# A Non-Cooperative Power Control Game in Delay-Constrained Multiple-Access Networks

Farhad Meshkati, H. Vincent Poor, and Stuart C. Schwartz Department of Electrical Engineering Princeton University Princeton, NJ 08544 USA Email: {meshkati, poor, stuart}@princeton.edu

Abstract—A game-theoretic approach for studying power control in multiple-access networks with transmission delay constraints is proposed. A non-cooperative power control game is considered in which each user seeks to choose a transmit power that maximizes its own utility while satisfying the user's delay requirements. The utility function measures the number of reliable bits transmitted per joule of energy and the user's delay constraint is modeled as an upper bound on the delay outage probability. The Nash equilibrium for the proposed game is derived, and its existence and uniqueness are proved. Using a largesystem analysis, explicit expressions for the utilities achieved at equilibrium are obtained for the matched filter, decorrelating and minimum mean square error multiuser detectors. The effects of delay constraints on the users' utilities (in bits/Joule) and network capacity (i.e., the maximum number of users that can be supported) are quantified.

# I. INTRODUCTION

In wireless networks, power control is used for resource allocation and interference management. In multiple-access CDMA systems such as the uplink of cdma2000, the purpose of power control is for each user terminal to transmit enough power so that it can achieve the desired quality of service (QoS) without causing unnecessary interference for other users in the network. Depending on the particular application, QoS can be expressed in terms of throughput, delay, battery life, etc. Since in many practical situations, the users' terminals are battery-powered, an efficient power management scheme is required to prolong the battery life of the terminals. Hence, power control plays an even more important role in such scenarios.

Consider a multiple-access DS-CDMA network where each user wishes to locally and selfishly choose its transmit power so as to maximize its utility and at the same time satisfy its delay requirements. The strategy chosen by each user affects the performance of other users through multiple-access interference. There are several questions to ask concerning this interaction. First of all, what is a reasonable choice of a utility function that measures energy efficiency and takes into account delay constraints? Secondly, given such a utility function, what strategy should a user choose in order to maximize its utility? If every user in the network selfishly and locally picks its utility-maximizing strategy, will there be a stable state at which no user can unilaterally improve its utility (Nash equilibrium)? If such an equilibrium exists, will it be unique? What will be the effect of delay constraint on the energy efficiency of the network?

Game theory is the natural framework for modeling and studying such a power control problem. Recently, there has been a great deal of interest in applying game theory to resource allocation is wireless networks. Examples of gametheoretic approaches to power control are found in [1-8]. In [1-5], power control is modeled as a non-cooperative game in which users choose their transmit powers in order to maximize their utilities. In [8], the authors extend this approach to consider a game in which users can choose their uplink receivers as well as their transmit powers. All the power control games proposed so far assume that the traffic is not delay sensitive. Their focus is entirely on the trade-offs between throughput and energy consumption without taking into account any delay constraints. In this work, we propose a non-cooperative power control game that does take into account a transmission delay constraint for each user. Our focus here is on energy efficiency. Our approach allows us to study networks with both delay tolerant and delay sensitive traffic/users and quantify the loss in energy efficiency due to the presence of users with stringent delay constraints.

The organization of the paper is as follows. In Section II, we present the system model and define the users' utility function as well as the model used for incorporating delay constraints. The proposed power control game is described in Section III, and the existence and uniqueness of Nash equilibrium for the proposed game is discussed in Section IV. In Section V, we extend the analysis to multi-class networks and derive explicit expressions for the utilities achieved at Nash equilibrium. Numerical results and conclusions are given in Sections VI and VII, respectively.

#### **II. SYSTEM MODEL**

We consider a synchronous DS-CDMA network with K users and processing gain N (defined as the ratio of symbol duration to chip duration). We assume that all K user terminals transmit to a receiver at a common concentration point, such as a cellular base station or any other network access point. The signal received by the uplink receiver (after chip-matched filtering) sampled at the chip rate over one symbol duration

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can be expressed as

$$\mathbf{r} = \sum_{k=1}^{K} \sqrt{p_k} h_k \ b_k \mathbf{s}_k + \mathbf{w},\tag{1}$$

where  $p_k$ ,  $h_k$ ,  $b_k$  and  $\mathbf{s}_k$  are the transmit power, channel gain, transmitted bit and spreading sequence of the  $k^{th}$  user, respectively, and  $\mathbf{w}$  is the noise vector which is assumed to be Gaussian with mean **0** and covariance  $\sigma^2 \mathbf{I}$ . We assume random spreading sequences for all users, i.e.,  $\mathbf{s}_k = \frac{1}{\sqrt{N}} [v_1 \dots v_N]^T$ , where the  $v_i$ 's are independent and identically distributed (i.i.d.) random variables taking values in  $\{-1, +1\}$  with equal probabilities.

### A. Utility Function

To pose the power control problem as a non-cooperative game, we first need to define a suitable utility function. It is clear that a higher signal to interference plus noise ratio (SIR) level at the output of the receiver will result in a lower bit error rate and hence higher throughput. However, achieving a high SIR level requires the user terminal to transmit at a high power which in turn results in low battery life. This tradeoff can be quantified (as in [1]) by defining the utility function of a user to be the ratio of its throughput to its transmit power, i.e.,

$$u_k = \frac{T_k}{p_k} . (2)$$

Throughput is the net number of information bits that are transmitted without error per unit time (sometimes referred to as *goodput*). It can be expressed as

$$T_k = \frac{L}{M} R_k f(\gamma_k), \tag{3}$$

where L and M are the number of information bits and the total number of bits in a packet, respectively.  $R_k$  and  $\gamma_k$  are the transmission rate and the SIR for the  $k^{th}$  user, respectively; and  $f(\gamma_k)$  is the "efficiency function" which is assumed to be increasing and S-shaped (sigmoidal) with  $f(\infty) = 1$ . We also require that f(0) = 0 to ensure that  $u_k = 0$  when  $p_k = 0$ . In general, the efficiency function depends on the modulation, coding and packet size. A more detailed discussion of the efficiency function can be found in [8]. Note that for a sigmoidal efficiency function, the utility function in (2) is a quasiconcave function of the user's transmit power. The throughput  $T_k$  in (3) could also be replaced with any increasing concave function such as the Shannon capacity formula as long as we make sure that  $u_k = 0$  when  $p_k = 0$ .

Based on (2) and (3), the utility function for user k can be written as

$$u_k = \frac{L}{M} R \frac{f(\gamma_k)}{p_k} .$$
(4)

For the sake of simplicity, we have assumed that the transmission rate is the same for all users, i.e.,  $R_1 = ... = R_K = R$ . All the results obtained here can be easily generalized to the case of unequal rates. The utility function in (4), which has units of *bits/Joule*, captures very well the tradeoff between throughput and battery life and is particularly suitable for applications where energy efficiency is crucial.

# B. Delay Constraints

Let X represent the (random) number of transmissions required for a packet to be received without any errors. The assumption is that if a packet has one or more errors, it will be retransmitted. We also assume that retransmissions are independent from each other. It is clear that the transmission delay for a packet is directly proportional to X. Therefore, any constraint on the transmission delay can be equivalently expressed as a constraint on the number of transmissions. Assuming that the packet success rate is given by the efficiency function  $f(\gamma)^1$ , the probability that exactly m transmissions are required for the successful transmission of the packet is given by

$$\Pr\{X = m\} = f(\gamma) \left(1 - f(\gamma)\right)^{m-1},$$
(5)

and, hence,  $E\{X\} = \frac{1}{f(\gamma)}$ . We model the delay requirements of a particular user (or equivalently traffic type) as a pair  $(D, \beta)$ , where

$$\Pr\{X \le D\} \ge \beta. \tag{6}$$

In other words, we would like the number of transmissions to be at most D with a probability larger than or equal to  $\beta$ . For example, (2,0.9), i.e., D = 2 and  $\beta = 0.9$ , implies that 90% of the time we need at most two transmissions to successfully receive a packet. Note that (6) can equivalently be represented as an upper bound on the delay outage probability, i.e.,

$$P_{delay \ outage} \triangleq \Pr\{X > D\} \le 1 - \beta.$$
(7)

Based on (5), the delay constraint in (6) can be expressed as

$$\sum_{m=1}^{D} f(\gamma) \left(1 - f(\gamma)\right)^{m-1} \ge \beta,$$

where

or

$$\eta(D,\beta) = 1 - (1-\beta)^{\frac{1}{D}}.$$
(9)

Here, we have explicitly shown that  $\eta$  is a function of D and  $\beta$ . Since  $f(\gamma)$  is an increasing function of  $\gamma$ , we can equivalently express (8) as

 $f(\gamma) \ge \eta(D,\beta),$ 

$$\gamma \ge \tilde{\gamma},$$
 (10)

(8)

where

$$\tilde{\gamma} = f^{-1}\left(\eta(D,\beta)\right). \tag{11}$$

Therefore, the delay constraint in (6) translates into a lower bound on the SIR. Since different users could have different delay requirements,  $\tilde{\gamma}$  is user dependent. We make this explicit by writing

$$\tilde{\gamma}_k = f^{-1}\left(\eta_k\right),\tag{12}$$

where  $\eta_k = 1 - (1 - \beta_k)^{\frac{1}{D_k}}$ . A more stringent delay requirement, i.e., a smaller *D* and/or a larger  $\beta$ , will result in a higher value for  $\tilde{\gamma}$ . Without loss of generality, we have assumed that all the users in the network have the same efficiency function. It is straightforward to relax this assumption.

<sup>1</sup>This assumption is valid in many practical systems (see [8] for further details).

# III. POWER CONTROL GAME WITH DELAY CONSTRAINTS

We propose a power control game in which each user decides how much power to transmit in order to maximize its own utility and at the same time satisfy its delay requirements. We have shown in Section II-B that the delay requirements of a user translate into a lower bound on the user's output SIR. Therefore, each user will seek to maximize its utility while satisfying its SIR requirement. This can be captured by defining a *delay-constrained* utility for user k as

$$\tilde{u}_k = \begin{cases} u_k & \text{if } \gamma_k \ge \tilde{\gamma}_k \\ 0 & \text{if } \gamma_k < \tilde{\gamma}_k \end{cases},$$
(13)

where  $u_k$  and  $\tilde{\gamma}_k$  are given by (4) and (12), respectively.

Let  $\tilde{G} = [\mathcal{K}, \{A_k\}, \{\tilde{u}_k\}]$  denote the proposed noncooperative game where  $\mathcal{K} = \{1, ..., K\}$ , and  $A_k = [0, P_{max}]$ , which is the strategy set for the  $k^{th}$  user. Here,  $P_{max}$  is the maximum allowed power for transmission. We assume that only those users whose delay requirements can be met are admitted into the network. For example, for the conventional matched filter, this translates into having

$$\sum_{k=1}^{K} \frac{1}{1 + \frac{N}{\tilde{\gamma}_k}} < 1$$

This assumption makes sense because admitting a user that cannot meet its delay requirement only causes unnecessary interference for other users.

The resulting non-cooperative game can be expressed as the following maximization problem:

$$\max_{p_k} \tilde{u}_k \text{ for } k = 1, ..., K,$$
(14)

where the  $p_k$ 's are constrained to be non-negative. The above maximization can equivalently be written as

$$\max_{p_k} u_k \text{ subject to } \gamma_k \ge \tilde{\gamma}_k \text{ for } k = 1, \dots, K.$$
(15)

Let us first solve the above maximization by ignoring the constraints on SIR. For all linear receivers, we have

$$\frac{\partial \gamma_k}{\partial p_k} = \frac{\gamma_k}{p_k} \ . \tag{16}$$

Taking the derivative of  $u_k$  with respect to  $p_k$  and taking advantage of (16), we obtain

$$\frac{\partial u_k}{\partial p_k} = \frac{f'(\gamma_k)}{p_k} \frac{\partial \gamma_k}{\partial p_k} - \frac{f(\gamma_k)}{p_k^2} = \frac{\gamma_k f'(\gamma_k) - f(\gamma_k)}{p_k^2}.$$

Therefore, the unconstrained utility function for user k is maximized when the user's output SIR is equal to  $\gamma^*$ , where  $\gamma^*$  is the (positive) solution to

$$f(\gamma) = \gamma \ f'(\gamma). \tag{17}$$

It can be shown that for a sigmoidal efficiency function,  $\gamma^*$  always exists and is unique. In addition, for all  $\gamma_k < \gamma^*$ ,  $u_k$  is increasing in  $p_k$  and for all  $\gamma_k > \gamma^*$ ,  $u_k$  is decreasing in  $p_k$  [9]. Therefore,  $\tilde{u}_k$  is maximized when user k transmits at a power level that achieves  $\tilde{\gamma}_k^*$  at the output of the uplink receiver, where

$$\tilde{\gamma}_k^* = \max\{\tilde{\gamma}_k, \gamma^*\}.$$
(18)

In the next section, we investigate the existence and uniqueness of Nash equilibrium for our proposed game.

## IV. NASH EQUILIBRIUM FOR THE PROPOSED GAME

The Nash equilibrium for the proposed game is a set strategies (power levels) for which no user can unilaterally improve its own (delay-constrained) utility function. We now state the following proposition.

Proposition 1: The Nash equilibrium for the noncooperative game  $\tilde{G}$  is given by  $\tilde{p}_k^* = \min\{p_k^*, P_{max}\}$ , for  $k = 1, \dots, K$ , where  $p_k^*$  is the transmit power that results in an SIR equal to  $\tilde{\gamma}^*$  at the output of the receiver with  $\tilde{\gamma}_k^* = \max\{\tilde{\gamma}_k, \gamma^*\}$ . Furthermore, this equilibrium is unique.

**Proof:** Based on the arguments presented in Section III,  $\tilde{u}_k$  is maximized when the transmit power  $p_k$  is such that  $\gamma_k = \tilde{\gamma}_k^* = \max\{\tilde{\gamma}_k, \gamma^*\}$ . If  $\tilde{\gamma}_k$  cannot be achieved, the user must transmit at maximum power level to maximize its utility. Let us define  $\tilde{p}_k$  as the power level for which the output SIR for user k is equal to  $\tilde{\gamma}_k$ . Since user k is admitted into the network only if it can meet its delay requirements, we have  $\tilde{p}_k \leq P_{max}$ . In addition, because  $\tilde{u}_k = 0$  for  $p_k < \tilde{p}_k$ , there is no incentive for user k to transmit at a power level smaller than  $\tilde{p}_k$ . Therefore, we can restrict the set of strategies for user k to  $[\tilde{p}_k, P_{max}]$ . In this interval,  $\tilde{u}_k = u_k$  and hence the utility function is continuous and quasiconcave. This guarantees existence of a Nash equilibrium for the proposed power control game.

Furthermore, for a sigmoidal efficiency function,  $\gamma^*$ , which is the (positive) solution of  $f(\gamma) = \gamma f'(\gamma)$ , is unique and as a result  $\tilde{\gamma}_k^*$  is unique for k = 1, 2, ..., K. Because of this and the one-to-one correspondence between the transmit power and the output SIR, the Nash equilibrium is unique.

The above proposition suggests that at Nash equilibrium, the output SIR for user k is  $\tilde{\gamma}_k^*$ , where  $\tilde{\gamma}_k^*$  depends on the efficiency function through  $\gamma^*$  as well as user k's delay constraint through  $\tilde{\gamma}_k$ . Note that this result does not depend on the choice of the receiver and is valid for all linear receivers including the matched filter, the decorrelator and the (linear) minimum mean square error (MMSE) detector.

#### V. MULTI-CLASS NETWORKS

Let us now consider a network with C classes of users. The assumption is that all the users in the same class have the same delay requirements characterized by the corresponding D and  $\beta$ . Based on Proposition 1, at Nash equilibrium, all the users in class c will have the same output SIR,  $\tilde{\gamma}^{*(c)} = \max{\{\tilde{\gamma}^{(c)}, \gamma^*\}}$ , where  $\tilde{\gamma}^{(c)} = f^{-1}(\eta^{(c)})$ . Here,  $\eta^{(c)}$  depends on the delay requirements of class c, namely  $D^{(c)}$  and  $\beta^{(c)}$ , through (9). The goal is to quantify the effect of delay constraints on the energy efficiency of the network or equivalently on the users' utilities.

In order to obtain explicit expressions for the utilities achieved at equilibrium, we use a large-system analysis similar to the one presented in [10] and [11]. We consider the asymptotic case in which  $K, N \to \infty$  and  $\frac{K}{N} \to \alpha < \infty$ . This allows us to write SIR expressions that are independent of the spreading sequences of the users. Let  $K^{(c)}$  be the number of users in class c, and define  $\alpha^{(c)} = \lim_{K,N\to\infty} \frac{K^{(c)}}{N}$ . Therefore, we have  $\sum_{c=1}^{C} \alpha^{(c)} = \alpha$ . It can be shown that for the matched filter (MF), the

It can be shown that for the matched filter (MF), the decorrelator (DE), and the MMSE detector, the minimum

power required by user k in class c to achieve an output SIR equal to  $\tilde{\gamma}^{*(c)}$  is given by the following equations:

$$p_{k}^{MF} = \frac{1}{h_{k}^{2}} \frac{\tilde{\gamma}^{*(c)} \sigma^{2}}{1 - \sum_{c=1}^{C} \alpha^{(c)} \tilde{\gamma}^{*(c)}}$$
for  $\sum_{c=1}^{C} \alpha^{(c)} \tilde{\gamma}^{*(c)} < 1,(19)$ 
$$p_{k}^{DE} = \frac{1}{h_{k}^{2}} \frac{\tilde{\gamma}^{*(c)} \sigma^{2}}{1 - \alpha}$$
for  $\alpha < 1,$  (20)

and

$$p_{k}^{MMSE} = \frac{1}{h_{k}^{2}} \frac{\tilde{\gamma}^{*(c)} \sigma^{2}}{1 - \sum_{c=1}^{C} \alpha^{(c)} \frac{\tilde{\gamma}^{*(c)}}{1 + \tilde{\gamma}^{*(c)}}}$$
for  $\sum_{c=1}^{C} \alpha^{(c)} \frac{\tilde{\gamma}^{*(c)}}{1 + \tilde{\gamma}^{*(c)}} < 1.(21)$ 

Note that we have implicitly assumed that  $P_{max}$  is sufficiently large so that the target SIRs (i.e.,  $\tilde{\gamma}^{*(c)}$ 's) can be achieved by all users. Furthermore, since  $\tilde{\gamma}^{*(c)} \geq \tilde{\gamma}^{(c)}$  for  $c = 1, \dots, C$ , we have  $\tilde{u}_k = u_k = \frac{L}{M} R \frac{f(\tilde{\gamma}^{*(c)})}{p_k}$ . Therefore, for the matched filter, the decorrelator, and the MMSE detector, the utilities achieved at the Nash equilibrium are given by

$$\tilde{u}_{k}^{MF} = \frac{LR}{M\sigma^{2}}h_{k}^{2}\left(1 - \sum_{c=1}^{C}\alpha^{(c)}\tilde{\gamma}^{*(c)}\right)\frac{f(\tilde{\gamma}^{*(c)})}{\tilde{\gamma}^{*(c)}}$$
  
for  $\sum_{c=1}^{C}\alpha^{(c)}\tilde{\gamma}^{*(c)} < 1$ , (22)

$$\tilde{u}_{k}^{DE} = \frac{LR}{M\sigma^{2}}h_{k}^{2}\left(1-\sum_{c=1}^{C}\alpha^{(c)}\right)\frac{f(\tilde{\gamma}^{*(c)})}{\tilde{\gamma}^{*(c)}}$$
  
for 
$$\sum_{c=1}^{C}\alpha^{(c)} < 1, \quad (23)$$

and

$$\begin{split} \tilde{u}_{k}^{MMSE} &= \frac{LR}{M\sigma^{2}} h_{k}^{2} \left( 1 - \sum_{c=1}^{C} \alpha^{(c)} \frac{\tilde{\gamma}^{*(c)}}{1 + \tilde{\gamma}^{*(c)}} \right) \frac{f(\tilde{\gamma}^{*(c)})}{\tilde{\gamma}^{*(c)}} \\ & \text{for } \sum_{c=1}^{C} \alpha^{(c)} \frac{\tilde{\gamma}^{*(c)}}{1 + \tilde{\gamma}^{*(c)}} < 1. \end{split}$$
(24)

Note that, based on the above equations, we have  $\tilde{u}_k^{MMSE} > \tilde{u}_k^{DE} > \tilde{u}_k^{MF}$ . This means that the MMSE reciever achieves the highest utility as compared to the decorrelator and the matched filter. Also, the network capacity (i.e., the number of users that can be admitted into the network) is the highest when the MMSE detector is used. For the specific case of no delay constraints,  $\tilde{\gamma}^{*(c)} = \gamma^*$  for all c and (22)–(24) reduce to

$$u_k^{MF} = \frac{LR}{M\sigma^2} h_k^2 \left(1 - \alpha \gamma^*\right) \frac{f(\gamma^*)}{\gamma^*} \text{ for } \alpha \gamma^* < 1,(25)$$

$$u_k^{DE} = \frac{LR}{M\sigma^2} h_k^2 \left(1 - \alpha\right) \frac{f(\gamma^*)}{\gamma^*} \quad \text{for} \quad \alpha < 1, \qquad (26)$$

and

$$u_{k}^{MMSE} = \frac{LR}{M\sigma^{2}}h_{k}^{2}\left(1 - \alpha \frac{\gamma^{*}}{1 + \gamma^{*}}\right)\frac{f(\gamma^{*})}{\gamma^{*}}$$
  
for  $\alpha \frac{\gamma^{*}}{1 + \gamma^{*}} < 1.$  (27)

Comparing (22)–(24) with (25)–(27), we observe that the presence of users with stringent delay requirements results not only in a reduction in the utilities of those users but

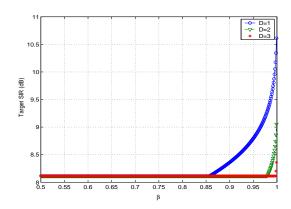


Fig. 1. Target SIR,  $\tilde{\gamma}^*$ , as a Function of  $\beta$  for D = 1, 2, and 3.

also a reduction in the utilities of other users in the network. A stringent delay requirement results in an increase in the user's target SIR (remember  $\tilde{\gamma}_k^* = \max\{\tilde{\gamma}_k, \gamma^*\}$ ). Since  $\frac{f(\gamma)}{\gamma}$  is maximum when  $\gamma = \gamma^*$ , a target SIR larger than  $\gamma^*$  results in a reduction in the utility of the corresponding user. In addition, because of the higher target SIR for this user, other users in the network experience a higher level of interference and hence are forced to transmit at a higher power which in turn results in a reduction in their utilities (except for the decorrelator, in which case the multiple-access interference is completely removed). Also, since  $\tilde{\gamma}_k^* \geq \gamma^*$  and  $\sum_{c=1}^C \alpha^{(c)} = \alpha$ , the presence of delay-constrained users causes a reduction in the system capacity (again, except for the decorrelator). Through (22)-(24), we have quantified the loss in the utility (in bits/Joule) and in network capacity due to users' delay constraints for the matched filter, the decorrelator and the MMSE receiver. The sensitivity of the loss to the delay parameters (i.e., D and  $\beta$ ) depends on the efficiency function,  $f(\gamma)$ .

#### VI. NUMERICAL RESULTS

Let us consider the uplink of a DS-CDMA system with processing gain 100. We assume that each packet contains 100 bits of information and no overhead (i.e., L = M =100). The transmission rate, R, is 100Kbps and the thermal noise power,  $\sigma^2$ , is  $5 \times 10^{-16}Watts$ . A useful example for the efficiency function is  $f(\gamma) = (1 - e^{-\gamma})^M$ . This serves as an approximation to the packet success rate that is very reasonable for moderate to large values of M [3]. We use this efficiency function for our simulations. Using this, with M = 100, the solution to (17) is  $\gamma^* = 6.48 = 8.1dB$ .

Fig. 1 shows the target SIR as a function of  $\beta$  for D = 1, 2, and 3. It is observed that, as expected, a more stringent delay requirement (i.e., a higher  $\beta$  and/or a lower D) results in a higher target SIR.

We now consider a network where the users can be divided into two classes: delay sensitive (class A) and delay tolerant (class B). For users in class A, we choose  $D_A = 1$  and  $\beta_A = 0.99$  (i.e., delay sensitive). For users in class B, we let  $D_B = 3$  and  $\beta_B = 0.90$  (i.e., delay tolerant). Based on these choices,  $\tilde{\gamma}_A^* = 9.6dB$  and  $\tilde{\gamma}_B^* = \gamma^* = 8.1dB$ . Without loss of generality and to keep the comparison fair, we also assume that all the users are 100 meters away from the uplink

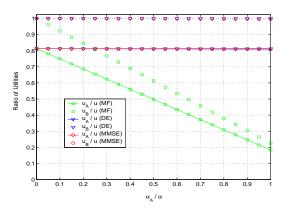


Fig. 2. Loss in Utility Due to Presence of Users with Stringent Delay Requirements ( $\alpha = 0.1$ )

receiver. The system load is assumed to be  $\alpha$  (i.e.,  $\frac{K}{N} = \alpha$ ) and we let  $\alpha_A$  and  $\alpha_B$  represent the load corresponding to class A and class B, respectively, with  $\alpha_A + \alpha_B = \alpha$ .

We first consider a lightly loaded network with  $\alpha = 0.1$ (see Fig. 2). To demonstrate the performance loss due to the presence of users with stringent delay requirements (i.e., class A), we plot  $\frac{u_A}{u}$  and  $\frac{u_B}{u}$  as a function of the fraction of the load corresponding to class A users (i.e.,  $\frac{\alpha_A}{\alpha}$ ). Here,  $u_A$  and  $u_B$  are the utilities of users in class A and class B, respectively, and u represents the utility of the users if they all had loose delay requirements which means  $\tilde{\gamma}_k^* = \gamma^*$  for all k. Fig. 2 shows the loss for the matched filter, the decorrelator, and the MMSE detector. We observe from the figure that for the matched filter both classes of users suffer significantly due to the presence of delay sensitive traffic. For example, when half of the users are delay sensitive, the utilities achieved by class A and class B users are, respectively, 50% and 60% of the utilities for the case of no delay constraints. For the decorrelator, only class A users suffer and the reduction in utility is smaller than that of the matched filter. For the MMSE detector, the reduction in utility for class A users is similar to that of the decorrelator, and the reduction in utility for class B is negligible.

We repeat the experiment for a highly loaded network with  $\alpha = 0.9$  (see Fig. 3). Since the matched filter cannot handle such a significant load, we have shown the plots for the decorrelator and MMSE detector only. We observe from Fig. 3 that because of the higher system load, the reduction in the utilities is more significant for the MMSE detector compared to the case of  $\alpha = 0.1$ . It should be noted that for the decorrelator the reduction in utility of class A users is independent of the system load. This is because the decorrelator completely removes the multiple-access interference.

It should be further noted that in Figs. 2 and 3 we have only plotted the ratio of the utilities (not the actual values). As discussed in Section V, the achieved utilities for the MMSE detector are larger than those of the decorrelator and the matched filter.

#### VII. CONCLUSIONS

We have proposed a game-theoretic approach for studying power control in multiple-access networks with (transmission) delay constraints. We have considered a non-cooperative game

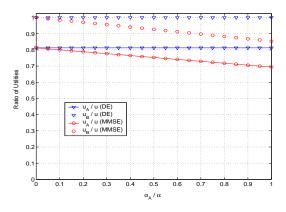


Fig. 3. Loss in Utility Due to Presence of Users with Stringent Delay Requirements ( $\alpha = 0.9$ )

where each user seeks to choose a transmit power that maximizes its own utility while satisfying the user's delay requirements. The utility function measures the number of reliable bits transmitted per joule of energy. We have modeled the delay constraint as an upper bound on the delay outage probability. We have derived the Nash equilibrium for the proposed game and have shown that it is unique. The results are applicable to all linear receivers. In addition, we have used a large-system analysis to derive explicit expressions for the utilities achieved at equilibrium for the matched filter, decorrelator and MMSE detector. The reductions in the users' utilities (in bits/Joule) and network capacity due to the presence of users with stringent delay constraints have been quantified.

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