} \int_{t_c}^{t_d} \frac{\mathbf{f}_m dt}{\|\mathbf{q}_m - \mathbf{q}_s\|}$$

where $[t_a, t_b] = [11, 29]$ [s] and $[t_c, t_d] = [55, 145]$ [s] are intervals of time for which $\dot{\mathbf{q}}_m \in \mathcal{R}$ and $\mathbf{q}_m - \mathbf{q}_s \in \mathcal{R}$ do not cross zero. In general, lower values of impedance and larger values of stiffness are desirable. Observe from Fig. 7 that the best overall results are obtained when a time-varying wave impedance is employed.

VI. CONCLUSION

In this paper, we presented a novel wave-based bilateral control architecture that passively improves the transparency of the teleoperation system regardless of the presence of time-varying communication delays. The control framework is built using concepts of passivity and exploits the effect that the wave impedance has on transparency. The main contribution lies on the provision of an update policy that

smoothly adjusts the wave impedance value according to the current environmental dynamics without compromising the stability of the closed-loop system. We showed that the proposed control framework guarantees asymptotic convergence of the velocities and boundedness of the coordination error if the human operator and environment are passive. Then, we relaxed the passivity assumption on the human operator and showed that the master-slave position error and velocities are ultimately bounded. Furthermore, we illustrated via simulations the effectiveness of the control strategy and compared its performance against the use of different constant wave impedance values. The simulation results demonstrated that the proposed controller can effectively adjust the transmitted impedance to the operator according to the remote dynamics.

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