Power Spectra of Multipath Faded Pulse Trains

Andrea Ridolfi
School of Computer and Communication Sciences
École Polytechnique Fédérale de Lausanne - EPFL
Lausanne, Switzerland
Email: andrea.ridolfi@epfl.ch

Moe Z. Win
Laboratory for Information and Decision Systems - LIDS
Massachusetts Institute of Technology - MIT
Cambridge, U.S.A.
Email: moewin@mit.edu

***Abstract*— We address the problem of modeling received pulse trains in a multipath fading channel and of computing their exact spectrum. We propose a model based on point processes. Such model is very general, simple and tractable and it allows to account for various phenomena that affect the transmission. We then give the exact spectrum of the output of such a model. Spectral formula of specific configurations are then derived from a singular general formula, where the various features of the channel and the pulse transmission appear explicitly.**

I. Introduction

In several communication systems the information is transmitted in the form of a train of pulses. In particular, this is the case of Ultra-Wide Bandwidth (UWB) radio [1], which characteristics is to communicate with pulses of very short duration, thereby spreading the energy of the radio signal over several GHz.

While the spectrum computation of pulse trains, and in particular of UWB signals, has benefit of several contributions (see for instance [2], [3], [4]), results on the exact spectrum computation of the output of a general multipath fading channel, when fed by an UWB signal or more generally a pulse train, are not available in the scientific literature.

Here, we address two main problems. The first one is to derive a very general, yet tractable, model for the output of a multipath fading channel when the input is a general pulse train or, as a special case, a UWB signal. The second is to provide the exact spectrum for such a model.

We propose a model based on point processes, where

- the positions of the pulses and of their replica due to multipaths are realizations of a point processes;
- the pulse shape is a random function;
- fluctuations of pulse amplitudes are taken into account through the modulation with a w.s.s. process;
- the different multipath components are attenuated by random functions.

From the point process perspective, such a model corresponds to a shot noise with random excitation, where the basic point process is modulated by a w.s.s. process. It allows to finely account for various phenomena that affects the transmission, such as jitter, attenuation, losses and distortion of the pulses. As a special case, it provides the classical double Poisson model of Saleh and Valenzuela [5]. A shot noise perspective of the output of multipath fading channels has already been proposed by [6]. However, the model proposed is macroscopic in the sense that it does not allow a fine characterization of the multipaths since the random instants of the transmitted pulses and of the reflected ones are undistincly modeled with the same point process. Moreover, the model is limited to specific basic point processes and specific filtering functions, and the computation of the spectral properties is restrained to the Poisson case.

Within the framework of our model, the power spectra of received pulse trains in multipath fading channels can be derived, in a systematic and rigorous manner, by exploiting recent results on the spectrum of point processes and shot noises [7], [8].

The exact power spectrum formula we obtain is easy to understand since the contributions corresponding to various features such as pulse modulation, multipath repetitions, fading effects as well as other random effects, *appear clearly and separately* in the power spectrum expression. Such a characteristics has a tremendous impact for the design and the analysis of the model, allowing to "tune" model features from the spectrum expression.

Power spectrum of a pulse train through a general multipath fading channel is, to the best of our knowledge, a novel result. Here, we focus on a one-dimensional model of signals taking real values, which is sufficient for our purpose. Extension to signals with complex values is straightforward while the extension to multi dimensions can be made following the general definitions of spike fields and related processes given in [7].

II. The Model

We develop the model of a pulse train transmitted through a multipath fading channel step by step, modularly adding the type of pulse modulation, the fading, the multipath repetitions and additional random effects.

A. Pulse Modulation

As described in [9], [10], [8], pulse modulated signals, and more generally, UWB signals, can be modeled as a shot noise with random excitation

$$X(t) = \sum_{n\in\mathbb{Z}} w(t - T_n, Z_n) \ , \tag{1}$$

where:
- $w(s,z)$ is a function on s that depends on a random

parameter z, where $w(s,\cdot)$ represents the pulse shape;
- $N = \{T_n\}_{n\in\mathbb{Z}}$ is a collection of random points (point process or stream of random spikes) characterizing the temporal structure of the modulation. We denote with λ the average intensity of the point process, *i.e.*, the average number of random points per unit of time;
- $\{Z_n\}_{n\in\mathbb{Z}}$ is a collection of random i.i.d. parameters associated to the random points $\{T_n\}_{n\in\mathbb{Z}}$, but independent of them. Such random parameters, that we shall call i.i.i.d. *marks* (the triple "i" denote the i.i.d. characteristic and the independence w.r.t. the random points), allows to account for random effects on the pulses, such as random amplitude, jitter and thinning (see [9], [10], [8] for more details).

In particular, we call the couples $\bar{N} = \{T_n, Z_n\}_{n\in\mathbb{Z}}$ a *marked point process* (where $\{T_n\}_{n\in\mathbb{Z}}$ is the basic or underlying point process). Examples of specific modulations modeled as shot noise can be found in [9], [10], [8].

B. Fading

We consider fading as random modifications of the pulse introduced by the channel, such as attenuation and/or distortion. We shall consider the fading process to be stationary, at least in wide sense.

Pulse Attenuation: We model the random attenuation through the multiplication of the train of pulses with a correlated w.s.s. stochastic process $\{V(t)\}_{t\in\mathbb{R}}$, independent of the train itself. Equivalently, considering a relatively short duration of the pulses, we can multiply the stream of random spikes by the stochastic process $\{V(t)\}_{t\in\mathbb{R}}$ and then convolute the so obtained modulated point process with the pulse shape. Using the latter approach, the effect of the random attenuation on the input is

$$X(t) = \sum_{n\in\mathbb{Z}} w(t-T_n, Z_n)\, V(T_n)\ .$$

Pulse Distortion: Random i.i.d. distortions of the pulses can be modelled through

- a random parameter characterizing the pulse shape, *e.g.*, random amplitude or width;
- the convolution of the pulse with a random function.

In both cases, the randomness is captured through the i.i.d. marks $\{Z_n\}_{n\in\mathbb{Z}}$ (see [8] for more details).

C. Multipaths

The multipath effects consist of attenuated repetitions of the transmitted pulse due to reflections by surrounding objects. It has been shown that reflected pulses arrive in clusters [5]. This suggests a model where the reflected pulses are divided into *principal reflections* and *secondary scattering*. To each pulse corresponds a sequence of repetitions due to principal reflections, that we shall call *principal pulse repetitions*. Then for each principal repetition (also including the pulse itself) we have a sequence of additional repetitions due to the secondary scattering, that we shall call *secondary pulse repetitions*. Principal repetitions have lower arrival rate and amplitudes attenuating less rapidly than the secondary repetitions.

We will first focus the principal reflections and then introduce the secondary scattering.

Principal Reflection Multipaths: The modelling of the principal reflection multipaths requires the introduction of:
- a sequence of collections of random points $\{T^{\mathrm{p}}_{n;l}, l\geq 1\}$, $n\in\mathbb{Z}$, where $\{T^{\mathrm{p}}_{n;l}\}_{l\geq 1}$ describes the relative positions of the primary repetitions of the n-th pulse;
- a function $g^{\mathrm{p}}(t)$, $t\in\mathbb{R}_+$ describing the attenuation of the repeated pulses.

Commonly, $\{T^{\mathrm{p}}_{n;l}\}_{l\geq 1, n\in\mathbb{Z}}$ are called *propagation delays*, and $\{g^{\mathrm{p}}(T^{\mathrm{p}}_l)\}_{l\geq 1}$ *gains*. Using the propagation delays and gains, a general sequence of pulses (depending on a random parameter) and their attenuated repetitions can be written as

$$X(t) = \sum_{n\in\mathbb{Z}} w(t-T_n, Z_n) + \sum_{n\in\mathbb{Z}}\sum_{l\geq 1} w(t-T_n-T^{\mathrm{p}}_l, Z_n)\, g^{\mathrm{p}}(T^{\mathrm{p}}_l)\ ,$$

where $\{T_n\}_{n\in\mathbb{N}}$ is the sequence of the random positions of the pulses at the input of the channel.

The model can be straightforwardly complexified by assuming that the attenuating function g^{p} depends on a random parameter. More precisely, we define:
- $g^{\mathrm{p}}(s,z)$ to be a function on s depending on a random parameter z (it determines the attenuation of the pulses corresponding to the principal repetitions);
- $\bar{N}^{\mathrm{p}}_n = \{(T^{\mathrm{p}}_{n;l}, Z^{\mathrm{p}}_{n;l}), l\geq 1\}$, $n\in\mathbb{Z}$, to be an i.i.d. sequence of marked point process (where the i.i.i.d. marks have common distribution $Q_{Z^{\mathrm{p}}}$). We call λ^{p} the common average intensity of the basic point processes.

Then, the output of the channel reads

$$X(t) = \sum_{n\in\mathbb{Z}} w(t-T_n, Z_n) + \sum_{n\in\mathbb{Z}}\sum_{l\geq 1} w\left(t-T_n-T^{\mathrm{p}}_{n;l}, Z_n\right) g^{\mathrm{p}}\left(T^{\mathrm{p}}_{n;l}, Z^{\mathrm{p}}_{n;l}\right)\ . \tag{2}$$

Figure 1 depicts two Dirac pulses and the corresponding principal repetitions.

Now, by letting $Z^{\mathrm{c}}_n = \left(Z_n, \bar{N}^{\mathrm{p}}_n\right)$, $n\in\mathbb{Z}$, and

$$H(t-T_n, Z^{\mathrm{c}}_n) = w(t-T_n, Z_n) + \sum_{l\geq 1} w\left(t-T_n-T^{\mathrm{p}}_{n;l}, Z_n\right) g^{\mathrm{p}}\left(T^{\mathrm{p}}_{n;l}, Z^{\mathrm{p}}_{n;l}\right)\ , \tag{3}$$

we can interpret expression (2) as the convolution of the random impulse response $H(t,z)$ with a marked point process $\bar{N}^{\mathrm{c}}$ with basic random points $\{T_n\}_{n\in\mathbb{Z}}$ and i.i.i.d. marks $\{Z^{\mathrm{c}}_n\}_{n\in\mathbb{Z}}$, *i.e.*,

$$X(t) = \sum_{n\in\mathbb{Z}} H(t-T_n, Z^{\mathrm{c}}_n)\ ,$$

Hence, a pulse train with multipath repetitions can be modeled as a shot noise with random excitation. Notice that here the marked point process $\bar{N}^{\mathrm{p}}$ plays the role of a *random cluster*.

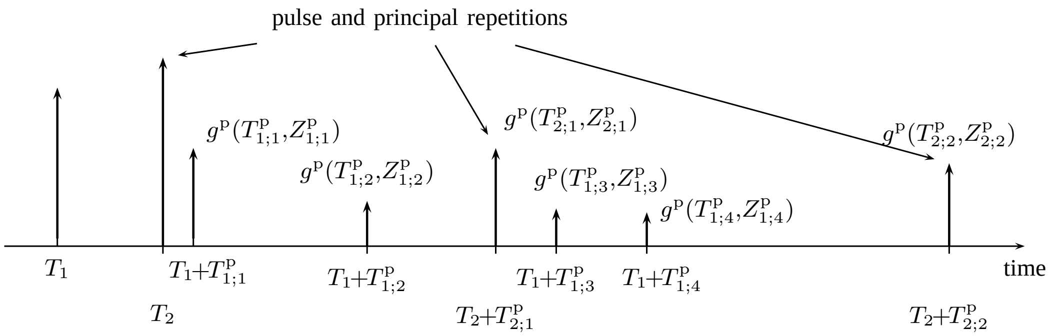

Fig. 1. Two Dirac pulses (at random times T_1 and T_2) with the corresponding principal repetitions.

In this context, $\bar{N}^{\mathrm{c}}$ is a *cluster point process* [11] (indeed, cluster point processes can be seen as a particular case of shot noises with random excitation - see [7], [8]).

Principal Reflections and Secondary Scattering Multipaths: We now introduce the repetitions due to secondary scattering multipaths. We follow the same approach adopted for the principal repetitions. However, we remark that secondary scattering multipaths are generated by both the pulses and their primary repetitions. Therefore, in order to introduce the secondary repetitions we associate to each pulse and to each of its primary repetitions a sequence of random points. More precisely, we introduce
- a double sequence of collections of random points

$$\{T^{\mathrm{s}}_{n,l;k}, k \geq 1\}, \quad n \in \mathbb{Z}\,, \quad l \geq 1\,,$$

where $\{T^{\mathrm{s}}_{n,l;k}\}_{k\geq 1}$ describes the relative positions of the secondary repetitions that where generated by the l-th primary repetition of the n-th pulse; We shall denote with $\{T^{\mathrm{s}}_{n,0;k}\}_{k\geq 1}$ the secondary repetitions directly generated by the n-th pulse; The double sequence, including $\{T^{\mathrm{s}}_{n,0;k}\}_{k\geq 1}$, is supposed to be i.i.d. (with respect to both indexes $n \in \mathbb{Z}$ and $l \in \mathbb{N}$);
- a function $g^{\mathrm{s}}(s, z)$ that depends on a random parameter, where $g^{\mathrm{s}}(s, \cdot)$ represents the attenuation, or path-loss, of the secondary repetitions;
- a double sequence of collections of i.i.d. random variables

$$\{Z^{\mathrm{s}}_{n,l;k}, k \geq 1\}, \quad n \in \mathbb{Z}, \quad l \in \mathbb{N}\,,$$

where each random variable $Z^{\mathrm{s}}_{n,l;k}$ represent the attenuating function random parameter that is associated to the relative position $T^{\mathrm{s}}_{n,l;k}$ of the secondary repetitions;
$\{Z^{\mathrm{s}}_{n,l;k}\}_{n\in\mathbb{Z}, l\in\mathbb{N}, k\geq 1}$ are supposed to be i.i.d. with respect to all three indexes.

If we consider that the input of the channel is a sequence of pulses, each depending on a random parameter Z, the output, with attenuated primary and secondary repetitions, then reads

$$\begin{aligned}
X(t) = & \sum_{n\in\mathbb{Z}} w(t - T_n, Z_n) \\
& + \sum_{n\in\mathbb{Z}} \sum_{k\geq 1} w\left(t - T_n - T^{\mathrm{s}}_{n,0;k}, Z_n\right) g^{\mathrm{s}}\left(T^{\mathrm{s}}_{n,0;k}, Z^{\mathrm{s}}_{n,0;k}\right) \\
& + \sum_{n\in\mathbb{Z}} \sum_{l\geq 1} w\left(t - T_n - T^{\mathrm{p}}_{n;l}, Z_n\right) g^{\mathrm{p}}\left(T^{\mathrm{p}}_{n;l}, Z^{\mathrm{p}}_{n;l}\right) \\
& + \sum_{n\in\mathbb{Z}} \sum_{l\geq 1} \sum_{k\geq 1} w\left(t - T_n - T^{\mathrm{p}}_{n;l} - T^{\mathrm{s}}_{n,l;k}, Z_n\right) \\
& \qquad g^{\mathrm{p}}\left(T^{\mathrm{p}}_{n;l}, Z^{\mathrm{p}}_{n;l}\right) g^{\mathrm{s}}\left(T^{\mathrm{s}}_{n,l;k}, Z^{\mathrm{s}}_{n,l;k}\right) .
\end{aligned}$$

From the top to the bottom,

- the first term corresponds to the sequence of pulses,
- the second term corresponds to the secondary repetition due to the scattering multipaths of each pulse,
- the third term models the primary repetition due to the principal multipaths of each pulse,
- the last term model the secondary repetitions due to the scattering multipaths of each primary repetition.

Using the point process formalism, we can define $\bar{N}^{\mathrm{s}}_{n,l} = \{(T^{\mathrm{s}}_{n,l;k}, Z^{\mathrm{s}}_{n,l;k}), k \geq 1\}$ to be a double sequence of marked point processes with i.i.i.d. marks (where the double sequence of marked point process is i.i.d. with respect to both indexes n and l). We call λ^{s} the average intensity of the corresponding basic point processes (λ^{s} is the same for all basic point processes since the sequence is i.i.d.). We can then define the random impulse response as in equation (4) where now $Z^{\mathrm{c}}_n = (Z_n, \bar{N}^{\mathrm{p}}_n, \{\bar{N}^{\mathrm{s}}_{n,l}, l \in \mathbb{N}\})$. Then, the output of a channel with principal secondary multipaths can be treated as a shot noise with random excitation

$$X(t) = \sum_{n\in\mathbb{Z}} H(t - T_n, Z^{\mathrm{c}}_n)$$

with random impulse response given by (4), and marked point process $\bar{N}^{\mathrm{c}}$ having basic point process $N = \{T_n, n \in \mathbb{Z}\}$ and i.i.i.d. marks

$$Z^{\mathrm{c}}_n = \left(Z_n, \bar{N}^{\mathrm{p}}_n, \{\bar{N}^{\mathrm{s}}_{n,l}, l \in \mathbb{N}\}\right)\,, \quad n \in \mathbb{Z}\,.$$

$$H\left(t-T_n, Z_n^{\mathrm{c}}\right)=w\left(t-T_n, Z_n\right)+\sum_{k \geq 1} w\left(t-T_n-T_{n,0;k}^{\mathrm{s}}, Z_n\right) g^{\mathrm{s}}\left(T_{n,0;k}^{\mathrm{s}}, Z_{n,0;k}^{\mathrm{s}}\right) + \sum_{l \geq 1} w\left(t-T_n-T_{n;l}^{\mathrm{p}}, Z_n\right) g^{\mathrm{p}}\left(T_{n;l}^{\mathrm{p}}, Z_{n;l}^{\mathrm{p}}\right) + \sum_{l \geq 1} \sum_{k \geq 1} w\left(t-T_n-T_{n;l}^{\mathrm{p}}-T_{n,l;k}^{\mathrm{s}}, Z_n\right) g^{\mathrm{p}}\left(T_{n;l}^{\mathrm{p}}, Z_{n;l}^{\mathrm{p}}\right) g^{\mathrm{s}}\left(T_{n,l;k}^{\mathrm{s}}, Z_{n,l;k}^{\mathrm{s}}\right), \quad n \in \mathbb{Z}, \tag{4}$$

We notice that now $\bar{N}^{\mathrm{c}}$ corresponds to a double cluster point process, with nested random clusters $\bar{N}^{\mathrm{p}}$ and $\bar{N}^{\mathrm{s}}$. Obviously, the impulse response (3) is a special case of (4).

D. Output of the Multipath Fading Channel

Combining all the effects together, we obtain the output $\{X(t)\}_{t \in \mathbb{R}}$ of a multipath fading channel as a shot noise with random excitation and modulated basic point process.

$$X(t)=\sum_{n \in \mathbb{Z}} H\left(t-T_n, Z_n^{\mathrm{c}}\right) V\left(T_n\right) \tag{5}$$

where H is given by (3) or by (4), depending on whether we are considering the secondary scattering or not.

III. Power Spectrum

We are interested in computing the power spectrum of a pulse train over a multipath, fading channel, modeled as in (5). In the following, we assume that the point processes describing the pulse positions and the primary and secondary repetitions are stationary.

As we have discussed in the previous section, the output of the multipath fading channel (5) corresponds to a shot noise with random excitation and modulated basic point processes. We recall that the power spectrum S_X of a shot noise with random excitation $\{X(t)\}_{t \in \mathbb{R}}$ modulated by a w.s.s. process $\{V(t)\}_{t \in \mathbb{R}}$ is given by equation (6) (see [7], [8]), where S_N is the spectral pseudo density of the point process, S_V is the spectral density of the modulating process, $\widehat{h}(\nu, Z)$ is the Fourier transform of the random filtering function. Then, the power spectral measure of received pulse trains in multipath, fading channels is given by equation (7). where, denoting with $\widehat{}$ the Fourier transformation,

$$\widehat{H}\left(\nu, Z^{\mathrm{c}}\right)=\widehat{w}(\nu, Z) F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right), \tag{8}$$

and

$$F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)=1+\sum_{l \geq 1} e^{-i 2 \pi \nu T_l^{\mathrm{p}}} g^{\mathrm{p}}\left(T_l^{\mathrm{p}}, Z_l^{\mathrm{p}}\right) + \sum_{k \geq 1} e^{-i 2 \pi \nu T_{0;k}^{\mathrm{s}}} g^{\mathrm{s}}\left(T_{0;k}^{\mathrm{s}}, Z_{0;k}^{\mathrm{s}}\right) + \sum_{l \geq 1}\left(e^{-i 2 \pi \nu T_l^{\mathrm{p}}} g^{\mathrm{p}}\left(T_l^{\mathrm{p}}, Z_l^{\mathrm{p}}\right) \sum_{k \geq 1} e^{-i 2 \pi \nu T_{l;k}^{\mathrm{s}}} g^{\mathrm{s}}\left(T_{l;k}^{\mathrm{s}}, Z_{l;k}^{\mathrm{s}}\right)\right).$$

The above result deserves a few comments:

- the expression of $F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)$ is purposely kept in a general form in order to accomodate various situation of gains and propagation delays;
- in the absence of multipaths $F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right) \equiv 1$ and we obtain the standard form of the spectrum of a shot noise with random excitation and filtering function w.

Concerning the first and second order moments of $\widehat{H}\left(\nu, Z^{\mathrm{c}}\right)$ we obtain

$$\mathrm{E}\left[\widehat{H}\left(\nu, Z^{\mathrm{c}}\right)\right]=\mathrm{E}[\widehat{w}(\nu, Z)] \mathrm{E}\left[F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right], \tag{9}$$

with

$$\mathrm{E}\left[F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right]=1+\lambda^{\mathrm{p}} \widehat{g}^{\mathrm{p}}(\nu)+\lambda^{\mathrm{s}} \widehat{g}^{\mathrm{s}}(\nu) + \lambda^{\mathrm{p}} \lambda^{\mathrm{s}} \widehat{g}^{\mathrm{p}}(\nu) \widehat{g}^{\mathrm{s}}(\nu), \tag{10}$$

and

$$\operatorname{Var}\left(\widehat{H}\left(\nu, Z^{\mathrm{c}}\right)\right)=\mathrm{E}\left[|\widehat{w}(\nu, Z)|^2\right] \operatorname{Var}\left(F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right) + \operatorname{Var}(\widehat{w}(\nu, Z))\left|\mathrm{E}\left[F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right]\right|^2 \tag{11}$$

with $\operatorname{Var}\left(F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right)$ given by equation (12), where $S_{N^{\mathrm{p}}}$ and $S_{N^{\mathrm{s}}}$ are the pseudo spectral densities of the point processes that model the primary and the secondary repetitions, respectively. From the above expressions of the first and second order moments, we can remark the modularity of the model: expressions (9) and (11) evidence the separate contribution of

- $\widehat{w}(\nu, z)$, which takes into account the characteristics of the input pulse train, such as the pulse shape and pulse modulation, the jittering, the thinning and the pulse distortion introduced by the channel;
- $F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)$ which takes into account the type of multipath repetitions;

while expressions (10) and (12) clearly shows the distinct contributions of the primary and secondary repetitions and their attenuating functions.

A straightforward application of the spectrum formula (7) gives the spectrum of the double Poisson model of Saleh and Valenzuela [5], where both the principal and secondary delays are Poisson processes (with intensities $\lambda^{\mathrm{p}}<\lambda^{\mathrm{s}}$) and the attenuating functions are deterministic. In such a case we have

$$\mathrm{E}\left[F\left(\nu,\left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right]=1+\lambda^{\mathrm{p}} \widehat{g}^{\mathrm{p}}(\nu)+\lambda^{\mathrm{s}} \widehat{g}^{\mathrm{s}}(\nu) + \lambda^{\mathrm{p}} \lambda^{\mathrm{s}} \widehat{g}^{\mathrm{p}}(\nu) \widehat{g}^{\mathrm{s}}(\nu)$$

$$S_X(\nu) = \left| \mathrm{E}\left[\widehat{h}\left(\nu, Z\right) \right] \right|^2 \left(S_N * S_V\left(\nu\right) + \lambda^2 S_V\left(\nu\right) + \mathrm{E}\left[V\right]^2 S_N\left(\nu\right) \right) + \lambda \mathrm{E}\left[\left| V\left(t\right) \right|^2 \right] \mathrm{Var}\left(\widehat{h}\left(\nu, Z\right) \right) . \tag{6}$$

$$S_X(\nu) = \left| \mathrm{E}\left[\widehat{H}\left(\nu, Z^{\mathrm{c}}\right) \right] \right|^2 \left(S_N * S_V\left(\nu\right) + \lambda^2 S_V\left(\nu\right) + \mathrm{E}\left[V\right]^2 S_N\left(\nu\right) \right) + \lambda \mathrm{E}\left[\left| V\left(t\right) \right|^2 \right] \mathrm{Var}\left(\widehat{H}\left(\nu, Z^{\mathrm{c}}\right) \right) . \tag{7}$$

$$\begin{aligned} \mathrm{Var}\left(F\left(\nu, \left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right) = &\Bigg\{ \left(\int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{p}}\left(\nu+\upsilon\right) \right|^2 S_{N^{\mathrm{p}}}\left(\upsilon\right) d\upsilon + \lambda^{\mathrm{p}} \int_{\mathbb{R}} \mathrm{Var}\left(\widehat{g}^{\mathrm{p}}\left(\nu+\upsilon, Z^{\mathrm{p}}\right)\right) d\upsilon \right) \\ &\times \left(\int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{s}}\left(\nu+\upsilon\right) \right|^2 S_{N^{\mathrm{s}}}\left(\upsilon\right) d\upsilon + \lambda^{\mathrm{s}} \int_{\mathbb{R}} \mathrm{Var}\left(\widehat{g}^{\mathrm{s}}\left(\nu+\upsilon, Z^{\mathrm{s}}\right)\right) d\upsilon \right) \Bigg\} \\ &+ \left| 1 + \lambda^{\mathrm{s}} \widehat{g}^{\mathrm{s}}\left(\nu\right) \right|^2 \left(\int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{p}}\left(\nu+\upsilon\right) \right|^2 S_{N^{\mathrm{p}}}\left(\upsilon\right) d\upsilon + \lambda^{\mathrm{p}} \int_{\mathbb{R}} \mathrm{Var}\left(\widehat{g}^{\mathrm{p}}\left(\nu+\upsilon, Z^{\mathrm{p}}\right)\right) d\upsilon \right) \\ &+ \left| 1 + \lambda^{\mathrm{p}} \widehat{g}^{\mathrm{p}}\left(\nu\right) \right|^2 \left(\int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{s}}\left(\nu+\upsilon\right) \right|^2 S_{N^{\mathrm{s}}}\left(\upsilon\right) d\upsilon + \lambda^{\mathrm{s}} \int_{\mathbb{R}} \mathrm{Var}\left(\widehat{g}^{\mathrm{s}}\left(\nu+\upsilon, Z^{\mathrm{s}}\right)\right) d\upsilon \right) . \end{aligned} \tag{12}$$

and

$$\begin{aligned} &\mathrm{Var}\left(F\left(\nu, \left(\bar{N}^{\mathrm{p}}, \bar{N}^{\mathrm{s}}\right)\right)\right) = \\ &\quad \lambda^{\mathrm{p}} \int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{p}}\left(\upsilon\right) \right|^2 d\upsilon \lambda^{\mathrm{s}} \int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{s}}\left(\upsilon\right) \right|^2 d\upsilon \\ &\qquad + \lambda^{\mathrm{p}} \int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{p}}\left(\upsilon\right) \right|^2 d\upsilon + \lambda^{\mathrm{s}} \int_{\mathbb{R}} \left| \widehat{g}^{\mathrm{s}}\left(\upsilon\right) \right|^2 d\upsilon \end{aligned}$$

IV. Conclusion

We have provided a general model for a pulse transmission through a multipath fading channel. Based on a point process approach, such a model is very general: it aptly captures the random artifacts introduced by a multipath fading channel as well as other random effects that may affects the transmission. Moreover, as a special case, it provides the classical Saleh and Valenzuela's double Poisson model. The power spectrum of the channel output is computed using the tools provided by point process spectral theory. In the resulting formulas, the contribution of the various features of the model appears explicitly, easily allowing for a detailed analysis of the impact of each feature on the spectrum. This is an extremely important characteristic that can have a tremendous impact in model design and analysis.

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