Antenna Array Geometry and Coding Performance

Weijun Zhu, Heechoon Lee, Daniel Liu, and Michael P. Fitz
UnWiReD Laboratory
University of California Los Angeles
Email: unwired@ee.ucla.edu

***Abstract*— This paper provides details about experiments in realistic, urban, and frequency flat channels with space-time coding that specifically examines the impact of the number of receive antennas and the design criteria for code selection on the performance. Also the performance characteristics are examined of the coded modulations in the presence of finite size array geometries. This paper gives some insight into which of the theories are most useful in realistic deployments.**

I. Introduction

Over the past several years, there has been a great deal of research to improve performance of wireless communications in fading environments by exploiting transmitter and/or receiver diversity. The pioneering work by Telatar [1], Foschini and Gans [2] showed that multiple antennas in a wireless communication system can greatly improve performance. For L_t transmit antennas and L_r receive antennas in Rayleigh fading, it was shown that with spatial independence there are essentially L_tL_r levels of diversity available and there are $\min(L_t, L_r)$ independent parallel channels that could be established. These information theoretic studies spawned two lines of work; one where the number of independent channels is large [3] and one where the number of independent channels is small [4], [5]. With eight years of intensive engineering research and development effort after these insights, MAR techniques are making a significant impact on how wireless services are provided. Examples include the nascent 802.11n standard and 3G and 4G mobile telecommunications systems. The efforts in this area have reached the point where researchers are calling the area mature.

A. Open Problems in Space-Time Signaling

The open problems in MAR communications relate to situations where more sophisticated and detailed aspects of communication systems need to be modeled and understood. For example performance is not easily understood in channel models that are not well modeled as Gaussian/Rayleigh, or where the scattering is not rich or isotropic, or where time-varying parameters, or system non-idealities impact system performance. These problems are not well addressed by simulation or analysis as the sophistication of the problem prohibits analysis in most cases and the utility of simulation is limited to the accuracy of the models used for simulation.

This paper examines a small subset of the open issues in the literature and reports on experiments that attempt to resolve these issues on real systems and real channels. The focus here is on the following systems

1) **Land Mobile Wireless –** Mobility and multipath typical of this environment will be the focus of the study presented in this paper.
2) **Frequency flat channels –** A vast majority of the work in space-time signaling has used frequency flat models. This corresponds to relatively narrowband transmission in a traditional land mobile wireless channels.
3) **Linear Modulations** – Only linear modulations will be the focus of the study presented in this paper.
4) **Short Packet Communication –** The packet lengths of the system presented in the paper will be approximately 300 symbols. This type of system is typical of speech communication systems or short packet data (paging).

Within this fairly focused area this paper will address:

1) **Signal Design and Number of Receiver Antennas –** With a small number of receive antennas the theory indicates signal design is dominated by the Hamming distance and the product measure of the pair–wise signal error matrix. With a large number of receive antennas the signal design is dominated by the Euclidean distance of the pair–wise codeword difference. The question at what number of receive antennas is the transition between these two design environments manifested and how significant is the difference in realistic environments.
2) **Impact of Spatial Correlation** – Code performance is very much a function of the spatial correlation between the transmission paths [6]. Consequently it is useful to see if any interesting characteristics are produced in realistic array geometries that impact the choice of coded modulations in practice.

This paper is organized with Section II overviewing the models, Section III detailing the design paradigms, Section IV presenting the experimental system, Section V providing the experimental results, and Section VI concludes.

II. Signal Models

For linear modulation the signal at the i^{th} transmit antenna is modeled as

$$X_i(t) = \sum_{l=1}^{N_f} X_i(l)u(t-(l-1)T) \tag{1}$$

where $u(t)$ is a Nyquist pulse shape and $X_i(l)$ is the modulation symbol on the i^{th} antenna at symbol l and N_f is the length of the frame. If the fading is slow enough, the sampled matched filter outputs are the sufficient statistics for the demodulation and the output samples of the matched filter for the k^{th} symbol are given as a $L_r \times 1$ vector

$$\vec{Y}(k) = \mathbf{H}(k)\sqrt{E_s}\vec{X}(k) + \vec{N}(k) \tag{2}$$

where E_s is the energy per transmitted symbol; $H_{ij}(k)$ is the complex path gain from transmit antenna j to receive antenna i at time kT; $\vec{X}(k)$ is the $L_t \times 1$ vector of symbols transmitted at symbol time k; $\vec{N}(k)$ is the additive white Gaussian noise vector of size $L_r \times 1$. The noise is modeled as an independent circularly symmetric zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension.

Coherent demodulation refers to the case of finding the most likely transmitted word when the channel is known. The optimum word demodulator denoted maximum likelihood (ML) word demodulator. Denote $\vec{B}$ as the transmitted word. For orthogonal modulations when $\mathbf{H}(k) = \mathbf{h}(k)$ the optimum word demodulator has a simpler form given as

$$\hat{\vec{B}} = \arg\min_n \sum_{k=1}^{N_f} \left(\vec{Y}(k) - \vec{s}(n)\right)^H \left(\vec{Y}(k) - \vec{s}(n)\right) \quad (3)$$

where $\vec{s}(n) = \sqrt{E_s}\mathbf{h}(k)\vec{x}^{(n)}(k)$ is the vector of noiseless received points on each of the antennas, $\vec{x}^{(n)}(k)$, a $L_t \times 1$ vector, is used to denote the transmitted codeword at k^{th} symbol for transmitted bit sequence n, $\vec{Y}(k)$ is the received matched filter output for the k^{th} symbol. The ML demodulator essentially finds the transmitted symbol or code matrix, $\mathbf{X} = \mathbf{x}_n$ that produces the minimum distance between the matched filter outputs, $\vec{Y}(k)$ and the channel output, $\sqrt{E_s}\mathbf{h}(k)\vec{x}^{(n)}(k)$. If the modulation is defined on a trellis then the Viterbi algorithm can be used to find this minimum distance transmitted codeword and if the transmitted codeword is defined by a lattice then a lattice search algorithm can be used to find the best codeword.

III. Overview of Code Design Paradigms

This section will discuss the different design paradigms for wireless communications that are often invoked by researchers. Since this experiment is focussed on frequency flat MIMO signalling the standard assumption in this field is that the channel is well modeled by Rayleigh fading so the brief discussion here will focus on the results for Rayleigh fading. Let $\mathbf{X}$ be the two dimensional code word matrix transmitted by the space-time modem, and the space-time code $\mathcal{X}$ be the collection of these code words.

The code design criteria for **coherent** demodulation in spatially white Rayleigh fading for systems with a small receive array size are [5], [4]

- *Diversity Advantage*: Maximize $\Delta_H(n_1, n_2) = \text{rank}\left(\mathbf{x}_{n_1} - \mathbf{x}_{n_2}\right)$ over all pairs of code words, $\mathbf{x}_{n_1} \neq \mathbf{x}_{n_2}$ and $\mathbf{x}_{n_1}, \mathbf{x}_{n_2} \in \mathcal{X}$.
- *Coding Gain*: Maximize the geometric mean of the nonzero eigenvalues of the signal matrix $\mathbf{C}_s = \left(\mathbf{x}_{n_1} - \mathbf{x}_{n_2}\right)\left(\mathbf{x}_{n_1} - \mathbf{x}_{n_2}\right)^H$ over all distinct pairs of code words $\mathbf{x}_{n_1},\ \mathbf{x}_{n_2} \in \mathcal{X}$.

The rank is often denoted the Hamming distance and the geometric mean is often denoted the product measure to show the relation to single antenna Rayleigh fading design [7]. A great deal of work has gone into designing codes based on these design criteria.

For a large receive array size the design criteria changes to be focussed more on Euclidean distance [8], [9], [10] and this design criteria can be stated succinctly as

- *Euclidean Distance*: Maximize over all distinct pairs of code words $\mathbf{x}_{n_1},\ \mathbf{x}_{n_2} \in \mathcal{X}$ the arithmetic mean of the eigenvalues of $\mathbf{C}_s = \left(\mathbf{x}_{n_1} - \mathbf{x}_{n_2}\right)\left(\mathbf{x}_{n_1} - \mathbf{x}_{n_2}\right)^H$.

A reader should note that the boundary between the two scenarios is not well defined but has been seen in simulation to be around 3 or 4 receive antennas where the channels are modeled as spatially white.

IV. Experimental System

The experimental system that has been deployed for this experiment is a narrowband 3×4 MAR system. We have chosen a carrier frequency of 220MHz and a bandwidth of around 4kHz. All modulations are linear modulation with a spectral raised cosine pulse shape with an excess bandwidth of 0.2 and a symbol rate of 3.2kHz. This carrier frequency and bandwidth allow us to do realistic land mobile testing and still be confident that the frequency flat assumption will be valid.

A. Radio System

The UnWiReD narrowband testbed is a software defined real-time 3×4 multi-antenna testbed. The information bits are encoded and pulse shaped by two Analog Devices (ADI) fixed point digital signal processors (DSP). The baseband signals are then digitally up converted to 10MHz IF signals. The 3-TX up converter radio further up converts the IF signals to the 220MHz RF and amplifies it for transmission with a maximum transmission power of 35dBm.

The receiver chain provides a high performance system for narrowband MIMO processing. The received signals are down converted from RF to 10MHz IF signals by a 4-channel down converter radio and then digitally down converted to baseband by a 4-channel digital receiver. The 4-channel digital receiver over–samples the input signals at 64MHz. Overall receiver dynamic range is greater than 80dB. The overall error vector magnitude through both the transmit and receive chains is less than 2%. The demodulation is performed by two floating point ADI DSPs. The demodulated data, as well as other important test information, is transferred to a laptop for data recording and displaying real-time test results. This data provides a near complete characterization of the system performance.

B. Packet Format

The frame for the transmitted signals of this experimental system was designed to allow many modulations to be tested in a time interleaved fashion. This comparison is enabled at the transmitter by implementing a superframe that is repeated about every 4 seconds. During this superframe a preamble is sent and 42 different frames of space-time modulations can be transmitted. The preamble has a signal format that allows high performance symbol time estimation (a dotting pattern) so that accurate timing and a course frequency offset can be acquired. Each of the subsequent frames or data packets are 300 symbols in length (93.75ms). Modulations are independent from frame

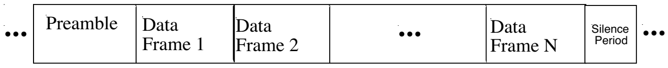

Fig. 1. The superframe used in the field experiments.

to frame for the experiments documented in this paper. At the end of the superframe there is a silence period of about 70 symbols. The noise power which can vary significantly at 220MHz in various scenarios due to man–made noise is measured every frame and averaged to get a good estimate of the SNR.

C. Receiver Processing Overview

All of the receiver functions are implemented in real-time in a digital signal processor. Time estimation is derived by using a nonlinear open loop timing estimator. Frequency estimation and frame synchronization are achieved during the first preamble portion of the frame during the decoding. Having a unique word for frame synchronization allows pilot symbols to be inserted and used for channel estimation and provides block boundary synchronization for all coded modulations during demodulation. The details of the pilot symbol processing are given in Section IV-D. For coherent demodulation ML receivers based on a trellis search and on a sphere/lattice decoder have been implemented. This great flexibility allows many algorithms to be compared to understand the complexity–performance trade-offs in real implementations. One of the powerful characteristics of the programmable implementation is that the same transmitted data can be used to compare decoding with the number of receive antennas. In almost all modulation formats, decoding for any number of receive antennas (from $L_r = 1$ to $L_r = 4$) can be accomplished in real time. The notable exception to this was the sphere decoder, real-time sphere/lattice decoding was only able to be implemented for the 2×2 case.

D. Channel Estimation

Accurate estimation of the channel is crucial for reliable decoding of coherent coding schemes. Pilot symbol assisted demodulation (PSAD) is employed when good performance in high-mobility situations is desired at a reasonable complexity. Pilot symbol based frame design and channel estimation is essentially an exercise in sampling and optimal interpolation of Gaussian processes [11], [12], [5]. Due to the Gaussian nature of the assumed Rayleigh fading, linear interpolation is optimal. For a finite frame size, interpolation at the frame edges performs worse hence it is important to have more samples at the frame edges. Uniform pilot sampling in the middle of the frame is optimal as long as the sampling is above the Nyquist rate of the channel. Guey et al. [5] showed that orthogonal pilot elements on each transmit antenna have many desirable characteristics. An orthogonal pilot symbol pattern maintains good performance (but not orthogonality at the receiver) even with high mobility.

The pilot symbol frame structure for this experiment is optimized for the short frame structure and the rapid fading that is possible with high mobility. The pilot symbol frame

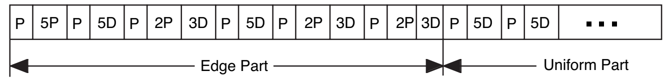

Fig. 2. The frame design for the pilot symbol processing for $L_t = 2$.

(a) Tx deployment. (b) Rx deployment.

Fig. 3. Example deployment for the outdoor tests.

is optimized separately for different number of transmit antennas. For example, the 2 Tx frame is shown in Figure 2. In this example 72 out of 300 total symbols are used for training. Hence to maintain a fair comparison with a modulation/demodulation not needing training for channel estimation, a code rate increase of roughly 4/3 needs to be implemented for the coherent coding and decoding. The channel gains between any transmitter-receiver pair are assumed to be spatially independent for interpolation filter design. Also, the channel coefficients are assumed to be constant over a symbol period but vary from symbol to symbol according to Clarke's model [13] which has $R_H(m) = J_0(2\pi f_D T m)$ where J_0 is the zeroth order Bessel function of the first kind and f_D is the Doppler spread of the channel. An FIR Wiener filter optimized for $E_b/N_0 = 30dB$ and Doppler fading rate $f_D T = 0.01$ is used for pilot interpolation in the experiments reported in this paper.

V. Experimental Results

The experimentation was done on the UCLA campus and the surrounding West Los Angeles area. The testing reported in this paper was limited to the scenario where one radio (TX) was deployed on the top of a 5 story building and one radio (RX) on a vehicle (a cart or a van). The test consisted of the receiver radio being driven around the campus area. The speed of the driving was maintained at a rate of less than 5 miles per hour as the codes that were tested were all designed for quasi-static fading. The UCLA campus area is heavily urbanized and a line of sight was not achieved in any significant portion of the testing. Unless otherwise specified the receiver array was square with a $\lambda/2$ spacing on each side and the transmitter array was linear with a 2λ spacing. An example of the testbed deployment is shown in Fig. 3.

The major findings reported will be the bit error rate and frame error frame versus E_b/N_0. The measured E_b/N_0 reported in the experiments are computed by the averages over the entire superframe and all the receive antennas. This measure gives something closer to the average SNR in high mobility tests and something closer to the instantaneous SNR in static testing but the measurement was viewed as the best compromise in reporting the data. The transmitted power from all the antennas in each of the experiments was roughly 15dBm.

A. Code Design and Number of Receive Antennas

Here a comparison is made of the performance of the various proposed design methodologies and resulting $R = 2$ QPSK $L_t = 2$ and $L_t = 3$ space-time codes on real channels. Specifically we would like to understand the impact of the number of receiver antennas on performance. The codes that are considered are

1) 16 state Yan and Blum (YB) [14] is a code optimized for Hamming distance and product measure.
2) 32 state Chen, Yuan, and Vucetic (CYV) [10] code is optimized for Euclidean distance.
3) 32 state superorthogonal code (SO) [15], [16] codes were optimized for simultaneously for Hamming distance, Euclidean distance, and product measure.
4) 32 state spatially multiplexed traditional (SMT) codes [9] were optimized simultaneously for Hamming distance, Euclidean distance, and product measure.
5) 32 state universal trellis codes [17] were optimized to give good performance on any channel that can has a capacity above $R = 2$ bits per channel use.

The general expectation derived from the theory before the experiment was that codes that were designed for Hamming distance would work well at small number of receive antennas and codes that are designed for Euclidean distance would work well with a large number of receive antennas.

A wide variety of data versus measured average SNR has been compiled for the drive tests. The results of the frame error rates (FER) for $L_t = 2$ are summarized in Fig. 4 and Fig 5. In Fig. 4 as expected the designs which have optimized Hamming distance and product measure (YB and SO) seem to do comparatively well. In fact all codes seem to perform very close to the same level with the universal code having a slight advantage in performance. This is perhaps not unexpected as this is the only code not designed under the assumption of spatially white Rayleigh fading in the group tested. In Fig. 5 we observe some unexpected results. The universal code shows the best performance followed by the SO code. Noticeably worse performance is observed with both the YB codes and the CYB codes. This result is also curious in that these codes have used different design criteria in terms of number of receive antenna and yet on real channels seem to produce close to the same performance. Also surprising was the performance of the (SMT) codes as they did not seem to be able to use the joint design of both Euclidean or Hamming distance to beat codes that optimized either metric individually (YB or CYV). Bit error performance and results for $L_t = 3$ coding schemes can be markedly different, but this data is not reported here due to space constraints.

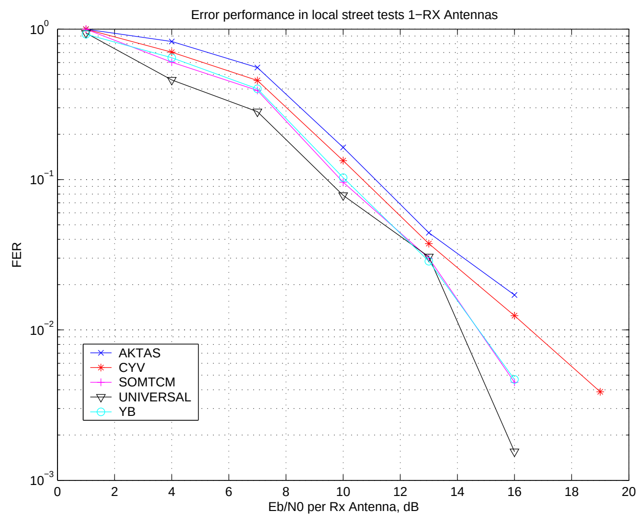

Fig. 4. FER performance for $L_r = 1$.

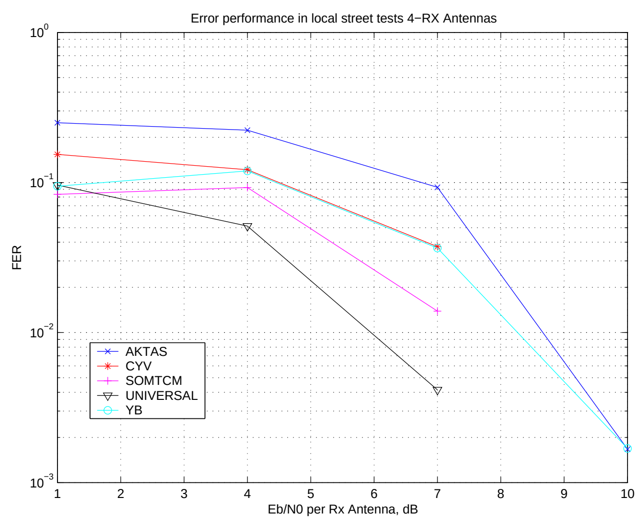

Fig. 5. FER performance for $L_r = 4$.

B. Antenna Spacing Impacts on Code Design

This section reports on a series of experiments whose goal is to evaluate the impact of antenna separation on code performance and code design. Two types of signalling is considered in this section: 1) multiplexing type schemes (space-time constellations and precoding) and 2) space-time codes designed to harvest diversity or performance. Three antenna configurations are considered: 1) 2 λ spacing at the transmitter with 0.5λ spacing at the receiver, 2) 2 λ spacing at the transmitter with 0.25λ spacing at the receiver, and λ spacing at the transmitter with 0.25λ spacing at the receiver.

For the case of precoding and space time constellations the Alamouti [18] coding scheme has the most robustness to different antenna geometries. All constellations considered here use $R = 4$ bits per symbol and the comparison for frame error rate is in Fig. 6 for $L_t = 2$ and $L_r = 2$. The Alamouti code must use a 16QAM constellation to achieve this rate while spatial multiplexing and the threaded architectures

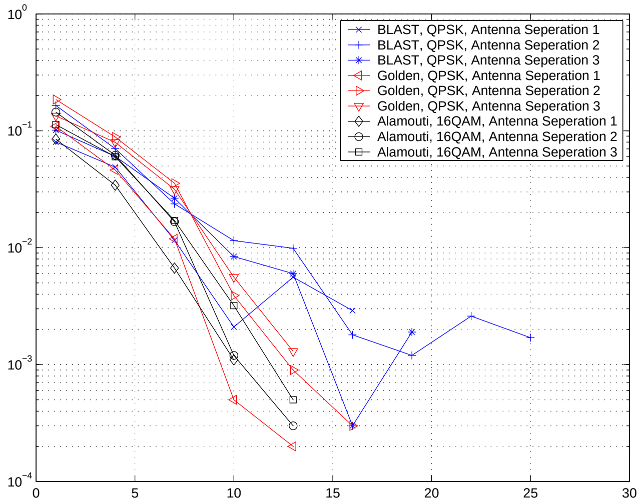

Fig. 6. FER performance for $L_t = 2$ and $L_r = 2$.

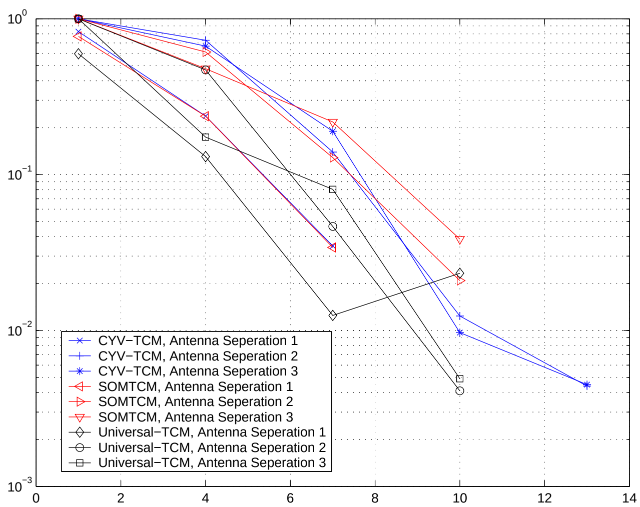

Fig. 7. FER performance for $L_r = 4$.

can use QPSK constellations. When the spacing of the array is larger (closer to spatial independence) the Golden Code [19] has the advantage due to the smaller constellation point differences. As the spacing become closer and the channels become correlated the Alamouti code has the advantage. Direct spatial multiplexing with a BLAST like architecture is clearly lower performance than either of the two considered architectures and suffers from not achieving full diversity.

For the case of trellis codes, the universal code has the most robustness to different antenna geometries. All constellations considered here use a $R = 2$ bits per symbol and the comparison for frame error rate is in Fig. 7 for $L_t = 2$ and $L_r = 2$. The universal code has good performance in most cases with moderate degradation due to spatial correlation. The SO and CYV codes show more significant degradation due to spatial correlation.

VI. Conclusion

The paper has presented field tests and the conclusions that can be drawn from the field tests for space–time coding with a variable number of antennas and varying antenna array size.

Acknowledgment

The National Science Foundation has provided support for this effort which had a time constant of not months but years. Analog Devices provided hardware and software support in the construction of this testbed.

References

[1] E. Teletar, “Capacity of multi-antenna Gaussian channels,” AT&T–Bell Labs, Tech. Rep., June 1995.

[2] G. J. Foschini and M. Gans, “On the limits of wireless communication in a fading environment,” *Wireless Personal Comm.*, vol. 6, pp. 311–355, March 1998.

[3] G. J. Foschini, “Layered space-time architecture for wireless communications in a fading environment when using multiple antennass,” *Bell Labs Technical Journal*, pp. 41–59, Autumn 1996.

[4] V. Tarokh, N. Seshadri, and A. Calderbank, “Space-time codes for high data rate wireless communication: Performance criterion and code construction,” *IEEE Info. Theory*, vol. vol. IT-44, pp. 744–765, March 1998.

[5] J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, “Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels,” in *1996 IEEE Vehicular Technology Conference*, Atlanta, GA, 1996, pp. 136–140.

[6] D. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, “Fading correlation and its effect on the capacity of multielement antenna systems,” *IEEE Trans. Commun.*, vol. 48(3), pp. 502–513, March 2000.

[7] D. Divsalar and M. K. Simon, “The design of trellis coded MPSK for fading channels: performance criteria,” *IEEE Trans. Commun.*, vol. COM-36, pp. 1004–1012, September 1988.

[8] E. Biglieri and A. M. Tulino, “Designing space-time codes for large number of receiving antennas,” *Electronics Letters*, vol. 37, pp. 1073–1074, August 2001.

[9] D. Aktas, H. E. Gamal, and M. P. Fitz, “On the design and maximum-likelihood decoding of space–time trellis codes,” *IEEE Trans. Commun.*, vol. vol.51, no.6, pp. 854–859, June 2003.

[10] Z. Chen, J. Yuan, and B. Vucetic, “Improved space-time trellis coded modulation scheme on slow Rayleigh fading channels,” *Electronics Letters*, vol. 37, pp. 440–441, March 2001.

[11] J. K. Cavers, “An analysis of pilot symbol assisted modulation for Rayleigh faded channels,” *IEEE Trans. Veh. Tech.*, vol. vol. VT-40, pp. 686–693, November 1991.

[12] W. Y. Kuo and M. P. Fitz, “User slot design and performance analysis for burst mode communication with fading and frequency uncertainty,” *International Journal on Wireless Information Networks*, vol. 1, pp. 239–252, October 1994.

[13] R. H. Clarke, “A statistical theory for mobile radio reception,” *Bell Systems Technical Journal*, vol. vol. 47, pp. 957–1000, 1968.

[14] Q. Yan and R. Blum, “Optimum space-time convolutional codes,” in *IEEE Wireless Communications and Networking Confernce*, September 2000, pp. 1351–1355.

[15] S. Siwamogsatam and M. Fitz, “Improved high rate space–time TCM via concatenation of expanded orthogonal block codes and MTCM,” in *IEEE International Conference on Communication*, New York, April 2002.

[16] ——, “Improved high rate space–time TCM via orthogonality and set partitioning,” in *International Symposium on Wireless Personal Multimedia Communications*, Alborg, Denmark, September 2001.

[17] C. Kose and R. Wesel, “Universal space-time trellis codes,” *IEEE Transactions on Information Theory*, vol. 49, pp. 2717 – 2727, Oct. 2003.

[18] S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, October 1998.

[19] J.-C. Belfiore, G. Rekaya, and E. Viterbo, “The golden code: A 2x2 full-rate space-time code with non-vanishing determinants,” in *IEEE International Symposium on Information Theory*, Chicago, IL, June 2004, p. 308.