A Comparison of Equalizers for Compensating Polarization-Mode Dispersion in 40-Gb/s Optical Systems

Jonathan Sewter and Anthony Chan Carusone
Edward S. Rogers Sr. Department of Electrical and Computer Engineering
University of Toronto
Email: tcc@eecg.utoronto.ca

***Abstract*— A system-level analysis of first-order polarization-mode dispersion (PMD) effects in a 40-Gb/s optical system is used to compare quantitatively different electronic equalizer architectures as potential PMD compensators. It is found that a decision feedback equalizer (DFE) consisting of a 3-tap feedforward equalizer (FFE) and a 1-tap feedback equalizer (FBE) is able to increase the useful length of an optical system by more than eight times when PMD is the dominant limiting factor.**

I. Introduction

Optical fibers have been used for decades for high-volume data communication. Until recently, long-haul links over single-mode fiber (SMF) could be designed without concern for the bandwidth limitations of the fiber. To satiate the demand for greater network capacity, the data rate of current optical systems has been pushed to 10 and 40 Gb/s (OC-192 and OC-768). At these data rates, it is no longer possible to neglect the bandwidth limitations of SMF, as several dispersion mechanisms lead to frequency-dependent loss [1].

The two most important dispersion mechanisms for SMF are chromatic dispersion (CD) and polarization-mode dispersion (PMD). CD is a result of the wavelength-dependency of the refractive index of the fiber. PMD results from the variation in the refractive index of the fiber with respect to the polarization of the light signal. Since CD is mitigated by the use of narrow spectral-width sources and proper choice of optical fiber, PMD has been identified as a major limiting factor in high-speed optical systems [2].

To mitigate the effects of PMD, optical systems must include some form of PMD compensation. This compensation can be achieved either optically or electronically. Electronic PMD compensation schemes are attractive because they allow higher integration with existing circuitry, leading to more compact, less expensive solutions. This is especially true for wavelength-division multiplexed (WDM) systems, in which every channel needs PMD compensation [3]. Also, because PMD fluctuates with changes in temperature and environment, PMD compensators must be able to adapt to varying channel conditions within milliseconds [4]. Fast and accurate adaptation is more easily performed in the electronic domain. Successful electronic equalization has been demonstrated at 10 Gb/s [5], [6], [7] and more recently, at 40 Gb/s [8], [9].

While it has been shown that nonlinear equalization using a decision feedback equalizer (DFE) is required to reduce the power penalty caused by PMD to acceptable levels [10], the tradeoffs between different equalizer architectures are not evident. In this paper, a system-level analysis of PMD effects in a 40-Gb/s system using Matlab/Simulink is used to compare several equalizer architectures in terms of overall system performance, as has been done for optical compensation schemes [11]. Using this analysis we are able to quantify the increase in system reach that can be attained by equalization and show that effective compensation of PMD can be achieved using only a few equalizer taps.

Section II provides a description of PMD and its consequences for high-speed optical system design. In Section III, a system-level analysis of first-order PMD effects in a 40-Gb/s system is described. The results of this analysis are used to compare quantitatively the performance of different equalizer configurations.

II. Polarization-Mode Dispersion (PMD)

PMD is a result of the phenomenon of birefringence which affects all real optical fibers. Birefringence is the difference in refractive index experienced by light in different polarization modes, and is caused by ellipticity of the fiber cross-section due to asymmetric stresses applied to the fiber during or after manufacturing. Birefringence leads to fast and slow modes of propagation and consequently dispersion.

A. Consequences of PMD for Optical Systems

In terms of digital communications, PMD results to a first order in an input pulse being split into a fast and slow pulse which arrive at the receiver at different times. If the differential delay of the two pulses is significant compared to the bit period, intersymbol interference (ISI) and a corresponding increase in bit-error rate (BER) will result.

To a first order, the impulse response of an optical fiber with PMD is [12]:

$$h_{\mathrm{PMD}}(t) = \gamma\delta(t) + (1-\gamma)\delta(t-\Delta\tau) \quad (1)$$

where γ is the proportion of the optical power in the "fast" state of polarization (SOP), (1-γ) is the proportion of power in the "slow" SOP and $\Delta\tau$ is the differential group delay (DGD) between the fast and slow components.

γ and $\Delta\tau$ vary depending on the particular fiber and its associated stresses. γ can by its definition take any value from zero to one. $\Delta\tau$ varies statistically according to a Maxwellian distribution [13]. Therefore, though it can vary to large values, it will for the most part be close to some average value. The average DGD per unit length for a given fiber is defined as its PMD parameter, which has units of ps/$\sqrt{\text{km}}$. The PMD of a typical installed fiber is in the range of 0.5 - 2.0 ps/$\sqrt{\text{km}}$. New fibers can be manufactured with a PMD of as low as 0.05 ps/$\sqrt{\text{km}}$. Given the PMD parameter, the average DGD of a fiber of length L is given by:

$$\Delta\tau_{avg} = \mathrm{PMD} \times \sqrt{L} \quad (2)$$

It has been calculated that to prevent PMD from causing system outages amounting to more than thirty seconds per year, the average DGD must be less than approximately 15% of a bit period, T_B [2].

$$\Delta\tau_{avg} < 0.15T_B \quad (3)$$

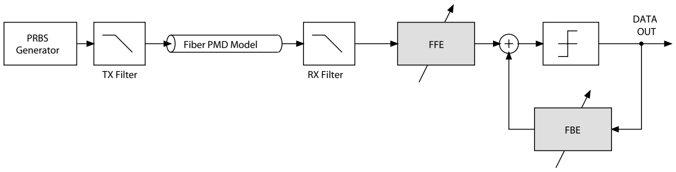

Fig. 1. System model used for Matlab/Simulink simulations.

This has severe implications as the data rate of these systems is increased to 10 and 40 Gb/s. As the data rate is increased on a given fiber, the maximum useful length of that fiber decreases according to the square of the increase. For example, given a fiber with a PMD of 1.0 ps/$\sqrt{\text{km}}$ and using (3), the maximum length of a 2.5-, 10- and 40-Gb/s system is 3600, 225 and 14 km, respectively.

III. Analysis of PMD Effects in Optical Systems

A. System Model

A simplified block diagram of the system model used in the Matlab/Simulink simulations is given in Figure 1. A pseudorandom bit sequence (PRBS) generator is used to generate input data at a rate (R) of 40 Gb/s. This data is passed through a lowpass filter ($f_{3dB} = 0.7 \times R$) which is used to simulate the effects of the finite bandwidth of the transmitter (TX). It is then passed through an optical fiber model which corrupts the data with PMD. The fiber is modelled using (1). At the output of the fiber model the data is filtered with another lowpass filter ($f_{3dB} = 0.7 \times R$) to simulate the finite bandwidth of the receiver (RX). Equalization is then performed and the equalized waveform is sliced to generate the output data. Not shown in Figure 1 are the clock recovery and adaptation components of the system. The sampling phase was determined by automatically selecting the clock phase corresponding to the largest eye opening at the output of the channel. Coefficient adaptation for both the feedforward equalizer (FFE) and feedback equalizer (FBE) was performed using the least mean square (LMS) algorithm.

Figure 1 shows the equalizer as a DFE, though several equalizer architectures were considered.

B. Equalizer Architectures

1) Analog Equalizer: Analog or “peaking” equalizers have been used in the past for equalizing simple lowpass channels [14]. The potential advantage of this architecture is its relatively simple implementation. However, the analog equalizer is unsuitable as a PMD compensator because it is not flexible enough to adapt to the wide range of potential PMD conditions. Also, because it is a linear circuit it is unable to compensate for the deep null in the frequency spectrum caused by PMD with γ values near 0.5.

2) Infinite Impulse Response (IIR) Filter: The infinite impulse response (IIR) equalizer would seem to have great potential as a PMD compensator. Taking the Fourier transform of (1), the frequency response of the PMD channel is given by:

$$H_{\text{PMD}}(f) = \gamma + (1-\gamma)e^{-j2\pi f\Delta\tau} \tag{4}$$

Therefore, the inverse of the channel transfer function is:

$$H_{\text{PMD}}^{-1}(f) = \frac{1}{\gamma + (1-\gamma)e^{-j2\pi f\Delta\tau}} \tag{5}$$

$H_{\text{PMD}}^{-1}(f)$ also relates the input $X(f)$ and output $Y(f)$ of the inverse filter:

$$H_{\text{PMD}}^{-1}(f) = \frac{Y(f)}{X(f)} \tag{6}$$

Solving for $Y(f)$ we get the input-output relationship:

$$Y(f) = \frac{1}{\gamma}X(f) + \frac{\gamma-1}{\gamma}e^{-j2\pi f\Delta\tau}Y(f) \tag{7}$$

Taking the inverse Fourier transform of (7) we get the difference equation:

$$y(t) = \frac{1}{\gamma}x(t) + \frac{\gamma-1}{\gamma}y(t-\Delta\tau) \tag{8}$$

This difference equation describes an IIR filter. While this architecture would seem to offer perfect (zero-forcing) equalization of a PMD channel, the nature of the feedback loop creates problems in practice. Specifically, for $\gamma \leq 0.5$, the equalizer loop gain, which is equal to $\frac{\gamma-1}{\gamma}$ by inspection of (8) is less than -1, meaning that the equalizer is unstable. Thus, since it is unable to compensate PMD for all values of γ, the IIR filter is unsuitable for implementation as a PMD compensator.

3) Finite Impulse Response (FIR) Filter: The FIR filter is a versatile equalizer architecture which is widely used. FIR filters can, given enough taps, approximate any linear transfer function, making them attractive because of their flexibility. However, the usefulness of an FIR filter as a PMD compensator is severely limited because, as a linear filter, it is unable to compensate for the deep nulls caused by PMD with γ values near 0.5.

4) Decision Feedback Equalizer (DFE): Figure 1 illustrates the basic DFE topology. The DFE consists of an FFE and an FBE, both of which can be implemented as FIR filters for maximum flexibility. The most important advantage of the DFE architecture in terms of PMD compensation is that the use of an FBE introduces nonlinear equalization, allowing compensation of the nulls resulting from γ values near 0.5. Because of this, the DFE is the only architecture surveyed that meets the requirements for an electronic PMD compensator.

C. Simulation Methodology

Having identified the DFE architecture as the most suitable, simulations were performed to identify the performance tradeoffs with respect to number of equalizer taps (FFE and FBE). All simulations were performed with symbol-spaced equalizer taps.

For each equalizer configuration, it was necessary that a wide range of PMD conditions were considered. $\Delta\tau$ was varied from 0 to 100 ps (4 bit periods at 40 Gb/s) and γ was varied from 0 to 1. For each ($\Delta\tau$, γ) pair the equalizer was allowed to converge to the ideal tap weights as determined by the LMS algorithm. Then, the ISI penalty was determined by calculating the amount of eye closure using:

$$\text{ISI penalty (dB)} = 10 \times \log_{10}\left(\frac{\textit{max. eye opening}}{\textit{min. eye opening}}\right) \tag{9}$$

Figure 2 shows representative eye diagrams for the unequalized and equalized case for one particular ($\Delta\tau$, γ) pair. Figure 3 shows surface plots of the ISI penalty for the unequalized and one equalized case over a range of ($\Delta\tau$,γ) pairs. It demonstrates the elimination of the penalty pole at $\Delta\tau$ = 25 ps, γ = 0.5 by equalization with a DFE.

Once the ISI penalty had been calculated for all ($\Delta\tau$,γ) pairs, the cumulative probability (CP) of a system outage given a particular power margin was calculated using:

$$\text{CP} = \sum_{(\Delta\tau,\gamma)'} \rho_1(\Delta\tau)\rho_2(\gamma) \tag{10}$$

where $\rho_1(\Delta\tau)$ is the probability distribution of $\Delta\tau$ (Maxwellian), $\rho_2(\gamma)$ is the probability distribution of γ (uniform) and $(\Delta\tau,\gamma)'$ is the set of $(\Delta\tau,\gamma)$ pairs for which the ISI penalty is greater than the power margin. Power margin represents the ratio of the transmitted

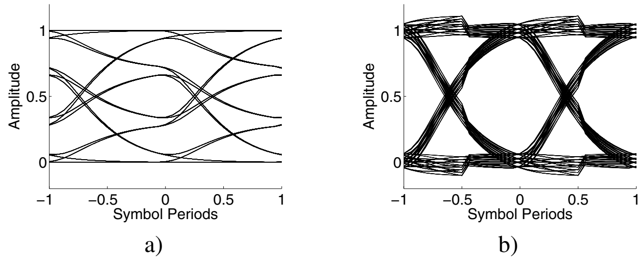

Fig. 2. Eye diagrams for $\Delta\tau$ = 25 ps, γ = 0.3. a) No equalization. b) Equalization by 3-tap FFE and 1-tap FBE.

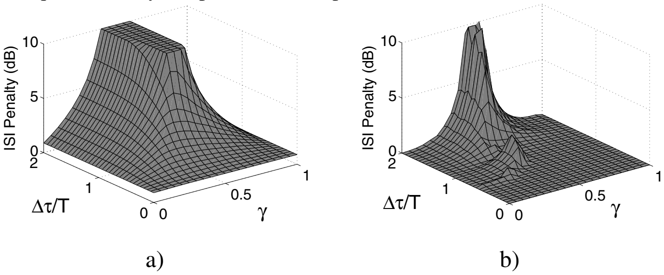

Fig. 3. ISI penalty vs. $\Delta\tau$ and γ. ISI penalty is truncated at 10dB. a) No equalization. b) Equalization by 3-tap FFE and 1-tap FBE.

power to the transmitted power required for a given BER (e.g. 10^{-12}). When the ISI penalty exceeds the power margin, a system outage occurs because the excess transmitted power cannot overcome the eye closure caused by the ISI, and the BER increases above the specified maximum level.

As described in Section II-A, $\rho_1(\Delta\tau)$ depends on the average DGD of the particular fiber. For each equalizer configuration, the probability distribution was varied by adjusting the average DGD to find the maximum average DGD that would result in a CP of less than 10^{-6} (30 seconds per year).

D. Simulation Results

Figures 4, 5 and 6 show the results of these simulations for DFEs with no FBE, a 1-tap FBE and a 2-tap FBE, respectively. In each case, the maximum average DGD that is tolerable from a system point of view is plotted against the power margin for different numbers of FFE taps. In addition, the unequalized case is included as a reference for comparison. Figure 4 shows that using an FFE only, a modest increase in maximum average DGD is possible, from roughly $0.15T_b$ to $0.25T_b$. Only minor improvements are achievable by increasing the number of FFE taps because regardless of the number of taps the FFE is unable to compensate for the case $\Delta\tau$ = 25 ps, γ = 0.5. Figure 5 demonstrates that by using a 1-tap FBE, a significant performance increase is possible, with the maximum average DGD increasing to approximately $0.5T_b$. For this case, the optimal number of FFE taps is dependent on the power margin. For power margins below 3dB, four taps are optimal, while three taps are optimal for power margins above 3dB. Figure 6 shows that a further increase in maximum average DGD is possible by using a 2-tap FBE, but significant gains are limited to power margins above 4dB. Once again, four FFE taps are optimal for power margins below 3dB, while three taps are optimal for power margins above 3dB.

Figure 7 shows the maximum average DGD plotted against the number of FFE taps for a power margin of 3dB. Once again, the unequalized case is included for comparison. This plot more clearly shows the performance of each of the equalizer architectures. It is clear from this plot that for a power margin of 3dB, a 3-tap FFE offers

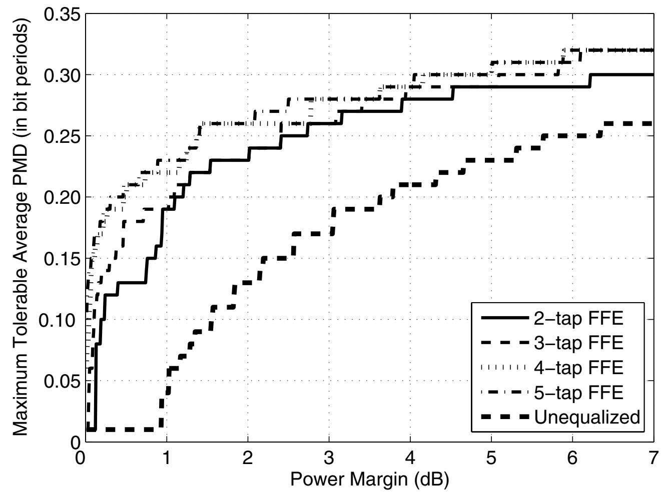

Fig. 4. Plot of maximum tolerable PMD vs. power margin for FFEs with varying number of taps (No FBE).

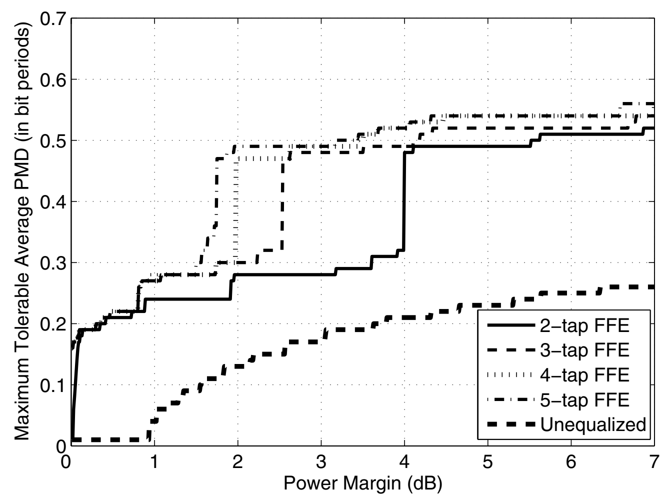

Fig. 5. Plot of maximum tolerable PMD vs. power margin for FFEs with varying number of taps (1-tap FBE).

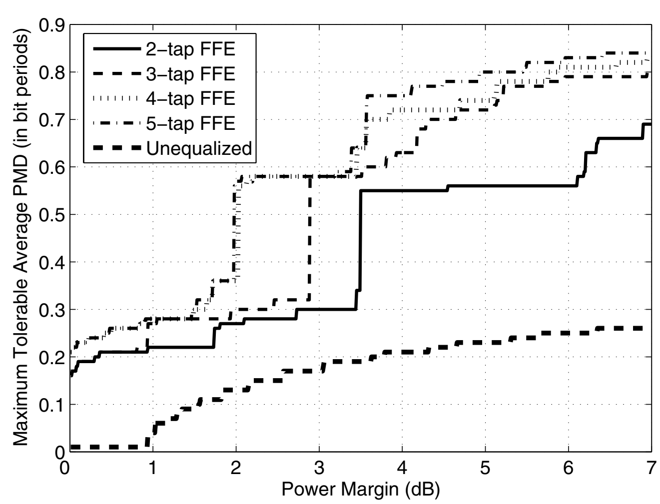

Fig. 6. Plot of maximum tolerable PMD vs. power margin for FFEs with varying number of taps (2-tap FBE).

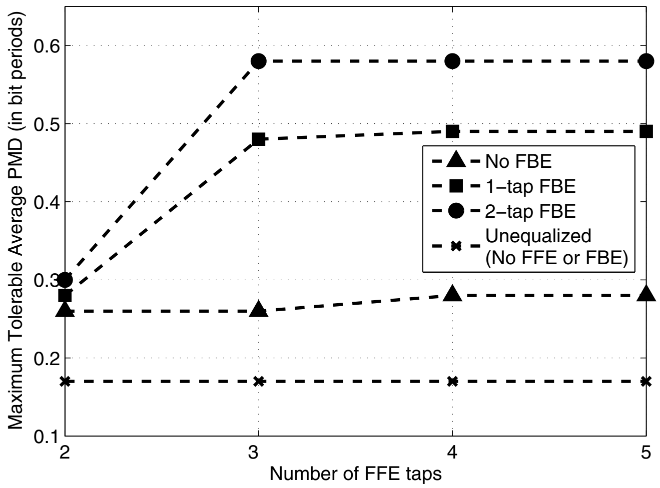

Fig. 7. Plot of maximum tolerable PMD vs. number of FFE taps for no FBE, 1-tap FBE and 2-tap FBE at a power margin of 3dB.

performance nearly equal to the more complex 4- and 5-tap FFEs. As expected, the 2-tap FBE offers a modest performance increase over the 1-tap FBE. However, the 1-tap FBE may be a more attractive choice when this performance increase is weighed against the added complexity of a second tap.

These results are significant because they imply that the useful length of high-speed optical systems affected by PMD can be greatly increased by including electronic PMD compensation in the form of a DFE. To keep system outage levels at an acceptable level, $\Delta\tau_{avg}$ must be less than the maximum average DGD. Therefore, using (2) it is found that the useful length of the fiber increases with the square of the increase in maximum average DGD. As an example, consider a 40-Gb/s system for which the PMD of the fiber is 1.0 ps/$\sqrt{\text{km}}$, and the power margin is 3dB. From Figure 7, the maximum average DGD for an unequalized system at a power margin of 3dB is $0.17T_b$, corresponding to a maximum system length of 18 km, using (2). The maximum average DGD for a system using a DFE with a 3-tap FFE and 1-tap FBE is $0.49T_b$, corresponding to a maximum system length of 150 km. Therefore, an increase in maximum length of more than eight times is achieved by equalization.

While considering the increase in system length due to equalization, it must be noted that this calculation assumes that PMD is the dominant factor limiting the length of the system. In practice, other impairments (noise, CD) would likely replace PMD as the limiting factors once PMD had been compensated (although equalization would also help to compensate CD). As a result, the increase in system length would be less than predicted. The main conclusion, however, is still valid: electronic equalization using only a few taps can significantly reduce the impact of PMD on 40-Gb/s optical systems, resulting in an increase in system reach and the elimination of PMD as the dominant length-limiting factor.

IV. Conclusion

A system-level analysis using Matlab/Simulink has been performed to compare the performance of different electronic PMD compensator architectures at 40 Gb/s. It has been demonstrated that equalization by a DFE with a 3-tap FFE and a 1-tap FBE is able to increase by nearly three times the maximum average DGD that is tolerable from a system point of view, from $0.17T_b$ to $0.49T_b$. This is significant because it implies an increase in the useful length of a given PMD-limited system of more than eight times (e.g. from 18 km to 150 km for a fiber with a PMD of 1.0 ps/$\sqrt{\text{km}}$).

Acknowledgment

This work has been supported by grants from the Natural Sciences and Engineering Research Council of Canada (NSERC), Gennum Corporation and Micronet.

References

[1] K. Azadet, E. F. Haratsch, H. Kim, F. Saibi, J. H. Saunders, M. Shaffer, L. Song, and M. L. Yu, "Equalization and FEC techniques for optical transceivers," *IEEE J. Solid-State Circuits*, vol. 37, no. 3, pp. 317–327, Mar. 2002.

[2] H. Bülow, "Polarisation mode dispersion (PMD) sensitivity of a 10 Gbit/s transmission system," in *Optical Communication, 1996. ECOC '96. 22nd European Conference on*, vol. 2, 1996, pp. 211–214.

[3] ——, "PMD mitigation techniques and their effectiveness in installed fiber," in *Optical Fiber Communication Conference, 2000*, vol. 3, 2000, pp. 110–112.

[4] H. Bülow, W. Baumert, H. Schmuck, F. Mohr, T. Schulz, F. Kuppers, and W. Weiershausen, "Measurement of the maximum speed of PMD fluctuation in installed field fiber," in *Optical Fiber Communication Conference, 1999, and the International Conference on Integrated Optics and Optical Fiber Communication. OFC/IOOC '99. Technical Digest*, vol. 2, 1999, pp. 83–85.

[5] H. Bülow, F. Buchali, W. Baumert, R. Ballentin, and T. Wehren, "PMD mitigation at 10 Gbit/s using linear and nonlinear integrated electronic equaliser circuits," *Electronics Letters*, vol. 36, no. 2, pp. 163–164, 2000.

[6] H. Wu, J. A. Tierno, P. Pepeljugoski, J. Schaub, S. Gowda, J. A. Kash, and A. Hajimiri, "Integrated transversal equalizers in high-speed fiber-optic systems," *IEEE J. Solid-State Circuits*, vol. 38, no. 12, pp. 2131–2137, Dec. 2003.

[7] C. Pelard, E. Gebara, A. J. Kim, M. G. Vrazel, F. Bien, Y. Hur, M. Maeng, S. Chandramouli, C. Chun, S. Bajekal, S. E. Ralph, B. Schmukler, V. M. Hietala, and J. Laskar, "Realization of multigigabit channel equalization and crosstalk cancellation integrated circuits," *IEEE J. Solid-State Circuits*, vol. 39, no. 10, pp. 1659–1670, Oct. 2004.

[8] M. Nakamura, H. Nosaka, M. Ida, K. Kurishima, and M. Tokumitsu, "Electrical PMD equalizer ICs for a 40-Gbit/s transmission," in *Optical Fiber Communication Conference and Exhibit, 2004. OFC 2004*, vol. TuG4, 2004.

[9] A. Hazneci and S. P. Voinigescu, "A 49-Gb/s, 7-tap transversal filter in 0.18μm SiGe BiCMOS for backplane equalization," in *IEEE Compound Semiconductor Integrated Circuits Symposium*, Monterey, CA, Oct. 2004.

[10] H. Bülow and G. Thielecke, "Electronic PMD mitigation - from linear equalization to maximum-likelihood detection," in *Optical Fiber Communication Conference and Exhibit, 2001. OFC 2001*, vol. 3, 2001, pp. WAA3–1–WAA3–3.

[11] H. Sunnerud, C. J. Xie, M. Karlsson, R. Samuelsson, and P. A. Andrekson, "A comparison between different PMD compensation techniques," *J. Lightwave Technol.*, vol. 20, no. 3, pp. 368–378, Mar. 2002.

[12] C. D. Poole, R. W. Tkach, A. R. Chraplyvy, and D. A. Fishman, "Fading in lightwave systems due to polarization-mode dispersion," *IEEE Photon. Technol. Lett.*, vol. 3, no. 1, pp. 68–70, Jan. 1991.

[13] G. J. Foschini and C. D. Poole, "Statistical-theory of polarization dispersion in single-mode fibers," *J. Lightwave Technol.*, vol. 9, no. 11, pp. 1439–1456, Nov. 1991.

[14] G. Zhang, P. Chaudhari, and M. Green, "26.7 - a BiCMOS 10Gb/s adaptive cable equalizer," in *Solid-State Circuits Conference, 2004. Digest of Technical Papers. ISSCC. 2004 IEEE International*, 2004, pp. 482–491.