Extensions of ACC Technology Toward Ad Hoc Platooning with Guaranteed String Stability and Improved Tracking Performance

Gábor Rödönyi
Systems and Control Laboratory, Institute for Computer Science and Control, Hungarian Academy of Sciences
H-1111. Kende u. 13-17. Budapest, Hungary. Email: rodonyi.gabor@sztaki.mta.hu

***Abstract*—The proliferation of autonomous driving and Adaptive Cruise Controlled (ACC) vehicles in particular increases the probability of the ad hoc formation of unorganized platoons. This rises new demands on control design beyond safe tracking: string stability must be ensured, which is a challenging problem in a heterogeneous platoon with unknown members. String stability, the ability of the vehicles to attenuate disturbances as they are propagating along the string, improves traffic stability and tracking performance. It is shown how the requirement of string stability can be formulated in terms of conditions that can be satisfied locally. The proposed approach opens ways to design a great variation of vehicle following control architectures including multiple predecessor and leader following architectures with guaranteed string stability even in heterogeneous, ad hoc platoons. The presented control methods may motivate developments on other fields of autonomous driving technology as well, such as sensing, communication, and embedded computation.**

I. Introduction

The ultimate goals of efforts in the field of Intelligent Transportation Systems (ITS) are to achieve the following properties

1) increased traffic performance (prevent shockwaves), and
2) increased road capacity and safety.

One research area in this field aims to provide *Intelligent Driving System* applications where the following trend can be observed in the developments. The degree of automation, from the manual to the fully autonomous driving, and the degree of cooperation between vehicles (and infrastructure) increases [1]. Between the two ends of this trend, from manual driving to the fully synchronized and automated vehicle platooning technologies, there is a great variety of developments, such as Adaptive Cruise Control (ACC) systems, cooperative (CACC) and extended ACC systems [2]–[5]. Each car following technology can, and should be evaluated in the view of the above-mentioned properties 1) and 2).

The main requirements against ACC technology is regarding individual vehicle safety and collision avoidance (property 2)) [6], [7]. With the proliferation of vehicles equipped with such car following functionality, the possibility of their meeting and forming unintended, unorganized platoons increases in dense traffic. For synchronized vehicle platoons, however, a further requirement is specified: *string stability*, the ability of the platoon to attenuate transients as they are propagating upstream [8]–[12]. Direct relation between this microscopic property and traffic stability (property 1)) have been shown, e.g., in [13]. Without string stability the upstream vehicles initiate stronger and stronger reactions, and finally, emergency braking of a follower vehicle becomes a likely event, causing shockwave in dense traffic. This implies that any car following application must ensure also string stability, as in [2]–[5]. In order to achieve increased road capacity and safety (property 2)), small inter-vehicle gaps should be maintained. This can be achieved by a control design that minimizes both the spacing error with respect to the predecessor vehicle and the time-headway.

Proving string stability for general nonlinear, time-varying and heterogeneous interconnected systems is a challenging and unsolved problem. In this paper we restrict our attention to linear time-invariant (LTI) systems and unidirectional information flow, but allow general component dynamics, and heterogeneous strings. It is shown that in most of the applications, homogeneous and heterogeneous string stability requirements lead to equivalent conditions. For general longitudinal car following problems, it is shown that the two objectives – small spacing errors and string stability,– can be achieved individually, by solving local control design problems for each vehicles independently, where the string stability requirement is transformed to a design constraint, and a norm for the spacing error is an objective function.

The design approach is illustrated on a series of car following problems. Starting from a simple string stable ACC design, more and more available information is utilized from predecessor vehicles, and the increase in tracking performance is demonstrated. Finally, an adaptive spacing policy is presented, which allows a vehicle to join an unknown, heterogeneous unorganized string of vehicles, with spacing performance similar to that of a synchronized platoon with leader and predecessor following communication architecture and constant spacing policy.

II. Vehicle Following Control Problem

The modeling for the longitudinal control of a single vehicle is adopted from [14], where a static output-feedback control structure is presented that is able to capture several communication architectures ranging from linear (C)ACC controllers, to *Leader and Predecessor Following* (LPF) platoon controllers with arbitrary spacing policies.

Assuming the existence of low level controllers, the dynamics of the ith vehicle in a string can be approximated by a first order linear system (here we neglect delay)

$$\dot{a}_i(t) = -\frac{1}{\tau_i} a_i(t) + \frac{1}{\tau_i}(u_i(t) + d_i(t)), \tag{1}$$

that is designed to track acceleration demand u_i. Vehicle acceleration, speed and position are denoted by a_i, v_i and p_i, respectively, τ_i denotes time constant, and $d_i(t)$ denotes disturbance to describe modeling inaccuracy and outer effects [2], [11], [15], [16].

In the car following control problem acceleration demand u_i is the control signal to vehicle i and may in general depend on the kinetic variables of multiple other vehicles. An LPF architecture is the following

$$\begin{aligned} u_i(t) \;\; = \;\; & K_{a,i}^{i-1}(a_{i-1}(t) - a_i(t)) \\ & + K_{v,i}^{i-1}(v_{i-1}(t) - v_i(t)) + K_{p,i}^{i-1} e_{i,i-1}(t) \\ & + K_{a,i}^{0}(a_0(t) - a_i(t)) \\ & + K_{v,i}^{0}(v_0(t) - v_i(t)) + K_{p,i}^{0} e_{i,0}(t), \end{aligned} \tag{2}$$

where $K_{*,i}^{j}$s are constant controller parameters ($*$ refers to a, v or p), $e_{i,i-1}(t)$ and $e_{i,0}(t)$ denote spacing errors of vehicle i with respect to the predecessor vehicle $i-1$ and the platoon leader, respectively.

Equipped with radars/lidars that measure the distance and relative speed of the predecessor vehicle, an ACC controller can be obtained from (3) by setting $K_{*,i}^{0} = 0$ and $K_{a,i}^{i-1} = 0$. If also V2V communication is available, the measured acceleration of the predecessor can be utilized in a CACC vehicle where $K_{*,i}^{0} = 0$.

The spacing errors are defined by

$$e_{i,j}(t) \triangleq p_j(t) - p_i(t) - R_{i,j}(t), \quad j \in \{0, i-1\}, \tag{3}$$

where $R_{i,j}(t)$ is the desired distance to vehicle j and is called *spacing policy with respect to vehicle* j. One of the most common spacing policies in vehicle following is the *Constant time-headway* (CTHSP) spacing policy

$$R_{i,j}(t) = L_{i,j} + h_{i,j} v_i(t), \quad h_{i,j} > 0 \tag{4}$$

that allows for string stability of (C)ACC architectures with sufficiently large headway parameter $h_{i,i-1}$. Standstill distance $L_{i,j}$ plays no role in stability and performance analysis.

It is shown in Section III and [14] that (C)ACC controllers can always be designed for heterogeneous string stability without respecting the properties of other vehicles, and they may join an arbitrary ad hoc, unorganized string of vehicles. The situation is different when, in order to improve tracking performance (smaller safe gaps), spacing policies are defined with respect to multiple preceding vehicles as in the case of LPF architectures. The spacing policy with respect to a distant vehicle, say $R_{i,0}(t)$, could be defined based on the knowledge of the spacing policy, $R_{i-1,0}(t)$, between vehicle $i-1$ and the leader as $R_{i,0}(t) \triangleq R_{i-1,0}(t) + R_{i,i-1}(t)$. But in an arbitrary vehicle string where even human driven vehicles are present $R_{i-1,0}(t)$ is in general undefined. To resolve this problem an *Adaptive Spacing Policy* (ASP) was presented in [14] to replace $R_{i-1,0}(t)$ by a *virtual spacing policy*, $R_{i-1,0}^{v}$, computed as an output of a dynamic system E_i driven by the kinetic variables (a, v, p) of the predecessor and the leader vehicles. It is defined in the frequency-domain for the brevity of notation as follows

$$R_{i-1,0}^{v}(j\omega) = \begin{bmatrix} E_{i,0}(j\omega), & E_{i,i-1}(j\omega) \end{bmatrix} \begin{bmatrix} a_0(j\omega) \\ a_{i-1}(j\omega) \end{bmatrix} \tag{5}$$

The choice of this virtual spacing policy influences both the string stability property and the tracking performance of vehicle i [14]. The general controller with (2) and (5) is referred by the abbreviation: LPF-ASP.

For the analysis of string stability and performance, we need the closed-loop dynamics including vehicle dynamics (1), driven by the controller (2) with spacing policy (4) or (5). The closed-loop vehicle model can be characterized in the frequency-domain by

$$\left[\begin{array}{c} a_i(j\omega) \\ \hline z_i(j\omega) \end{array}\right] = \left[\begin{array}{c|c} \mathcal{A}_i(j\omega) & \mathcal{B}_i(j\omega) \\ \hline \mathcal{C}_i(j\omega) & \mathcal{D}_i(j\omega) \end{array}\right] \left[\begin{array}{c} a_{i-1}(j\omega) \\ \hline w_i(j\omega) \end{array}\right], \tag{6}$$

where $z_i(j\omega) = \begin{bmatrix} u_i(j\omega) \\ e_{i,i-1}(j\omega) \end{bmatrix}$ and $w_i(j\omega) = \begin{bmatrix} a_0(j\omega) \\ d_i(j\omega) \end{bmatrix}$. For more details of the derivation, please see [14].

Definition 1 (Heterogeneous String Stability (HSS) [14]): Assume that $a_0, d_i \in \mathcal{L}_2$ and the initial conditions $a_i(0), v_i(0), e_{i,j}(0)$ are zero. If the sequence of vehicle accelerations $a_i \in \mathcal{L}_2$, $i = 1, 2, ...$, is uniformly bounded for all string length and vehicle ordering, then the vehicle string defined by the component models (6) is *heterogeneous string stable*, otherwise it is *heterogeneous string unstable*.

Definition 2 (Heterogeneous String Performance (HSP) [14]): Assume that $a_0, d_i \in \mathcal{L}_2$ with $\|a_0\|_2 \leq 1$, $\|d_i\|_2 \leq 1$, and the initial conditions $a_i(0), v_i(0), e_{i,j}(0)$ are zero. The interconnected system defined by the component models (6) has *heterogeneous string performance of level* γ if the sequence of performance signals $z_i \in \mathcal{L}_2$ is uniformly bounded by $\|z_i\|_2 < \gamma$, $i = 1, 2, ...$, for all string length and vehicle ordering.

The formal goal of this paper is threefold:

1) to present a general design procedure for a system described by (6) such that heterogeneous string stability and performance can be achieved;
2) to illustrate the design trade-off between string stability, tracking performance, and time-headway in the case of (C)ACC;
3) to illustrate the increase in the tracking performance, as more and more information is utilized for control. We restrict our attention to controllers that are applicable in ad hoc platoons, i.e., ACC, CACC and LPF-ASP methods.

String stability is a notion that is related to the whole string, but in the cases presented in this paper, the condition for string stability is distributed, i.e., each vehicle has to satisfy a local condition (a condition for homogeneous string stability). If one vehicle happens to violate its string stability condition, and the string happens to consist mostly of this kind of vehicle, transients will amplify upstream.

III. Conditions for HSS and HSP

In contrast to the mainstream literature on string stability, it is not defined as a relation between the consecutive spacing errors, since in general, spacing errors appear as outputs of the component models, see (6). It is the acceleration that is propagating along the string. Component model (6) can be viewed also as a description for the whole string: for all fixed frequency, ω, (6) is a linear discrete(-time) system in state-space form with independent variable i, where the variables and coefficients are complex, the coefficients are varying with i, $a_i(j\omega)$ is the state variable, $z_i(j\omega)$ and $w_i(j\omega)$ are the (performance) output and input, respectively. String stability is related to the convergence or boundedness of state a_i, while string performance is related to the evolution of output z_i. It can be seen that similarly to the relation between stability and performance of ordinary systems, HSS is necessary to HSP.

A. General Conditions

Without elaborating the technical details, which partly can be found in [14], the following can be said about HSS, i.e., the uniform $\mathcal{L}_2$ boundedness of the accelerations.

When (6) is viewed as a discrete-time system with scalar state variable and varying bounded coefficients, it is immediate that the sequence $a_i(j\omega)$ is uniformly bounded if, and only if $|\mathcal{A}_i(j\omega)| < 1$ (exponential stability) and the input $|\mathcal{B}_i(j\omega)w_i(j\omega)|$ is bounded (BIBO stability), or for frequencies where $|\mathcal{A}_i(j\omega)| = 1$ (actually for $\omega = 0$, marginal stability), the input must be zero (transfer function $\mathcal{B}_i(j\omega)$ having at least one zero in $j\omega = 0$). For all a_i belonging to $\mathcal{L}_2$, quadratic integrability of $a_i(j\omega)$ is required, which is guaranteed if the inputs satisfy $\mathcal{B}_i w_i \in \mathcal{L}_2$.

Whenever HSS is established, HSP is implied, for example, by the boundedness of $\mathcal{C}_i$ and $\mathcal{D}_i$ in $\mathcal{H}_\infty$, since $z_i = \mathcal{C}_i a_{i-1} + \mathcal{D}_i w_i$. [1]

B. Controller Design

The above discussion suggests a possible design procedure for a single vehicle. The control structure (2) and stability of the closed-loop vehicle system guarantees the required properties for $|\mathcal{B}_i(j\omega)w_i(j\omega)|$. For string stability, it is required that $|\mathcal{A}_i(j\omega)| < 1$ for $\omega > 0$. In case of the presented control structures $|\mathcal{A}_i(0)| = 1$ guarantees appropriate steady-state tracking properties. In order to improve the spatial damping of the transients that are propagating upstream, a more stringent string stability requirement can be introduced as follows

$$|\mathcal{A}_i(j\omega)| \le \eta_i(\omega) \triangleq \frac{1}{|\tau_{\eta,i} j\omega + 1|}, \tag{7}$$

[1]The performance does not depend on z_{i-1}! It is common to derive a spacing error transfer function to establish string stability. This can be done only for some special cases. It can be seen that this transfer function is also a transfer function between a_{i-1} and a_i, so the approach presented here can be viewed as a generalization of those methods.

where $1/\tau_{\eta,i}$ can be considered as a bandwidth constraint for $\mathcal{A}_i$. Function $\eta_i(\omega)$ characterizes the damping/string stability property of a vehicle. As string stable behavior and transient damping helps in avoiding shockwaves and increases the comfort of the following vehicles, the choice of a large $\tau_{\eta,i}$ serves the "common good".

The "selfish" part of the design is related to string performance where the goal is to minimize the spacing errors and control effort subject to disturbances and the behavior of the preceding vehicle string. This part of the design can be formulated as a $\mathcal{H}_\infty$ optimization problem

$$\min \|W_{z,i}(s) \begin{bmatrix} \mathcal{C}_i(s) & \mathcal{D}_i(s) \end{bmatrix} W_{w,i}(s)\|_\infty, \tag{8}$$

where $W_{z,i}$ and $W_{w,i}$ are appropriate (diagonal) weighting functions characterizing the inputs and the performance requirements [17]. The maximal spacing error over the whole operating region determines the minimal inter-vehicle gap required to avoid collisions.

The inter-vehicle gap is also determined by the spacing policy with respect to the predecessor, $R_{i,i-1}(t)$. The minimization of the inter-vehicle gap allows for increasing traffic density and road capacity. It follows that minimizing the time-headway parameter, $h_{i,i-1}$, serves again the "common good".

The control design algorithm can be summarized as follows. Let θ_i denote the vector of design variables for a particular controller including parameters, such as $K_{a,i}^{i-1}, K_{v,i}^{i-1}, K_{p,i}^{i-1}, \ldots$ etc.

1) Fix the specifications for the "common good" determining design constraints, i.e., choose a time-headway parameter $h_{i,i-1}$ and a string stability parameter $\tau_{\eta,i}$ defined, respectively, by (4) and (7).
2) Minimize the local performance criteria (8) in variable θ_i subject to the string stability constraint (7).

IV. Design Trade-offs Among String Stability, Performance and Spacing Policy

In the case of ACC and CACC design, trade-offs among string stability, control performance and spacing policy are illustrated. String stability is characterized by parameter $\tau_{\eta,i}$. A large $\tau_{\eta,i}$ is advantageous by improving the ith vehicle's high-frequency damping of transient propagation. The vehicle's performance criterion is the maximum of the weighted sum of control energy and spacing error energy when disturbances are arbitrary but bounded signals. The minimum of this criterion that can be achieved with the constraints is denoted by $\gamma_{CD} \triangleq \min_{\theta_i} \|W_{z,i}(s) \begin{bmatrix} \mathcal{C}_i(s) & \mathcal{D}_i(s) \end{bmatrix} W_{w,i}(s)\|_\infty$. The spacing policy is characterized by the time-headway parameter $h_{i,i-1}$. A smaller value is advantageous by prescribing smaller inter-vehicle gap.

Figure 1 shows the achievable performance values in terms of fixed time-headway and string stability parameters. It can be observed that increasing the damping of the transient propagation (τ_η increasing) implies performance degradation and finally loss of stability that can be regained for the price of increased time-headway.

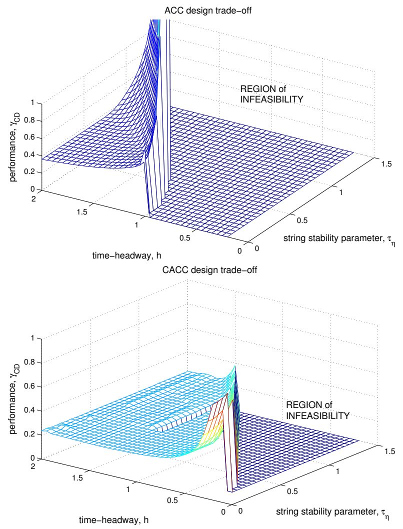

Fig. 1. Design trade-offs for ACC and CACC controlled vehicles.

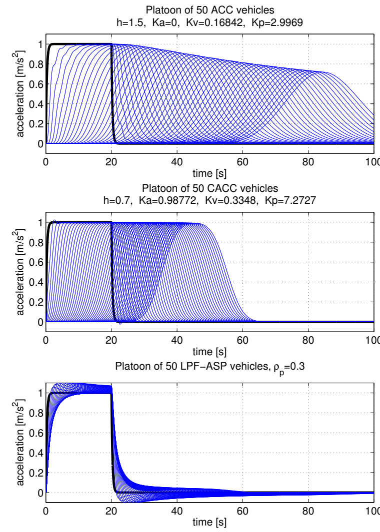

Fig. 2. Simulated ACC, CACC and LPF-ASP accelerations

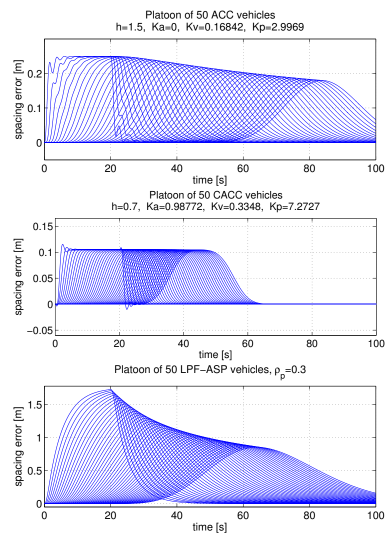

Fig. 3. Simulated ACC, CACC and LPF-ASP spacing errors

V. Comparison of ACC, CACC and LPF-ASP Performance

Fig. 1 shows also that acceleration information of the predecessor vehicle strongly improves control performance (by about 40%), and allows a significantly smaller time-headway.

An LPF-ASP vehicle design was presented in [14]. Its spacing with respect to the predecessor is constant, $R_{i,i-1}(t) = L_{i,i-1}$. The resulted controller performed well in a heterogeneous ad hoc vehicle string [14]. In the following it is compared with ACC and CACC vehicles by simulation examples. Homogeneous platoons are compared without disturbances, because the damping and spacing properties can be better observed in this clean scenario.

Fig. 2 and 3 show the accelerations and spacing errors of 50 identical vehicles. Leader acceleration is plotted by thick black solid line in Fig. 2. The vehicles start from standstill and accelerate up to $20m/s^2$. The damping parameter is set to $\tau_\eta = 0.6$ for all type of vehicles. ACC and CACC vehicles require a time-headway of $h_{i,i-1} = 1.5s$ and $h_{i,i-1} = 0.7s$, respectively, to satisfy the prescribed damping constraint and the spacing performance shown in the figures.

The LPF-ASP vehicles follow a small constant spacing with respect to the predecessors, yet they are string stable. It can be observed that the LPF-ASP vehicles react almost simultaneously to the changes in the leader motion. The relatively larger spacing errors are compensated with the small required constant safety gap (1.7m). Although the ACC and CACC vehicles have smaller spacing errors, they follow their predecessors with a speed dependent gap, which is 30m for the ACC and 14m for the CACC vehicle at speed of 20m/s.

VI. Conclusion

Three vehicle following controllers are presented, each is able to work in ad hoc unorganized platoons, even with human driven vehicles. A general systematic procedure is provided to the control design for heterogeneous string stability and performance, which can be performed locally, without knowledge about other vehicles. The design problem is solved by minimization of a $\mathcal{H}_\infty$ performance criterion subject to stability and string stability constraints. It is shown that utilizing kinetic information from multiple vehicles is possible and preferable in an ad hoc unorganized string of vehicles, string stability and a small inter-vehicle safety gap can be achieved. In this way LPF-ASP control architecture as an autonomous car following technology is useful in increasing traffic stability, road capacity and safety.

Acknowledgment

This work was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

References

[1] G. Silberg and R. Wallace, “Self-driving cars: The next revolution,” Center for Automotive Research, Transportation Systems Analysis Group, Tech. Rep., 2012.

[2] S. Öncü, N. van de Wouw, and H. Nijmeijer, “Cooperative adaptive cruise control: Tradeoffs between control and network specifications,” in *14th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, 2011, pp. 2051 –2056.

[3] G. Naus, R. Vugts, J. Ploeg, R. van de Molengraft, and M. Steinbuch, “Cooperative adaptive cruise control, design and experiments,” *Proc. 2010 American Control Conference, Baltimore, MD*, pp. 6145–6150, 2010.

[4] S. E. Shladover, C. Nowakowski, D. Cody, F. Bu, J. O’Connell, J. Spring, S. Dickey, and D. Nelson, “Effects of cooperative adaptive cruise control on traffic flow: Testing drivers’ choices of following distances,” *PATH Research Report UCB-ITS-PRR-2009-23*, 2009.

[5] U. Montanaro, M. Tufo, G. Fiengo, M. di Bernardo, and S. Santini, “On convergence and robustness of the extended cooperative cruise control,” in *53rd IEEE Conference on Decision and Control*, Dec 2014, pp. 4083–4088.

[6] S. Moon, I. Moon, and K. Yi, “Design, tuning, and evaluation of a full-range adaptive cruise control system with collision avoidance,” *Control Engineering Practice*, vol. 17, no. 4, pp. 442 – 455, 2009.

[7] G. N. Bifulco, L. Pariota, F. Simonelli, and R. D. Pace, “Development and testing of a fully adaptive cruise control system,” *Transportation Research Part C: Emerging Technologies*, vol. 29, no. Supplement C, pp. 156 – 170, 2013.

[8] L. Peppard, “String stability of relative-motion pid vehicle control systems,” *IEEE Transactions on Automatic Control*, vol. 19, No. 5, pp. 579–581, 1974.

[9] K. ching Chu, “Decentralized control of high-speed vehicular strings,” *Transportation Science*, vol. 8, no. 4, pp. 361–384, 1974.

[10] D. Swaroop and J. Hedrick, “String stability of interconnected systems,” *Automatic Control, IEEE Transactions on*, vol. 41, no. 3, pp. 349–357, Mar 1996.

[11] D. Swaroop, “String stability of interconnected systems: An application to platooning in automated highway systems,” *PhD dissertation*, 1994.

[12] C.-Y. Liang and H. Peng, “Optimal adaptive cruise control with guaranteed string stability,” *Vehicle System Dynamics*, vol. 32, no. 4-5, pp. 313–330, 1999.

[13] G. Orosz, R. E. Wilson, and G. Stépán, “Traffic jams: dynamics and control,” *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 368, no. 1928, pp. 4455–4479, 2010.

[14] G. Rödönyi, “An adaptive spacing policy guaranteeing string stability in multi-brand ad hoc platoons,” *to appear in IEEE Transactions on Intelligent Transportation Systems*, vol. ?, no. ?, pp. ?–?, ? ?

[15] J. Ploeg, N. van de Wouw, and H. Nijmeijer, “$\mathcal{L}_p$ string stability of cascaded systems: Application to vehicle platooning,” *Control Systems Technology, IEEE Transactions on*, vol. 22, no. 2, pp. 786–793, March 2014.

[16] E. Shaw and J. Hedrick, “String stability analysis for heterogeneous vehicle strings,” in *American Control Conference, 2007. ACC ’07*, 2007, pp. 3118–3125.

[17] K. Zhou, J. Doyle, and K. Glover, *Robust and optimal control*. Prentice-Hall Inc., Upper Saddle River, New Jersey, 1996.