Optimum Design of Very Low Distortion Class E Power Amplifiers

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***Abstract*— In this paper[1] the optimum design of a class E power amplifier with resonant tank being symmetrically driven by two class E circuits is studied. The optimally designed symmetrical class E circuit has extremely low harmonic distortion, and the load matching network (if required) is non-critical and can be designed with ease. Practical steady-state design equations are derived and graphically presented. Experimental circuits are constructed for distortion evaluation.**

I. Introduction

Class E zero-voltage-switching (ZVS) resonant power amplifiers are widely regarded as a very efficient type of converters [1]–[3], which have found applications in high-frequency power amplification, and in particular radio-frequency (RF) transceiver systems. In the original single-ended design, the class E power amplifier is driven asymmetrically. The energy of higher harmonics generated gives rise to harmonic distortions, which have to be filtered. Any practical filtering method, however, incurs a penalty on the overall efficiency and the design cost. Low distortion CMOS complementary class E power amplifiers have been proposed [4], in which two identical resonant circuits are used. This design, however, poses a practical problem of matching inductors and capacitors. Recently, a symmetrically driven push-pull class E amplifier has been proposed for high power applications, with its output voltage doubled over the original class E circuit [5]. In addition to the high power capability and high efficiency, this symmetrically driven amplifier offers very low harmonic distortions and hence can ease the design of the matching filter for applications requiring very high quality sinewave outputs. Despite the many potential advantages this amplifier offers, very little work has been done to analyze its operation and harmonic distortion performance. In this paper, we study a particular form of this symmetrically driven class E circuit. In particular, we will derive the optimum operating conditions and design equations of this circuit, and evaluate its distortion performance experimentally.

II. Principle of Operation

The symmetrically driven class E circuit under study is shown in Fig. 1. It consists of an LCR parallel resonant tank which is driven symmetrically by two MOSFET switches, Q_1 and Q_2. The switches are driven on and off alternately within each of the half operating period $T/2$ ($T = 1/f = 2\pi/\omega$). The components are chosen to satisfy the symmetry conditions, i.e., $Q_1 = Q_2$ ($D_1 = D_2$), $L_1 = L_2$ and $C_1 = C_2$. Thus, the waveforms of I_1 and v_{C_1} should resemble those of I_2 and v_{C_2}, respectively, but with a phase shift of $\omega T/2$, i.e., π. The parallel resonant tank is then driven by a nearly sinusoidal voltage given by $v_T = v_{C_1} - v_{C_2}$.

Depending upon the driving frequency ω, three sets of steady-state waveforms are shown in Fig. 2, denoted as operating modes 1, 2 and 3. By symmetry, we need to consider only the half switching period. The circuit for states 1, 2 and 3 are given in Figs. 3, 4 and 5, respectively. Note that Fig. 2 (b) is a special case of Figs. 2 (a) and (c) with $\kappa = 0$ and $\lambda = 0$, respectively. It should be obvious that operations in the vicinity of operating mode 2, as shown in Fig. 2 (b), should give the least distortion. We will focus on this case.

Analysis of such class E zero-voltage-switching (ZVS) circuits can be done using the *fundamental frequency component method* [6]. The basic assumptions of this method are: (i) The transistors, capacitors and inductors are ideal and lossless; (ii) I_1 and I_2 are identical constant current sources in the steady state; (iii) the fundamental component of the tank inductor current is

$$i_L(t) = I_L \sin(\omega t + \phi) \tag{1}$$

where I_L and ϕ are the amplitude and phase of i_L.

Referring to the circuits shown in Figs. 3, 4 and 5 for the various switching states, the steady-state equations are

$$i_{C_1}(t) = \begin{cases} I_1 - i_L(t) & -\frac{T}{2} \le t < \tau \\ 0 & \tau \le t < \frac{T}{2} \end{cases} \tag{2}$$

where $\tau = -\kappa T$ for mode 1, $\tau = 0$ for mode 2, and $\tau = \lambda T$ for mode 3; and

$$i_{C_2}(t) = \begin{cases} 0 & -\frac{T}{2} \le t < 0 \text{ and} \\ & (\frac{1}{2} - \kappa)T \le t < \frac{T}{2} \text{ (mode 1)} \\ & -\frac{T}{2} \le t < 0 \text{ (mode 2)} \\ & -\frac{T}{2} \le t < (\lambda - \frac{1}{2})T \text{ and} \\ & (-\frac{1}{2} + \lambda)T \le t < 0 \text{ (mode 3)} \\ I_2 - i_L(t) & 0 \le t < (\frac{1}{2} - \kappa)T \text{ (mode 1)} \\ & 0 \le t < \frac{T}{2} \text{ (mode 2)} \\ & -\frac{T}{2} \le t < (-\frac{1}{2} + \lambda)T \text{ and} \\ & 0 \le t < \frac{T}{2} \text{ (mode 3)} \end{cases} \tag{3}$$

where $\kappa < \frac{1}{2}$ and $\lambda < \frac{1}{2}$.

The capacitor voltage v_{C_1} can be found by integrating (2), i.e.,

$$v_{C_1}(t) = \begin{cases} \frac{1}{\omega C_1} [I_1 (\omega t + \pi) \\ \quad + I_L (\cos(\omega t + \phi) \\ \quad\quad + \cos\phi)] & -\frac{T}{2} \le t < \tau \\ 0 & \tau \le t < \frac{T}{2} \end{cases} \tag{4}$$

where $\tau = -\kappa T$ for mode 1, $\tau = 0$ for mode 2, and $\tau = \lambda T$ for mode 3.

The boundary condition for ZVS can be found by solving

$$v_{C_1}(\sigma T) = 0 \tag{5}$$

where $\sigma = -\kappa$ for mode 1, $\sigma = 0$ for mode 2, and $\sigma = \lambda$ for mode 3. This gives

$$I_L \cos(\pi\sigma)\,(-\cos(\pi\sigma + \phi)) = I_1 \pi \left(\frac{1}{2} + \sigma\right) \tag{6}$$

where $\cos(\pi\sigma) > 0$ for $|\sigma| < \frac{1}{2}$ and $-\cos(\pi\sigma + \phi) > 0$ for some $\pi\sigma + \phi$.

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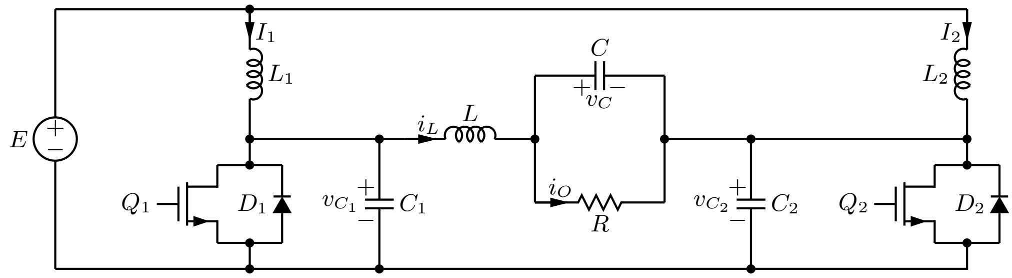
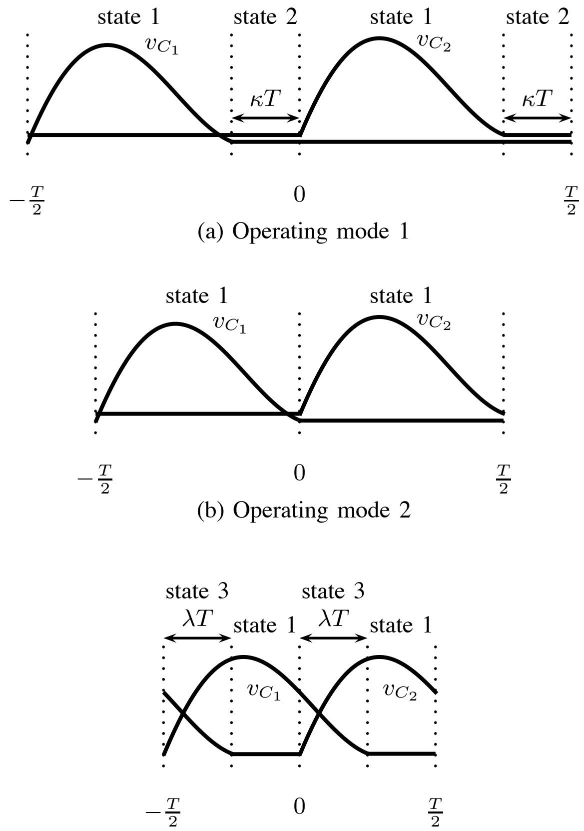


Fig. 1. Symmetrically driven class E power amplifier.



(a) Operating mode 1

(b) Operating mode 2

(c) Operating mode 3

Fig. 2. Three possible operating modes of the symmetrically driven class E power amplifier.

Equation (6) is often referred to as "sub-optimum" condition in class E amplifiers, where $i_{C_1}(\sigma T) < 0$ is utilized to achieve soft switching. The anti-parallel diode D_1 of transistor Q_1 is let conduct while v_{C_1} has reversed its polarity. The voltage across Q_1 will then be held at a low level equal to the diode drop. Transistor Q_1 is then turned on by the control circuit to take over the current of the body diode and keep the capacitor v_{C_1} at near 0 V. The switching loss can be further reduced if the current at the switching instant is also low. If the condition of zero-current-switching (ZCS), $i_{C_1}(\sigma T) = 0$, is also enforced, we have the optimum operation. This corresponds to

$$I_L \sin(2\pi\sigma + \phi) = I_1. \tag{7}$$

Combining (6) and (7), we obtain a condition for optimum operation, i.e.,

$$-\pi\left(\frac{1}{2} + \sigma\right)\left[\tan(\pi\sigma + \phi) + \tan(\pi\sigma)\right] = 1. \tag{8}$$

TABLE I
DESIGN EXAMPLES WITH $Q_L = 1.3$ AND 1.9

Components/Parameters	Values	
Q_L	1.3	1.9
R	50 Ω	50 Ω
Z_o	38.4 Ω	26.3 Ω
ω_1/ω_o	1.51	1.43
ω/ω_o	1.101	1.124
C_1/C	0.78	0.96
L	6 μH	4.27 μH
C	4.06 nF	6.17 nF
C_1	3.17 nF	5.90 nF
f_o	1.02 MHz	0.98 MHz
f	1.12 MHz	1.10 MHz
P	30–150 W	50–270 W
E	14.63–32.72 V	13.95–32.41 V

The resonant tank is driven by $v_T(t) = v_{C_1}(t) - v_{C_2}(t)$ as shown in Fig. 2, which is given by

$$v_T(t) = \begin{cases} v_{C_1}(t) & \text{state 1 and} \\ & \quad t \in \left[-\frac{1}{2}T, 0\right), \\ 0 & \text{state 2 and} \\ & \quad t \in \left[-\kappa T, 0\right), \\ v_{C_1}(t) - v_{C_1}(t + \frac{1}{2}T) & \text{state 3 and} \\ & \quad t \in \left[-\frac{1}{2}T,\right. \\ & \quad \left.(-\frac{1}{2} + \lambda)T\right), \\ -v_T(t - \frac{1}{2}T) & t \in \left[0, \frac{1}{2}T\right) \end{cases} \tag{9}$$

Putting (6) in (4) gives

$$v_{C_1}(t) = \begin{cases} A\left[\left(\frac{\omega t}{\pi} + 1\right)\right. & \\ \quad -B\left(\cos(\omega t + \phi)\right. & \\ \quad \left.\left. +\ \cos\phi\right)\right] & -\frac{T}{2} \le t < \tau \\ 0 & \tau \le t < \frac{T}{2} \end{cases} \tag{10}$$

where $\tau = -\kappa T$ for mode 1, $\tau = 0$ for mode 2, and $\tau = \lambda T$ for mode 3, and $A = \frac{I_1 \pi}{\omega C_1}$ and $B = \frac{\frac{1}{2}+\sigma}{\cos(\pi\sigma+\phi)\cos(\pi\sigma)}$. Using Fourier analysis, it can be shown that the harmonic components of (9) is lowest when operating near mode 2, i.e., $\sigma \approx 0$. In our analysis, we will assume $\sigma = 0$.

An equivalent circuit for analysis is shown in Fig. 6 [6]. This equivalent circuit is driven by a sinusoidal source $i(\omega) = \frac{4}{\pi} I_1 \sin(\omega t)$. At optimum operation, the magnitude and phase of the inductor current of the parallel resonant tank at ω are given as

$$\left(\frac{I_L}{I_1}\right)^2(\omega) = \frac{(4/\pi)^2\left[(Q_L\omega)^2 + \omega_o^2\right]\left(\omega_o^2 - \omega_1^2\right)^2}{\left[\omega Q_L\left(\omega^2 - \omega_1^2\right)\right]^2 + \left[\omega_o\left(\omega^2 - \omega_1^2 + \omega_o^2\right)\right]^2} \tag{11}$$

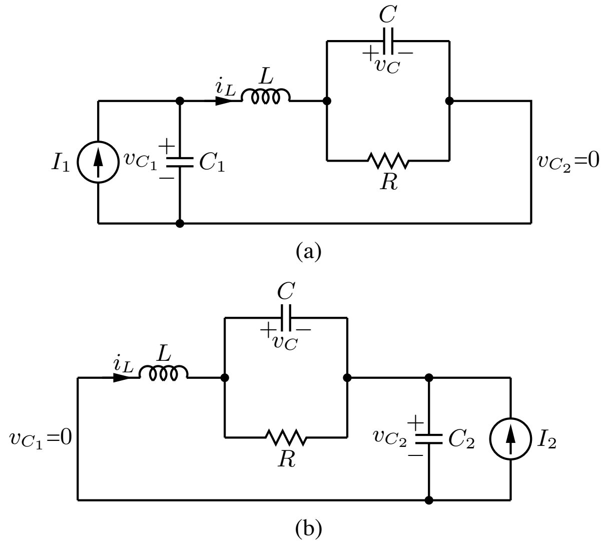


Fig. 3. State 1 of the symmetrically driven class E amplifier.

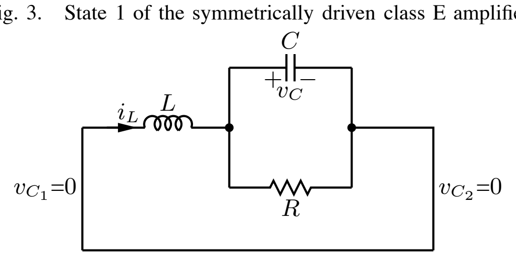


Fig. 4. State 2 of the symmetrically driven class E amplifier.

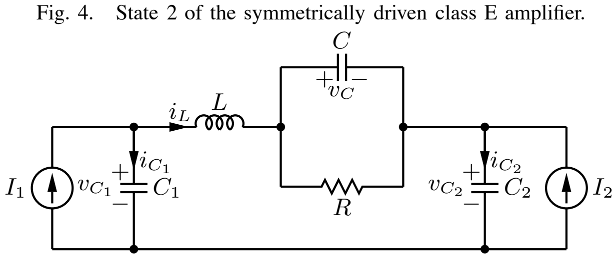


Fig. 5. State 3 of the symmetrically driven class E amplifier.

and

$$\tan(\phi(\omega)) = \frac{Q_L \omega \omega_o^3}{Q_L^2 \omega^2 \left(\omega^2 - \omega_1^2\right) + \omega_o^2 \left(\omega^2 - \omega_1^2 + \omega_o^2\right)}, \tag{12}$$

respectively, where $Z_o = \sqrt{L/C}$, $Q_L = \omega_o CR$, $\omega_o = 1/\sqrt{LC}$, $\omega_1 = 1/\sqrt{LCC_1/C + C_1}$. Conversely, the component values of the resonant tank are represented as $L = Z_o/\omega_o$, $C = 1/\omega_o Z_o$, $R = Q_L Z_o$, $C_1 = 1/\omega_o Z_o((\omega_1/\omega_o)^2 - 1)$. Combining (7), (8), (11) and (12), the optimum class E operation point for mode 2 can be found by solving the following equations numerically.

$$\begin{aligned} -\frac{2}{\pi} &= \tan(\phi(\omega)) \\ &= \frac{Q_L \omega \omega_o^3}{Q_L^2 \omega^2 \left(\omega^2 - \omega_1^2\right) + \omega_o^2 \left(\omega^2 - \omega_1^2 + \omega_o^2\right)} \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{2^2}{2^2 + \pi^2} &= \frac{1}{\sin^2(\phi(\omega))} \\ &= \frac{(4/\pi)^2 \left[(Q_L \omega)^2 + \omega_o^2\right] \left(\omega_o^2 - \omega_1^2\right)^2}{\left[\omega Q_L \left(\omega^2 - \omega_1^2\right)\right]^2 + \left[\omega_o \left(\omega^2 - \omega_1^2 + \omega_o^2\right)\right]^2}. \end{aligned} \tag{14}$$

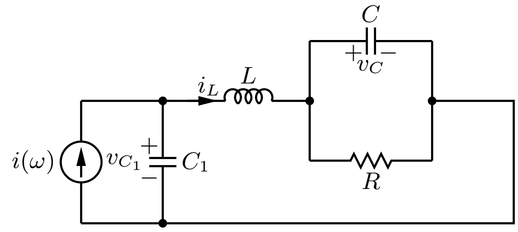


Fig. 6. Equivalent circuit for analysis.

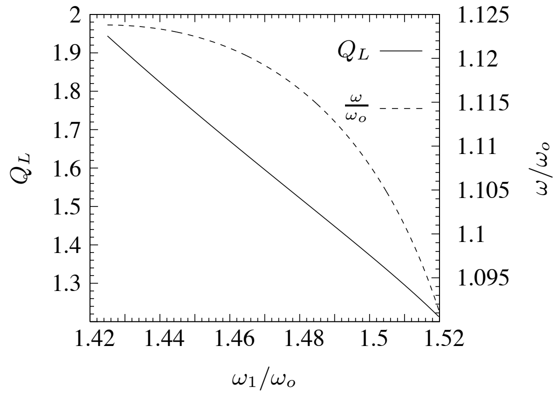


Fig. 7. Quality factor and switching frequency design curves for optimum operation of the symmetrically driven class E amplifier.

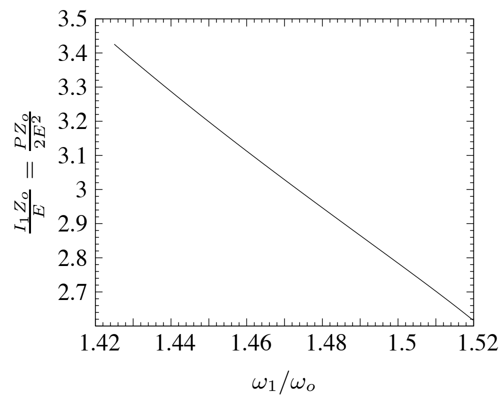


Fig. 8. Normalized input current design curve for optimum operation of the symmetrically driven class E amplifier.

In the steady state, the volt-time product of L_1 should be zero, i.e.,

$$2E = \frac{2}{T} \int_{-\frac{T}{2}}^{0} v_{C_1}(t) dt. \tag{15}$$

Solving (15), we obtain

$$\frac{\pi\omega}{\omega_o} \Bigg/ \left[\left(\frac{\omega_1}{\omega_o}\right)^2 - 1\right] = \frac{I_1 Z_o}{E} = \frac{PZ_o}{2E^2} \tag{16}$$

where P is the power given by $P = 2I_1E$. Figure 7 shows the graphical representations for (13) and (14), and Fig. 8 shows the corresponding plot for (16).

III. Design Examples and Experiments

Suppose $Q_L = 1.3$, $Z_o = 38.4\ \Omega$, and $R = 50\ \Omega$. Then, from Fig. 7, $\frac{\omega_1}{\omega_o} = 1.51$ and $\frac{\omega}{\omega_o} = 1.101$. If we wish to use a switching frequency of about 1 MHz and a convenient range of input voltage

TABLE II

COMPARISON OF HARMONIC CONTENTS OF INDUCTOR CURRENT i_L

Harmonic of i_L	$Q_L = 1.3$				$Q_L = 1.9$			
	Symmetrically driven		Conventional		Symmetrically driven		Conventional	
	Simulation	Measured	Simulation	Measured	Simulation	Measured	Simulation	Measured
2^{nd}	-	-	17.07%	16.06%	-	-	12.53%	12.02%
3^{rd}	3.45%	2.75%	2.33%	2.00%	1.82%	2.31%	1.46%	2.75%
4^{th}	-	-	1.10%	0.95%	-	-	0.85%	1.20%
5^{th}	0.64%	0.58%	0.41%	0.33%	0.22%	0.43%	0.28%	0.48%
6^{th}	-	-	0.29%	0.32%	-	-	0.22%	0.34%
Total (rms)	**3.51%**	**2.81%**	**17.27%**	**16.75%**	**1.83%**	**2.35%**	**12.65%**	**12.41%**

TABLE III

COMPARISON OF HARMONIC CONTENTS OF OUTPUT CURRENT i_o

Harmonic of i_o	$Q_L = 1.3$				$Q_L = 1.9$			
	Symmetrically driven		Conventional		Symmetrically driven		Conventional	
	Simulation	Measured	Simulation	Measured	Simulation	Measured	Simulation	Measured
2^{nd}	-	-	9.64%	8.71%	-	-	6.64%	5.50%
3^{rd}	1.36%	0.91%	0.89%	0.91%	0.82%	0.76%	0.52%	0.78%
4^{th}	-	-	0.32%	0.50%	-	-	0.23%	0.26%
5^{th}	0.15%	0.26%	0.09%	0.25%	0.07%	0.16%	0.06%	0.19%
6^{th}	-	-	0.06%	0.20%	-	-	0.04%	0.19%
Total (rms)	**1.37%**	**0.95%**	**9.69%**	**8.78%**	**0.82%**	**0.78%**	**6.66%**	**5.56%**

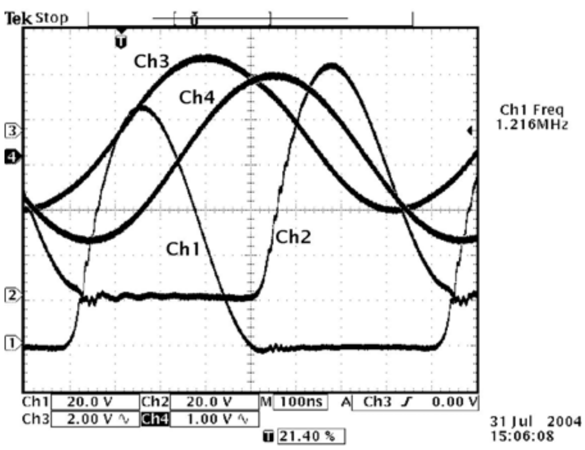


Fig. 9. Waveforms from the symmetrically driven class E power amplifier. Trace Ch1 is the voltage of v_{C_1}, trace Ch2 is the voltage of v_{C_2}, trace Ch3 is the tank inductor current and trace Ch4 is the load resistor current.

from around 15 to 32 V, then using Fig. 8, we arrive at the component and parameter values shown in Table I. The power level ranges from 30 to 150 W, depending upon the input voltage value. Likewise, we can obtain the component and parameter values for the case $Q_L = 1.9$, $Z_o = 26.3\ \Omega$ and $R = 50\ \Omega$, as shown in Table I.

Experiments have been carried out for the two design examples mentioned in the previous section. For the purpose of comparison, two additional conventional class E power amplifier circuits have also been implemented. For a fair comparison, the circuits use the same Q-factor and resonant tank as in the symmetrically driven circuit. A large capacitor (0.1 μF) has been added to block the dc to the output load. To achieve class E operation, the value of the parallel capacitor C_1 has been reduced to 1.47 nF (for $Q_L = 1.3$) and 4.2 nF (for $Q_L = 1.9$), and the corresponding switching frequency has been increased to 1.36 MHz and 1.22 MHz.

Figure 9 shows the waveforms from the symmetrically driven class E amplifier for $Q_L = 1.3$. The harmonic contents of the currents are measured using the FFT function of a digital storage oscilloscope (DSO). Tables II and III compare the harmonic contents of the symmetrically driven class E amplifier and conventional single-ended counterpart. It should be noted that the even harmonics of the symmetrically driven class E circuit are too small to be measured, and are thus omitted in the tables. It is found that the symmetrically driven class E amplifier circuit has distortion of nearly an order of magnitude lower than that of the conventional class E circuit. The measured efficiency from our prototypes is about 85%, the loss being mainly dissipated in the heating of the inductor core and the on-resistance of the switches.

IV. CONCLUSION

A symmetrically driven class E power amplifier has been studied in detail. Analysis has been performed for three possible modes of operation. Optimum design equations and look-up graphs have been presented to facilitate design and implementation. Experimental circuits have been built for harmonic distortion evaluation. By virtue of the symmetrical driving voltage which is already close to a sinewave, the amplifier achieves very low harmonic distortion while maintaining similar power efficiency.

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