Connectivity and Critical Point Behavior in Mobile Ad hoc and Sensor Networks

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***Abstract*— A well-known approach to increase the resilience of mobile ad hoc networks (MANETs) and unstructured sensor networks is to ensure a network topology where there are at least *k* disjoint routes in the network between each pair of network nodes (usually called *k*-connectivity). Asymptotic analyses of node density requirements for *k*-connectivity have been considered in the literature. In this paper, we present the results of a simulation study investigating the relationship between asymptotic results in the literature and *k*-connectivity under varying nodal density and nodal degree. The numerical results illustrate where the asymptotic approximations breakdown and we show that this largely due to the existence of critical connectivity points in the topology. Using a critical point identification algorithm we examine how the number of critical points varies with nodal degree, nodal density and node mobility. In addition, critical point is evaluated its effectiveness on the network caused by failure.**

***Index Terms*—Connectivity; Critical points; MANETs; Survivability**

I. INTRODUCTION

A fundamental problem in MANETs and unstructured sensor networks is achieving and maintaining connectivity. A network is connected if all nodes have a communication route (typically multi-hop) to each other. Maintaining connectivity is a challenge due to the unstructured nature of the network topology and the frequent occurrence of link and node failures due to interference, mobility, radio channel effects and battery limitations.

There can be three types of failures of the end-to-end communication in MANETs, which are node faults, power faults, and link faults [1-3]. A node fault occurs when an intermediate node, acting as a router, is not available due to hardware/software failure or the node moving out of the network itself. A power fault occurs when the battery charge on a node is too low making it unable to serve as a router. Link faults occur due to a variety of factors such as node mobility, obstacles between communicating nodes, fading, and excessive interference. Figure 1 (a) illustrates the effect of a node or power fault. Here, nodes A1 and B1 are communicating through node C1. If node C1 fails then an alternate route must be discovered and communications restored by rerouting the traffic through node C2. Figure 1 (b) illustrates a link fault that occurs when node B moves to a different position, resulting in the communication link between A and B degrading (e.g., because of an obstacle). Obviously, these failures break the local connectivity between nodes and can disrupt the end-to-end connectivity as well.

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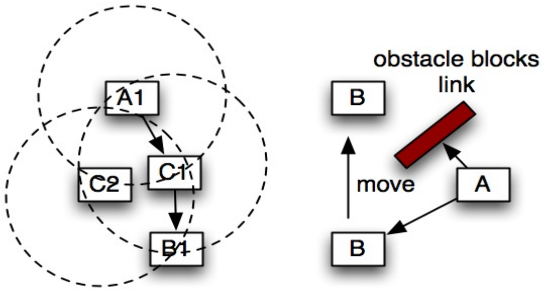
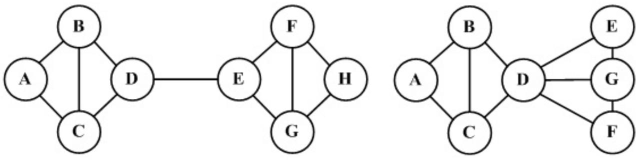


(a) Node failure (b) Link failure

Fig. 1. Example failures in mobile ad hoc network: (a) node failure and (b) link failure due to node movement.

In order to prevent failures from partitioning the network as a whole, many researchers have recommended that the network topology be *k-connected,* that is, the network topology be connected such that there are at least *k disjoint* routes between each node pair. These *k* routes may be link (i.e., edge) disjoint or node disjoint. Since both node and link failures are likely in MANETs and sensor networks, the focus of the research literature has been mostly on node disjoint *k*-connectivity. Ensuring that the network has *k*-node disjoint connectivity results in the network being able to survive the failure of *k*-1 nodes and still remain connected (i.e., at least one route between each node pair).

Recently several papers have looked at determining conditions under which *k-node connectivity* can be inferred probabilistically or assured asymptotically [4-6]. The focus has largely been on what combination of node density and transmission range are required to provide *k*-node disjoint connectivity in a specific deployment scenario for a homogenous network (all nodes are identical). Bettstetter [4] considered a uniform distribution of homogeneous nodes in a rectangular deployment area and derived a relationship between the minimum transmission range and the probabilistic behavior of the minimum node degree (i.e., number of neighbor nodes). Furthermore, he notes that the minimum node degree in



(a) Bridge link DE and Critical nodes D, E (b) Critical node D

Fig. 2. Example of link and nodes whose failure partitions the network.

the network d_{min} can be related to the probability that the network graph G is k-connected (node disjoint) through (1) below.

$$P(G is k-connected) \le P(d_{\min} \ge k) \quad (1)$$

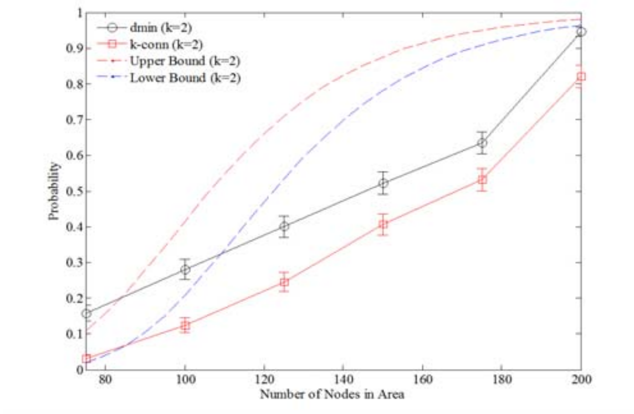
Through simulations, these two probabilities were determined for different transmission ranges over 100,000 random topologies. According to the results, those two probabilities are very close when the network is very dense (e.g., 500 nodes over 1000×1000 m^2 with 125 m transmission range) [4]. However, the simulation results and analysis used to justify (1) in the literature use very high node densities which would lead to interference and low throughput in real networks. Why such results are theoretically important, a weakness of much of the current literature is the assumption that an equality relationship holds between a k-connected network and k-minimum node degree (or in some cases k-average node degree) regardless of the scenario. For example, Ling and Tian use a k-minimum node degree assumption to derive the upper and lower bounds on k-connected [6]. It is worth noting that it is well known in graph theory that ensuring every network node has k neighbors is a *necessary condition* for k-connectivity *but not a sufficient condition*.

This is because the network graph may have critical connectivity points. For example, the link D-E in Figure 2 (a) is a critical point. If link D-E fails, the network partitions into 2 clusters. In the literature, links whose failure results in partition of the network are termed “bridge links”. Similarly, an articulation or critical node is defined as a node that partitions the network due to its failure. In Figure 2(b), node D is a critical node because the network is partitioned if node D fails.

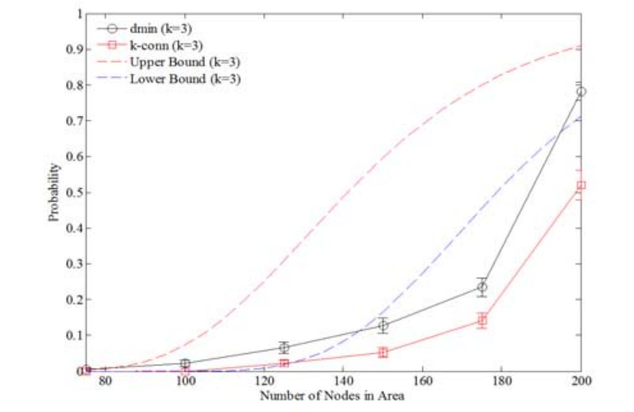
In this paper we present the results of a simulation based study on k-connectivity and its behavior. We first examine the relation between node degree and its behavior and how good metrics such as the minimum and average node degree are at ensuring k-connectivity in Section II. We also compare our simulation results with the results in [6]. We then discuss and consider the behavior of critical connectivity points and how they are affected by node density and mobility in Section III. Section IV studies the performance impact of the failure of critical nodes in terms of the size of the network partitions and the packet loss rate. We conclude the paper in Section V.

II. Relation Between Node Degree and Connectivity

Here we use different simulations to explore the relationship between the node degree and k-connectivity of the network. In our simulation models we assume identical nodes with omni-directional antennas and transmission is modeled as a disk of radius R. Links between a node and its neighbors will exist only if they fall within the disk (i.e., distance between



(a) $k = 2$



(b) $k = 3$

Fig. 3. Probabilities of minimum node degree and k-connected in different network densities.

nodes less than or equal R). First, we study the relationship between minimum node degree and k-connectivity versus network density and we compare probability of k-connectivity with its upper and lower bounds derived in [6].

A. Minimum node degree and k-connectivity

We use the ns2 to generate random topologies with different number of nodes, (75, 100, 125, 150, 175, and 200), in an area of 1500×1500 m^2. Nodes are identical with transmission range of 250m. Once the topologies are randomly generated, a C++ program that we developed was used to evaluate the node degree. The program also implements the k-shortest path algorithm [11], to test the number of k-node disjoint routes between each node pair. In each case, we test 1000 randomly generated connected. The probabilities are computed by the fraction of topologies that satisfy the minimum node degree (i.e., $P(d_{min} \ge k)$) or k-connectivity (i.e., $P(G \text{ is } k\text{-connected})$).

The simulation results are given in Figure 3 and show the probabilities of having a network with minimum node degree of k (i.e., black solid line) and k-connectivity (i.e., red solid line) versus the network density. Note, that the error bars on the results represent 95% confidence intervals. Note, that the probability of k-connectivity never reaches the probability of minimum node degree. For example, at network density of 175 nodes for the $k = 2$ case, $P(d_{min} \geq 2) = 0.635 \pm 0.03$, whereas $P(G \text{ is 2-connected}) = 0.532 \pm 0.031$), this is in a network with average node degree = 18.7. Note that, as the k value increases, both the probability of a minimum node degree of k and the probability of k-connectivity decrease. For example, in Figure 3(a), the $P(d_{min} \geq 2) = 0.522$ in a network with 150 nodes, while $P(d_{min} \geq 3) = 0.128$. The results in Figure 3(a) and 3(b) illustrate that the probabilities of achieving a minimum node degree of k and k-connectivity increase as the network density increases. The upper and lower bounds are plotted in Figure 3 as dotted line, using the equations in [6] with the parameters adopted in our simulation. It shows that the bounds are not a good predictor of $P(k\text{-connected})$ in the tested network conditions. Therefore, we observe that the assumption of minimum node degree being k implying k-connectivity is not valid in sparse networks and even in the network with medium density.

B. Average node degree and k-connectivity

We now study the relationship between average node degree of the network and k-connectivity (i.e., number of disjoint paths).

In next set of simulations, we used ns2 to generate random 50 node ad hoc network topologies. Again, we use a transmission range of 250m in area of 1000×1000 m^2. For values of k = 2, 3, 4, and 5, we generate random network topologies until 100 connected topologies are found with minimum node degree k. We analyze the 100 topologies found for each k value of 2, 3, 4, and 5. Table 1 shows the observed and calculated data from the obtained topologies. Specifically, Table 1 includes the average minimum node degree (*Ave Min ND*), the average minimum number of disjoint paths (*Ave Min DJP*), average node degree (*Ave ND*), and average number of disjoint paths (*Ave DJP*) for each of the 100 network topologies. *Ave Min ND* is computed by average minimum node degrees of 100 topologies. *Ave Min DJP* means the average of the minimum number of disjoint paths of 100 topologies. *Ave ND* is an average of the average node degree from each topology and *Ave DJP* is an average of the average number of disjoint paths from each topology.

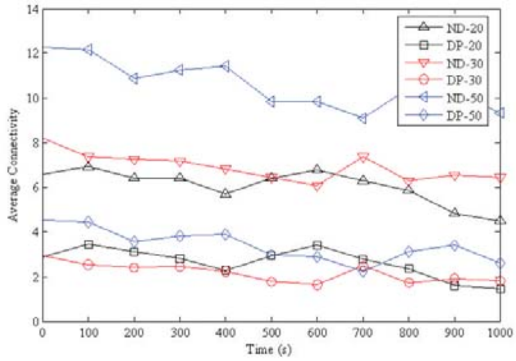
According to Table 1, the average number of disjoint paths is always lower than the average node degree and they do not increase greatly as the k value increases. When the average minimum node degree changes from 2.33 to 5.00, the average minimum number of disjoint paths does not change significantly (it goes from 1.41 to 2.21). The difference between average minimum node degree and average minimum number of disjoint paths slightly increases when the k value is larger. The difference is 39.5% at k = 2 and 55.8% at k = 5. This indicates that at any given k value, on average, the minimum node degree requirements do not ensure k-connectivity.

TABLE I

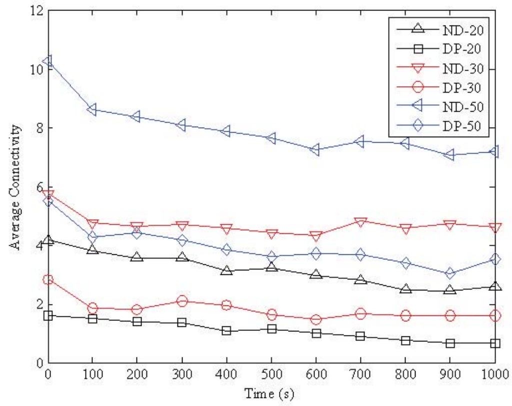
TABLE 1: MINIMUM AND AVERAGE NODE DEGREE AND NUMBER OF DISJOINT PATHS IN DIFFERENT MINIMUM NODE DEGREE REQUIREMENTS

Req'd ND	Ave Min ND	Ave Min DJP	Ave ND	Ave DJP
2	2.33	1.41	7.65	3.89
3	3.09	1.50	7.63	3.86
4	4.02	1.83	7.83	4.03
5	5.00	2.21	8.39	4.34

Ave Min ND – Average of minimum node degree of 100 satisfied topologies; ***Ave Min DJP*** – Average of minimum numbers of disjoint paths of 100 satisfied topologies; **Ave *ND*** – Average of the average node degree of each topology; ***Ave DJP*** – Average of the average number of disjoint paths of each topology.



(a) RPGM Model



(b) RWM Model

Fig. 4. Average node degree and average number of disjoint paths in different network density, 20, 30, 50 nodes in 1000x1000 m^2, over 1000 seconds of simulation time in two mobility models, (a) RPGM and (b) RWM; ND – Average node degree, DP – Average number of disjoint paths.

C. Average node degree and k-connectivity under mobility

In this simulation, we add the node mobility in order to observe the average node degree and the average number of disjoint paths in sparse ad hoc networks with mobility. We use the BonnMotion simulation tool with two mobility models – the Reference Point Group Mobility (RPGM) Model and Random Waypoint Mobility (RWM) Model. The maximum speed is 10 m/s and minimum speed is 0.5 m/s. A deployment area of 1000×1000 m^2 is chosen with 20, 30, and 50 nodes to understand the impact of node density. The transmission range is fixed at 250m and we use the unit disk model for link connectivity (i.e., two nodes have a link of they are within transmission range of each other and no link exists otherwise). The simulation is run for 1000 seconds and we capture a snap shot every 100 seconds. We observe the sequence of 10 topologies starting from the initial topology at time 0. Based on the node positions in each snapshot, we can obtain the network connectivity.

First, we evaluate the relationship between the average node degree (ND) and average number of disjoint paths (DP) of all pairs of nodes with the two mobility models, RPGM and RWM, at different node densities (20, 30, and 50 nodes in the 1000×1000 m^2 area). We compare these two averages in Figure 4. We compute the average nodal degree and the average number of disjoint paths for each topology captured every 100 seconds in different scenarios. In RPGM model as shown in Figure 4 (a), the average number of disjoint path does not show significant difference among 3 different network densities, (i.e., 20, 30, and 50 nodes in 1000×1000 m^2 area) while the RWM model shows that 50 nodes in area of 1000×1000 m^2 holds averagely twice more number of disjoint paths than 20 or 30 nodes network has as shown in Figure 4 (b). This phenomenon may be caused by the node mobility model. In RPGM, once the node involves in group, it moves along with its group leader and this prevent from creating connection between nodes that are in different group.

According to Figure 4, the average number of disjoint paths is always lower than the average node degree. The difference between the two averages increases when the network density increases. Another observation is that the behaviors of both average node degree and the average number of disjoint paths have similar tendencies over the simulation time. When the average node degree increases, the average number of disjoint path increases and it decreases when the average node degree decreases in time.

From these results, the average node degree always has a higher number than the average number of disjoint paths; the average number of disjoint paths is about 50% less than the average node degree. In other words, the average number of disjoint paths is smaller than the average node degree at all times. *The simulation results indicate that maintaining an average node degree cannot guarantee an equal average number of disjoint paths.* The average number of disjoint paths is approximated to only 50% of average node degree.

We examine the partition check every 100 seconds during simulation time of 1000 seconds in order to compute how many times the network partitioned during the simulation for both mobility models. RWM partitioned the network averagely 2.7 times with a maximum of 6. In case of RPGM, since nodes are gathered around the leader initially, network is partitioned already at time 0s. Then, we compute how many time the number of partitioned network during the simulation. Average is 8.4 with a maximum of 10. From this observation, it is found that the average node degree cannot provide network partition information. Even if the network is partitioned during the simulation, the average node degree does not change noticeably and neither does the average number of disjoint paths.

III. Discussions

From above results, we find that neither the average nor the minimum node degree can represent the number of disjoint paths (k-connectivity). The problem is more severe than simple inability to predict k -connectivity as illustrated in Figures 5 and 6. In Figure 5, the minimum node degree is 3 and the maximum node degree is 13. However, there exists a node that partitions the network i.e., a critical node. In Figure 6, the maximum node

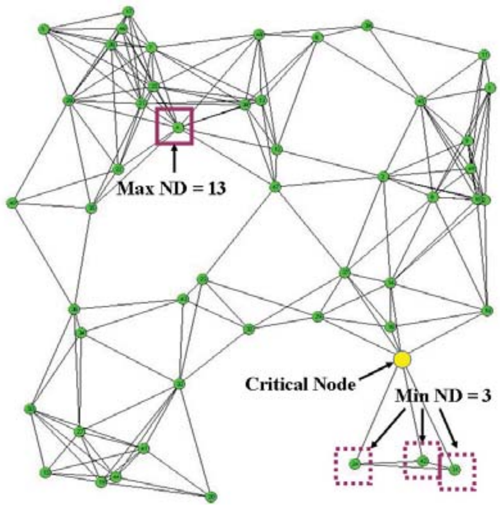


Fig. 5. Sample Fifty Node Network Topology with Critical node.

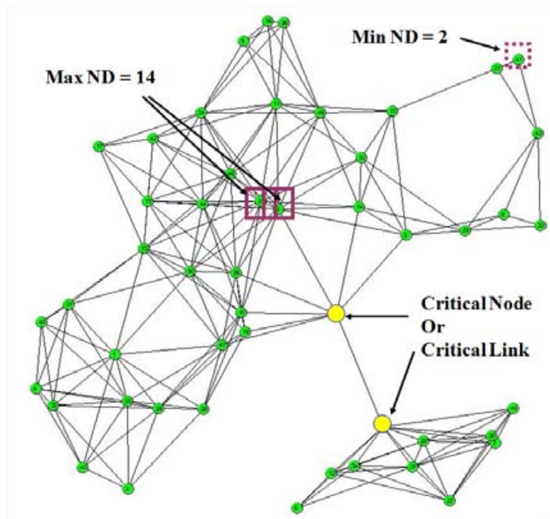


Fig. 6. Sample Fifty Node Network Topology with Critical nodes or Critical Link.

degree is 14 and minimum node degree is 2. But there still exist critical nodes or critical links. In such cases, the network is partitioned when the link between critical nodes fails or the critical node itself fails. This shows that if the critical nodes are connected in one hop then, it is also bridge link or critical link. This lead us that all critical links are included if we find all critical nodes.

If we consider that the network is alive as long as the network is not partitioned. Based on this definition, the critical node is a weak point in the network and the network can be more survivable when those critical nodes are strengthened in many ways such as increasing transmission range, recharge the battery, and etc. This definition also indicates that either average or minimum node degree does not suitable survivability metric because it cannot sense the network partition. We consider the network partition as a network failure in our new approach. Weak point of the network is considered and it could be either bridge link or articulation node. We provide 3 of 50 nodes network scenarios in 1000x100

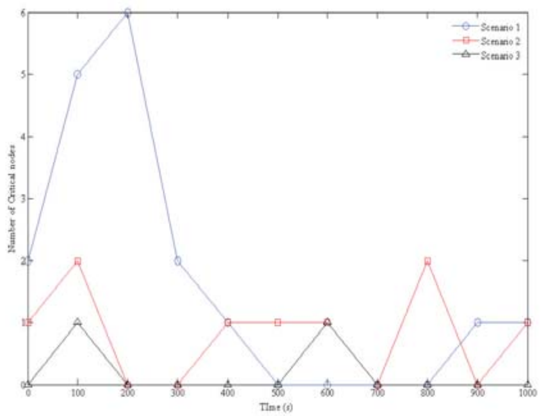


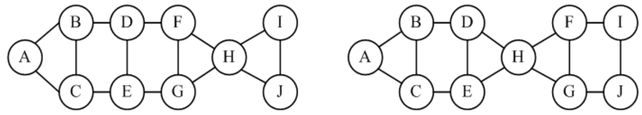
Fig. 7. Number of Critical nodes behavior vs. simulation time.

m^2 with 250 m transmission range. Random Way Point Mobility model is chosen for this simulation due to the difficulty of being connected in Reference Point Group Mobility model. We collect 3 scenarios that the network is connected (no partition) during the simulation time of 1000 seconds. Figure 7 shows that the number of critical nodes during simulation time for all 3 scenarios. Figure 7 shows that the number of critical node is not consistent or pattern, but random. This result indicates that the network has to observe the critical nodes periodically for the survivability of the network at each time due to its topology changing. To identify the critical nodes, several algorithms are introduced [8-10] and they can be used to find critical nodes to make the network more survivable by strengthen them to prevent network partition.

IV. IMPACT OF CRITICAL NODE FAILURE

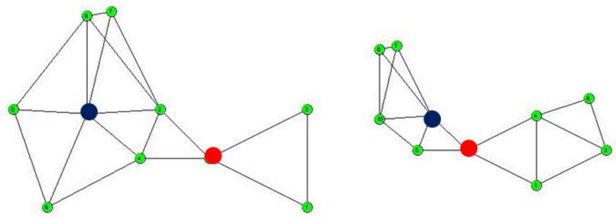
Here, we found out that the critical point plays a big role in network connectivity. In this section, we study the comparison of the impacts on network performance from failures of the maximum degree nodes and critical nodes. The network performance measure we use for comparison study is packet loss rate.

When the critical node fails, the network is partitioned into more than two clusters. Among existing traffics, if the source and destination nodes are associated in different clusters after critical node failure, packets will be lost. Here, we introduce the partition rate that is the ratio of the number of nodes in partitioned small cluster to the number of total nodes of the network before partition. For example in Figure 8, two nodes are isolated when the node H fails in (a) and its partition rate is 20%. Similarly, the topology in Figure 8 (b) has partition rate of 40%. Then, partition rate may affect the packet loss. When the partition rate increases, the more possible traffic sessions exist between partitioned clusters. And this may increase the packet loss rate. For the network with n nodes, it has total of $n(n-1)$ possible traffic sessions. If the partition rate is r_p, the possible disconnecting traffics due to critical node H failure will be $2r_pn[(1-r_p)n-1]$. Therefore, the possible disconnecting traffics increases as the partition rate of r_p increases with fixed number of nodes n. For example in Figure 8, both topologies, (a) and (b),



(a) 20% partition rate network (b) 40% partition rate network

Fig. 8. Number of Critical nodes behavior vs. simulation time.



(a) 10 nodes density with 20% PR (b) 10 nodes density with 40% PR

Fig. 9. 10 nodes network density topologies with 20% and 40% partition rate.

have 10 nodes and possible traffic is 90. But when critical node H fails, 28 traffic sessions in (a) and 40 traffic sessions in (b) are disconnected. Therefore, the larger partition rate may lead the higher packet loss rate with high probability.

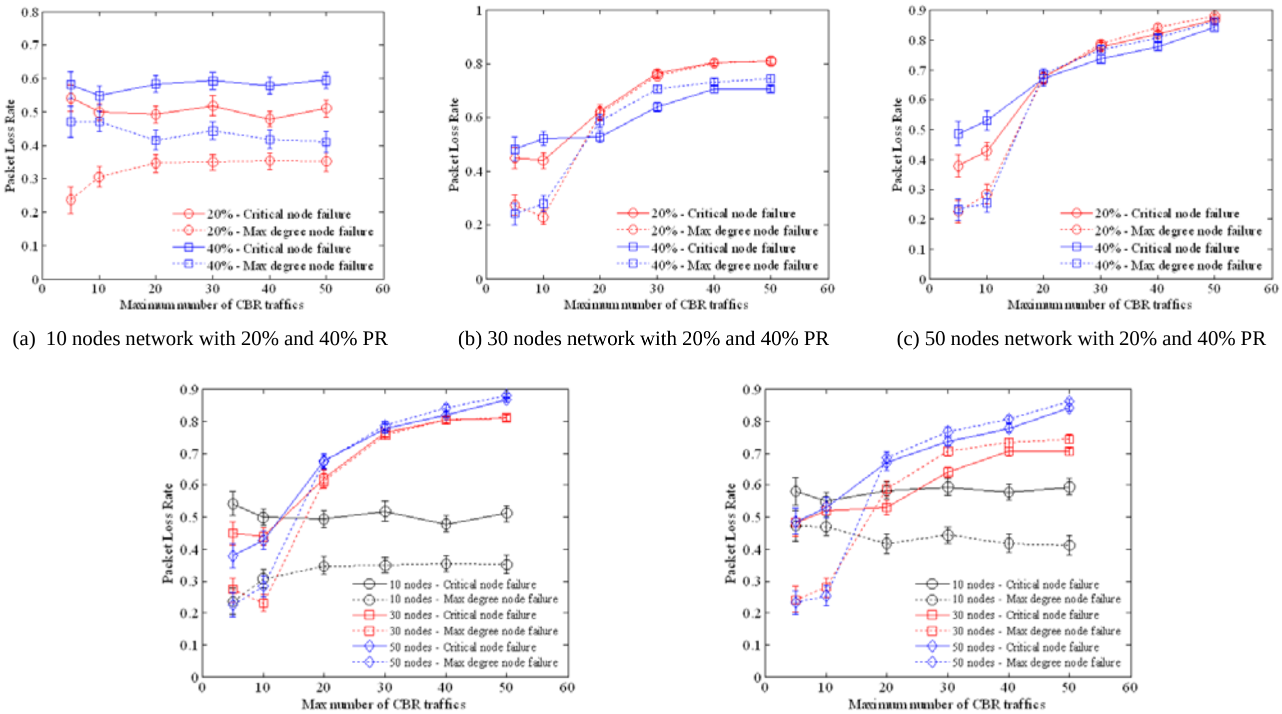
A. *Simulation Study*

In simulation study, we vary the partition rate with fixed network density. We select 20% and 40% partition rate and 10, 30, and 50 nodes for the network density over 5, 10, 20, 30, 40, and 50 maximum numbers of traffics. Traffics are 512 bytes of random Constant Bit Rate (CBR) with an interval of 0.25 seconds CBRs. NS2 is used to observe the packet loss. We fail the critical or maximum degree node at 400 simulation seconds out of total simulation time of 600 seconds. Thus, packet loss is observed during 200 seconds.

The topologies used for 10 nodes network density are shown in Figure 10. The topology in Figure 9 (a) indicates 20% of partition rate and (b) indicates 40% of partition rate. The red nodes represent critical nodes and blue nodes represent maximum degree nodes for both topologies. Figure 9 (a) isolates 2 nodes out of 10 nodes and Figure 9 (b) does 4 nodes out of 10 nodes when critical node fails. However, failure of maximum degree node in 10 nodes topology with 40% partition rate produces more bottle necks or critical points. This may increase higher packet loss rate when maximum degree node fails. All other topologies of 30 and 50 nodes do not produce bottle necks or critical points due to maximum degree node failure such as Figure 9 (a).

B. *Results and Discussions*

The simulation results are shown in Figure 10. The solid line indicates the packet loss rate when critical node fails and dotted line represents the packet loss rate when maximum degree node fails. Figure 10 (a) shows packet loss rate for 10 nodes network with 20% and 40% partition rate. The packet loss rate is higher when critical node fails for both cases. However, packet loss rate in 40% partition rate is higher than 20% partition rate in case of maximum degree node failure. As mentioned, 10 nodes topology with 40% partition rate creates more bottle necks or critical points when maximum degree node fails and it increase the packet loss when compare to that with 20%. Although maximum degree node failure produces more bottle necks or



(a) 10 nodes network with 20% and 40% PR

(b) 30 nodes network with 20% and 40% PR

(c) 50 nodes network with 20% and 40% PR

(d) 20% PR of 10, 30, and 50 nodes networks

(e) 40% PR of 10, 30, and 50 nodes networks

Fig. 10. Packet Loss Rates (PLRs) in different partition rate, 20% and 40%, with same network densities, 10 nodes, 30 nodes, and 50 nodes.

critical points, critical node failure is worse in packet loss. All other network densities also show that packet loss is more severe in critical node failure at low traffic load. The higher partition rate loses more packets and it becomes more significant when network is more dense at low traffic load by comparing Figure 10 (b) and (c).

When the network density varies with fixed partition rate, it does not affect the packet loss rate except for 10 nodes network density because 5 maximum traffic loads is relatively heavy. This is shown in Figure 10 (d) and (e) and packet loss is similar for 30 and 50 nodes network in both 20% and 40% partition rate cases.

V. Conclusions

The connectivity is fundamental principle of the survivability in MANETs. Node degree represents the connectivity of the network in general. However, node degree information cannot guarantee the network to be connected because critical node exists even with high node degree. Maximum degree node and critical node are compared in packet loss rate as a performance measure. It shows that the packet loss rate increases more when the critical node fails. Therefore, the network can be more resilient by finding the critical nodes and strengthening them. There are several algorithms to identify the critical nodes.

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