Adaptive Time Delay Estimation for Sinusoidal Signal

H. C. So

Department of Electronic Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

e-mail: hcso@ee.cuhk.edu.hk

ABSTRACT

An adaptive algorithm is proposed for time delay estimation between sinusoidal signals received at two spatially separated sensors. The idea is to model the differential delay by an FIR filter whose coefficients are samples of a sine function. The delay estimate is updated directly on a sample-by-sample basis using the least mean square method and its convergence behavior and mean square delay error are analyzed. The delay estimation performance of the algorithm can be further improved when the signal and noise powers are available. Computer simulations are presented to validate the theoretical derivations of the proposed estimator for static and linearly varying delays.

I. Introduction

The problem of estimating the propagation delay between two noisy versions of the same signal received at spatially separated sensors has attracted much attention in the literature [1]. It is widely used in passive sonar where the bearing of a moving target can be determined from the time delay measurements by triangulation [2]. Other applications include determination of the center of earthquakes, navigation, speed sensing [3] and speech enhancement [4].

In discrete-time form, the two sensor outputs can be expressed as

$$r_1(k) = s(k) + n_1(k)$$
$$r_2(k) = s(k - D) + n_2(k) \tag{1}$$

where $s(k)$ is the signal of interest, $n_1(k)$ and $n_2(k)$ are uncorrelated zero-mean noises which are statistically independent of $s(k)$ and D is the differential delay to be determined. For simplicity but without loss of generality, it is assumed that the sampling period is 1 second.

When the time difference of arrival is nonstationary due to either relative source/receiver motion or time-varying characteristics of the transmission medium, adaptive tracking of D is necessary. However, most of the existing adaptive delay estimators [5]-[10] assume

that $s(k)$ is a stochastic process and thus it is not appropriate for use in situations where the source signal is deterministic.

The aim of this paper is to devise an adaptive delay estimation algorithm for a moving source that emits a constant tone in radar and certain types of underwater acoustic systems [11]-[12]. The source signal is expressed as $s(k) = A\cos(\omega_0 k + \phi)$ where A and ϕ represent the unknown tone amplitude and phase, respectively, and $\omega_0 \in (0, \pi)$ is the known radian frequency. Section II derives the FIR filter that can generate the time shifted version of a pure sinusoid and its delay modeling error is investigated. An adaptive delay estimator for sinusoid (ADES) is then developed in Section III. In particular, learning behavior and mean square delay error of the algorithm for both static and linearly varying delays are analyzed. In Section IV, the ADES is improved to provide unbiased delay estimates for all ω_0, assuming that the signal and noise powers are known. Simulation results are presented in Section V to corroborate the theoretical analyses and to evaluate the delay estimation performance of the proposed approach. Finally, conclusions are drawn in Section VI.

II. Modeling of Delay for Sinusoid

Using inverse discrete-time Fourier transform and noting that $s(k)$ is a pure sinusoid, the FIR filter coefficients $\{h_l\}$ that can time shift $s(k)$ by a delay D are derived as

$$
\begin{aligned}
h_l &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j\omega(l-D)} \cdot \pi(\delta(\omega + \omega_0) + \delta(\omega - \omega_0)) d\omega \\
&= \cos(\omega_0(l - D)), \qquad l = \cdots, -1, 0, 1, \cdots \tag{2}
\end{aligned}
$$

where $\delta(\cdot)$ is the Dirac delta function. From the convolution theorem and with proper scaling of h_l, $s(k - D)$ can thus be expressed as

$$s(k - D) = \lim_{L \to \infty} \frac{2}{L} \sum_{l=0}^{L-1} s(k - l) \cos(\omega_0(l - D)) \tag{3}$$

Let $\hat{s}_L(k - D)$ be the finite filter length representation of (3). Making use of trigonometric identities, the delay

modeling error, $\epsilon(D)$, is given by

$$\epsilon(D) \triangleq \hat{s}_L(k-D) - s(k-D)$$

$$= \frac{A}{L} \sum_{l=0}^{L-1} \cos(\omega_0(2l-k-D)+\phi) \quad (4)$$

It can be observed that $\epsilon(D) = 0$ when $\omega_0 L$ is an integral multiple of π and its magnitude decreases as L increases.

III. Adaptive Delay Estimator for Sinusoid

Based on (3), the ADES is devised to compensate the time difference between $r_1(k)$ and $r_2(k)$ and its system block diagram is depicted in Figure 1. The filter coefficients given by $\{2/L \cos(\omega_0(l - \hat{D}(k))\}$, $0 \le l \le L-1$, are expressed as a function of the delay estimate, $\hat{D}(k)$, only. In the ADES, the output error function $e(k)$ is computed from

$$e(k) = r_2(k) - \frac{2}{L} \sum_{l=0}^{L-1} r_1(k-l) \cos(\omega_0(l-\hat{D}(k))) \quad (5)$$

By differentiating $e^2(k)$ with respect to $\hat{D}(k)$, stochastic gradient estimate which is similar to that in the Widrow's least mean square (LMS) algorithm [13] is obtained. The estimated delay is adapted iteratively to minimize the mean square output error, $E\{e^2(k)\}$, according to the following equation

$$\hat{D}(k+1) = \hat{D}(k) - \mu \frac{\partial e^2(k)}{\partial \hat{D}(k)}$$

$$= \hat{D}(k) - \frac{2\mu}{L} e(k) \sum_{l=0}^{L-1} r_1(k-l)$$

$$\cdot \sin(\omega_0(l-\hat{D}(k))) \quad (6)$$

where μ is a positive scalar that controls convergence rate and ensures system stability of the algorithm. To reduce computation, values of the sine and cosine functions are retrieved from a pre-stored sine vector. For each sampling interval, the algorithm requires $(2L+4)$ multiplications, $2L$ additions and $2L$ look-up operations.

Considering that $\omega_0 L / \pi$ is an integer and taking the expected value of (6), we obtain

$$E\{\hat{D}(k+1)\}$$
$$= E\{\hat{D}(k)\} + \mu\sigma_s^2 E\{\sin(\omega_0(D - \hat{D}(k))\}$$
$$\approx E\{\hat{D}(k)\} + \mu\sigma_s^2\omega_0(D - E\{\hat{D}(k)\})$$
$$= D + (1-\mu\sigma_s^2\omega_0)^{k+1}(\hat{D}(0) - D) \quad (7)$$

where σ_s^2 and $\hat{D}(0)$ represent the signal power and the initial delay estimate, respectively. Provided that $2/(\sigma_s^2\omega_0) > \mu > 0$, $\hat{D}(k)$ will converge to D in the mean sense with a time constant of $1/(\mu\sigma_s^2\omega_0)$.

Assuming that $n_1(k)$ and $n_2(k)$ are white Gaussian processes with variance σ_n^2 and using (6), the delay variance of the ADES algorithm, denoted by $\mathrm{var}(\hat{D})$, can be shown to be

$$\mathrm{var}(\hat{D}) \triangleq \lim_{k\to\infty} E\{(\hat{D}(k) - D)^2\}$$

$$\approx \frac{\mu\sigma_s^2(L^2\mathrm{SNR} + L + 2)}{\omega_0 L^2 \mathrm{SNR}^2} \quad (8)$$

where $\mathrm{SNR} = \sigma_s^2/\sigma_n^2$. It can be seen that $\mathrm{var}(\hat{D})$ is proportional and inversely proportional to μ and ω_0, respectively, and its value decreases with increasing L for small SNR. Moreover, the delay variance has zero value in the absence of noise.

For nonstationary delays, we can still use (6) and (7) to obtain the mean delay estimates, although closed form expressions are generally not available. In particular, when the time delay is linearly varying, that is, $D(k) = D_0 + \lambda k$, where D_0 and λ represent the delay at $k = 0$ and Doppler time compression, respectively, the mean value of (6) can be approximated as

$$E\{\hat{D}(k+1)\}$$
$$\approx E\{\hat{D}(k)\} + \mu\sigma_s^2\omega_0(D(k) - E\{\hat{D}(k)\}) \quad (9)$$

Solving (9) yields the tracking behavior for a linearly varying delay which is expressed as

$$E\{\hat{D}(k)\} = D(k) - \frac{\lambda}{\mu\sigma_s^2\omega_0} + (1-\mu\sigma_s^2\omega_0)^k$$

$$\cdot(\hat{D}(0) - D + \frac{\lambda}{\mu\sigma_s^2\omega_0}) \quad (10)$$

The second term of the right hand side represents the steady state time lag which is directly proportional to λ and inversely proportional to μ, σ_s^2 and ω_0 while the last term is a transient factor that converges to zero as k goes to infinity. In this case, the mean square delay error, $\mathrm{mse}(\hat{D})$, is equal to the delay variance in (8) plus the square of time lag [14] and has the form

$$\mathrm{mse}(\hat{D}) = \frac{\mu\sigma_s^2(L^2\mathrm{SNR} + L + 2)}{\omega_0 L^2 \mathrm{SNR}^2} + \frac{\lambda^2}{\mu^2\sigma_s^4\omega_0^2} \quad (11)$$

Since the first term increases with μ but the second term decreases with μ^2, μ must be selected appropriately in order to achieve the best performance. As a rule of thumb, when noise dominates, a smaller value of μ should be used. Otherwise, a larger value of μ is preferred particularly when the delay is changing rapidly with time.

IV. An Impro ved ADES

When $\omega_0 L$ is of any real value, the mean value of (6) is given by

$$E\{\hat{D}(k+1)\} - E\{\hat{D}(k)\}$$
$$= \mu\sigma_s^2 E\{\sin(\omega_0(D - \hat{D}(k)))\}$$
$$-2\mu(L\sigma_s^2 + \sigma_n^2)E\{\sum_{l=0}^{L-1}\sin(2\omega_0(l - \hat{D}(k)))\}/L^2$$
$$+\mu\sigma_s^2 E\{\sum_{l=0}^{L-1}\sin(\omega_0(2l - \hat{D}(k) - D))\}/L \quad (12)$$

If $\omega_0 L/\pi$ is an integer, (12) becomes (7). Otherwise, $\hat{D}(k)$ is a biased estimate of D, although its delay error decreases with increasing L. If the signal and noise powers are known, we make use of (12) to remove the delay bias by modifying the ADES algorithm to

$$\hat{D}(k+1)$$
$$= \hat{D}(k) - \frac{2\mu}{L}e(k)\sum_{l=0}^{L-1}r_1(k - l)\sin(\omega_0(l - \hat{D}(k)))$$
$$+\mu\left(\frac{\sigma_s^2}{L} + \frac{2\sigma_n^2}{L^2}\right)\sum_{l=0}^{L-1}\sin(2\omega_0(l - \hat{D}(k))) \quad (13)$$

Notice that the convergence behavior and mean square delay error analyses of the improved algorithm are identical to those of (6).

V. Sim ulation Results

Extensive computer sim ulations had been conducted to evaluate the delay estimation performance of the proposed approach for sinusoidal signals in the presence of white Gaussian noise. The amplitude and phase of the sinusoid were $\sqrt{2}$ and 1.0, respectively, while SNR = 10dB. The filter length L was chosen to be 16 and the initial delay estimate was set to 0. The results provided were averages of 200 independent runs.

Figure 2 shows the trajectory for the delay estimate of the ADES when D was a static delay with a value of 4.5s. The sinusoidal frequency was assigned to be 0.125πrad/s which made $\omega_0 L/\pi$ an integer while μ was selected to 0.005. It is seen that $\hat{D}(k)$ converged to the desired value at approximately the 2500th iteration. The initial convergence rate was slightly slower than the theoretical calculation because the delay estimate was not close to D at the beginning of adaptation. Moreo ver, the measured variance of $\hat{D}(k)$ was found to be $1.282 \times 10^{-3}s^2$, which conformed very well to the theoretical value as given by (8).

Figures 3 illustrates the tracking performance of the ADES when the actual delay was a linearly time-varying function of the form $D(k) = 0.0005k$ at $\omega_0 = 0.125\pi$rad/s. The step size μ was increased to 0.05 for a faster convergence speed. We observe that the algorithm tracked the delay satisfactorily with a time lag of approximately 2.343×10^{-2}s and this value agreed with (10). Furthermore, the measured mean square dela y error had a value of $1.288 \times 10^{-2}s^2$ which was quite close to that derived from (11).

The first test was repeated for a non-integer $\omega_0 L/\pi$, namely, $\omega_0 = 0.136\pi$rad/s and the learning characteristics of the ADES and the improved ADES are shown in Figure 4. It can be seen that both methods con verged at approximately the 2000th iteration. However, the ADES gave a biased delay estimate of 4.414s while the estimate of D in the improved ADES was very accurate provided that σ_s^2 and σ_n^2 were available. The measured delay variance of the improved ADES was $1.267\times10^{-3}s^2$ which also conformed to (8).

VI. Conclusions

The ADES has been proposed for estimating and tracking the time difference of arrival of a sinusoid received at two separated sensors. It uses an adaptive FIR filter whose coefficients are sample of a sine function to model the delay. Using an LMS-st yle algorithm, the delay estimate is adjusted explicitly on a sample-b y-sample basis. Learning behavior and mean square delay error of the ADES for both static and linearly varying delays are derived. When the signal and noise po wers are known, the ADES algorithm can be modified to pro vide unbiased delay estimation for all finite L and ω_0. Numerical examples are included to validate the theoretical analysis and to demonstrate the effectiveness of the proposed approach.

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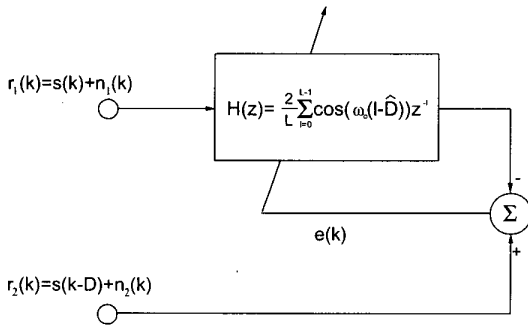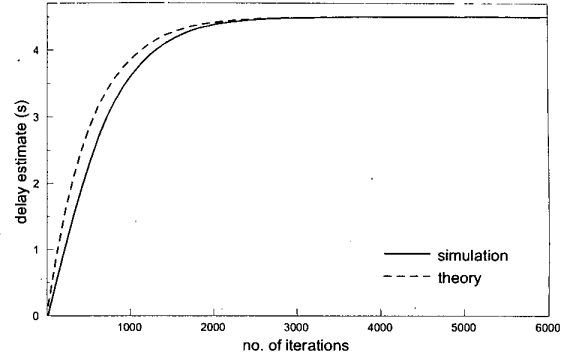
Figure 2: Delay estimate of ADES for static delay at $\omega_0 = 0.125\pi\mathrm{rad}/s$

Figure 3: Delay estimate of ADES for linearly varying delay at $\omega_0 = 0.125\pi\mathrm{rad}/s$

Figure 1: System block diagram for adaptive time delay estimation

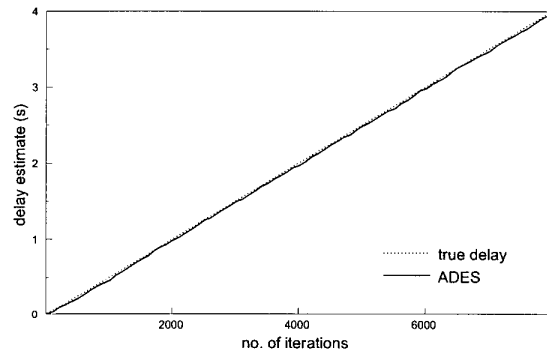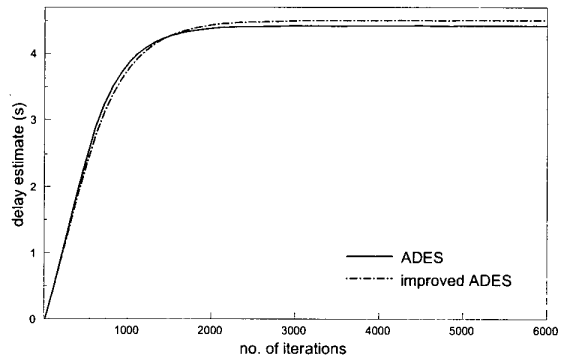Figure 4: Delay estimates for static delay at $\omega_0 = 0.136\pi\mathrm{rad}/s$