

PERFECT RECONSTRUCTION MODULATED FILTER BANKS WITH SUM OF POWERS-OF-TWO COEFFICIENTS

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ABSTRACT

In this paper, a new family of multiplier-less modulated filter banks, called the SOPOT MFB, is presented. The coefficients of the proposed filter banks consist of sum of powers-of-two coefficients (SOPOT), which require only simple shifts and additions for implementation. The modulation matrix and the prototype filter are derived from a fast DCT-IV algorithm of Wang [10] and the lattice structure in [1]. The design of the SOPOT MFB is performed using the genetic algorithm (GA). An 16-channel SOPOT MFB with 34 dB stopband attenuation is given as an example, and its average number of terms per SOPOT coefficient is only 2.6.

I. INTRODUCTION

Perfect Reconstruction (PR) critically decimated filter banks have important applications in speech, audio and image processing. Fig. 1(a) shows the block diagram of a critically decimated uniform M -channel filter banks. The theory and design of M -channel PR maximally decimated uniform filter bank has been extensively studied [1]. The cosine modulated filter banks (CMFB) [2] and the lapped transforms [3], [4], [5] are two efficient classes of filter banks with low implementation complexity and good performance. Recently, there is increasing interest in designing filter banks with low implementation complexity. Approaches based on the sum of power-of-two (SOPOT) coefficients [6,9] or integer coefficients [7] have been proposed. With the use of SOPOT, coefficient multiplications can be implemented with simple shifts and additions. In [7], a subspace approach was proposed to design the prototype filter of the CMFB with integer coefficients. Since integer coefficient filter banks require only integer arithmetic (additions and possibly multiplications), the implementation of the filter banks is greatly simplified. Also, if sufficient word length is used to represent the intermediate data, the round-off error can completely be eliminated.

In this paper, a new family of modulated filter bank with sum of powers-of-two coefficients, called SOPOT-MFB, is developed. The SOPOT-MFB is based on our previous work in [8], where it was shown that modulated filter banks with perfect reconstruction can be obtained by using modulations other than the conventional cosine modulation. Unfortunately, satisfactory design of the modulations have not been obtained. In this work, the SOPOT modulation matrix and prototype filter are

derived from a fast DCT-IV algorithm of Wang [10] and the lattice structure in [1]. The design of the SOPOT MFB is performed using the genetic algorithm. An 16-channel SOPOT MFB with 34 dB stopband attenuation is given as an example, and its average number of terms per SOPOT coefficient is 2.6.

The rest of the paper is organized as follows: Section II is a brief overview of the theory of modulated filter banks. The construction of the SOPOT modulation matrix and the design of the SOPOT MFB will be described in Section III and IV, respectively. A design example is shown in Section V. A summary of the results is given in Section VI, the conclusion.

II. THEORY OF MFB

In CMFB, the analysis filters $f_k(n)$ and synthesis filters $g_k(n)$ are derived from a prototype filter $h(n)$ by cosine modulation $c_{k,n}$ ($\bar{c}_{k,n}$):

$$\begin{aligned} f_k(n) &= h(n)c_{k,n}, \quad g_k(n) = h(n)\bar{c}_{k,n} \\ k &= 0, 1, \dots, M-1; \quad n = 0, 1, \dots, N-1, \end{aligned} \quad (1)$$

where M is the number of channels and $N=2mM$ is the length of the filter. Two different modulations can be used and without loss of generality, here we will consider the modulation proposed in [2]:

$$\begin{aligned} c_{k,n} &= 2 \cos \left((2k+1) \frac{\pi}{2M} \left(n - \frac{N-1}{2} \right) + (-1)^k \frac{\pi}{4} \right) \\ \bar{c}_{k,n} &= 2 \cos \left((2k+1) \frac{\pi}{2M} \left(n - \frac{N-1}{2} \right) - (-1)^k \frac{\pi}{4} \right) \end{aligned} \quad (2)$$

Let $H(z) = \sum_{q=0}^{2M-1} z^{-q} H_q(z^{2M})$ be the type-I polyphase

decomposition of the prototype filter and $F_k(z)$ the z -transform of $f_k(n)$. It can be shown that

$$F_k(z) = \sum_{q=0}^{2M-1} c_{k,q} z^{-q} H_q(-z^{2M}). \quad (3)$$

The analysis filter bank can be expressed in matrix form

$$\text{as: } f(z) = \hat{C}_A \begin{bmatrix} h_0(z^{2M}) \\ z^{-M} h_1(z^{2M}) \end{bmatrix} e_M(z) = E(z^M) e_M(z), \quad (4)$$

where $e_M^T(z) = [1 \quad z^{-1} \quad \dots \quad z^{-(M-1)}]$,

$$f(z) = [F_0(z), F_1(z), \dots, F_{M-1}(z)]^T,$$

$$h_0(z) = \text{diag}[H_0(-z), H_1(-z), \dots, H_{M-1}(-z)]$$

$$h_1(z) = \text{diag}[H_M(-z), H_{M+1}(-z), \dots, H_{2M-1}(-z)]$$

$E(z)$ is the polyphase matrix and \hat{C}_A is a $(M \times 2M)$ modulation matrix given by $[\hat{C}_A]_{k,n} = c_{k,n}$. \hat{C}_A can be further partitioned as $\hat{C}_A = (-1)^q [A_0 \ A_1]$ ($q=m/2$ for m even and $q=(m-1)/2$ for m odd) [2]:

$$A_0 = \sqrt{M} C_M^{IV} (I_M + (-1)^{m-1} J_M), \quad (5a)$$

$$A_1 = (-1)^{m-1} \sqrt{M} C_M^{IV} (I_M - (-1)^{m-1} J_M), \quad (5b)$$

where $C_M^{IV}(k, n) = \sqrt{2/M} \cos((k+1/2)(n+1/2)\pi/2M)$ is the type-IV DCT. It can be shown that the system is PR if the prototype filter satisfies

$$H_k(z)H_{2M-k-1}(z) + H_{M+k}(z)H_{M-k-1}(z) = cz^{-sk}. \quad (6)$$

It has been shown in [8] that the PR condition of the modulated filter bank is still given by (6) if

$$\hat{C}_A^T \hat{C}_A = 2M \begin{bmatrix} I_M \pm J_M & 0 \\ 0 & I_M \mp J_M \end{bmatrix}, \quad (7)$$

where I_M and J_M are $(M \times M)$ identity and exchange matrices, respectively. The result in [8] was originally derived for orthogonal MFB but it is still valid for the more general biorthogonal PR condition (6). The general solutions for A_0 and A_1 are (up to a scaling)

$$A_0 = \sqrt{M} U_M (I_M \pm J_M), \quad (8a)$$

$$\text{and } A_1 = \pm \sqrt{M} U_M (I_M \mp J_M), \quad (8b)$$

for some unitary matrix U_M . If the modulation is not orthogonal, (6) will still be valid if the analysis modulation matrix \hat{C}_A and synthesis modulation matrix satisfy \hat{C}_s satisfy

$$\hat{C}_s^T \hat{C}_A = 2M J_{2M} \begin{bmatrix} I_M \pm J_M & 0 \\ 0 & I_M \mp J_M \end{bmatrix}, \quad (9)$$

where \hat{C}_s is the modulation matrix of the type-I polyphase decomposition of the synthesis filters:

$$\mathbf{g}(z) = [G_0(z), \dots, G_{M-1}(z)]^T = \hat{C}_s \begin{bmatrix} h_0(z^{2M}) \\ z^{-M} h_1(z^{2M}) \end{bmatrix} \mathbf{e}_M(z).$$

It can be shown that \hat{C}_A and \hat{C}_s are given by (up to a scaling)

$$C_A = \sqrt{M} U_M [I_M \pm J_M, \pm(I_M \mp J_M)], \quad (10a)$$

$$C_s = \sqrt{M} (U_M^{-1})^T [I_M \pm J_M, \pm(I_M \mp J_M)] J_{2M}, \quad (10b)$$

where U_M is now an arbitrary nonsingular matrix. Without loss of generality, we will consider the case where

$$C_A = \sqrt{M} U_M [I_M + J_M, (I_M - J_M)],$$

$$C_s = \sqrt{M} (U_M^{-1})^T [I_M + J_M, (I_M - J_M)] J_{2M}.$$

III. SOPOT MODULATION

To derive the SOPOT modulation, the simplest way is to quantize the coefficients in the type-IV DCT. The inverse of the quantized matrix, however, cannot in general be expressed in terms of SOPOT coefficients. Let's consider the simple example:

$$R_\theta = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}. \quad (11)$$

If $\cos \theta$ and $\sin \theta$ are expressed directly in terms of SOPOT coefficients, say α and β . The inverse of

$$\tilde{R}_\theta = \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix} \text{ is } \tilde{R}_\theta^{-1} = \frac{1}{\alpha^2 + \beta^2} \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}. \text{ As } \alpha \text{ and } \beta$$

are SOPOT coefficients, the term $(\alpha^2 + \beta^2)^{-1}$ cannot in general be expressed as a SOPOT coefficient. The basic idea of the proposed multiplier-less modulation is based on the following factorization of R_θ and its inverse

$$R_\theta = R_\theta^{-1} = \begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin \theta & 1 \end{bmatrix} \begin{bmatrix} \tan(\theta/2) \\ 0 & -1 \end{bmatrix}. \quad (12)$$

Since the factorizations of R_θ and R_θ^{-1} involve the same set of coefficients, i.e. $\tan(\theta/2)$ and $\sin \theta$, they can be directly quantized to SOPOT coefficients. To get the SOPOT modulation, we start with the fast algorithm of Wang [10], which factorizes the DCT-IV into a product of $2\ell + 1$ sparse matrices as follows:

$$C_M^{IV} = Q_M V_M(\ell) \left[\prod_{i=1}^{\ell-1} K_M(\ell-i) V_M(\ell-i) \right] H_M, \quad \ell = \log_2 M, \quad (13)$$

where

$$H_M = P_M (P_{M/2} \oplus \bar{P}_{M/2}) \cdot \text{diag}(P_{M/4}, \bar{P}_{M/4}, P_{M/4}, \bar{P}_{M/4}) \cdots \text{diag}(P_4, \bar{P}_4, \dots, P_4, \bar{P}_4), \quad N \geq 4,$$

$$P_M = \begin{bmatrix} 1 & 0 & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 1 & \dots \\ 0 & 1 & 0 & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 1 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 1 & 0 & \dots \\ 0 & \dots & 1 & 0 & \dots & \dots \\ 0 & \dots & 1 & 0 & 0 & 0 \end{bmatrix}, \quad Q_M = \begin{bmatrix} 1 & 0 & \dots & \dots & \dots & 0 \\ 0 & \dots & \dots & \dots & 0 & 1 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & \dots & \dots & 1 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \end{bmatrix}$$

\bar{P}_M is obtained by reversing both the rows and columns of P_M . Q_M is a permutation matrix that changes the odd-numbered components of the vector into a reversed order. $K_M(i)$, $i=1,2,\dots,\ell-1$, are block diagonal matrices given by

$$K_M(i) = (1/\sqrt{2}) \cdot \text{diag}(B(i), B(i), \dots, B(i), B(i)), \quad (14)$$

where $B(i) = \begin{bmatrix} I_{2^i} & I_{2^i} \\ I_{2^i} & -I_{2^i} \end{bmatrix}$. $V_M(\ell)$ is given by

$$V_M(\ell) = \text{diag}(T_{1/4M}, T_{5/4M}, \dots, T_{(2M-3)/4M}), \quad (15)$$

where $T_r = \begin{bmatrix} \cos r\pi & \sin r\pi \\ \sin r\pi & -\cos r\pi \end{bmatrix}$. The other matrices

$V_M(i)$, $i=1,2,\dots,\ell-1$, are obtained by alternating the submatrices I_{2^i} and $E(i)$ in the main diagonal as follows

$$V_M(i) = \text{diag}(I_{2^i}, E(i), I_{2^i}, \dots, E(i)) \quad (16)$$

where $E(i) = \text{diag}(T_{1/2^{i+1}}, T_{5/2^{i+1}}, \dots, T_{(2^{i+1}-3)/2^{i+1}})$.

Replacing T_r by $S_r = \begin{bmatrix} 1 & -\beta_r & 1 & 0 \\ 0 & 1 & \alpha_r & 1 \end{bmatrix} \begin{bmatrix} 1 & \beta_r \\ 0 & -1 \end{bmatrix}$, with α_r and β_r SOPOT coefficients, the desired SOPOT modulation matrix is obtained. Let $R_M(i)$ be the matrix obtained by replacing T_r in $V_M(i)$ by S_r . Because $S_r^{-1} = S_r$, $R_M^{-1}(i) = R_M(i)$. The SOPOT modulation matrix U_M and its inverse are given by:

$$U_M = Q_M R_M(\ell) \left[\prod_{i=1}^{\ell-1} K_M(\ell-i) R_M(\ell-i) \right] H_M \quad (17)$$

$$\text{and } U_M^{-1} = H_M^T \left[\prod_{i=1}^{\ell-1} R_M(i) K_M(i) \right] R_M(\ell) Q_M \quad (18)$$

IV. DESIGN OF THE MFB

To design the prototype filter with SOPOT coefficients, we use the lossless lattice structure proposed in [1] (Figure 2). The lattice coefficients are represented as:

$$\alpha_{q,n} = \sum_{k=1}^{p_n} a_k 2^{b_k} \quad (19)$$

$a_k \in \{-1, 1\}$ $b_k \in \{l, \dots, 1, 0, -1, \dots, -l\}$.

l is a positive integer, which determines the range of the coefficient. p_n is the number of terms used in each coefficient. The minimax objective function is used in the design of the prototype filter:

$$D_h = \max \| |H(e^{j\omega})| - |H_d(e^{j\omega})| \|, \quad (20)$$

where $H(z)$ and $H_d(z)$ are respectively the actual and ideal frequency responses of the prototype filter. The modulation matrix is designed using a similar minimax objective function

$$D_M = \max_{0 \leq k \leq M-1} \{ E_k = \max \| F_k(e^{j\omega}) - |\hat{F}_k(e^{j\omega})| \| \}, \quad (21)$$

where $F_k(e^{j\omega})$ and $\hat{F}_k(e^{j\omega})$ are respectively the actual and ideal frequency responses of the k -th analysis filter of the SOPOT MFB. The SOPOT coefficients are found by minimizing D_h and D_M using the Genetic Algorithm.

V. DESIGN EXAMPLES

Fig. 3(a) and 3(b) show, respectively, the frequency responses of the prototype and the analysis filters of an 16-channel SOPOT MFB. The cutoff frequency of the prototype filter is $\omega_c = 0.075\pi$ and its stopband attenuation is 37 dB. Because of the linear-phase property of the prototype filter, there are altogether eight different lattices. Polyphase components $H_q(z)$ and $H_{q+16}(z)$ are derived from the q -th lattice, and all of them have two lattice coefficients $\alpha_{q,1}, \alpha_{q,2}$, i.e. ($m=2$).

q	$\alpha_{q,1}$	$\alpha_{q,2}$
0	$2^2 - 2^{-1}$	$-2^2 + 2^{-4}$
1	$2^1 + 2^{-1} + 2^{-5}$	$-2^2 - 2^0 - 2^{-2}$
2	$2^1 + 2^{-1}$	$-2^2 - 2^0 - 2^{-1} - 2^{-3}$
3	2^1	$-2^3 - 2^0$
4	$2^0 + 2^{-1} + 2^{-4}$	$-2^4 - 2^2$

5	$2^0 + 2^{-1} - 2^{-6}$	$-2^5 + 2^1$
6	$2^0 + 2^{-2}$	$-2^4 - 2^1$
7	$2^0 + 2^{-4}$	$-2^5 - 2^4$

The SOPOT modulation matrix, U_{16} , is given by

$$U_{16} = Q_{16} R_{16}(4) K_{16}(3) R_{16}(3) K_{16}(2) R_{16}(2) K_{16}(1) R_{16}(1) H_{16}, \quad (22)$$

where $R_{16}(4) = \text{diag}\{S_{1/64}, S_{5/64}, S_{9/64}, S_{13/64}, S_{17/64}, S_{21/64}, S_{25/64}, S_{29/64}\}$,

$$R_{16}(3) = \text{diag}\{I_8, S_{1/16}, S_{5/16}, S_{9/16}, S_{13/16}\},$$

$$R_{16}(2) = \text{diag}\{I_4, S_{1/8}, S_{5/8}, I_4, S_{1/8}, S_{5/8}\},$$

$$R_{16}(1) = \text{diag}\{I_2, S_{1/4}, I_2, S_{1/4}, I_2, S_{1/4}\}.$$

As mentioned earlier, each S_r is characterized by two parameters α_r and β_r . Noting that

$$T_{x+1/2} = T_x \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \text{then } S_{x+1/2} = S_x \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

($x=1/16, 5/16, 1/8$), $S_{9/16}$, $S_{13/16}$, and $S_{5/16}$ can be derived from $S_{1/16}$, $S_{5/16}$, and $S_{1/8}$, respectively. The number of variables is therefore reduced and their details are as follows:

$r(S_r)$	α_r	β_r
1/64	2^{-4}	2^{-5}
5/64	$2^{-2} - 2^{-6} - 2^{-7}$	$2^{-3} - 2^{-7}$
9/64	$2^{-1} - 2^{-4}$	$2^{-2} - 2^{-6} - 2^{-7}$
13/64	$2^{-1} + 2^{-4} + 2^{-6} + 2^{-7}$	$2^{-2} + 2^{-4} + 2^{-6}$
17/64	$2^{-1} + 2^{-2} + 2^{-6} + 2^{-7}$	$2^{-1} - 2^{-4} + 2^{-7}$
21/64	$2^0 - 2^{-3} - 2^{-7}$	$2^{-1} + 2^{-4} - 2^{-7}$
25/64	$2^0 - 2^{-4}$	$2^{-1} + 2^{-2} - 2^{-5}$
29/64	$2^0 - 2^{-7}$	$2^0 - 2^{-3} - 2^{-6}$
1/16	$2^{-3} + 2^{-4} + 2^{-6} + 2^{-7}$	$2^{-3} - 2^{-6}$
5/16	$2^{-1} + 2^{-2} + 2^{-4}$	$2^{-1} + 2^{-5}$
1/8	$2^{-2} + 2^{-3}$	$2^{-3} + 2^{-4} + 2^{-7} + 2^{-8}$
1/4	$2^{-1} + 2^{-2} - 2^{-4} + 2^{-6}$	$2^{-1} - 2^{-4} - 2^{-6} - 2^{-7}$

The average number of terms per SOPOT coefficient is 2.6.

VI. CONCLUSION

In this paper, a new family of multiplier-less modulated filter banks, called the SOPOT MFB, is presented. The coefficients of the proposed filter banks consist of sum of powers-of-two coefficients (SOPOT), which require only simple shifts and additions for their implementation. The modulation matrix and the prototype filter are derived from a fast DCT-IV algorithm of Wang [10] and the lattice structure in [1]. The genetic algorithm is used to search for the SOPOT coefficients of the filter banks. An 16-channel SOPOT MFB with 34 dB stopband attenuation is given as an example, and its average number of terms per SOPOT coefficient is only 2.6.

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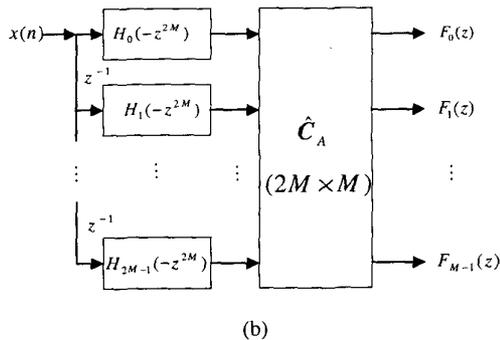
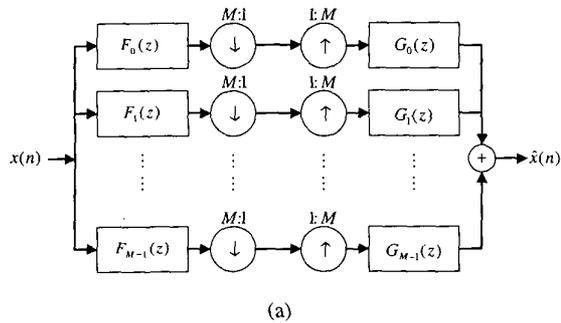


Fig. 1. Block diagrams of (a) M -channel uniform filter banks, (b) polyphase implementation of the cosine-modulated analysis filter bank.

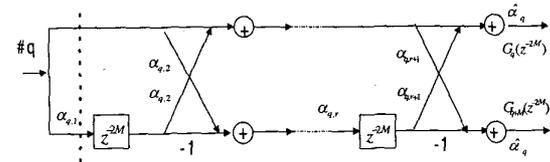
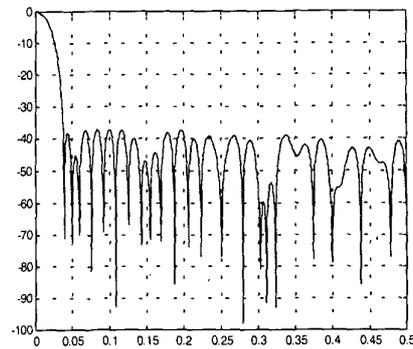
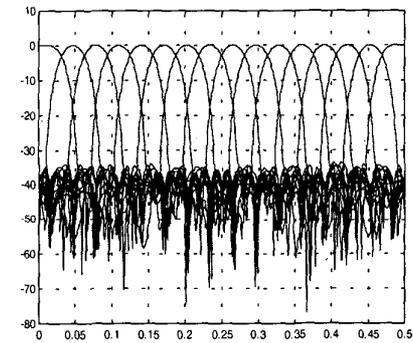


Fig. 2. Lossless lattice structure of the prototype filter in [1]

$$(\hat{\alpha}_q = \prod_{i=1}^{r+1} (1 + \alpha_{q,i}^2)^{-0.5}).$$



(a)



(b)

Fig. 3. Frequency responses of the 16-channel SOPOT MFB: (a) prototype filter (length=64, $\omega_s = 0.075\pi$, $A_s = 37dB$), (b) analysis filters, $A_s = 34dB$.