Truthful Spectrum Auction for Efficient Anti-Jamming in Cognitive Radio Networks

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Abstract

One significant challenge in cognitive radio networks is to design a framework in which the selfish secondary users are obliged to interact with each other truthfully. Moreover, due to the vulnerability of these networks against jamming attacks, designing anti-jamming defense mechanisms is equally important. In this paper, we propose a truthful mechanism, robust against the jamming, for a dynamic stochastic cognitive radio network consisting of several selfish secondary users and a malicious user. In this model, each secondary user participates in an auction and wish to use the unjammed spectrum, and the malicious user aims at jamming a channel by corrupting the communication link. A truthful auction mechanism is designed among the secondary users. Furthermore, a zero-sum game is formulated between the set of secondary users and the malicious user. This joint problem is then cast as a randomized two-level auctions in which the first auction allocates the vacant channels, and then the second one assigns the remaining unallocated channels. We have also changed this solution to a trustful distributed scheme. Simulation results show that the distributed algorithm can achieve a performance that is close to the centralized algorithm, without the added overhead and complexity.

Index Terms

Cognitive Radio Network, Zero Sum Game, Auction, Learning, Anti-Jamming Scheme.

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I. INTRODUCTION

Spectrum scarcity has been a major problem for the existing wireless networks which motivated researchers to investigate new intelligent paradigm to manage available spectrum. Cognitive radio (CR) has thus emerged as a promising approach to improve spectral efficiency in wireless networks. In CR networks, secondary users (SUs) may cognitively access unused spectrum that is not currently occupied by licensed users, namely primary users (PUs) under the condition that the PUs' transmission will not be interfered [1].

Spectrum management in CR networks has been considered in many recent works such as [2] and [3] (and references therein). One important technique that enables CR-oriented spectrum allocation is to consider spectrum auction among SUs that seek to idle channels [4]. Auction theory, which is rooted in economics, offers a promising solution for intelligently allocating resources, such as power and spectrum, in CR networks. There are different approaches for implementing auction theory in wireless networks, which have been investigated in [5]. In general, in such scenarios, users are rational and have their own strategies in order to get more resources. Extensive existing works are available on different auction approaches for spectrum allocation (e.g., see [6]). For instance, the authors in [7] find the maximization of the PUs' expected profit by proposing the leasing based spectrum allocation for SUs. In addition, the first price auction to optimize both the total payoff of SUs and revenue of auctioneer is studied in [8]. One drawback of the suggested scheme is that SUs might reveal wrong to further improve their utilities. The work in [9] provides a spectrum allocation based upon a double-sided auction mechanism. In this scheme, an untruthful behavior also brings suboptimal solutions.

Competition among the selfish SUs is crucial to use rare resources in the spectrum market framework [10]. More importantly, non-cooperative users have intentions to cheat so as to gain more benefits. The Vickrey Clarke Groves (VCG) auction mechanism is commonly used in the auction games in order to provide not only the assurance of truthfulness but also the maximization of the social welfare [11]. For example, the authors in [12] and [13] proposed the incentive mechanism to encourage users to contribute truthfully their resources by forming coalitions. Moreover, because of selfishness of SUs, each user attending in the auction has incomplete information about the other users. Hence, selecting a proper learning task is a big challenge for designing the distributed game. A Bayesian nonparametric belief update scheme is suggested to

solve this issue in [14].

In CR networks, SUs are susceptible to several malicious attacks. Several anti-attack mechanisms have been proposed in existing literature [15]. For example, the problem of PU emulation attack on CR networks has been investigated in [16] in which a malicious user can send signals with the same PU transmission characteristics in order to mislead the SUs. Instead, SUs can recognize PUs' transmission by adapting a favorable verification protocol. In addition, a game-theoretic approach based upon the concept of secrecy capacity is proposed to model eavesdropping attacks on CR networks in [17]. In [18], a set of SUs is available in a stochastic medium and they select randomized channel hopping as the defensive strategy. This framework falls into the category of the zero sum stochastic game and the authors propose a minimax-Q learning to find the related solution. Besides, the randomized defense strategy for channel hopping and power allocation with learning algorithms is suggested in [19]. However, in a spectrum auction, users act selfishly and these defense strategies are not fully applicable.

The main contribution of this paper is to jointly consider truthful spectrum auction and the presence of a jamming attack. In this scenario, two types of users exist: selfish SUs participating the auction and a malicious jamming user that wishes to reduce the social welfare as much as possible. Our key contributions can therefore be summarized as follows:

- To model the mentioned scenario, we formulated two inter-related games: a zero-sum stochastic game between the CR network and the jammer, and an associated mechanism design among the SUs at each stage of the game. Indeed, the zero sum game exists between the CR network and the malicious user, while mechanism design is considered among the SUs. Using our proposed framework, the SUs do not show their selfishness and at the same time cooperate with each other to get higher profits against the malicious user.
- In order to realize the joint games, we propose an algorithm based on zero-sum game which can extensively reduce the complexity of solving the game with an asymmetric number of actions for the players. The proposition is a basis for the work because the malicious user and the SUs are unequal in the number of actions.
- Using the derived proposition, we show that the zero-sum stochastic game and spectrum auction game can be converted to a centralized two-level spectrum auction in which SUs send their bids to a coordinator and the coordinator confronts against the malicious user. More specifically, the coordinator initially allocates spectrum to the first level bids, and

then the remaining spectrum is allocated by the second auction. Indeed, the main idea of the centralized two-level auction is inspired from the randomized auction which is common in combinatorial auction theory such as [20] and [21]. However, our considered scenario significantly differs from those existing works.

- A decentralized method based upon the centralized two-level auction is examined. The
 proposed algorithm use the proven interesting properties of the centralized game which
 extremely reduces the complexity of the game. Simulation results show that the loss in performance for the decentralized method in comparison with the centralized one is negligible.
- Due to the fact that SUs have no knowledge about the states of other SUs and jammer, the parameters for the decentralized scheme must be learnt from a proper scheme like the one proposed in [22]. We propose a Boltzmann-Gibbs algorithm to estimate the unknown parameters for each users. Simulation results show that this method yields considerable performance gains. Moreover, the convergence of the proposed decentralized game can be controlled by learning parameters.

The rest of this paper is organized as follows. The system model is presented in Section II. In Section III, a centralized algorithm based on a two-level auction is described. In Section IV, we propose a truthful decentralized method in accordance with the proposed centralized auction. The simulation results are given in Section V. Finally, in Section VI, we conclude the paper.

II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

We consider a CR network consisting of M channels having a slotted-time structure indexed by $j \in \{1, 2, ..., M\}$. Moreover, the duration of each time-slot is assumed to be T_s . There are $N \ge M$ SUs that seek to access the vacant channels to send their data. Moreover, these users are selfish and non-cooperative. The primary network consists of a number of PUs who have a have priority to use the channels in a slotted-time manner. We consider an on-off scheme to model the channel usage, in which $y_j(t) = 1$ and $y_j(t) = 0$ indicate that channel j is idle and busy at time t, respectively [18] and [19]. The transition probabilities from on-to-off and off-to-on are $\alpha_{N2F,j}$ and $\alpha_{F2N,j}$, respectively. Without loss of generality, we assume that every SU can only use one channel at time t [23]. In order to avoid the conflict with the PUs transmission, each SU knows the availability of all the channels before transmitting. This can be done by using wideband sensing or cooperative sensing techniques [24].



The state of channel j for SU i is assumed to be the received signal-to-noise-ratio (SNR) $\gamma_{ij}(t)$, following an exponential distribution with mean of $\overline{\gamma_{ij}}$. Similar to [25], we represent $\gamma_{ij}(t)$ by discrete states to attain a finite Markov chain. In addition, let b_i^t indicate the buffer state of user i at time t and $b_i^t \in \{0, 1, \ldots, B_{\max}\}$ where B_{\max} is the maximum buffer size. Thus, the state of SU i at time t is $s_i(t) = (\gamma_{i1}(t), \gamma_{i2}(t), \ldots, \gamma_{iM}(t), b_i^t)$ and the state of the stochastic game is described as follows:

$$\boldsymbol{S}(t) = (y_1(t), \dots, y_M(t), \boldsymbol{s}_1(t), \dots, \boldsymbol{s}_N(t)),$$
(1)

where the state of the game S(t) consists of the state of each SU and the occupancy state of each channel. The assigned channel to the *i*-th SU is denoted by $A_i(t)$. Moreover, it is possible that no channel is assigned to the SU, i.e., $A_i(t) = 0$. Thus, we have $A_i(t) \in \{0, 1, ..., M\}$.

Assume there is a malicious attacker in this scenario which attempts to interrupt the communication links of the SUs by inserting interference. The action of malicious user is to jam L channels chosen from the vacant channels. Indeed, if the malicious user jams channel j, the communication link is assumed to be disrupted at that time. We assume that the jammer knows the channel occupancy states at each stage time. For simplicity, we assume L = 1, and our approach can be extended to L > 1 case. The action of jammer, $A_0(t) \in \{1, 2, ..., M\}$, indicates the jammed channel by the attacker. Fig. 1 shows the proposed system model and illustrates how users occupy the time-frequency resources.

Notice that the availabilities of the channels are only imposed by PUs, and hence, they are independent of the attacker's action and SUs' actions. Consequently, we can now derive the transition probability of the states as

$$P(\mathbf{S}(t+1) \mid \mathbf{S}(t), A_0(t), A_1(t), \dots, A_N(t)) =$$

$$P(y_1(t+1), \dots, y_M(t+1) \mid y_1(t), \dots, y_M(t)) \prod_{i=1}^N P(\mathbf{s}_i(t+1) \mid \mathbf{s}_i(t), A_0(t), \dots, A_N(t)),$$
(2)

 $s_i(t+1)$ includes information about the channels' conditions and the buffer state. The channel conditions do not depend on the SUs action. Besides, the buffer state, $b_i(t+1)$, is affected by the jammer action, $A_0(t)$, the action of SU *i*, $A_i(t)$, and $s_i(t)$. Hence, we can express the last term of (2) as

$$P(\mathbf{s}_{i}(t+1) \mid \mathbf{s}_{i}(t), A_{0}(t), A_{1}(t), \dots, A_{N}(t)) = P(\mathbf{s}_{i}(t+1) \mid \mathbf{s}_{i}(t), A_{0}(t), A_{i}(t))$$

= $P(b_{i}(t+1) \mid b_{i}(t), A_{0}(t), A_{i}(t)) \times \prod_{i=1}^{M} P(\gamma_{ij}(t+1) \mid \gamma_{ij}(t)).$ (3)

We denote the incoming traffic of SU *i* at time *t* as f_i^t where $f_i^t \in \{0, 1, ..., \infty\}$. It is assumed that f_i^t has the Poisson distribution with the average $\overline{f_i}$ [23]. Moreover, the buffer state is derived from $b_i(t+1) = \min((b_i(t) - g_{A_i,A_0}(t))^+ + f_i^t, B_{\max})$. Hence, we have the following expression for its transition probability

$$P(b_{i}(t+1) \mid b_{i}(t), A_{0}(t), A_{i}(t)) =$$

$$\begin{cases} \frac{\overline{(f_{i})^{x}}e^{\overline{f_{i}}}}{x!}, & 0 \le x < -(b_{i}(t) - g_{A_{i},A_{0}}(t))^{+} + B_{\max}, \\ \sum_{x=B}^{\infty} \frac{\overline{(f_{i})^{x}}e^{\overline{f_{i}}}}{x!}, & x = -(b_{i}(t) - g_{A_{i},A_{0}}(t))^{+} + B_{\max}, \end{cases}$$
(4)

where $(c)^+ = \max(c, 0)$ and $g_{A_i,A_0}(t)$ indicates the transmission bit rate if channel $A_i(t)$ is selected and channel $A_0(t)$ is jammed. Therefore, $g_{A_i,A_0}(t)$ can be calculated as [32]

$$g_{A_{i},A_{0}}(t) = \left[T_{s}W \log_{2} \left(1 + \frac{1.5\gamma_{i,j}}{\ln(\frac{0.2}{\text{BER}_{\text{tar}}})} \right) \right] I(A_{i} \neq A_{0}),$$
(5)

where T_s , W and BER_{tar} are the time duration, bandwidth of each channel and target bit error rate, respectively. In (5), $\lfloor X \rfloor$ and I(Y) indicate the largest integer number which is lower than X and the sign of Y, respectively. When the *i*-th SU selects channel $A_i(t)$ and the jammer selects the $A_0(t)$ -th channel at the same time, the utility function of user *i* at time *t* is characterized as follows

$$r_i(\mathbf{S}(t), A_i(t), A_0(t)) = -\left(b_i(t) - g_{A_i, A_0}(t) - B_{\max} + f_i^t\right)^+.$$
(6)

In our scenario, we consider the presence of a coordinator that allocates spectrum to the SUs according to the submitted bids while maximizing the worst-case social welfare corrupted by the attacker. Hence, the interactions between the coordinator and the SUs are cast as an *auction* with the following elements:

- The auctionees are the SUs which aim at using the vacant channels.
- The auctioneer is the coordinator which allocates the channels to SUs. Afterwards, the auctioneer and coordinator are used interchangeably.
- Each bid is denoted by a_{ij,k}, where 1 ≤ j, k ≤ M. Here, a_{ij,k} indicates the proper bid for SU i to use channel j while the attacker jams channel k.
- The following constraints must be satisfied at each stage of the auction:

$$\sum_{j=1}^{M} z_{ij}(t) \le 1$$

$$\sum_{i=1}^{N} z_{ij}(t) = 1, \text{ if channel } j \text{ is idle,}$$

$$\sum_{i=1}^{N} z_{ij}(t) = 0, \text{ if channel } j \text{ is busy,}$$
(7)

in which $z_{ij}(t) \in \{0, 1\}$ shows that channel j is allocated to the *i*-th SU if $z_{ij}(t) = 1$; and is not allocated otherwise.

In order to combat the jammer, the coordinator should assign the channels to the SUs via a random strategy. In the next section, we will investigate this optimal strategy.

III. ANTI-JAMMING DECENTRALIZED GAME BASED ON LEARNING PROCESS

In the previous section, the PC-game is proposed in order to extract the anti-jamming mechanism under the condition that all SUs and the auctioneer act as one player to defeat the malicious user. However, this assumption may not hold in general since the SUs are selfish and maybe untruthful. Unreliable information may lead to an improper strategy for protection of the SUs against the jammer. Besides, the SUs send their M'^2 bids to the coordinator, which has the high complexity. Due to these drawbacks, this section suggests a decentralized method according to the framework provided by the PC-game. In the PC-game, we use a two level auction, and our aim is to specify a distribution function to the actions. These actions can be recognized by the first and second preferences of all the SUs. First, pay attention to $p_1^{*T} U p_2^* = \left(\sum_{l=1}^{\frac{N!}{(N-M')!}} p_{1,l}^* U^l\right) p_2^*$ where p_1^* and p_2^* are the optimal policies of the auctioneer and the jammer, respectively. Moreover, $p_{1,l}^*$ and U^l are the *l*-th entry of p_1^* and the *l*-th row of payoff matrix U of the original game in *Definition 1*. If we extend each U^l into its elements, we have the following formulation:

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$$\left(\sum_{l=1}^{\frac{N}{(N-M')!}} \boldsymbol{p}_{1,l}^* U^l\right) \boldsymbol{p}_2^* = \left(\sum_{i=1}^{N} \sum_{j=1}^{M'} \boldsymbol{p}_{u(i,j)}^* [a_{i,j,1}, \dots, a_{i,j,M'}]\right) \boldsymbol{p}_2^*,$$
(8)

in which $p_{u(i,j)}^*$ is equal to the probability of selection of the *j*-th channel for the *i*-th user. Every policy, which yields the same $p_{u(i,j)}^*$, is the optimal strategy against the attacker. This fact motivates us to move from the PC-game to a distributed game. In the PC-game, we specify a probability to each action distinguished by the first auction or equivalently the first preferences of the SUs. By truthfulness assumption and help of the mentioned fact, if each SU individually estimate the probabilities connected with the preferences over the channels, then the value of the PC-game obtained from (14) can be approximated by the following formulation:

$$\sum_{l_1=1}^{M'} \dots \sum_{l_N=1}^{M'} Q_{l_1} \dots Q_{l_N} \boldsymbol{U}_{l_1,\dots,l_N} \boldsymbol{p}_2^* \cong \boldsymbol{p}_1^{*T} \boldsymbol{U} \boldsymbol{p}_2^*,$$
(9)

where Q_{l_i} and $U_{l_1,...,l_N}$ are the estimated probability related to the first preference by the *i*-th SUand the value of the game when the SUs's preferences are $l_1, ..., l_N$, respectively.

Each auction consists of M' allocations to the SUs. Note that from *Proposition* 1, we only need M' auctions to reach to the best response against the jammer. Thus, there are at most ${M'}^2$ important probabilities, $p_{u(i,j)}^*$, at each stage of game. Moreover, it can be easily demonstrated that every policy, which has these M'^2 probabilities, is optimal from the perspective of the zero-sum game. On the other side, each SU has control over M' probabilities for stating its first preference over the channels. From this point of view, the SUs have $N \times M'$ variables for estimations of M'^2 important probabilities which are improved with increasing N compared to M.

At this time, by applying the auction feature to the game, the coordinator can get payments from the SUs. The payment of each SU is constructed from two parts. One payment part is related to the first-auction and the other part is associated with the second-auction. The computation approach of the payment for the first-auction which is similar to [23]is stated as

$$p_i^t = \sum_{(k=1,k\neq i)}^N \sum_{j=1}^{M'} z_{kj}^{t,opt} a_{kj}'(t) - \max_{(z_{kj}|a_{ij}'=0,\forall j)} \sum_{(k=1,k\neq i)}^N \sum_{j=1}^{M'} z_{kj}^t a_{kj}'(t),$$
(10)

in which $z_{kj}^{t,opt}$ is the solution of the first auction. For the second-auction, this payment can also be computed by the same procedure while the selected SUs in the first-auction and their corresponding announced bids are omitted by the coordinator. The PD-game procedure is described in Table I. We show these payments oblige the SUs to bid truthfully. In order to prove that the proposed distributed game (PD-game) contains the truthful mechanism, first we define the concept of truthfulness in expectation.

Definition 1: Assume v_i , v'_i , v_{-i} and p^t_i are the real value of bid for user *i*, the announced value of bid for user *i*, the value of bids for other users and the payment assigned to user *i*, respectively. A mechanism is truthful in expectation when for any user *i* and any $v_{-i} \in V_{-i}$ of other users, the expectation of profit attained by user *i*, $\mathbb{E}\{v'_i - p^t_i(v'_i, v_{-i})\}$ is maximum if $v'_i = v_i$ [28].

We now focus on a proposition which states that the PD-game is truthfulness in expectation.

Proposition 1: The proposed procedure for assigning payment satisfies truthfulness in the expectation criterion.

Proof: The proof is given in Appendix C.

Note that the payment of each SU, which is dependent on all the SUs' bids, converts the profit gained by each SU into a notion of the overall value of the zero-sum game. Thus, we are trying to model the game between each SU and the attacker as the zero-sum game separately so that the separate game for each SU has some external factors related to other SUs, and each SU is effective only on a certain amount of the profit.

By doing so, every SU computes the distribution of stating its preference over the channels. In addition, the communication burden of stating its bids obviously plummets. Since, the SU only sends M' bids instead of stating M'^2 . Duties of the coordinator decreases since it only computes the first and second-auctions and their related payments. Indeed, the utility matrix of the separate game between each SU and the attacker is modeled as a $(M' \times M')$ matrix because the SU has M' choices for the announcement of its first preference. Note that our algorithm is distinct from work suggested in [30] in which authors employ a factored approximation of

Step 1. The SUs submit the bid based upon (12) to the coordinator. At the same time, the SUs announce their preferences over channels in order to be used in the first and second auctions.

Step 2. First auction is computed for the first preferences of the SUs. Then, allocation and payment for each SU is assigned to

them by using (7) and (16).

Step 3. Similarly, the second auction is computed for the remaining channels and the SUs.

the overall Q-function based upon the linear combination of users' Q-function for the stochastic game. The proposed algorithm is not applicable in our scenario because the SUs are selfish and interested in benefiting further. Indeed, the payment structure makes the profit of SUs' network directly relevant to each individual profit due to *Proposition* 3. Instead, p_1^* is estimated by SUs' probabilities, Q_{l_1}, \ldots, Q_{l_N} .

The fundamental difficulty of the PD-game is that each SU does not know enough about its related separate utility matrix. Remembering that the game will be repeated infinitely, and therefore, the SUs can learn their utilities by a certain learning scheme. We employ the scheme proposed in [22]. The advantage of this scheme is that each SU can adapt different patterns of learning. The probabilistic strategy over the actions and utility of each stage can be learned through the game. First, we apply an iterative Boltzmann-Gibbs strategy which is stated as

$$\boldsymbol{\sigma}_{i} \big(\boldsymbol{q}_{1i}^{t}, \widehat{\boldsymbol{u}}_{1i,t}, \boldsymbol{S}(t) \big)(j) = \frac{\boldsymbol{q}_{1i}^{t} \big(j, \boldsymbol{S}(t) \big) e^{\frac{\widehat{\boldsymbol{u}}_{1i,t} \big(j, \boldsymbol{S}(t) \big)}{\epsilon}}{\sum_{j=1}^{M'} \boldsymbol{q}_{1i}^{t} \big(j, \boldsymbol{S}(t) \big) e^{\frac{\widehat{\boldsymbol{u}}_{1i,t} \big(j, \boldsymbol{S}(t) \big)}{\epsilon}}$$
(11)

where $q_{1i}^t(j, S(t))$ and $\hat{\mathbf{u}}_{1i,t}$ are distribution of selecting channel j as the first preference of SU i and the estimated average payoffs updated at iteration t, respectively [22]. Next, we update distribution and payoff, respectively, as

$$\boldsymbol{q}_{1i}^{(t+1)}(\boldsymbol{S}(t)) = (1 - \lambda_{1i,t})\boldsymbol{q}_{1i}^{t}(\boldsymbol{S}(t)) + \lambda_{1i,t}\boldsymbol{\sigma}_{i}(\boldsymbol{q}_{1i}^{t}, \widehat{\boldsymbol{u}}_{1i,t}, \boldsymbol{S}(t))(j)$$
(12)

$$\widehat{\mathbf{u}}_{1i,t+1}\big(\boldsymbol{S}(t)\big) = \widehat{\mathbf{u}}_{1i,t}\big(\boldsymbol{S}(t)\big) + \frac{\mu_{1i,t}}{\boldsymbol{q}_{1i}^t\big(j,\boldsymbol{S}(t)\big)} \bigg(U_{1i,t}\big(\boldsymbol{S}(t)\big) - \widehat{\mathbf{u}}_{1i,t}\big(\boldsymbol{S}(t)\big)\bigg).$$
(13)

in which $U_{1i,k,t}$ is the profit gained by SU *i* at time *t* when selecting channel *j* as its preference, which is zero when no channel is assigned to it, and is $a'_{ij,k} - p_i(t)$ when channel *j* is assigned. Furthermore, $\mu_{1i,t}$ and $\lambda_{1i,t}$ are the learning rates indicating players' capabilities of information retrieval and update. Therefore, each SU can learn the distribution over its preference from implementing a Q-learning based method. It can be proved that Q-learning method converges to the optimal solution for only single-agent case; However, there is no such a guarantee for multi-agent cases [29]. In the next section, simulation results illustrate the convergence of the PD-game to the sub-optimal solutions.

IV. SIMULATION RESULTS

In this section, we provide simulation results to verify the truthful anti-jamming network. We consider a cognitive radio environment with M channels, N secondary users and a malicious user. We assume that the state of signal to noise ratio for SU i and channel j, γ_{ij} , has three values 10, 30 and 50. The probability of state transitions from these states are $p(\gamma_{ij} = 10|\gamma_{ij}^1 = 10) = 0.4$, $p(\gamma_{ij} = 30|\gamma_{ij}^1 = 10) = 0.3$, $p(\gamma_{ij} = 10|\gamma_{ij}^1 = 30) = 0.3$, $p(\gamma_{ij} = 30|\gamma_{ij}^1 = 30) = 0.4$, $p(\gamma_{ij} = 10|\gamma_{ij}^1 = 50) = 0.3$, and $p(\gamma_{ij} = 30|\gamma_{ij}^1 = 50) = 0.3$. In addition, $\alpha_{N2F,j} = 0.3$ and $\alpha_{F2N,j} = 0.4$ for $1 \le j \le M$. We set also BER_{tar} for all the users in (5) as 10^{-5} .

A. Convergence

The convergence speed of the PC-game and the PD-game for three SUs are investigated in Fig. 2 and Fig. 3 when M = 2 and M = 3, respectively. Besides, $B_{\text{max}} = 2$ and $\overline{f_i} = 0.5$ for all SUs for the either case. The normalized cumulative value of SUs is used as a convergence comparison tool. As Fig. 2 and Fig. 3 report, both algorithms converge; however, the PD-game takes longer time to reach the stable solution. The PD-game is done in the decentralized scheme with incomplete information. Therefore, it needs more times to learn the unknown parameters.



Fig. 2. The convergence of the normalized cumulative value of SUs in the PD-game and PC-game in a networks with M = 2 and N = 3.

In particular, the convergence rates in Fig. 3 for both the PC-game and PD-game are quite slower than those in Fig. 2. Indeed, increase in M leads to rises in the numbers of the states and the complexity of the system. Consequently, the required numbers of iterations in Fig. 3 explicitly becomes greater.

The learning parameters $\lambda_{1i,t}$, $\mu_{1i,t}$ and ϵ in (17), (18) and (19) play important roles in the convergence of the PD-game. In [22], it is shown that $\frac{\lambda_{1i,t}}{\mu_{1i,t}} \to 0$ for assurance of the convergence. Hence, we consider $\frac{\lambda_{1i,t}}{\mu_{1i,t}} = \frac{\frac{1}{T_S^{(1+\beta)}}}{\frac{1}{T_S}}$ where T_s is the repetition numbers of state occurrence, where $\beta > 0$. Fig. 4 depicts the effect of different β and ϵ on the iterations required for the convergence under the mentioned condition when M = 2. It is clear that when these parameters increase, the convergence speed decrease, since the impact of instantaneous utilities on current strategy decreases.

B. The effects of SU parameters on performance

In this part, the effects of the maximum allowable B_{max} , the number of channels M, and the number of users N on the PD-game and the PC-game are evaluated. In order to have a similar



Fig. 3. The convergence of the normalized cumulative value of SUs in the PD-game and PC-game in a networks with M = 3 and N = 3.



Fig. 4. The effect of different β and ϵ on the performance of the PD-game.

benchmark for comparison of two methods, we define a new parameter θ based on (6) as,

$$\theta = \sum_{t=0}^{N} \sum_{i=1}^{N} -r_i(t)/N.$$



Fig. 5. The effect of different B_{max} s and ϵ s on the performance of the PD-game.

Fig. 5 and Fig. 6 illustrate the performance of the PC-game and the PD-game by θ for variable B_{max} and N when M = 2 and M = 3, respectively. The other parameters are set alike to the previous part. In Fig. 5, the SU with the greater B_{max} is able to hold the data for a longer time. Thus, the increment in B_{max} decreases θ . In other words, it can improve the performance of the system. However, increase in N has opposite impact on the θ which is result of increasing the dropping probability of data. Moreover, Fig. 6 shows the performance when M = 3. Note that both the PC-game and PD-game in Fig. 6 have lower θ rather than those in Fig. 5 for the same condition. Indeed, M = 3 increases the opportunities of available vacant channels for each SU; therefore, decreases the numbers of unsent buffered information.

The performance of the scenario for different average of incoming traffic $\overline{f_i}$ and the numbers of SUs is shown in Fig. 7 and Fig. 8. The results are obtained for M = 2 and M = 3, respectively. Rise in $\overline{f_i}$ means that the average of incoming traffic increase. The outcome of the rise is to receive more traffic data at each stage of the game; as a result, the average unsent traffic θ increase. Finally, Fig. 9 displays θ versus $\overline{f_i}$ when N = M. Notice that increase in N along with M causes θ to be lower which validates or otherways about the performance of the scheme.

Spectrum management among the SUs is a vital issue for CR networks, and auction theory provides a helpful tool to allocate spectrum to SUs. In this article, first, we proposed a centralized



Fig. 6. The effect of different B_{max} and N for M = 3 on the performance of the PD-game.



Fig. 7. The effect of different $\overline{f_i}$ and N for M = 2 on the performance of the PD-game and the PC-game.

two-level auction which combined both the advantages of efficient resource assignment to SUs and acting against the malicious user. Next, a proposition for the zero-sum game was given which can be applied in a game with the non-uniform number of users' actions. More importantly, we introduced a decentralized protocol based upon the centralized method properties and the



Fig. 8. The effect of different $\overline{f_i}$ and N for M = 3 on the performance of the PD-game and the PC-game.



Fig. 9. The effect of different $\overline{f_i}$ on the performance of the PD-game.

mentioned proposition. The decentralized scheme obliges SUs to bid truthfully because SUs can gain higher profit in expectation for the long-term interaction. Simulation studies show that both the centralized and decentralized scheme converge in the limited numbers of stages. Moreover, the performance of the proposed approach are comparable with the efficient centralized solution.

Appendix A

PROOF OF PROPOSITION 1

Consider a zero-sum game with payoff matrix O as follows

$$\boldsymbol{O} = \begin{pmatrix} o_{1,1} & \cdots & o_{1,l_2} \\ \vdots & \ddots & \vdots \\ o_{l_1,1} & \cdots & o_{l_1,l_2} \end{pmatrix}$$
(14)

in which $o_{n,m}$ shows that player 1 and player 2 obtain $o_{n,m}$ and $-o_{n,m}$ profit her they select their *n*-th and *m*-th actions, respectively. To attain the optimal solution [26], we should consider mixed strategy with the help of the following equation:

$$\max_{\boldsymbol{p}_1} \min_{\boldsymbol{p}_2} \boldsymbol{p}_1^T \boldsymbol{O} \boldsymbol{p}_2 = \min_{\boldsymbol{p}_2} \max_{\boldsymbol{p}_1} \boldsymbol{p}_1^T \boldsymbol{O} \boldsymbol{p}_2 = v$$
(15)

where p_1 and p_2 indicate the probability distributions over the related actions of player 1 and player 2, and v is the value of the game. Moreover, O can be expressed as,

$$\boldsymbol{O} = \left[\boldsymbol{o}_1^T, \boldsymbol{o}_2^T, \dots, \boldsymbol{o}_{l_1}^T\right]^T$$

where o_i is $1 \times l_2$ vector for $i \in (1, ..., l_1)$. Hence, $v_1 = p_1^T O = \sum_{i=1}^N p_{1,i} o_i$ and $v = v_1 p_2$. In addition, we consider all the entries of matrix are more than zero. The value of the game, which contains l_1 actions with vectors $o_1, ..., o_{l_1}$, is denoted by $\operatorname{zerosum}(o_1, ..., o_{l_1})$. First, we state a lemma in order to prove the proposition.

Lemma 1: If the following relationship exists between o_1, \ldots, o_{l_1} , player 1 can play the game without the l_1 -th action while it gets the same value,

$$\boldsymbol{o}_{l_1} = \lambda_1 \boldsymbol{o}_1 + \lambda_2 \boldsymbol{o}_2 + \ldots + \lambda_{l_1 - 1} \boldsymbol{o}_{l_1 - 1}$$
$$\sum_{i=1}^{l_1 - 1} \lambda_i = 1, \quad -\infty < \lambda_i < \infty, \forall i.$$
(16)

Proof: First, assume that player 1 has optimal probabilities $p_{1,1}^*, \ldots, p_{1,l_1-1}^*$ over o_1, \ldots, o_{l_1-1} , respectively. Equation (22) can be rewritten by the following representation,

$$\boldsymbol{o}_{l_1} = \left(\left(\left((h_{11}\boldsymbol{o}_1 + h_{12}\boldsymbol{o}_2)h_{21} + h_{22}\boldsymbol{o}_3 \right)h_{31} + h_{32}\boldsymbol{o}_4 \right) + \dots \right) h_{l_1-2,1} + h_{l_1-2,2}\boldsymbol{o}_{l_1-1} \right),$$

where the following relationships exist between the set of $h_{k,1}$ s and $h_{k,2}$ s

$$h_{k,1} + h_{k,2} = 1, \ 1 \le k \le l_1 - 2.$$

Moreover, we have the next equations between $\{h_{k,1}, h_{k,2}\}$ and $\{\lambda_k\}$ for $1 \le k \le l_1 - 2$,

$$h_{l_{1}-2,2} = \lambda_{l_{1}-1}, \qquad h_{l_{1}-2,1} = 1 - \lambda_{l_{1}-1},$$

$$h_{l_{1}-3,2}(h_{l_{1}-2,1}) = \lambda_{l_{1}-2}, \qquad h_{l_{1}-3,1} = 1 - h_{l_{1}-3,2},$$

$$\dots \qquad \dots \qquad \dots \qquad \dots$$

$$h_{2,2} \prod_{i=3}^{l_{1}-2} h_{i,1} = \lambda_{3}, \qquad h_{2,1} = 1 - h_{2,2},$$

$$h_{1,2} \prod_{i=2}^{l_{1}-2} h_{i,1} = \lambda_{2}, \qquad h_{1,1} \prod_{i=2}^{l_{1}-2} h_{i,1} = \lambda_{1}.$$
(17)

Afterwards, we introduce a game containing l_1 actions with vectors o_1, \ldots, o_{l_1-1} and $o'_{l_1} = h_{1,1}o_1 + h_{1,2}o_2$. Besides, the optimal probabilities of the new game are assumed as $q^*_{1,1}, \ldots, q^*_{1,l_1-1}$ and $q'^*_{1l_1}$. The value of game to which the l_1 -th action is added is not less than the game without the l_1 -th action according to [31], meaning that,

$$\operatorname{zerosum}(o_1, \dots, o_{l_1-1}) \leq \operatorname{zerosum}(o_1, \dots, o_{l_1-1}, o'_{l_1} = h_{11}o_1 + h_{12}o_2).$$
 (18)

In other words, for the new game, we have the following results,

$$\operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1}) \leq \operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},\boldsymbol{o}_{l_{1}}' = h_{11}\boldsymbol{o}_{1} + h_{12}\boldsymbol{o}_{2})$$
$$= \min_{v}(q_{11}^{*}\boldsymbol{o}_{1} + \ldots + q_{1,l_{1}-1}^{*}\boldsymbol{o}_{l_{1}-1} + q_{1l_{1}}^{'*}\boldsymbol{o}_{l_{1}}')$$
$$= \min_{v}((q_{11}^{*} + h_{11}q_{1l_{1}}^{'*})\boldsymbol{o}_{1} + (q_{12}^{*} + h_{12}q_{1l_{1}}^{'*})\boldsymbol{o}_{2} + \ldots + q_{1,l_{1}-1}^{*}\boldsymbol{o}_{l_{1}-1}),$$

where \min_{v} finds the entry with the minimum value of vector v. If both $h_{1,1}$ and $h_{1,2}$ are not less than zero, set $(q_{11}^* + h_{1,1}, q_{12}^* + h_{1,2}, \dots, q_{1,l_{1}-1}^*)$ can be interpreted as a distribution vector over $l_1 - 1$ actions of player 1. Notice that each probability distribution over these selected actions brings the value not greater than v. Thus, we can conclude that

$$\operatorname{zerosum}(\boldsymbol{o}_1,\ldots,\boldsymbol{o}_{l_1-1}) \geq \operatorname{zerosum}(\boldsymbol{o}_1,\ldots,\boldsymbol{o}_{l_1-1},\boldsymbol{o}_{l_1}'=h_{11}\boldsymbol{o}_1+h_{12}\boldsymbol{o}_2). \tag{19}$$

Due to (24) and (25), we have

$$\operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1}) = \operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},\boldsymbol{o}_{l_{1}}' = h_{11}\boldsymbol{o}_{1} + h_{12}\boldsymbol{o}_{2}).$$
(20)

In other words, if the action l_1 with vector $o'_{l_1} = h_{1,1}o_1 + h_{1,2}o_2$ is eliminated, we will gain the same value. However, if one of them is less than zero, we cannot get the above formulation. Without loss of generality, we assume that $h_{1,1} < 0$ and $-\alpha = q^*_{11} + h_{1,1}q'^*_{1l_1} < 0$. Remind that

 $h_{1,1}+h_{1,2}=1,$ thus $h_{1,2}>0$ and therefore $q_{1,2}^*+h_{1,2}q_{1,l_1}^{'*}>0$. Because the summation over probabilities is 1, hence,

$$\sum_{i=1}^{l_1-1} q_{1,i}^* + q_{1,l_1}^{'*} = 1$$

$$(q_{1,1}^* + h_{1,1}q_{1l_1}^{'*}) + (q_{1,2}^* + h_{1,2}q_{1l_1}^{'*}) + \dots + q_{1,(l_1-1)}^* = 1,$$

$$(q_{1,2}^* + h_{1,2}q_{1,l_1}^{'*}) + \dots + q_{1,(l_1-1)}^* = 1 + \alpha.$$

Now, consider distribution vector $[T_2, T_3, \ldots, T_{l_1-1}]$ which is constructed by the following,

$$\begin{cases} \frac{(q_{1,2}^*+h_{1,2}q_{1,l_1}')}{(1+\alpha)} = T_2 \\ \frac{q_{1,3}^*}{(1+\alpha)} = T_3 \\ \cdots \\ \frac{q_{1,(l_1-1)}^*}{(1+\alpha)} = T_{l_1-1} \end{cases}$$
(21)

where $T_2 + T_3 + \cdots + T_{l_1-1} = 1$. Again, we have the following inequality:

$$\min_{v}(\boldsymbol{o}_{2}T_{2}+\ldots+\boldsymbol{o}_{l_{1}-1}T_{l_{1}-1}) \leq \operatorname{zerosum}(\boldsymbol{o}_{2},\ldots,\boldsymbol{o}_{l_{1}-1}) \leq \operatorname{zerosum}(\boldsymbol{o}_{1},\boldsymbol{o}_{2},\ldots,\boldsymbol{o}_{l_{1}-1}).$$
(22)

To put it differently, (28) can be reformulated as

$$\min_{v} \left(\frac{o_2 p_{12}^* + \dots + o_{l_1 - 1} p_{1(l_1 - 1)}^*}{1 - p_{11}^*} \right) \le \operatorname{zerosum}(o_2, \dots, o_{l_1 - 1}) \le \operatorname{zerosum}(o_1, o_2, \dots, o_{l_1 - 1}).$$

Besides, (24) gives that

$$\min_{v} \left((1+\alpha)(\boldsymbol{o}_2 T_2 + \dots + \boldsymbol{o}_{l_1-1} T_{l_1-1}) - \alpha \boldsymbol{o}_1 \right) \geq \operatorname{zerosum}(\boldsymbol{o}_1, \boldsymbol{o}_2, \dots, \boldsymbol{o}_{l_1-1}),$$

and,

$$\min_{v} \left(\left(\frac{\boldsymbol{o}_{2} p_{12}^{*} + \dots + \boldsymbol{o}_{l_{1}-1} p_{1(l_{1}-1)}^{*}}{1 - p_{11}^{*}} \right) (1 - p_{11}^{*}) + p_{11}^{*} \boldsymbol{o}_{1} \right) \geqslant \operatorname{zerosum}(\boldsymbol{o}_{1}, \boldsymbol{o}_{2}, \dots, \boldsymbol{o}_{l_{1}-1}).$$

If b_k , c_k and d_k are the k-th the entries of $(o_2T_2 + \dots + o_{l_1-1}T_{l_1-1})$, $\left(\frac{o_2p_{12}^* + \dots + o_{l_1-1}p_{1(l_1-1)}^*}{1-p_{11}^*}\right)$ and o_1 , respectively, we could obtain the following result,

$$b_{k} > \frac{d_{k}}{\alpha + 1} + \frac{\operatorname{zerosum}(o_{1}, o_{2}, \dots, o_{l_{1} - 1})}{\alpha + 1} \geqslant \frac{\alpha(\operatorname{zerosum}(o_{1}, o_{2}, \dots, o_{l_{1} - 1}) - p_{11}^{*}c_{k})}{(\alpha + 1)(1 - p_{11}^{*})} + \frac{\alpha(\operatorname{zerosum}(o_{1}, o_{2}, \dots, o_{l_{1} - 1}))}{\alpha + 1} = \operatorname{zerosum}(o_{1}, o_{2}, \dots, o_{l_{1} - 1}) \left(\frac{\alpha}{(1 + \alpha)(1 - p_{11}^{*})} + \frac{1}{\alpha + 1}\right) - \frac{\alpha p_{11}^{*}c_{k}}{(1 + \alpha)(1 - p_{11}^{*})}.$$
 (23)

Consequently,

$$\operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1})\left(\frac{\alpha}{(1+\alpha)(1-p_{11}^{*})}+\frac{1}{\alpha+1}\right) < \frac{\alpha p_{11}^{*}c_{k}}{(1+\alpha)(1-p_{11}^{*})}.$$
(24)

This suggests the following inequality for b_k , c_k and d_k

 $w_1c_k + w_2b_k > d_k$ st. $w_1 + w_2 = 1.$ (25)

Therefore, consideration of both (30) and (31) gives us the following inequality,

$$\operatorname{zerosum}(\boldsymbol{o}_{1}, \boldsymbol{o}_{2}, \dots, \boldsymbol{o}_{l_{1}-1}) < \\ \min_{v} \left(w_{1} \left(\frac{\boldsymbol{o}_{2} p_{12}^{*} + \dots + \boldsymbol{o}_{l_{1}-1} p_{1,l_{1}-1}^{*}}{1 - p_{11}^{*}} \right) + w_{2}(\boldsymbol{o}_{2} T_{2} + \dots + \boldsymbol{o}_{l_{1}-1} T_{l_{1}-1}) \right) \\ = \min_{v} (\beta_{2} \boldsymbol{o}_{2} + \dots + \beta_{l_{1}-1} \boldsymbol{o}_{l_{1}-1})$$
(26)

in which $\beta_2 + \beta_3 + \cdots + \beta_{l_1-1} = 1$, β_2 , β_3 , \ldots , $\beta_{l_1-1} \ge 0$. We know that the minimum entry of vector $\beta_2 o_2 + \cdots + \beta_{l_1-1} o_{l_1-1}$ is not higher than $\operatorname{zerosum}(o_1, o_2, \ldots, o_{l_1-1})$ for any set of β_2 , β_3 , \ldots , β_{l_1-1} . Therefore, our initial assumption is not correct. In other words, $q_{1,1}^* + h_{1,1}q'^*h_{1,l_1}$ is not less than zero, and we can obviate o'_{l_1} . It means that

zerosum
$$(o_1, \ldots, o_{l_1-1})$$
 = zerosum $(o_1, \ldots, o_{l_1-1}, o'_{l_1} = h_{1,1}o_1 + h_{1,2}o_2).$ (27)

Returning to the general case in (22), it can be concluded from (33) that

$$\operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1}) = \operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},\boldsymbol{o}_{l_{1}}' = h_{1,1}\boldsymbol{o}_{1} + h_{1,2}\boldsymbol{o}_{2}) =$$

$$\operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},\boldsymbol{o}_{l_{1}}',h_{2,2}\boldsymbol{o}_{3} + h_{2,1}\boldsymbol{o}_{l_{1}}') = \operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},h_{2,2}\boldsymbol{o}_{3} + h_{2,1}\boldsymbol{o}_{l_{1}}') =$$

$$\operatorname{zerosum}\left(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},h_{22}\boldsymbol{o}_{3} + h_{21}\boldsymbol{o}_{l_{1}}',h_{3,1}(h_{2,2}\boldsymbol{o}_{3} + h_{2,1}\boldsymbol{o}_{l_{1}}') + h_{3,2}\boldsymbol{o}_{4}\right) = \ldots =$$

$$\operatorname{zerosum}\left(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},h_{31}(h_{22}\boldsymbol{o}_{3} + h_{21}\boldsymbol{o}_{l_{1}}') + h_{32}\boldsymbol{o}_{4}\right) =$$

$$\operatorname{zerosum}\left(\left(\left((h_{11}\boldsymbol{o}_{1} + h_{12}\boldsymbol{o}_{2})h_{21} + h_{22}\boldsymbol{o}_{3}\right)h_{31} + h_{32}\boldsymbol{o}_{4}\right) + \ldots\right)h_{(l_{1}-2)1} + h_{(l_{1}-2)2}\boldsymbol{o}_{l_{1}-1}\right) =$$

$$\operatorname{zerosum}(\boldsymbol{o}_{1},\ldots,\boldsymbol{o}_{l_{1}-1},\boldsymbol{o}_{l_{1}}). \tag{28}$$

The expression in (34) states that if o_{l_1} satisfies the conditions of *Lemma* 1, we can omit it. Here, we return to prove *Proposition* 1. Each o_i has l_2 entries, so we can represent all of the l_1 vectors by at most l_2 vectors of them. These basic vectors are linear and independent. Without loss of generality, we assume that these vectors are $o_1, \ldots, o_{l'_2}$, where $l'_2 \leq l_2$. Based upon the vector representation, all $\{o_i\}$ s are classified into three groups.

Now, we assume that each o_i can be displayed by $o_i = \lambda_{1,i}o_1 + \cdots + \lambda_{l'_2,i}o_{l'_2}$. Also, the coefficients are unique due to linearity and independency. These groups are stated as follows:

Group I: If $\sum_{j=1}^{l'_2} \lambda_{j,i} = 1$, we can obviate o_i and get the same value as in *Lemma* 1. **Group II:** If $\sum_{j=1}^{l'_2} \lambda_{j,i} < 1$, we have the following facts:

We assume that the optimal probability distributions over the l_1 actions are $p_{1,1}^*, \ldots, p_{1,l_1}^*$.

$$\operatorname{zerosum}(o_1,\ldots,o_{l_1}) \geq \operatorname{zerosum}(o_1,\ldots,o_{i-1},o_{i+1},\ldots,o_{l_1})$$

where the second term does not include o_i . From [26], we know that

zerosum
$$(o_1, \ldots, o_{l_1}) = \min_v (p_{1,1}^* o_1 + \cdots + p_{1,l_1}^* o_{l_1}).$$
 (29)

Now, we extend (35) as

$$p_{1,1}^* \boldsymbol{o}_1 + \dots + p_{1,i}^* \boldsymbol{o}_i + \dots + p_{1,l_1}^* \boldsymbol{o}_{l_1} = p_{1,1}^* \boldsymbol{o}_1 + \dots + p_{1,i}^* (\lambda_{1i} \boldsymbol{o}_1 + \dots + \lambda_{l'_{2i}} \boldsymbol{o}_{l'_{2}}) + \dots + p_{1,l_1}^* \boldsymbol{o}_{l_1}$$

$$< p_{1,1}^* \boldsymbol{o}_1 + \dots + p_{1,i}^* (\lambda_{1,i} \boldsymbol{o}_1 + \dots + \lambda_{l'_{2i}} \boldsymbol{o}_{l'_{2}}) + \dots + p_{1,l_1}^* \boldsymbol{o}_{l_1} + \left(1 - \sum_{j=1}^{l'_{2}-1} \lambda_{ji} - \lambda_{l'_{2i}}\right) \boldsymbol{o}_{l'_{2}}$$

$$= p_{1,1}^* \boldsymbol{o}_1 + \dots + p_{1,i}^* \left(\lambda_{1,i} \boldsymbol{o}_1 + \left(1 - \sum_{j=1}^{l'_{2}-1} \lambda_{i,j}\right) \boldsymbol{o}_{l'_{2}}\right) + \dots + p_{1,l_1}^* \boldsymbol{o}_{l_1}$$

in which we call the expression stated in parenthesis in the last term as o'_i . The value of the game when playing with o'_i instead of o_i is given via

$$\operatorname{zerosum}(o_1, \dots, o'_i, \dots, o_{l_1}) \ge \min(p_{11}^* o_1 + \dots + p_{1i}^* o'_i + \dots + p_{1l_1}^* o_{l_1}) >$$
$$\min(p_{11}^* o_1 + \dots + p_{1l_1}^* o_{l_1}) = \operatorname{zerosum}(o_1, \dots, o_{l_1}).$$

Now, o_i can be represented by basic vectors $\{o_1, \ldots, o'_i, \ldots, o_{l_1}\}$ in which the sum of coefficients becomes 1. Thus, we can obviate o'_i and at the same time get the same value. In other words, we have

$$\operatorname{zerosum}(\boldsymbol{o}_1,\ldots,\boldsymbol{o}_{i-1},\boldsymbol{o}_i',\boldsymbol{o}_{i+1},\ldots,\boldsymbol{o}_{l_1})=\operatorname{zerosum}(\boldsymbol{o}_1,\ldots,\boldsymbol{o}_{i-1},\boldsymbol{o}_{i+1},\ldots,\boldsymbol{o}_{l_1}).$$

Moreover, we have

$$\operatorname{zerosum}(\boldsymbol{o}_1,\ldots,\boldsymbol{o}_{l_1}) \geq \operatorname{zerosum}(\boldsymbol{o}_1,\ldots,\boldsymbol{o}_{i-1},\boldsymbol{o}_{i+1},\ldots,\boldsymbol{o}_{l_1}).$$

Then, we can remove all of vectors which have coefficients satisfying the following inequality without loss in the value of the game, v.

$$\sum_{j=1}^{l_2'} \lambda_{ji} < 1.$$
 (30)

Group III: If $\sum_{j=1}^{l'_2} \lambda_{ji} > 1$, we can show o_i by the following equation

$$\boldsymbol{o}_{i} = \lambda_{1,i}\boldsymbol{o}_{1} + \lambda_{2,i}\boldsymbol{o}_{2} + \dots + \lambda_{l_{2}',i}\boldsymbol{o}_{l_{2}'-1}.$$
(31)

In this case, there exists at least one coefficient, e.g., $\lambda_{l'_2,i}$, which is greater than zero. Now, we try to show $o_{l'_2}$ by $o_1, \ldots, o_{l'_2-1}$ including o_i . Indeed,

$$o_{l'_{2}} = \frac{-(\lambda_{1i}o_{1} + \lambda_{2i}o_{2} + \dots + \lambda_{l'_{2}-1i}o_{l'_{2}-1})}{\lambda_{l'_{2}i}} + \frac{1}{\lambda_{l'_{2}i}}o_{i} = \mu_{1}o_{1} + \mu_{2}o_{2} + \dots + \mu_{l'_{2}-1}o_{l'_{2}-1}.$$

However, we know that

$$\mu_1 + \dots + \mu_{l'_2 - 1} = \frac{-(\lambda_{1i} + \lambda_{2i} + \dots + \lambda_{l_2' - 1, i}) + 1}{\lambda_{l'_2 i}} > 1.$$
(32)

Therefore, we can remove $o_{l_2'}$ according to the second group. As a result, we only need the l_2 actions among the l_1 onesand get the same value. Similar classification can be applied to vectors of o_1, \ldots, o_{l_1} and at each stage one vector is removed. Finally, $l'_2(l'_2 \leq l_2)$ actions (vectors) remain for playing the game.

APPENDIX B

PROOF OF PROPOSITION 2

First, we consider an assumption for each allocation to continue our proof.

Assumption 1: If the following relation exist between vectors of allocations,
$$\{\boldsymbol{h}_1, \boldsymbol{h}_2, \dots, \boldsymbol{h}_{M'}, \boldsymbol{h}_{M'+1}\},$$

$$\sum_{i=1}^{M'} \lambda_i \boldsymbol{h}_i = \boldsymbol{h}_{M'+1}, \sum_{i=1}^{M'} \lambda_i = 1,$$
for $1 \le i \le M' + 1, EL\{\boldsymbol{h}_i\} \nsubseteq EL\{\boldsymbol{h}_1, \dots, \boldsymbol{h}_{i-1}, \boldsymbol{h}_{i+1}, \dots, \boldsymbol{h}_{M'+1}\},$
(33)

then the occurrence probability of all relations is zero.

Indeed, each h_i has M' entries. Accordingly, the allocation vectors construct M' equations, and we have M' - 1 parameters involving $\lambda_1, \ldots, \lambda_{M'-1}$. For (39), we have M' - 1 parameters satisfying M' equations. This situation makes hard to yield these M' - 1 parameters out of M' equations. For instance, if each $a_{ij,k}$ is independent with respect to the other $a_{i'j',k'}$ with the uniform distribution, this assumption is precise. Also, our simulation result certifies the assumption. We assume that the attackers strategy is the same as the strategy of the original zero-sum game. In the original form, we have the following equation

in which p_1^* and p_2^* are the optimal action probabilities of the coordinator and attacker, respectively, and U is the payoff matrix in accordance with the zero-sum game between the coordinator and the attacker.

The related u_i 's to the allocations with the non-zero probabilities, which are named as the proper allocations, are the same as the overall value of the game and $\max(u_1, \ldots, u_{\frac{N!}{(N-M')!}})$. According to *Proposition* 1, we only need M' proper allocations, namely complete allocations, to obtain the similar value when using all actions. Hence, we must show that each complete allocation is surely selected as the solution of the PC-game at least one action by means of contradiction. Notice that if more than one proper allocations exist in the first-auction, only one of them is randomly selected as the solution of auction. Furthermore, we know that each allocation of channels can be found $M'^{(N-M')}$ times at the first auction. The worst case occurs for a complete allocation, for example J, when at least one of the other proper allocations always exists in the first auction including this allocation. For simplicity, we assume that J is $(1, 2, \ldots, M')$. Now, consider the following first auctions:

$$\begin{cases} 1: & (1, 2, \dots, M', 1, \dots, 1) \\ 2: & (1, 2, \dots, M', 2, \dots, 2) \\ \dots \\ M': & (1, 2, \dots, M', M', \dots, M') \end{cases}$$
(34)

where (1, 2, ..., M', j, ..., j) means that this first-auction includes allocation J, and the coordinator selects channel j for the remaining users as well. Hence, we have at least M' + 1proper allocations among the above actions. These vectors cannot be linearly independent since dimension of vectors is M'. Therefore, we have the following according to *Proposition 1*.

$$\boldsymbol{J} = \lambda_1 \boldsymbol{h}_{\boldsymbol{o}_1} + \lambda_2 \boldsymbol{h}_{\boldsymbol{o}_2} + \ldots + \lambda_{M'} \boldsymbol{h}_{\boldsymbol{o}_{M'}} \qquad \lambda_1 + \ldots + \lambda_{M'} = 1,$$
(35)

where h_{o_i} s are the proper allocations. Also, any two allocations of M' + 1 allocations differ from two elements so that the conditions of *Assumption 1* are satisfied, and the probability of this occurrence is zero. Hence, our initial assumption about the concurrent existence of these allocations, is not correct and the PC-game is equal to the original game.

APPENDIX C

PROOF OF PROPOSITION 3

In the PD-game, payment for user *i* has two parts, $p_{i,1}$ and $p_{i,2}$, which are related to first and second auctions. If we assume that the SUs choose preferences l_1, \ldots, l_N as their actions and channel *j* is dedicated to the *i*-th SU while the attacker jams channel *h*, we have the following formulation for the average profit of SU *i*.

$$\mathbb{E}\left(v_{i} - p_{i1}^{t}(\widehat{v}_{i}, v_{-i}) - p_{i2}^{t}(\widehat{v}_{i}, v_{-i}) \mid l_{1}, \dots, l_{N}\right) \\
= a_{ij,h}Q_{2,h}(t) + \sum_{h=1}^{M'} \left(\left[\sum_{k=1,k\neq i}^{N} z_{kl_{k}}^{t,opt} a'_{kl_{k}}(t) - \max_{Z\mid a_{ij'}=0\forall j'} \sum_{k=1,k\neq i}^{N} z_{kl_{k}}^{t} a'_{kl_{k}}(t) \right] \right) Q_{2,h}(t) \\
+ \sum_{h=1}^{M'} \left[\sum_{k=1,k\neq i,k\in S_{1}}^{N} \sum_{j'=1,j'\in S_{2}}^{M'} z_{kj'}^{t,opt} a'_{kj'}(t) - \max_{Z\mid a_{ij'}=0\forall j'} \sum_{k=1,k\neq i,k\in S_{1}}^{N} \sum_{j'=1,j'\in S_{2}}^{M'} z_{kj'}^{t,opt} a'_{kj'}(t) \right] Q_{2,h}(t) \right] Q_{2,h}(t) . (36)$$

In (42), v_i and \hat{v}_i are the actual and submitted bid for SU *i*. Moreover, $Q_{2,h}$, S_1 and S_2 are the probability for jamming of channel *h* by the jammer, the set of SUs and channels remained from first auction, respectively. To attain the *i*-th user's profit, we should apply the probability of preferences for all the SUs. Moreover, $p_2(t) \approx Q_2(t)$, therefore, $\sum_h Q_{2,h}(t) a_{ij,h} = a'_{ij}$ similar to (12). Hence, the expectation profit can be stated as follows,

$$\mathbb{E}\left(v_{i} - p_{i}^{t}(v_{i}, v_{-i})\right) = \sum_{\substack{i=1\\ o_{1}=1}}^{M'} Q_{l_{1}} \sum_{\substack{o_{2}=1\\ o_{2}=1}}^{M'} \dots \sum_{\substack{o_{N}=1\\ o_{N}=1}}^{M'} Q_{l_{2}} \dots Q_{l_{N}} \left[\sum_{\substack{k=1\\ k=1}}^{N} z_{kl_{k}}^{t,opt} a'_{kl_{k}}(t) - \max_{\substack{Z \mid a_{ij}=0 \forall j}} \sum_{\substack{k=1\\ k\in S_{1}}}^{N} \sum_{\substack{j=1, j\neq j'\\ j\in S_{2}}}^{N} z_{kj}^{t,opt} a'_{kj}(t) - \max_{\substack{Z \mid a_{ij}=0 \forall j}} \sum_{\substack{k=1\\ k\in S_{1}}}^{N} \sum_{\substack{j=1, j\neq j'\\ j\in S_{2}}}^{M'} z_{kj}^{t} a'_{kj}(t)\right].$$
(37)

Equivalently,

$$\mathbb{E}\left(v_{i} - p_{i}^{t}(v_{i}, v_{-i})\right) = \sum_{o_{1}=1}^{M'} Q_{l_{1}} \sum_{o_{2}=1}^{M'} \dots \sum_{o_{N}=1}^{M'} Q_{l_{2}} \dots Q_{l_{N}} \left[\sum_{\substack{k=1\\k=1\\k\neq k}}^{N} z_{kl_{k}}^{t,opt} a'_{kl_{k}}(t) + \sum_{\substack{k=1,k\neq i\\j\in S_{1}}}^{N} \sum_{\substack{j=1,j\neq j'\\j\in S_{2}}}^{M'} z_{kj}^{t,opt} a'_{kj}(t)\right] - \left[\max_{\substack{Z|a_{ij}'=0\forall j\\k\neq i}}^{N} \sum_{\substack{k=1\\k\neq i}}^{N} z_{kl_{k}}^{t} a'_{kl_{k}}(t) + \max_{\substack{Z|a_{ij}=0\forall j\\k\in S_{1}}}^{N} \sum_{\substack{j=1\\k\in S_{1}}}^{M'} z_{kj}^{t} a'_{kj}(t)\right]. \quad (38)$$

The third and fourth terms are not function of the *i*-th SU. Therefore, we can disregard them to further analysis. But, the summation over the first and second terms are equal to the total profit of the SUs according to (15). In other words, the individual profit is equivalent to the total profit. For this reason, the rational SUs must bid truthfully.

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