Optimizing Query Injection from Mobile Objects to Sensor Networks

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Abstract

To facilitate flexible data discovery, sensor networks can be supported by query processing, where a query is injected into the sensor network from some base station. In his paper we consider the problem of query injection by base stations that are mobile (mobile objects) and individual sensors are "location-ignorant." The idea is to have mobile objects take advantage of each other's independent motion plans to do a form of opportunistic query injection. We discuss methods to optimize query injection in terms of optimal injection points and transmission ranges. Numerical simulations on coverage rate metrics are provided to support the proposed methods.

Index Terms: mobile object, query injection, sensor networks.

1. Introduction

1.1 Background

Many applications can benefit from sensor nodes randomly scattered in some environmental space. Each of these scattered nodes has the capability to collect and route data either to other sensor nodes or back to an external base-station. A base station may be a fixed node or a mobile node capable of connecting the sensor network with an existing network, where a user has access to the reported data. To facilitate flexible data discovery, sensor networks can be supported by query processing – a query is injected into a sensor network from some base station, and query results are then sent back to a base station after some appropriate data routing among the sensor nodes.

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Most previous research assumes sensor networks have some type of GPS capability, either by GPS-enabled sensor nodes [1], application nodes (AN) [2], or beacon nodes [3]. However, it is generally not practical to integrate a GPS receiver with a sensor node for applications with a very large number of sensor nodes. The GPS receiver's antenna size, as well as the monetary and power costs associated with GPS components, contradicts with the basic sensor node's requirements of small size, low cost, and low power consumption, respectively. A beacon node or an application node (AN) can be two orders of magnitude more expensive than a sensor node if a GPS receiver is available for the beacon or AN. This means that, even if only 10% of the sensor nodes are beacons or ANs, the cost of the network will increase at least tenfold. Also, GPS receivers of beacons do not serve much purpose after sensor node's localization has been achieved. Such capability can add a significant degree of resource wastefulness. Sichitiu and Ramadurai presents an approach that uses a localization technique based on a single mobile beacon node that is aware of its position [4]. The precision of the localization is achieved only when the trajectory of the beacon node covers the entire deployment area. However, for wide-area environment monitoring, it may not be natural (or feasible) for one beacon node to travel within the proximity of all sensor nodes in the environment. For example, consider the case of forest rangers (mobile objects) operating within a forest covering thousands of acres. To help monitor the environment, possibly assisting in the search for a lost person, sensors may be densely dispersed within this environment. If some ranger is interested in a sensor value for some remote region (a region not within the ranger's local transmission range), that ranger should not be required to travel to that region in order to collect the required data – the ranger should be able to disseminate the query to other mobile objects, allowing one or more of those objects to inject the query when/if they happen to reach an appropriate location with respect to the region to be sensed. Thus, we are not requiring travel plans of an individual mobile object to be dependent on queries that that object happens to be carrying.

To our knowledge, there is a lack of research on the problem of query processing for mobile objects operating in the context of sensor networks, especially for the case when individual sensors are accepted to be "location-ignorant" and mobile objects take advantage of each other's independent motion plans to do a form of opportunistic query injection.

For this work, we assume that a mobile object plays the dual roles associated with users and mobile base stations. Key properties of mobile objects, such as location-awareness, are used to offset the constraints associated with sensor nodes. Our research explores the problem of optimization of query injection from mobile objects to location-ignorant sensor nodes.

1.2 Mobile objects and sensor nodes

We assume that a mobile object moves within the environment – maybe with destinations that change over time or following a fixed or random motion path during "idle" periods. Further, it is assumed that such an object has high computational and communication ability, and is supported by a rechargeable battery, a GPS receiver and a synchronized clock system. Because of its GPS capability, a mobile object can calculate its own velocity. In addition to these just mentioned basic properties, a mobile object is assumed to have the following additional practical attributes:

1) A relatively large transmission range. A mobile object's transmission range, which we denote as *OTR*, is typically much larger than a sensor node's transmission range. For example, a typical sensor node's transmission range is on the order of 100 meters. However, IEEE 802.16 allows connectivity up to 40 kilometers between mobile devices without a direct line of sight.

2) An adjustable transmission range [5][6][7]. Each mobile object can adjust (decrease) its transmission range. This can be useful in order to limit the number of engaged sensor nodes in reaction to the injection of a query into a sensor network.

After being scattered in the environment (possibly by dispersal from a plane or other imprecise means), a sensor node operates with limited battery power and limited computational ability and is located at some unknown static position.

2. Query details

For this paper, we assume that some mobile object carries a query that seeks to obtain sensor data from a target region during some time interval. We refer to the target region as a *query region* and for simplicity we assume it is a circular region. The source query may have originated with the mobile object or have been disseminated among mobile objects in a peer-to-peer fashion – this is outside the scope of our concern. As an example, assume that a mobile object MO100 issues the following query at time 500: "During the next 30 time units report the temperature associated with the query region with center (45, 90) and radius 10."

A *query q* is a 5-tuple: *q* = (*q-id, source-id, q-region, and q-expiration*), where

q-id is a unique query identifier; *source-id* is an identifier (name) of the mobile object that generates the query; *q-region* is the query region; *q-region* = (*S, r),* where *S* is the (x, y) coordinate position of the center of the query region and *r* is the radius of the query region; *q-expiration is* the expiration time of the query and *q-expiration* = *q-time* + *q-duration*, where *q-time* is a time stamp for when the *source-id* object generates the query , and *q-duration* is the time period during which the query is valid.

In terms of the above example, the query can be expressed as the 5-tuple:

q = (*q1, MO100,* ((*45, 90*)*, 10*)*, 530*).

3. Query injection with fixed transmission range

Since we are considering the case where senor nodes are location-ignorant, there is limited value in performing query routing within the sensor network. In contrast, with our approach, a mobile object injects a query only when the query region is within the object's transmission range. This approach is practical if a large number of mobile objects, like vehicles or soldiers, move within the environment and a mobile object carries queries generated by other mobile objects or itself.[1] A mobile object can predict whether the query region will be within its transmission range, based on the object's own current speed and direction.

To determine the location to inject a query, the mobile object first needs to compute its future (predicted) path. Each mobile object knows its current location, *C* (*x, y*); current velocity, $\overline{V(v_x, v_y)}$; local time, *current-time*; and the query's expiration time, *q-expiration.* Based on this information, the object can easily compute an end point *E* (x_2, y_2), which is the predicted location of the mobile object when the query expires. The line segment $\overline{CE}$ is called the *active-query segment* since it represents points at which the query is still active, i.e., not expired. See Figure 1. Recall that *S* (*x, y*) is the center of query region.

[1] We assume that query routing among mobile objects (i.e., within the mobile-object network) is used, although details on this phase of overall query processing are outside the scope of this paper.

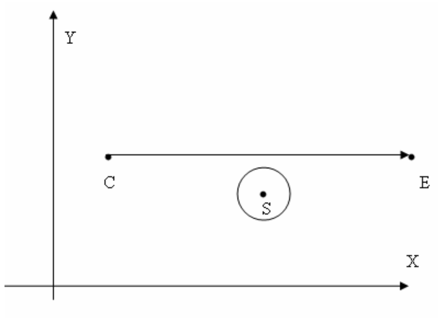

Figure 1. Active-query segment

Definition: For a mobile object with current position $C(x, y)$, carrying a query q and traveling with velocity $\overrightarrow{V(v_x, v_y)}$ at time t, the *active-query segment* $\overline{CE}$ is a segment with end points $C(x, y)$ and $E(x, y)$, where $E(x, y) = \overrightarrow{V(v_x, v_y)} * (q\text{-}expiration - t) + C(x, y)$.

An *injectable-query segment* is a sub-segment of the active-query segment. The injectable-query segment defines the locations during which the distance between the mobile object and the center $S(x, y)$ is less than or equal to ($OTR + r$) (the sum of the transmission range of the object and the radius of the query region). Thus the query can be injected to some sensor nodes in the query region. The injectable-query segment $\overline{C'E'}$ is a chord of the circle with center $S(x, y)$ and radius (OTR+ r), as shown in Figure 2.[2]

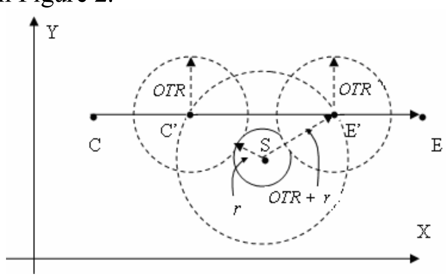

Figure 2. Injectable-query segment

Definition: For a mobile object that is carrying a query q and has an active-query segment $\overline{CE}$, the *injectable-query segment* $\overline{C'E'}$ is the following set of points:

$\overline{C'E'} = \{P(x, y)$, s.t. $P(x, y)$ belongs to $\overline{CE}$ and $|P - S| \leq (OTR + r)\}$.

A mobile object can inject a query to the sensor network when it reaches an *injection point* (IP), which can potentially be any point on $\overline{C'E'}$.

Definition: For a query injected at injection point IP, the *dissemination region* (DR) is the region formed by the following set of points:

$DR = \{P(x, y)$, s.t. $|P - IP| \leq OTR\}$.

The dissemination region is simply the region that is covered by an injected query. See Figure 3. The goal of sending (broadcast) a query at some injection point is to have the query directly reach sensor nodes within the query region. Depending on the specific injection point, a query will cover some portion of the query region, denoted as the *covered query region, CQR*, as shown in Figure 3.

Definition: For a query injection, the *covered query region (CQR)* is the region formed by the following set of points:

$CQR = \{P(x, y)$, s.t. $|P - S| \leq r$ and $|P - IP| \leq OTR\}$; or equivalently, $CQR = DR \cap QR$.

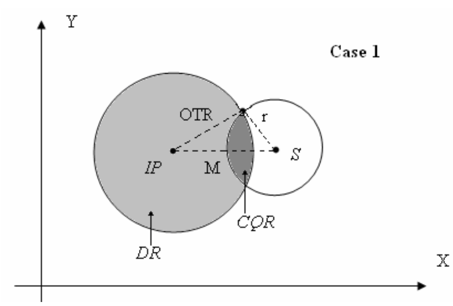

Figure 3. Partial coverage of query region

Intuitively, a mobile object should choose an *injection point* that will maximize the coverage of sensor nodes in the query region. We introduce a metric called the *query region coverage rate* (*QRCR*) to measure the effectiveness of query dissemination.

$$QRCR = \frac{\#\ of\ sensors\ in\ covered\ query\ region}{\#\ of\ sensors\ in\ query\ region}$$

Assuming a large number of sensors uniformly distributed in the environment, then:

$$QRCR = \frac{area\ of\ covered\ query\ region}{area\ of\ query\ region}$$

To analyze the coverage metric, we can identify four cases and then evaluate the relationship between the coverage metric and the position of the mobile object when it injects a query (at an injection point). Let M denote the distance between the injection point and the center of the query region, i.e., $M = |IP - S|$.

Case 1: The dissemination region (DR) and query region (QR) partially overlap, as shown in Figure 3. $CQR \neq \emptyset$ and $CQR \neq QR$.

Case 2: DR and QR do not overlap, as shown in Figure 4. $CQR = \emptyset$.

Case 3: QR is fully contained within DR, as shown in Figure 5. $CQR = QR$.

Case 4: DR is fully contained within QR, as shown in Figure 6. $CQR = DR$.

[2] Although in practice radio signal coverage may not be uniform in all directions from a source, we adopt a circular model, which is quite common in the literature, e.g., [8].

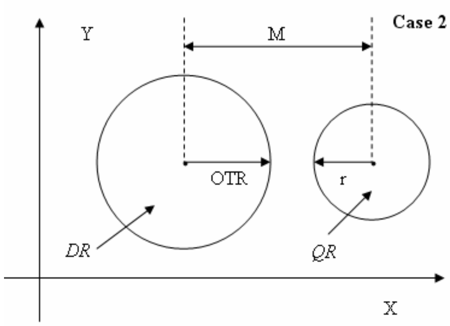

Figure 4. No coverage of query region

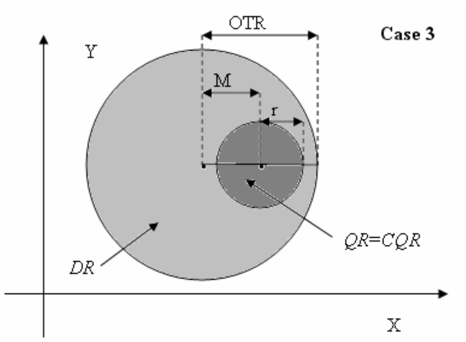

Figure 5. Total coverage of query region

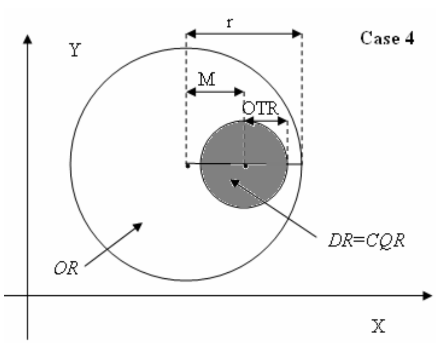

Figure 6. Total coverage of dissemination region

Case 1:

For this case, OTR – $r < M < OTR + r$. The boundaries of the regions *QR* and *DR* have two intersecting points, labeled as *G* and *H* in Figure 7.

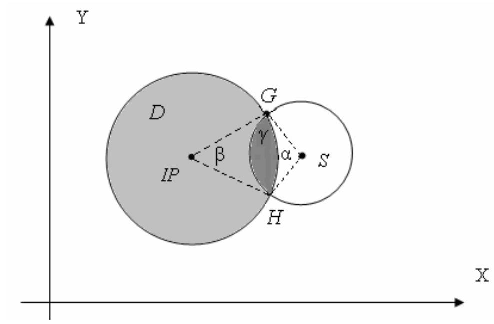

Figure 7. Intersecting boundary points

Assume α, β and γ are the angles of sector (G, S, H), sector (G, IP, H) and angle (IP, G, S), respectively. Given $OTR = |IP - G|$ *and* $|G - S| = r$, then α, β and γ are all functions of *M*. Using some basic geometry:

QRCR = (area of sector (G, S, H) – area of triangle (G, S, H) + Area of sector (G, IP, H) – area of triangle (G, IP, H))/area of QR

Area of sector (G, S, H) = $\frac{1}{2} r^2 * \alpha$

Area of triangle (G, S, H) = $\frac{1}{2} r * r * \sin(\alpha)$

Area of sector (G, IP, H) = $\frac{1}{2} OTR^2 * \beta$

Area of triangle (G, IP, H) = $\frac{1}{2} OTR * OTR * \sin(\beta)$

Area of *QR* = $\pi * r^2$

$$QRCR = \frac{\left[\frac{1}{2} r^2 * (\alpha - \sin(\alpha)) + \frac{1}{2} OTR^2 * (\beta - \sin(\beta))\right]}{\pi * r^2}$$

Lemma 1: For Case 1*, QRCR* is a decreasing function with respect to the distance measure *M*. Proof is in the appendix.

For Case 1, the mobile object can inject a query at any point on the injectable-query segment, but by Lemma 1, *QRCR* is maximized when *M* is minimized. This matches intuition – we achieve greater query region coverage when the injection point is close to the query region. So, the optimal injection point is the injection point that is closest to the center of the query region. Figure 8 shows an example situation.

For a mobile object that is carrying a query q and has an injectable-query segment $\overline{C'E'}$, the optimal injection point *OIP* is the following point: *IP* (x, y), s.t. *IP* (x, y) belongs to $\overline{C'E'}$ and Min ($|IP - S|$).

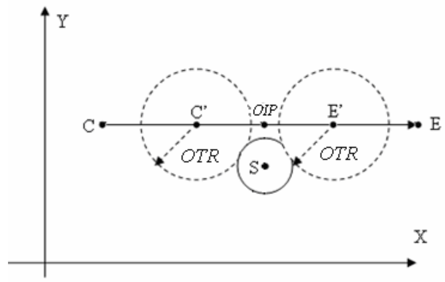

Figure 8. Optimal injection point

Case 2:

For Case 2, $M > OTR + r$. Thus, $QRCR = 0$, independent of M. Since the query region coverage rate is always zero, there is no optimal injection point. OIP = NULL.

Case 3:

For Case 3, $OTR \geq r$ and $M \leq OTR - r$. Thus, $QR = CQR$ and $QRCR = 1$, independent of M. In this case, there is complete coverage of the query region. So, any injection point is an optimal injection point; there is a set of optimal injection points.

Case 4:

For Case 4, $OTR < r$ and $M < r - OTR$. Thus, $DR = CQR$ and $QRCR = (\frac{\pi * OTR^2}{\pi * r^2}) = (\frac{OTR}{r})^2$, independent of M. Since $QRCR$ is not dependent on M, any injection point is an optimal injection point; there is a set of optimal injection points.

4. Query injection with adjustable transmission range

From the above analysis, we determined that a mobile object should inject a query when the object reaches the optimal injection point, thus achieving maximum *query region coverage rate*. We refer to those sensor nodes in the query region as "target sensor nodes".

But, maximizing this form of coverage may still result in query reception by some number (possibly many) sensor nodes that are *not* within the query region. In Figure 9, which represents a case when the coverage rate is optimal (equal to 1), we see many such sensor nodes that are being "reached" inadvertently. We refer to these sensor nodes as "unintended sensor nodes." Those sensor nodes outside the dissemination region are called "uncovered sensor nodes" (they are not covered/reached by an injected query).

Definition: A sensor node located at P (x, y), is a *target sensor node* (*TSN*) iff $|P - S| < r$.

Definition: A sensor node located at P (x, y), is an *unintended sensor node* (*UISN*) iff $|P - S| > r$ and $|P - IP| < OTR$.

Definition: A sensor node located at P (x, y), is an *uncovered sensor node* (*UCSN*) iff $|P - IP| > OTR$.

Note: Some sensor nodes can be both target sensor nodes and uncovered sensor nodes.

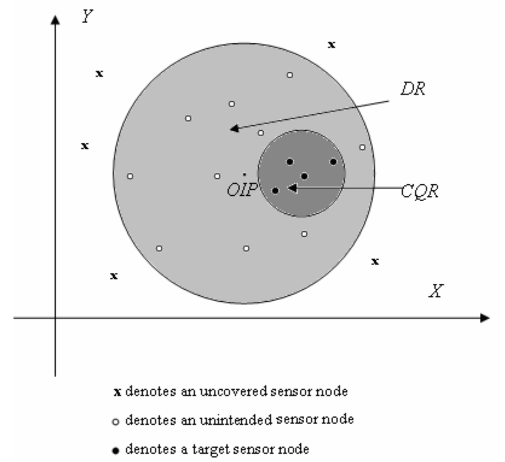

Figure 9. Three types of sensor nodes

As mentioned in Section 1, assume that a mobile object has an adjustable transmission range. Thus, the mobile object can utilize this ability to reduce the number of undesired target sensor nodes. This is illustrated in Figure 10, where the object's transmission range has been reduced, in comparison to the situation in Figure 8.

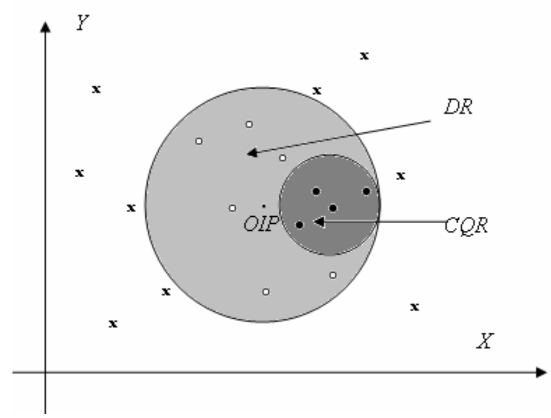

Figure 10. Reduced transmission range

Figure 11 shows a case where the transmission range has been reduced even further, to the point that there are only two undesired target sensor nodes. But this gain is at the sacrifice of also possibly reducing the number of sensors in the covered query region. Clearly, there is a tradeoff to be managed.

Thus, we now introduce a new coverage metric, *dissemination region coverage rate* (*DRCR)*, to measure the effectiveness of covering the query region with a reduced object transmission range.

$$DRCR = \frac{\#\ of\ sensors\ in\ covered\ query\ region}{\#\ of\ sensors\ in\ dissemination\ region}$$

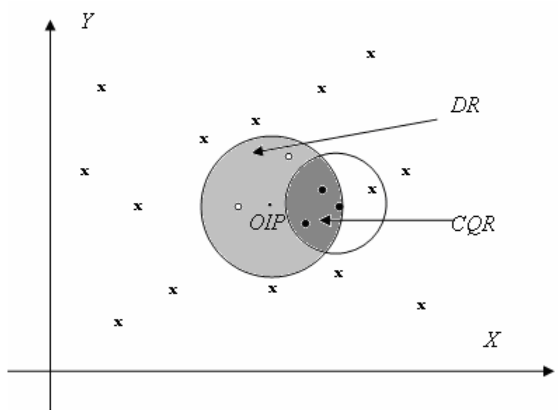

Figure 11. Further reduced transmission range

As before, if we assume a large number of uniformly distributed sensors, then

$$DRCR = \frac{area\ of\ covered\ query\ region}{area\ of\ dissemination\ region}$$

Now, *DRCR* can be analyzed in three cases, where we let *R* denote the object's (reduced) transmission range at the time of query injection.

Case A: The dissemination region and query region partially overlap, but the optimal injection point is not within the query region. In this case, $CQR \neq \emptyset$ and $|OIP\text{-}S| > r$; thus (similar to the previous analysis for QRCR)

$$DRCR = \frac{\left[\frac{1}{2}r^2 * (\alpha - \sin(\alpha)) + \frac{1}{2}R^2 * (\beta - \sin(\beta))\right]}{\pi * R^2}$$

Although the maximum dissemination region coverage rate with respect to *R* can be characterized by solving the ordinary differential equation $\frac{d(DRCR)}{dR} = 0$, we simplify the analysis by using numerical simulation, as presented in Section 5. The simulation reveals that *DRCR* increases as *R* decreases, but up to a limit.

Case B: The dissemination region and query region do not overlap, as shown in Figure 4. In this case, $CQR = \emptyset$; thus $DRCR = 0$, independent of *R*.

Case C: The dissemination region and query region overlap, and the optimal injection point is within the query region, as shown in Figure 12. $QR \cap DR \neq \emptyset$ and $|OIP - S| < r$. To maximize DRCR, the mobile object can reduce its transmission range so that *DR* is within *QR*, i.e., $R \leq r - M$. In this case, $CQR = DR$ and $DRCR = \pi R^2/\pi R^2 = 1$.

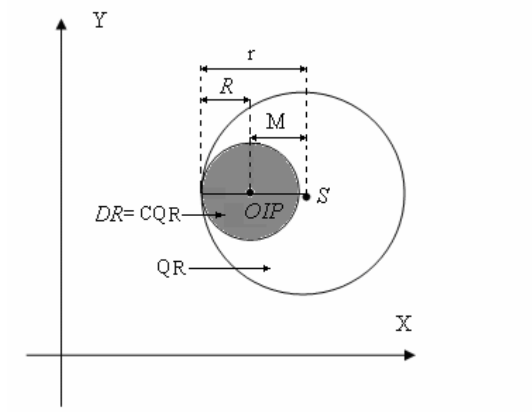

Figure 12. Covered query region, given *M* < r.

5. Numerical simulation

We analyze the query region coverage rate and the dissemination region coverage rate metrics using MATLAB 7.0.

Figure 13 shows the simulation results of *QRCR*, given *OTR*=500, *r*=400, M_{min}=500. This result is consistent with Lemma 1 for Case 1: *QRCR* is a decreasing function and it is maximized when the mobile object injects a query at the *optimal injection point* $M = M_{min}$ =500.

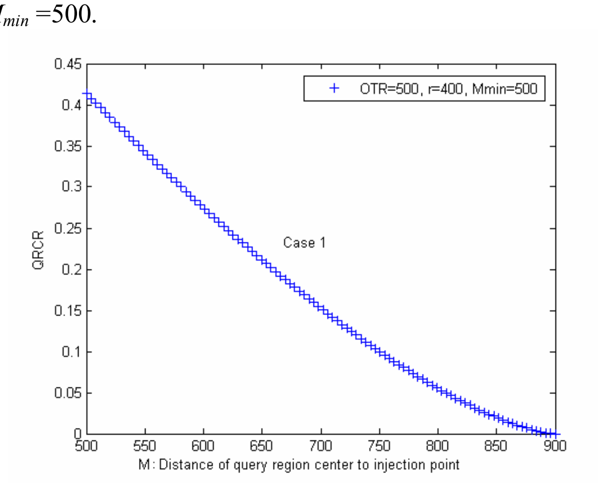

Figure 13. *QRCR*, given OTR=500, r=400, M_{min}=500.

Figure 14 shows the simulation results of *QRCR*, given *OTR*=500, *r* = 100, M_{min} = 200. This analysis result is consistent with our analysis for Case 2: when $M > OTR + r$, $QRCR = 0$; and also for Case 3: when $M < OTR - r$, $QRCR = 1$.

Figure 15 shows the simulation results of *QRCR*, given *OTR*=300, *r* = 500, M_{min} = 100. This result is consistent with our analysis for Case 4: when $OTR < r$ and $M < r - OTR$, $QRCR = (\frac{OTR}{r})^2 = 0.6^2 = 0.36$.

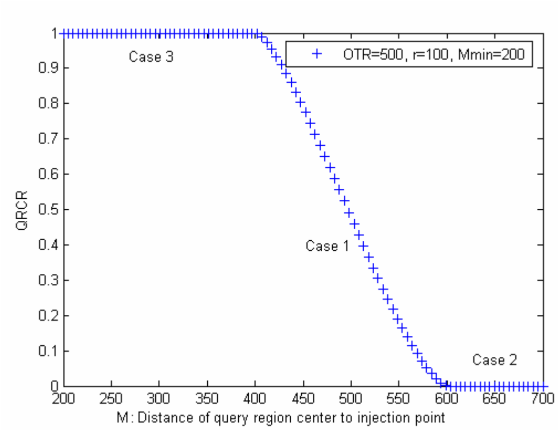

Figure 14. *QRCR*, given OTR=500, r=100, M_{min}=200.

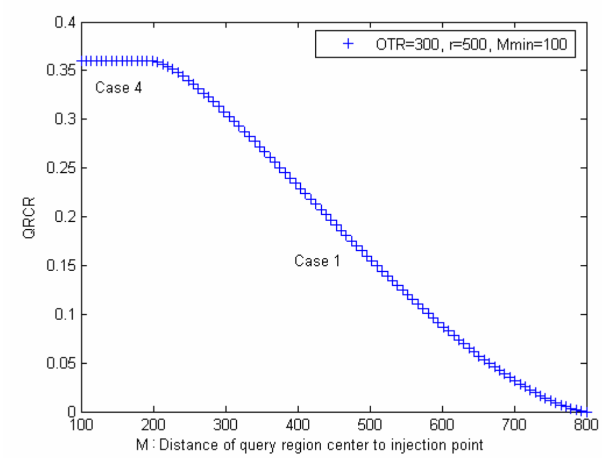

Figure 15. *QRCR*, given OTR=300, r=500, M_{min}=100.

Figure 16 gives simulation results for *DRCR,* with respect to *R* for varying values of *M*. The query region radius is 500. This analysis corresponds to Case A (*M*>*r*).

We can observe that the coverage rate improves initially when the transmission range (*R*) of the mobile object decreases from its initial range value of 2000. This is due to a reduction in the number of unintended sensor nodes. But then the coverage rate decreases after *R* goes below some value. This is due to the fact that the rate at which nodes change from target nodes to uncovered nodes is greater than the rate at which nodes change from unintended nodes to uncovered nodes. The peak value for *R* is the optimal transmission range to be used for query injection. For example, based on the simulation data of Figure 16, if a mobile object injects a query when the object is 550 units from the query region, then the optimal transmission range is 360 units and this achieves a dissemination region coverage rate of 34%.

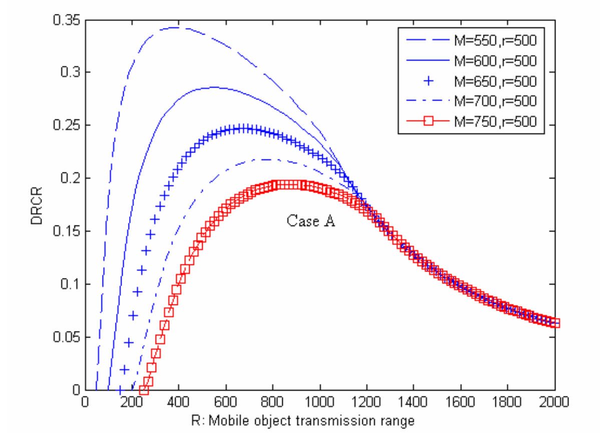

Figure 16. Coverage rate of dissemination region

Figure 17 gives analytical results for *DRCR* under different parameter values of *M*, where $M < r$ and *M* varies from 400 to 200. Again, the radius of the query region is assumed to be 500. When the mobile object adjusts its transmission range *R* to be less than or equal to $r - M$, *DRCR* is 1, corresponding to Case C.

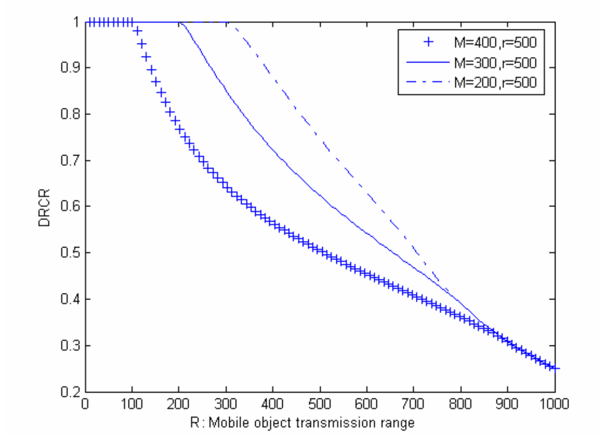

Figure 17. DRCR, given r = 500, M < r.

6. Conclusion and future work

This paper discussed the problem of optimizing query injection for mobile objects operating within the context of a sensor rich environment. Advanced properties of mobile objects, such as location-awareness, are used to offset the constraints associated with sensor nodes. We introduced query region coverage rate and dissemination region coverage rate as metrics to evaluate the effectiveness of a query injection action. To maximize the coverage metrics, we considered two key mobile object properties: optimal injection point and optimal transmission range.

Future research will expand the method and analysis to include the other aspects of complete query processing, not just query injection. For example, after a mobile object injects a query to a sensor network, all sensor nodes in the dissemination region should sense their environment and route a query result back to the source mobile object. One topic of future work will be query result routing from sensor nodes to a source mobile object. Another area of future work can be query routing among peer mobile objects – recall that some mobile object may carry a query that is generated by some other source object. Finally, we plan to perform more explicit simulations considering environment issues like simulation time and node density, and consider performance comparisons with related works.

Acknowledgments

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Appendix

Lemma 1: For case 1, *QRCR* is a decreasing function with respect to the distance measure *M*, where

$$QRCR = \frac{\left[\frac{1}{2}r^2 * (\alpha - \sin(\alpha)) + \frac{1}{2}OTR^2 * (\beta - \sin(\beta))\right]}{\pi * r^2}$$

Proof:

First, note that α, β and γ are all functions of *M*.

Take derivative over *M* on both sides:

$$\frac{d(QRCR)}{dM} = \frac{\left[\frac{1}{2}r^2 * (1-\cos(\alpha)) * \alpha' + \frac{1}{2}OTR^2 * (1-\cos(\beta)) * \beta'\right]}{\pi * r^2}$$

$$= \frac{\left[r^2 * (\sin(\frac{\alpha}{2}))^2 * \alpha' + OTR^2 * (\sin(\frac{\beta}{2}))^2 * \beta'\right]}{\pi * r^2}$$

$$= \frac{\left[\frac{1}{4} * (2r\sin(\frac{\alpha}{2}))^2 * \alpha' + \frac{1}{4}(2OTR * \sin(\frac{\beta}{2}))^2 * \beta'\right]}{\pi * r^2}$$

$$= \frac{\left[\frac{1}{4} * (|GH|)^2 * \alpha' + \frac{1}{4}(|GH|)^2 * \beta'\right]}{\pi * r^2}$$

$$= \frac{\left[(|GH|)^2 * (\alpha' + \beta')\right]}{4\pi r^2}$$

$$= \frac{\left[(|GH|)^2 * (2\pi - 2\gamma)'\right]}{4\pi * r^2}$$

$$= \frac{-\left[(|GH|)^2 * \gamma'\right]}{2\pi * r^2} \quad (1)$$

Now, consider γ'.

From Figure 7, we observe that

$M^2 = OTR^2 + r^2 - 2r * OTR * \cos(\gamma)$.

Take derivative over *M* on both sides:

$$\frac{d(M^2)}{dM} = \frac{d(OTR^2 + r^2 - 2r * OTR * \cos(\gamma))}{M}$$

$$2M = 2r * OTR * \sin(\gamma) * \gamma' \quad (2)$$

Since γ is one angle of triangle (IP, G,S), $0 \leq \gamma \leq \pi$.

Thus, $\sin(\gamma) \geq 0$. (3)

In (2), *M, r* and *OTR* are all greater than zero and $\sin(\gamma) \geq 0$ (from (3)). Therefore, $\gamma' > 0$. (4)

Now, (1) can be written as:

$$\frac{d(QRCR)}{d(M)} < 0$$

So, *QRCR* is a decreasing function with respect to the distance measure *M*.