OPTIMAL MULTI-CHANNEL TIME-SEQUENTIAL ACQUISITION IN DYNAMIC MRI WITH PARALLEL COILS

Behzad Sharif and **Yoram Bresler**

Coordinated Science Laboratory, Department of Electrical and Computer Engineering University of Illinois at Urbana Champaign, IL, USA

Abstract

Magnetic resonance imaging with parallel coils is commonly used as a method to speed up the acquisition by exploiting spatial encoding. Another, distinct acceleration method is spatio-temporal modeling with adaptive acquisition, which has been previously proposed for dynamic cardiac MRI with a single coil. We propose an optimal acquisition scheme for parallel dynamic MRI combining the two methods. It provides accelerations up to the product of the individual accelerations. The proposed scheme produces an artifact-free cine of the dynamic object, with high spatial and temporal resolution. Simulations demonstrate the method for cases where neither parallel imaging nor adaptive acquisition alone is feasible.

1. INTRODUCTION

Dynamic MRI (D-MRI) involves reconstruction of a spatio-temporal signal, $I(x, y, t)$, from samples of its spatial Fourier transform $I(k_x, k_y, t)$. The sampling strategy, which defines the acquisition procedure, is subject to an inherent time sequential sampling (TSS) constraint: only one sample in k-space can be acquired at any one time. Owing to limited speed of MR acquisition, it is usually not possible to sample each k-space (spatial Fourier transform) point at the required Nyquist temporal sampling rate. An approach to bypassing this problem is to effectively freeze cardiac and respiratory motion by ECG-gating and breathholding, or additional respiratory gating. However these methods can only provide a static view of the dynamic process time-averaged over several cardiac cycles.

Authors in [1, 2, 3] develop an adaptive dynamic imaging (ADI) scheme, which instead of attempting to freeze all motion, by sufficiently fast acquisition, explicitly accounts for time variation during acquisition. This is done by using adaptive minimally-redundant acquisition and reconstruction procedures, which are automatically optimized for the specific object being imaged. The approach builds upon optimum time-sequential sampling theory developed in [4].

Parallel MRI techniques such as SENSE [5] or GRAPPA [6] present a complementary hardware approach to accelerate the imaging process by using multiple receiver coils, with distinct spatial sensitivities, to simultaneously acquire k-space samples. This reduces the effective field-of-view (FOV) for data acquired by each coil and thus reduces the k-space Nyquist sampling rate requirement.

In this paper, we propose an optimal acquisition scheme for D-MRI by combining the two methods. The method, named parallel adaptive dynamic imaging (PADI), provides acceleration factors up to the product of the individual accelerations. An efficient

reconstruction algorithm is proposed which bounds the reconstruction error of the resulting inverse problem. The target application for the proposed scheme is 2-D or 3-D cardiac cine imaging with high spatial and temporal resolution. Although the derivation and analysis of the proposed method is based on a 2-D imaging scheme, it can be readily generalized to the 3-D case.

The paper is organized as follows. In Section 2, we briefly describe the banded spatio-temporal model and give an overview of single-coil ADI (SCADI). Section 3, introduces the new method of PADI. Section 4, discusses the sampling design algorithm and possible applications of the proposed method. Finally, Section 5 provides simulation results and demonstrates the effectiveness of the method.

2. SPATIO-TEMPORAL MODELING AND SCADI

The time-varying image (TVI) with d-spatial dimensions is denoted by $I(\vec{x}, t)$, $\vec{x} \in \mathbb{R}^d$. Fourier transforms (FT) of signals are indicated by the variables used, *e.g.* $I\left(\vec{k}, k_t\right)$ is the Fourier transform of $I(\vec{x}, t)$ *w.r.t.* $\vec{k}$ and k; $\vec{k}$ and k_t refer to the spatial and temporal frequencies respectively.

The banded spectral model, introduced in [1], characterizes the cardiac TVI (in the absence of respiratory motion), $I(\vec{x}, t)$ by its spatio-temporal spectral support $B = \text{supp } \{I(\vec{x}, k_t)\}$ of the form shown in Figure 1. This model captures (1) the finite spatial FOV outside which the function is zero, (2) the approximate periodicity of the cardiac motion reflected in the quasi-harmonic banded temporal spectrum with frequency spacing determined by the heart-rate, and (3) the spatial localization of the fast motion to the heart-region defining the dynamic field-of-view.

Given the banded spectral model, SCADI finds when and where to sample k-space so that alias-free reconstruction of the underlying TVI is possible, and the requirements on sampling speed are minimized. The sampling sequence must also satisfy the TSS constraint. This problem was solved [1, 2] by acquiring data in k-space in a scrambled order defined by a TSS schedule $\Psi = \left\{ \vec{k}^{TS}(n), nT_R \right\} n : : \vec{k}^{TS}(n)$ is the k-space location sampled at time nT_R. The TSS schedule is optimally adapted to B which is previously estimated directly from the MR data [3]. The TVI $I(\vec{x}, t)$ can be efficiently reconstructed from the TS-sampled data by filtering the acquired data with a multi-dimensional linear shift-invariant filter that has unit magnitude response over the B support.

3. PADI: RECONSTRUCTABILITY CONDITIONS

In a 2-D parallel imaging experiment, each receiver coil is characterized by its two-dimensional spatial sensitivity function $s_\ell(\vec{x})$. Given L coils, each with complex spatial sensitivity $s\ell(\vec{x})$, $\ell = 1, 2, \cdots, L$, the data acquired by the ℓ-th coil for a TVI is given by

$$D_l\left(\vec{k}, t\right) = \iint_{FOV} s_l(\vec{x}) I(\vec{x}, t) e^{-i2\pi \vec{k} \cdot \vec{x}} d\vec{x} \quad (1)$$

where $\vec{x} = [xy]^T$ for 2-D imaging. Acquisition along the frequency encoding (k_x) direction is sufficiently fast to be assumed instantaneous relative to the temporal dynamics of the image. Therefore, the effect of sampling along k_x is ignored in our analysis.

Consequently, we have a 2-D sampling problem in (k_y, t) domain where each point stands for a horizontal line in (k_x, k_y). Consider a temporally-uniform timesequential sampling lattice Λ with basis matrix $\mathbf{A}$. According to multi-dimensional sampling theory, sampling on a lattice Λ in (k_y, t) will result in replications of the spectrum in the reciprocal domain, i.e. (y, k_t), where kt denotes the temporal frequency. This is equivalent to convolution of the spectrum in (y, k_t) domain with the following point spread function:

$$h(y, k_t) = \frac{1}{|\det(\mathbf{A})|} \sum_{\vec{n} \epsilon Z^2} \delta\left(\begin{bmatrix} y \\ k_t \end{bmatrix} - \mathbf{A}^* \vec{n} \right) \quad (2)$$

where $\mathbf{A}^* = \mathbf{A}^{-T}$ is the basis matrix for the polar lattice Λ^*. Sampling on Λ in (k_y, t) and taking the inverse FT in k_y and forward FT in t of the ℓ-th coil measurement, we arrive at:

$$d_l(y, k_t) = \frac{1}{|\det(\mathbf{A})|} \sum_{\vec{n} \epsilon Z^2} s_l\left(\begin{bmatrix} y \\ k_t \end{bmatrix} - \mathbf{A}^* \vec{n} \right) I\left(\begin{bmatrix} y \\ k_t \end{bmatrix} - \mathbf{A}^* \vec{n} \right) \quad (3)$$

For a point $(y_0, k_{t0}) . \in \mathbf{B}$, define

$$Q(y_0, k_{t_0}) = \left\{ \begin{bmatrix} y \\ k_t \end{bmatrix} \epsilon B \middle| \exists \vec{n} | \epsilon \mathbb{Z}^2 s.t. \begin{bmatrix} y \\ k_t \end{bmatrix} = \begin{bmatrix} y_0 \\ k_{t_0} \end{bmatrix} - \mathbf{A}^* \vec{n} \epsilon B \right\} \quad (4)$$

to which we refer as the equivalence class (EC) of (y_0, k_{t0}). From (3), it follows that for a fixed $(y_0, k_{t0}) \in \mathbf{B}$, the set $Q(y_0, k_{t_0})$ contains all the points in B that are aliasing with (y_0, k_{t0}). It can be shown that the set of all ECs provides a partitioning of the (y, k_t) space. Now, for a fixed EC $Q(y_0, k_{t0})$ with a representative member $(y_0, k_{t0}) \in B$, (3) can be written in matrix form as follows:

$$\vec{d}(y_0, k_{y_0}) = \mathbf{S}(y_0, k_{t_0}) \vec{I}(y_0, k_{t_0}) \quad (5)$$

where $\vec{d}(y_0, k_{t0}) = [d_1(y_0, k_{t0}) \cdots d_L(y_0 k_{t0})]^T$ is the collection of measurements of all coils, and

$$\vec{I}(y_0, k_{t_0}) = \left[I\left(\begin{bmatrix} y_0 \\ k_{t_0} \end{bmatrix} - \mathbf{A}^* \vec{n}_1 \right) \cdots I\left(\begin{bmatrix} y_0 \\ k_{t_0} \end{bmatrix} - \mathbf{A}^* \vec{n}_R \right) \right]^T$$

where $R = |Q(y_0, k_{t0})|$ is the number of elements in the EC. The $S(_o, k_{t0})$ matrix is is of size $L \times R$ and its ℓ-th row is given by:

$$\frac{1}{|\det(A)|} \left[s_l\left(\begin{bmatrix} y_0 \\ k_{t_0} \end{bmatrix} - \mathbf{A}^* \vec{n}_1 \right) \cdots s_l\left(\begin{bmatrix} y_0 \\ k_{t_0} \end{bmatrix} - \mathbf{A}^* \vec{n}_R \right) \right]^T$$

In D-MRI, $\vec{d}(y_0, k_{t0})$ is known for all $(y_0, k_{t0}) \in B$. Solving the inverse problem in (5) will recover the main replica of the spectrum for all members of $Q(y_0, k_{t0})$. Doing so for all ECs will give a reconstruction of the object in the (y, k_t) domain. Following that, the object $I(y, t)$ can be recovered by an inverse FT along k_t. For a 2-D object, this procedure can be repeated for all kx to recover the whole object. Therefore, solving (5) is in effect unfolding the aliasing pattern. Being able to undo the aliasing means that we can handle denser sampling schemes in the (y, k_t) domain (i.e., sparser sampling in (k_y, t) domain) compared to the

aliasing-free schemes such as SCADI whereby the sampling pattern is designed such that no aliasing exists in (y, k_t) domain. Hence, sparser reconstructible sampling schemes in the (k_y, t) domain (i.e., lower sampling rates) are achievable.

A necessary and sufficient condition for the linear inverse problem in (5) to be invertible is for S(y_o, k_{t0}) to have a left inverse, i.e., to have full column rank. If all of the ECs for a lattice Λ and support B satisfy this condition, we call that lattice "B-unfoldable." In addition to the necessary and sufficient condition, the following necessary conditions are useful in the sampling design algorithm. A necessary condition for a sampling lattice to be B-unfoldable, is that $L \geq |Q(y_0, k_{t0})|$ for all $(y_0, k_{t0}) \in B$. That is, the maximum of $|Q(y, k_t)|$ over all $(y, k_t) \in B$ has to be smaller than the number of coils L. It follows that PADI can achieve up to an L-fold improvement in the required temporal sampling rate over SCADI. Therefore, PADI can give a gain in sampling rate equal to the product of the individual gains of SCADI or parallel MRI. The limits on acceleration factor of SCADI over conventional Nyquist (progressive) TSS are discussed in [2].

Furthermore, note that the coil profiles are typically assumed to be time-invariant and therefore their sensitivities are independent of k_t. Now, assume that there exits an EC such that two of its members have the same y position. From the structure of the **S** matrix above, it can be seen that in this case two of the columns will be identical and hence the inverse problem will not be perfectly invertible. Therefore, a second necessary condition is for all members of an EC to have different y values. In other words, translates of B in k_t only, are not allowed to overlap.

4. OPTIMALITY CRITERIA AND PROPOSED SAMPLING ALGORITHM

The goal in the sampling design algorithm is to find a time-sequential lattice Λ such that it is B-unfoldable. The proposed sampling scheme for PADI searches over all B-unfoldable lattices and finds the one that gives a predetermined acceleration factor over SCADI and is optimal in some specified sense. A reasonable optimality criterion is to minimize the time-averaged reconstruction error in noisy conditions. In a practical setting, the coil measurements are corrupted by additive noise. If an estimate of the noise covariance matrix is available (which is true in MRI), the optimal solution for (5) is no longer given by the left inverse and instead is the linear minimum variance estimate:

$$\widehat{\overrightarrow{I}}(y, k_t) = \left(\mathbf{S}(y, k_t)^H \Phi_n^{-1} \mathbf{S}(y, k_t)\right)^{-1} \mathbf{S}(y, k_t)^H \Phi_n^{-1} \overrightarrow{d}(y, k_t)$$

Defining the residual to be the difference of $\overrightarrow{I}(y, t)$ and $\overrightarrow{I}(y, t)$, the average reconstruction error is defined to be time-averaged expectation of the energy of the residual signal. Note that in general, the reconstruction error has two components, one due to noise and one due to signal distortion. By restricting the design of the sampling pattern to be B-unfoldable time-sequential lattices, the signal distortion component is completely eliminated.

The reconstruction error due to noise is computed next. Discretizing the B support and partitioning the set of pixels to ECs $\{Q_p\}_{p=1}^{c}$ as defined in the last section, it can be shown that the average reconstruction error is given by:

$$\bar{\epsilon} = \sum_{p=1}^{c} \text{trace}\left\{\left(\mathbf{S}_p^H \Phi_n^{-1} \mathbf{S}_p\right)^{-1}\right\} \quad (6)$$

where S_p is the corresponding S matrix for Q_p as in (5).

Note that, using SCADI, a high resolution and accurate spatio-temporal model could result in a very short maximal temporal sampling interval, which is below the allowed hardware constraints. In this case, distortion-free D-MRI is impossible using SCADI. The sampling requirement becomes even more demanding when 3-D imaging is desired. However, using the proposed PADI scheme, we will be able to reduce the required minimum sampling rate and obtain a distortion-free reconstruction while optimizing the reconstruction error due to noise amplification. In Section 5, we provide numerical experiments focusing on this aspect.

Furthermore, it has recently been shown that parallel MRI can be used as a means of improving the SNR performance in case of static imaging [7]. The gain in SNR is obtained since the noise power is inversely proportional to the length of the acquisition interval in each repetition of the MR pulse sequence. Similarly, in the dynamic case PADI can be used to offer the following improvements over SCADI:

- Enhance SNR with fixed spatial resolution and total scan time.

- Enhance spatial resolution with fixed SNR and total scan time.

- Reduce total scan time with fixed SNR and spatial resolution.

- All the three above can be alternatively considered for maximizing the contrast-to-noise ratio.

5. SIMULATION RESULTS

In the simulation scenario, a set of 8 phased array coils is assumed for the MR scanner. The coil profiles were obtained using numerical simulation of Bio-Savart's law for a spherical water phantom. The proposed method was tested using data generated from a 2-D phantom. The phantom, shown in Figure 2(a), consists of a circle at the center with changing amplitude and six stationary circles around it. The spatial resolution is taken to be 128×128 pixels and the width of FOV in both directions is assumed to be equal. Temporal variation of amplitude of the central circle was designed to follow the banded spectral model (Figure 1) with 10 harmonic bands. The fundamental frequency of the harmonics is taken to be 1 Hz and width of the bands to be 0.2 Hz. The energy in each band was designed to be inversely proportional to its corresponding temporal frequency. The spatio-temporal spectral support for a fixed x is shown in Figure 2(b).

The minimum feasible temporal sampling interval of the scanner is assumed to be $T_R^{min} = 5$ milliseconds (ms). This is the minimum possible temporal distance for two consecutive phase encodes (k_y positions). For the described phantom, the Nyquist sampling rate is 0.4 ms which is more than 10 times below the hardware limit T_R^{min} The maximum acceleration possible with parallel MRI is a factor of 8 (number of coils). So, even with full acceleration we will need a $T_R ee T_R^{min}$ which is infeasible. Note that in practice, accelerations equal to number of coils is never attempted because of the so called geometric factor noise amplification [5]. Most commonly, irrespective of the number of coils, the acceleration factors due to parallel coils are less than four.

The optimal (maximal) temporal sampling interval for the distortion-free SCADI scheme was computed and turned out to be T_R^{min} / T_R^{DFSC}, Therefore, distortion-free imaging using SCADI is impossible in this case. Finding the optimal sampling pattern with a T_R longer than T_R^{min} would require relaxing either the spatial or temporal resolution. Fixing the spatial resolution, the maximum number of harmonic bands which would give a feasible T_R using SCADI is 6 that gives an optimal T_R of 5.54 ms.

Simulated MRI data was generated by computing the spatial FT of the phantom at time instants of the phase encodes. A typical amount of noise was added in the *k*-space. Figure 3 shows the SCADI reconstructions[1] for two snapshots of the horizontal cut of the object shown by the dashed line in Figure 2(a). As can be seen from the figure, severe signal distortion is present in the reconstruction, which is due to aliasing from unmodeled high frequency bands into lower ones. Define the normalized average reconstruction error as

$$\sqrt{\sum_{t_i}\|I\left(\overrightarrow{x},t_i\right)-\widehat{I}\left(\overrightarrow{x},t_i\right)\|_2^2}/\sqrt{\sum_{t_i}\|I\left(\overrightarrow{x},t_i\right)\|_2^2}$$

where t_i's are the reconstruction instances. For SCADI with the reduced-resolution model (Figure 3), this is 51.6%.

Next, we demonstrate that distortion-free imaging is possible using the proposed PADI scheme. Setting the minimum acceleration to be T_R^{min}/T_R^{DFSC}, a search in performed among all *B*-unfoldable lattices to find the one that minimizes the average reconstruction error due to noise as computed in (6). The optimal sampling scheme has an acceleration factor of 1.45 in T_R over SCADI, that is a T_R of 5.1 ms. Figure 4 shows the reconstructions (as described in Section 4) at the same time instances as before using the PADI sampling scheme. As can be seen, no distortion is present and the error is only due to measurement noise. The normalized average reconstruction error for PADI is 2.52% which is almost 20 times better than that of SCADI.

It should be noted that a spatial resolution of 256×256 would roughly halve the achievable T_R for all the methods mentioned. In that case, even with today's modern MR scanners (with T_R^{min} of 2 to 3 ms), distortion-free imaging using SCADI or parallel MRI is not feasible. In conclusion, in this section we have demonstrated that, through the proposed PADI scheme, a combination of accelerations provided by multiple coils and the ADI scheme will result in distortion-free D-MRI with high temporal and spatial resolution, which is not feasible by any of the individual methods.

Acknowledgments

6. REFERENCES

1. Qi, Zhao; Aggarwal, Nitin; Bresler, Yoram. Dynamic imaging of time-varying objects. Proc. of ISMRM; 2001. p. 1776

2. Bresler, Yoram. Fast acquisition and sampling in MRI: Introduction to time-sequential sampling of spatio-temporal signals. Proc. ISBI; Jul. 2002 p. 713-716.

3. Aggarwal, Nitin; Zhao, Qi; Bresler, Yoram. Spatio-temporal modeling and minimum redundancy adaptive acquisition in dynamic MRI. Proc. ISBI; Jul. 2002 p. 737-740.

4. Willis NP, Bresler Y. Lattice-theoretic analysis of time-sequential sampling of spatio-temporal signals part i. IEEE Trans. Information Theory. Jan.1997 :190–207.

5. Pruessmann KP, et al. SENSE: Sensitivity encoding for fast MRI. Mag. Res. in Medicine. 1999; 42:952–962.

---

[1]For SCADI, the reconstructed image corresponding to each coil is computed using the data of that coil. Next, all images are optimally combined using minimum variance estimation (assuming knowledge of coil profiles).

6. Griswold MA, et al. Generalized autocalibrating partially parallel acquisitions (GRAPPA). Magnetic Resonance in Medicine. 2002; 6:1202–1210. [PubMed: 12111967]

7. Weiger M, et al. Sensitivity enconding as a means of enhancing the SNR efficiency in steady-state MRI. Mag. Res. in Medicine. 2005; 53:177–185.
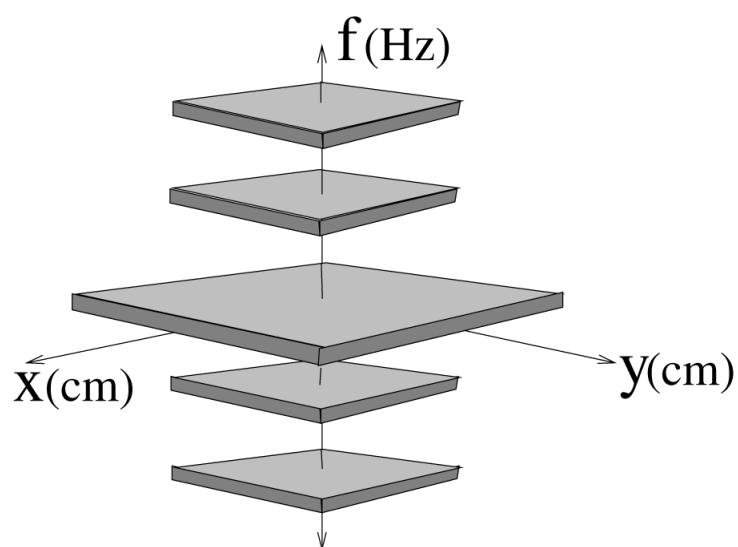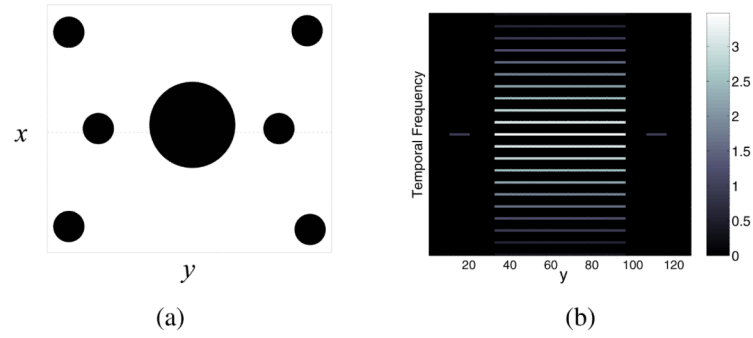
Fig. 1.
Banded spectral support *B*.

Fig. 2.
The 2-D phantom used in the simulations. (a) the (x, y)-domain object. (b) the (y, k_t)-domain spectral support for a fixed x shown by the dashed line in (a).
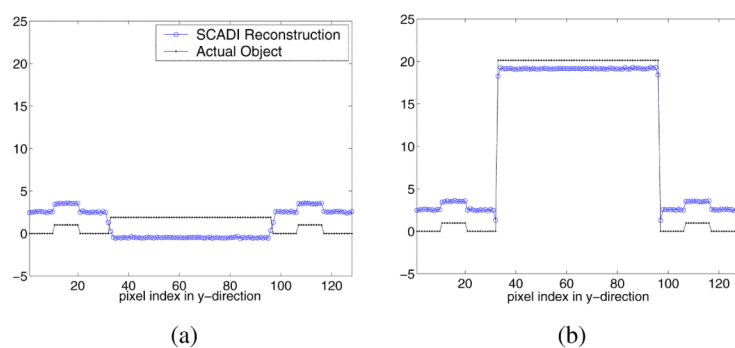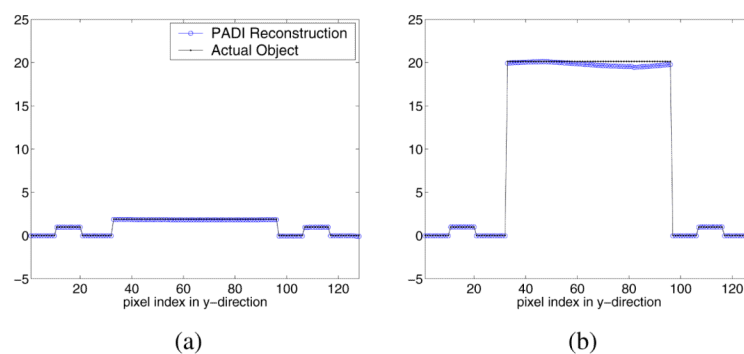
Fig. 3.
A cross section of the SCADI reconstruction for two time instances. The true phantom is shown in black.

(a)

(b)

Fig. 4.
A cross section of the PADI reconstruction for the same two time instances. The true phantom is shown in black.