

# Solving the Divergence Problem in AC-QSS Cascading Failure Model by Introducing the Effect of a Realistic UVLS Scheme

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**Abstract**—AC-Quasi-Steady-State (AC-QSS) models for cascading failure often suffer from divergence issues. Presently, this is handled by uniform load shedding (ULS) or its variants till convergence is achieved. In reality, a pre-defined undervoltage load shedding (UVLS) scheme sheds loads in blocks below a voltage threshold and the system reaches a post-UVLS equilibrium, if it exists. This may not be captured by the post-ULS equilibrium, which in turn can lead to a different path of cascade propagation. We use the ULS equilibrium as the initial state and pose the problem of reaching post-UVLS equilibrium as an optimization problem. Due to the nature of the problem, an exhaustive search can be used that has exponential complexity in inputs. To solve this, we propose a heuristic that reaches a post-UVLS candidate equilibrium. The proposed algorithm is tested on a 2383-bus Polish system.

**Index Terms**—AC-QSS model, Cascading failure, Load shedding, UVLS

## I. INTRODUCTION

CASCADING failure in power system is a low-probability-high-risk event. It has significantly negative impact on the economy and society in general. Therefore, it is necessary to carefully study this phenomena, which can help in preventing its occurrence. Broadly, there are three types of cascading failure models reported in literature - DC-quasi-steady-state (QSS), AC-QSS, and dynamic model. DC-QSS and dynamic models are first briefly discussed. However, the main focus of this paper is AC-QSS model.

DC-QSS models [1]–[6] are based on DC power flow (PF), which neglects the resistive losses in the system and assumes a uniform voltage profile. Authors in [1] proposed a stochastic model based on DC PF. Reference [2] investigates critical points and transitions in network for simulating cascading failures. In [3], cascading failure is studied under influence of load growth and power fluctuations as effects of renewable sources. Paper [4] proposed a new metric “critical moment” based on rotor angle and voltage stability, whereas authors in [5] and [6] modeled cascading failure in coupled power/communication networks. Although DC-QSS models are easy to implement

and computationally inexpensive, they cannot capture voltage stability/collapse and reactive power in the power system.

At the other extremity in terms of representation of details lie dynamic models of cascading failure, which have been proposed in [17]–[19]. Despite these models’ ability to capture extensive mechanisms of cascading outages, they are computationally expensive for large-scale networks.

AC-QSS models [7]–[15] are based on the AC PF, which can represent voltage collapse issues during cascade propagation. Reference [7] proposed an online control scheme to attain a post-cascade equilibrium while shedding minimum amount of load. Authors in [8] proposed the well-known Manchester model, which studies the overloaded system up to its critical load. Monte Carlo simulations are performed to analyze the behavior of the system during cascading failure. In [9] and [10], a predictive control model is proposed to mitigate the cascade propagation that uses both DC and AC PFs. The voltage collapse is assumed to happen if non-convergence of AC PF is observed.

Authors in [11] proposed to study voltage stability using the eigenvalues of reduced Jacobian matrix, however no rigorous study was actually performed. Reference [12] utilized AC PF and DC optimal PF (OPF) methods. If AC PF diverges during cascading failure, the DC OPF is applied followed by the AC PF. If no convergence is observed, a voltage collapse is assumed to occur. In [13], authors introduced an AC OPF model to improve the Manchester model [8]. In [14], a new AC OPF model, which considers frequency is introduced to include remedial control during system collapse.

Authors in [15] proposed a stochastic model for cascading failure in power systems. Divergence of AC PF is addressed by using continuation power flow (CPF) [16]. However, problems regarding starting point of CPF were not discussed. In our understanding, CPF is sensitive to the loading at its initial point and may result in completely different P-V curves due to different PV-PQ bus status at the starting point.

In the case of voltage collapse, literature have applied different load shedding methods, e.g. the Manchester model [8] sheds load in blocks uniformly at all buses; [11], [13], [14] applied similar methods until convergence of AC PF;

and [12] utilized DC OPF to find convergence for AC PF. In other words, these are different variants of uniform load shedding (ULS). In reality, there are pre-designed undervoltage load shedding (UVLS) relays in a network that shed a fixed fraction of loads when voltage goes below a certain threshold [20]. The most comprehensive UVLS architectures involving local protection relays and centralized remedial action schemes (RASs) are present in the WECC system [21]–[23]. *Unfortunately, none of the papers [7]–[15] ensure that given such a UVLS scheme, a post-UVLS equilibrium will be reached.*

We use the ULS equilibrium as the initial state and pose the problem of reaching post-UVLS equilibrium as an optimization problem. Due to the nature of the problem, an exhaustive search can be used that has exponential complexity in inputs. To solve this, we propose a heuristic that reaches a post-UVLS candidate equilibrium. The proposed algorithm is tested on a 2383-bus Polish system.

## II. AC-QSS MODEL

This section provides an overview of the AC-QSS cascading failure model [8], [11], [12], [14], which is shown in *Algorithm 1*. The cascading failure simulation starts with a set of initial node outages and disconnection of lines connected to the nodes. Our focus is not on the root cause of such failures – e.g. it can be a nation-wide state-sponsored nefarious cyber-attack opening breakers in substations. The focus rather is on improving existing AC-QSS models to better reflect the *ground truth* during cascading failure propagation.

Cascade propagates after tripping of overloaded lines/branches in each ‘tier’ of cascade. This continuously changes the topology of the system and might lead to formation of islands. Following each tier, the model uses Newton-Raphson method for solving AC power flow (PF) equations in each island. This process stops when none of the remaining lines are overloaded, or in the worst case no load are left to be served (i.e. total blackout). In addition to line overloading, AC-QSS models can capture voltage collapse phenomena [8]–[12], [14] leading to divergence of AC load flow, which can be regarded as complete blackout in the island. These are described next.

### A. Models for Tripping Overloaded Branches

Two branch tripping methods are used in modeling the cascading failure, which are explained next.

1) *Instantaneous line tripping (ILT)*: In this method, overcurrent relays are assumed to release tripping command immediately after identification of an overloaded branch, i.e. the trip time is defined to be equal to zero. In other words, this line tripping method is independent of time.

2) *Delayed line tripping (DLT)*: In the previous method, all the lines above the rating were identified and tripped in a single tier. On the contrary, in this method every overloaded line is assigned a trip time and the lines are tripped only when the trip time elapses. The trip time is computed based on the inverse-time overcurrent characteristics.

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### Algorithm 1: AC-QSS Model with ULS

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- 1 Given a set of initial node outages  $\Phi(N)$  as input.
  - 2 Disconnect all connected branches  $\lambda(L)$  to  $\Phi(N)$ .
  - 3 Identify resulting islands due to nodes and branch outages. Shed generation/load to restore generation - load balance within each island. Identify the islands that can be operated and neglect the rest. Call the final set of subnetworks  $\mathcal{F}$ .
  - 4 Choose an uninvestigated subnetwork from  $\mathcal{F}$  and run PF. If PF converges, add this subnetwork to set  $\Sigma$  and go to 6, if not, go to 5.
  - 5 Uniformly shed load in the subnetwork until PF converges. If PF converged, add the subnetwork to  $\Sigma$ , if not, declare voltage collapse in this subnetwork.
  - 6 If all the subnetworks in  $\mathcal{F}$  are investigated, go to 7, otherwise go to 4.
  - 7 If there is no overloaded line, go to 9. Otherwise go to 8.
  - 8 Trip overloaded lines considering one of the line tripping methods, i.e. ILT or DLT, and go to 3.
  - 9 Save data.
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Moreover, for a particular line to trip, it must remain overloaded for the duration of its trip time. This means it is possible that some lines can be relieved before their trip time has passed due to redistribution of power flow after other line trips. Similarly, following such a change, it is also possible that some lines, which are overloaded at one level, can be overloaded to a higher level, and can thus observe a reduction in their trip time.

Unlike ILT, there is a sense of elapsed time in this mechanism. In order to ascertain that a specific amount of trip time has passed, in the simulation, we consider flow of time in increments of a fixed time step. The smaller the time step, the higher the accuracy and computational burden. Therefore, to strike a balance between accuracy and computational burden, in this paper, a time step of 5 s is chosen.

### B. Divergence Issue in AC-QSS Models: Present Solution

The other aspect captured by AC-QSS model is the voltage collapse phenomenon. As shown in step 3 of Algorithm 1, a number of islands might form. Some of these islands are able to be operated, i.e. they have both generation and load, which are included in the set  $\mathcal{F}$ . Rest of the islands are assumed to be failed networks. However, PF may diverge in some of the subnetworks (step 4), which is usually caused by voltage collapse. Present algorithms consider some form of load shedding (uniform or otherwise) until convergence is achieved [8], [11]–[14]. For the purpose of comparison, we consider a simple case of uniform load shedding (ULS) as illustrated in step 5 of Algorithm 1.

In reality, within a subnetwork, a pre-defined undervoltage load shedding (UVLS) scheme sheds loads in blocks below a voltage threshold and the system reaches a post-UVLS equilibrium, if it exists - thereby avoiding voltage collapse. *This may not be captured by the post-ULS equilibrium, which in turn can lead to a different path of cascade propagation.* Thus motivated, we aim to improve existing AC-QSS models to reach a post-UVLS candidate equilibrium.

### III. PROBLEM FORMULATION & PROPOSED SOLUTION

We pose the problem of reaching post-UVLS equilibrium as an optimization problem shown below. We assume that for a given UVLS setting, the ground truth in the post-UVLS equilibrium represents the minimum possible load shedding.

$$\begin{aligned}
& \underset{\vec{v}_2, \vec{\gamma}_1}{\text{minimize}} \quad -\Delta P_L(\vec{v}_1, \vec{\gamma}_1) + \Gamma(\vec{v}_2) \\
& \text{subject to} \quad \vec{v}_1 = \vec{v}^0 \\
& \quad \vec{v}_2 = \vec{g}(\vec{v}_1, \vec{\gamma}_1) \\
& \quad P_{Li}^0 + jQ_{Li}^0 = r^{\gamma_i}(P_{Li}^{pre} + jQ_{Li}^{pre}), \forall i \in n_L \\
& \quad 0 \leq \gamma_i \leq \gamma_{max}, \forall i \in n_L \\
& \quad \Gamma(\vec{v}_2) = \sum_j w_j (v_{th} - |v_{2j}|), \\
& \text{where,} \quad w_j = 0, \quad \text{if } v_{th} < |v_{2j}| \\
& \quad \quad \quad = 10^{12}, \quad \text{otherwise} \\
& \quad \vec{\gamma}_k \in \mathbb{Z}^{n_L}, v_{th} \in \mathbb{R}, \vec{v}_k \in \mathbb{C}^n
\end{aligned} \tag{1}$$

The formulation is similar in nature with the so-called deterministic dynamic programming (DDP) problem, albeit the solution is obtained in a single step. The state variables are elements of the complex voltage vector  $\vec{v}_k$  in the  $n$ -bus subnetwork from  $\mathcal{F}$ . The input variables are integers and elements of an  $n_L$ -dimensional vector  $\vec{\gamma}_k$ . Here,  $n_L$  is the number of buses with loads and  $k$  is the step number. The optimization starts from a *known* initial state  $\vec{v}_1 = \vec{v}^0$ . The second constraint,  $g(\cdot, \cdot)$ , represents the PF equations.

*UVLS Setting and Architecture:* We assume that the goal of the UVLS scheme is to bring the voltages in all buses above  $v_{th}$ . A two-layered UVLS architecture is considered – (a) UVLS relay at each bus trips  $(1 - r)$ th fraction of the load when the bus voltage is below a threshold  $v_{th}$ . The maximum number of tripping allowed is  $\gamma_{max}$ . (b) A centralized Remedial Action Scheme (RAS) sheds the loads in the same fraction, when  $\gamma_{max}$  is reached at certain buses whose voltages are still below  $v_{th}$ .

*Initial Condition:* We propose to leverage the solution of AC-QSS model with ULS (Algorithm 1). After obtaining this solution, we shed loads at all of  $n_L$  buses by  $P_{Li} + jQ_{Li} - r^{\gamma_i}(P_{Li}^{pre} + jQ_{Li}^{pre})$  so that each bus satisfy the third equality constraint. Here,  $P_{Li}^{pre}$  is the load in the  $i$ th bus after restoring generation-load balance in step 3 of Algorithm 1. The load flow solution at this stage gives our initial state  $\vec{v}^0$ .

*Transition Cost:* The objective function or so-called *transition cost* considers the -ve value of total load increase from the initial state following a control action, i.e. applying  $\vec{\gamma}_1$  for load change. The terminal cost  $\Gamma(\vec{v}_2)$  penalizes a solution where voltage magnitudes in all buses are not above  $v_{th}$ .

*Discretization & Computational Cost:* One way to solve this problem is to use exhaustive search. The order of computational complexity is  $\mathcal{O}((\gamma_{max} + 1)^{n_L})$ , which is exponential in number of input variables. Although the search space can be reduced by disregarding control actions that do not meet  $\Delta P_L(\vec{v}_1, \vec{\gamma}_1) \geq 0$ , which is an implicit constraint – the computational burden is enormous for a large system with a few thousand buses and reasonably large  $\gamma_{max}$ . We propose

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#### Algorithm 2: AC-QSS Model with UVLS

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- 1 Steps 1 - 3 in Algorithm 1.
  - 2 Choose a subnetwork from  $\mathcal{F}$  and run PF. If PF converged, run Algorithm 3 and add the resulted subnetwork to set  $\Sigma$ , and go to 6, if not, go to 5.
  - 3 Uniformly shed load in the subnetwork until PF converges. Use the set of voltages  $\mathcal{V}$  and run Algorithm 4 with the intact network at the start of 5. If there is an output from Algorithm 4, add it to set  $\Sigma$ , if not, assume that the subnetwork has encountered complete blackout.
  - 4 Steps 6 - 9 in Algorithm 1.
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a heuristic to solve this problem, which might lead us to a suboptimal solution. In other words our modified objective is to reach a post-UVLS candidate equilibrium. The proposed heuristic is described next.

#### A. Proposed Heuristic with UVLS

The proposed Heuristic is presented in Algorithm 2, which also involves Algorithms 3 and 4 as sub-algorithms. The goal is to shed minimum load and maintain the following constraints at the equilibrium:  $P_{Li} + jQ_{Li} = r^{\gamma_i}(P_{Li}^{pre} + jQ_{Li}^{pre})$ ,  $\forall i \in n_L$  and  $|v_i| \geq v_{th}$ ,  $\forall i$ . Algorithm 3 is dedicated for shedding load in those islands with converged PF - step 1 is to reflect local UVLS action at individual buses while step 2 reflects RAS action.

Algorithm 4 is assigned to shed load in islands with diverged PF. *The core idea is to use the PF solution following ULS as an initial starting point and use logical arguments to iteratively reach a post-UVLS candidate equilibrium.* Algorithm 4, at first, tries to find a converged PF solution with load shedding in different buses. After attaining a convergence for PF, this algorithm sheds more load in order to satisfy that voltage magnitude of all buses are greater than  $v_{th}$ . Then, the algorithm starts to recover load in buses that are far from voltage collapse (buses with high voltage magnitudes) maybe with the cost of shedding load in the buses that are prone to voltage collapse.  $\Psi_U$  and  $\Psi_D$  are two sets of buses that the algorithm uses in this regard.

*Comments:* Most of the steps in Algorithm 4 are self-explanatory. Nevertheless, some of the logical arguments/explanations are elaborated in brief. (1) Initial state with ULS gives an indication of buses that are more likely to undergo load shedding and vice-versa. (2) Step 2 is an intermediate step to ensure convergence. Since shedding does not follow UVLS logic in this step, the loads at these buses should either be fully recovered or further reduced to satisfy criteria for post-UVLS equilibrium. (3) Set  $\Psi_U$  represents buses that are likely to undergo shedding when loads in buses with higher voltages are recovered. (4)  $\Psi_U^{\gamma_{max}}$  and  $\Psi_D^{\gamma_{max}}$  represent the buses in the corresponding sets where number of load shedding has reached its limit.

### IV. CASE STUDY

The Polish network during winter 1999 – 2000 peak condition from Matpower [24] is used as a test system. It includes

**Algorithm 3: UVLS Function for Converged PF**

- 1 If  $|v_i| < v_{th}$ , keep shedding  $(1-r)$ th fraction of load at **all** **such** buses until voltage magnitude of all buses are above  $v_{th}$ . If the number of shedding at all buses with  $|v_i| < v_{th}$  reached  $\gamma_{max}$  and there are some buses where  $|v_i| < v_{th}$ , go to 2, else go to 3.
- 2 Sort buses with  $|v_i| \geq v_{th}$  from lowest voltage magnitude to the highest voltage magnitude. Start shedding load individually in the bus with the lowest  $|v_i|$  till number of shedding reaches  $\gamma_{max}$ . Keep doing this until  $|v_i| \geq v_{th}, \forall i$ . Go to 3.
- 3 Save data.

**Algorithm 4: UVLS Function for Diverged PF**

- 1 Using  $\mathcal{V}$ , for all buses with  $|v_i| < v_{th}$  shed  $(1-r)$ th fraction of load (if  $\gamma_{max}$  is not reached) and run PF. If PF converged, go to 3, else go to 2.
- 2 For all buses with  $|v_i| \geq v_{th}$ , shed  $(1-p)$ th fraction of load until convergence of PF. Here,  $(1-p) \ll (1-r)$ . If PF converged, go to 3, else go to 1.
- 3 Step 1 of Algorithm 3: 'if' condition go to 4, 'else' go to 5.
- 4 Step 2 of Algorithm 3: Go to 5.
- 5 Form sets  $\Psi_U$  and  $\Psi_D$ , such that  $|v_i| \geq 0.95, i \in \Psi_U$ , and if  $v_{th} \leq |v_i| < 0.95, i \in \Psi_D$ . For all buses in  $\Psi_U$ , recover loads to attain  $P_{Li}^{pre} + jQ_{Li}^{pre}$  and run PF. If PF converged, go to 8, else go to 6.
- 6 For all buses in  $\Psi_D$ , start shedding load with parameters  $r$  till convergence or  $\gamma_{max}$  is reached - whichever comes earlier. If PF converged, go to 8, else go to 7.
- 7 Sort buses in  $\Psi_U$  from lowest voltage magnitude to the highest voltage magnitude and shed load as in step 2 of Algorithm 3 until a convergence of PF is reached. Go to 8.
- 8 Form  $\Psi_{red} = \{\Psi_U \setminus \Psi_U^{\gamma_{max}} \cup \Psi_D \setminus \Psi_D^{\gamma_{max}}\}$ . Sort buses in  $\Psi_{red}$  from lowest voltage magnitude to the highest voltage magnitude and shed load as in step 2 of Algorithm 3. Go to 9.
- 9 Save data.

2383 buses, 2896 branches, 327 generators, and 1826 constant power loads. The simulations are conducted in Matlab R2019a. To simulate the AC-QSS cascading failure model with ULS and UVLS, 5 cases with initial node outages varying from 1% – 5% of total nodes are considered. Each case includes 500 random set of node outages. The ULS scheme sheds load in blocks of 0.5% uniformly from all loads. The UVLS parameters are:  $r = 0.75$ ,  $p = 0.99$ ,  $v_{th} = 0.8645$  pu [18], and  $\gamma_{max} = 7$ .

We have used *box – whisker* plots to represent results in figures in this paper. On each box, the central mark indicates median. The bottom and the top edges of the box cover data in the range of  $Q_1 = 25$  and,  $Q_3 = 75$  percentiles, respectively; whereas the corresponding whiskers indicate  $Q_1 - 1.5(Q_3 - Q_1)$  and  $Q_3 + 1.5(Q_3 - Q_1)$ , respectively. The whiskers exclude outliers which are plotted individually using red “+” symbol.

Figure 1 represents the box-whisker plots of minimum voltage magnitudes in the network at the end of cascade using ILT. For each percentage of initial node outage, result of proposed UVLS algorithm is compared against ULS-based

method. The black dot for each box plot shows the mean value while the red line denotes the median. For UVLS, the minimum voltages stay above  $v_{th}$ , while most of those for ULS are smaller than  $v_{th}$ . Thus, the proposed heuristic provides a post-UVLS candidate equilibrium without facing any convergence problem.

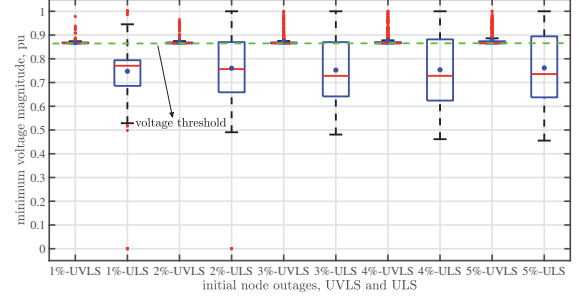


Fig. 1. Minimum voltage magnitudes in different initial node outages for UVLS and ULS, ILT case. black dots: mean values.

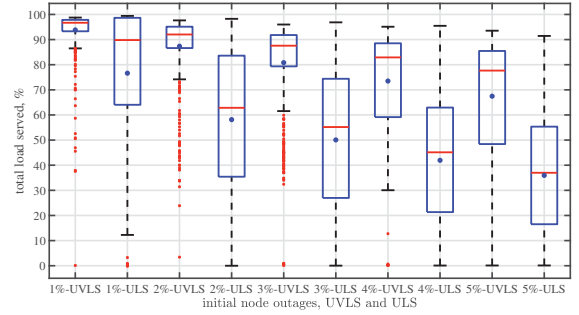


Fig. 2. Total load served in different initial node outages for UVLS and ULS, ILT case. black dots: mean values.

Figure 2 depicts boxplots of the total load served at the end of cascading failure modeled using ILT. According to Figs 1 and 2, the proposed model not only attains a post-UVLS candidate equilibrium, but also results in a higher mean and median of total load served. Similar conclusions can be drawn from Figs 3 and 4 that consider the DLT scenario. Finally, Table I shows some important measures that highlight contrasts in cascade propagation between ULS and UVLS cases. The table is self-explanatory – results indicate that the UVLS scheme results in less number of islands, tiers of cascades, total blackouts, and better voltage profile at the end

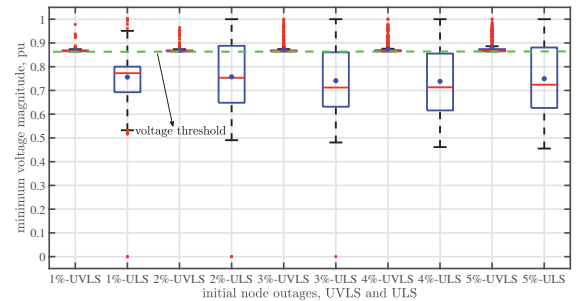


Fig. 3. Minimum voltage magnitudes in different initial node outages for UVLS and ULS, DLT case. black dots: mean values.



TABLE I  
CASCADE PROPAGATION COMPARISON FOR ULS AND UVLS FOR DLT

Case	Number of islands			Number of tiers of cascade			Minimum voltage		Number of blackouts
	Mean	Median	Max	Mean	Median	Max	Mean	Median	
1%—UVLS	31.75	32	43	1	1	1	0.868	0.866	1
1%—ULS	32.69	32	64	2.89	2	28	0.756	0.772	18
2%—UVLS	62.81	63	80	1	1	1	0.869	0.865	0
2%—ULS	64.73	64	145	4.16	3	36	0.757	0.753	36
3%—UVLS	94.41	94	111	1	1	1	0.873	0.865	3
3%—ULS	96.26	96	209	3.86	3	34	0.741	0.712	35
4%—UVLS	126.58	126	144	1	1	1	0.875	0.865	6
4%—ULS	128.48	128	211	3.93	3	33	0.738	0.713	34
5%—UVLS	158.71	159	180	1.02	1	11	0.879	0.866	5
5%—ULS	160.40	160	230	3.65	3	31	0.749	0.724	21

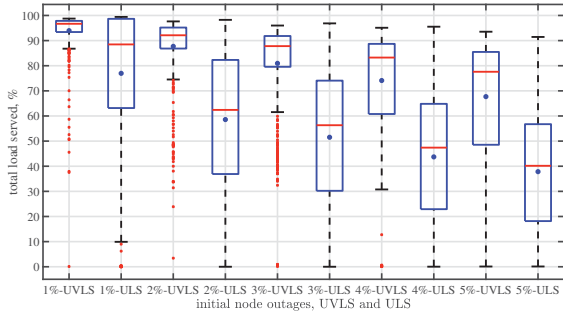


Fig. 4. Total load served in different initial node outages for UVLS and ULS, DLT case. black dots: mean values.

## V. CONCLUSION AND ONGOING WORK

This paper formulates an optimization-problem in AC-QSS cascading failure model to attain the post-UVLS equilibrium with minimum load shedding. Due to its exponential complexity in inputs, a heuristic is proposed that can reach a post-UVLS ‘candidate’ equilibrium. The method is tested on the 2383-bus Polish network and results show less number of islands, less tiers of cascade, and lower number of complete blackouts as compared to the uniform load shedding (ULS) case. Our current research focuses on improving this AC-QSS cascade model by considering bus voltage sensitivity measures and validating it against dynamic cascading failure simulations, which closely mimic the ground truth.

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