On the Capacity of Partially Cognitive Radios

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Abstract— This paper considers the problem of cognitive radios with partial-message information. Here, an interference channel setting is considered where one transmitter (the "cognitive" one) knows the message of the other ("legitimate" user) *partially*. An outer bound on the capacity region of this channel is found for the "weak" interference case (where the interference from the cognitive transmitter to the legitimate receiver is weak). This outer bound is shown for both the discrete-memoryless and the Gaussian channel cases. An achievable region is subsequently determined for a mixed interference Gaussian cognitive radio channel, where the interference from the legitimate transmitter to the cognitive receiver is "strong". It is shown that, for a class of mixed Gaussian cognitive radio channels, portions of the outer bound are achievable thus resulting in a characterization of a part of this channel's capacity region.

Note that results in this paper specialize to the case of the weak/mixed interference channel and the cognitive radio channel with full-message information.¹

I. INTRODUCTION

A cognitive radio is one that possesses information that allows it to tailor its transmission to maximize network throughput while meeting constraints imposed on it [1]. There are multiple notions of cognition in literature [1], [2] with an increasingly popular strategy known as overlay cognition, where both the cognitive and the legitimate users transmit their own messages in the same sub-band simultaneously, as in [3]. In this setting, the cognitive transmitter has access to (limited) information about the legitimate user so as to mitigate network interference and thus increase overall throughput.

In previous work, the class of interference channels with degraded message sets has been considered [6], where the cognitive user has access to the entire message of the legitimate user. Examples of this setting include [7], where the authors determine the capacity region of this channel for both the case of "weak" interference and for a class of "strong" interference channels. However, the paper's assumption of perfect and complete message information should be relaxed in order to apply the ideas and concepts to more general classes of cognitive radio channels.

This paper considers a cognitive radio channel model where the cognitive radio is not fully cognitive of the other transmitter's message set. In this setting, the cognitive radio has

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access only to a portion of the message. Note that as this portion varies from nothing to everything, it includes the interference channel (IFC) in literature [8], [9], [10], and IFC with fully-degraded message set [7] as special cases. This channel is referred to as an interference channel with a partially cognitive transmitter. Note that this channel model is motivated by practical constraints, where the cognitive transmitter is only able to garner limited information about the legitimate transmitter's message.

The interference channel with a partially cognitive transmitter has already been studied in [4], with a specific focus on strong interference settings. This paper focuses on the weak and mixed interference settings. Specifically, we derive an outer bound on the capacity region of this channel for both the discrete memoryless and Gaussian cases when the interference from the cognitive transmitter to the legitimate receiver is "weak". Subsequently, we show for the Gaussian case that Gaussian distributions satisfying the constraints on the inputs/auxiliary random variables which makes the outer bound extreme exist. Finally, for a special class of mixed interference channels (where the interference from the cognitive transmitter to the legitimate receiver is "weak" and that from the legitimate transmitter to the cognitive receiver is "strong"), we show that a portion of the capacity region can be characterized, i.e., a non-trivial subset of the outer bound is achievable.

This paper is organized as follows. The next section details the system model and notations used in the paper. In Section III, we describe an outer bound on the partially cognitive radio channel for the discrete memoryless case and for the Gaussian channel. In Section IV, we describe an achievable region for the Gaussian partially cognitive radio channel. In Section V, we derive channel conditions under which the achievable region is optimal. We conclude in Section IV.

II. SYSTEM MODEL AND PRELIMINARIES

The notation used in this paper is based largely on that of [7]. Random variables (RVs) are denoted by capital letters, and their realizations using the corresponding lower case letters. X_m^n denotes the random vector $(X_m, ..., X_n)$, X^n denotes the random vector $(X_1, ..., X_n)$, and $X^{n \setminus m}$ denotes the random vector $(X_1, ..., X_{m-1}, X_{m+1}, ..., X_n)$. Also, for any set S, \overline{S}



Fig. 1. The discrete memoryless partially cognitive radio model

denotes the convex hull of S, and \widetilde{S} means the complementary set of S. Finally, the notation $X \Rightarrow Y \Rightarrow Z$ is used to denote that X and Z are conditionally independent given Y.

A. Discrete Memoryless Partially Cognitive Radio Channels

A two user interference channel as in Fig. 1 is a quintuple $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2, p)$, where $\mathcal{X}_1, \mathcal{X}_2$ are two input alphabet sets; $\mathcal{Y}_1, \mathcal{Y}_2$ are two output alphabet sets; $p(y_1, y_2|x_1, x_2)$ is a transition probability. Since we confine channel to be memoryless, the transition probability of y_1^n, y_2^n given x_1^n, x_2^n is

$$p(y_1^n, y_2^n | x_1^n, x_2^n) = \prod_{i=1}^n p(y_{1,i}, y_{2,i} | x_{1,i}, x_{2,i})$$

This channel model is similar to that of an interference channel with the difference being the message sets at each transmitter. Transmitter 1 is the legitimate user, who communicates messages from the sets $W_0 \in \{1, ..., M_0\}$ and $W_1 \in \{1, ..., M_1\}$ to Receiver 1, the legitimate receiver. Transmitter 2, the cognitive transmitter communicates messages $W_2 \in \{1, ..., M_2\}$ to Receiver 2, the cognitive receiver. The unique feature of this channel is that the realization of W_0 is known to *both* Transmitters 1 and 2, which allows for partial unidirectional cooperation between the transmitters. An $(R_0, R_1, R_2, n, P_{e,0}, P_{e,1}, P_{e,2})$ code is any code with the rate vector (R_0, R_1, R_2) and block size *n*, where $R_t \triangleq \log(M_t)/n$ bits per usage for t = 0, 1, 2. As discussed above, W_0 , and W_1 are the messages from Receiver 1 which must be decoded with (average) probabilities of error of at most $P_{e,0}, P_{e,1}$ respectively, and W_2 must be retrieved at Receiver 2 while suffering an error probability of no more than $P_{e,2}$. Rate pair (R_0, R_1, R_2) is said to be achievable if the error probabilities $P_{e,t}$ for t = 0, 1, 2 can be made arbitrarily small as the block size n grows. The capacity region of the interference channel with partially cognitive transmitter is the closure of the set of all achievable rate pairs (R_0, R_1, R_2) . The main goal of the users, legitimate and cognitive, is to maximize in general the $\mu_0 R_0 + \mu_1 R_1 + \mu_2 R_2$ for some non-negative number μ_0, μ_1 , and μ_2 . We also have a restriction on the pair (R_0, R_1) , such that $R_1 \ge \mu R_0$ for some positive number μ . This restriction is to ensure that optimization of (μ_0, μ_1, μ_2) in order to maximize $\mu_0 R_0 + \mu_1 R_1 + \mu_2 R_2$ does not drive R_1 to zero, which results in a fully cognitive solution.

B. Gaussian Partially Cognitive Radio Channel

In the Gaussian IFC, input and output alphabets are the reals \mathbb{R} , and outputs are the linear combination of the inputs and

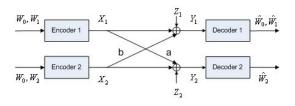


Fig. 2. The Gaussian partially cognitive radio channel

additive white Gaussian noise. A Gaussian IFC model in Fig 2. is characterized mathematically as follows:

$$Y_1 = X_1 + bX_2 + Z_1$$

$$Y_2 = aX_1 + X_2 + Z_2,$$
(1)

where a and b are real numbers and Z_1 and Z_2 are independent, zero-mean, unit-variance Gaussian random variables. Further, each transmitter has a power constraint

$$\frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[X_{t,i}^2] \le P_t, t = 1, 2.$$

This concludes our description of the models considered in this paper. The next section describes the outer bound on the capacity region for these channels under "weak" interference.

III. THE OUTER BOUND REGION

A. Discrete Memoryless Partially Cognitive Radio Channels

For a discrete memoryless channel, under the condition

$$X_2|X_1 \Rightarrow Y_1|X_1 \Rightarrow Y_2|X_1, \tag{2}$$

we say that the legitimate receiver is observing weak interference. For the Gaussian case, the weak interference constraint can be interpreted as the requirement of b < 1 in (1). First, we reproduce a useful lemma from [5].

Lemma 1 ([5]): The following forms a Markov chain for the partially cognitive radio channel:

$$W_0, W_t) \Rightarrow (W_0, X_t) \Rightarrow Y_t$$
 (3)

where t = 1, 2.

We present the outer bound in the following:

Theorem 1: The convex closure of the following inequalities defines an outer bound on the capacity region of "weak" partially cognitive radio channels:

$$R_0 \le I(U, X_1; Y_1 | V)$$
 (4)

$$R_0 + R_1 \le I(U, X_1; Y_1) \tag{5}$$

$$R_2 \le I(X_2; Y_2 | U, X_1) \tag{6}$$

$$R_1 \ge \mu R_0 \tag{7}$$

for any $p(u,v))p(x_1|u,v)p(x_2|u)$ such that:

- 1. V and X_2 are independent
- 2. X_1 is a function of U and V
- 3. $(U, V) \Rightarrow (X_1, X_2) \Rightarrow (Y_1, Y_2).$

Proof: First we prove the outer bound for R_0 in (5) and R_2 the sum rate $R_0 + R_1$ in (6). We have in (7). We have

$$nR_{0} = H(W_{0}|W_{1})$$

$$\leq I(W_{0};Y_{1}^{n}|W_{1}) + n\epsilon_{0}$$

$$= \sum_{i=1}^{n} [H(Y_{1,i}|Y_{1}^{i-1},W_{1}) - H(Y_{1,i}|Y_{1}^{i-1},W_{0},W_{1})] + n\epsilon_{0}$$

$$\leq \sum_{i=1}^{n} [H(Y_{1,i}|W_{1}) - H(Y_{1,i}|Y_{1}^{i-1},X_{1}^{n\setminus i},W_{0},W_{1},X_{1,i})] + n\epsilon_{0}$$

$$\stackrel{(a)}{\leq} \sum_{i=1}^{n} [H(Y_{1,i}|W_{1}) - H(Y_{1,i}|Y_{2}^{i-1},X_{1}^{n\setminus i},W_{0},W_{1},X_{1,i})] + n\epsilon_{0}$$

$$\stackrel{(b)}{=} \sum_{i=1}^{n} [H(Y_{1,i}|V_{i}) - H(Y_{1,i}|U_{i},V_{i},X_{1,i})] + n\epsilon_{0}$$

$$= \sum_{i=1}^{n} I(U_{i},X_{1,i};Y_{1,i}|V_{i}) + n\epsilon_{0}$$

where (a) results from the conditional Markov chain $Y_{2,i}|X_1^n \Rightarrow Y_1^{i-1}|X_1^n \Rightarrow Y_2^{i-1}|X_1^n$, which can be derived from the Markov chain for the weak interference channel, $X_2 \Rightarrow Y_1 \Rightarrow Y_2$, given X_1 in (3) as in the proof of Lemma 3.6 in [7]. (b) results from identifying auxiliaries $U_i = (Y_2^{i-1}, X_1^{n \setminus i}, W_0)$ and $V_i = W_1$. For R_2 ,

$$nR_{2} = H(W_{2}|W_{0})$$

$$\leq I(W_{2}; Y_{2}^{n}|W_{0}) + n\epsilon_{2}$$

$$\leq I(W_{2}; Y_{2}^{n}, X_{1}^{n}|W_{0}) + n\epsilon_{2}$$

$$\stackrel{(a)}{=} I(W_{2}; Y_{2}^{n}|X_{1}^{n}, W_{0}) + n\epsilon_{2}$$

$$= H(Y_{2}^{n}|X_{1}^{n}, W_{0}) - H(Y_{2}^{n}|X_{1}^{n}, W_{0}, W_{2}) + n\epsilon_{2}$$

$$\stackrel{(b)}{\leq} H(Y_{2}^{n}|X_{1}^{n}, W_{0}) - H(Y_{2}^{n}|X_{1}^{n}, W_{0}, X_{2}^{n}) + n\epsilon_{2}$$

$$\stackrel{(c)}{\leq} \sum_{i=1}^{n} [H(Y_{2,i}|U_{i}, X_{1,i}) - H(Y_{2,i}|U_{i}, X_{1,i}, X_{2,i})] + n\epsilon_{2}$$

$$= \sum_{i=1}^{n} I(X_{2,i}; Y_{2,i}|U_{i}, X_{1,i}) + n\epsilon_{2}$$

where (a) is due to the independence of W_2 and X_1^n , (b) is from Lemma 1, and (c) comes from the same definition above of $U_i = Y_2^{i-1}, X_1^{n \setminus i}, W_0$. Next, we prove the outer bound for

$$n(R_{0} + R_{1}) = H(W_{0}, W_{1})$$

$$\leq I(W_{0}, W_{1}; Y_{1}^{n}) + n\epsilon_{1}$$

$$= H(Y_{1}^{n}) - H(Y_{1}^{n}|W_{0}, W_{1}) + n\epsilon_{1}$$

$$\stackrel{(a)}{\leq} H(Y_{1}^{n}) - H(Y_{1}^{n}|W_{0}, X_{1}^{n}) + n\epsilon_{1}$$

$$= \sum_{i=1}^{n} \begin{bmatrix} H(Y_{1,i}|Y_{1}^{i-1}) \\ -H(Y_{1,i}|Y_{1}^{i-1}, X_{1}^{n\setminus i}, W_{0}, X_{1,i}) \end{bmatrix}$$

$$\stackrel{(b)}{=} \sum_{i=1}^{n} \begin{bmatrix} H(Y_{1,i}|Y_{1}^{i-1}) \\ -H(Y_{1,i}|Y_{2}^{i-1}, X_{1}^{n\setminus i}, W_{0}, X_{1,i}) \end{bmatrix}$$

$$\stackrel{(c)}{=} \sum_{i=1}^{n} [H(Y_{1,i}) - H(Y_{1,i}|U_{i}, X_{1,i})] + n\epsilon_{1}$$

$$= \sum_{i=1}^{n} I(U_{i}, X_{1,i}; Y_{1,i}) + n\epsilon_{1}$$

(a) results from (Lemma 1), (b) is due to the conditional Markov chain $Y_{2,i}|X_1^n \Rightarrow Y_1^{i-1}|X_1^n \Rightarrow Y_2^{i-1}|X_1^n$, and (c) follows from the definition above of $U_i = Y_2^{i-1}, X_1^{n \setminus i}, W_0$. Note that the choice of auxiliary random variables automatically satisfies the constraints imposed on them in Theorem 1. Finally, (8) comes from the restriction on the (R_0, R_1) , which is described in the section II.A.

B. Gaussian Partially Cognitive Radio Channel

First, note that similar proof will ensure the outer bound for the rate region defined in Theorem 1 to be valid for the Gaussian partially cognitive radio channel. The main details of proof are omitted here. Next, we establish three lemmas that will be essential in proving the optimality of a jointly Gaussian input distribution for the region defined in Theorem 1.

Lemma 2 (Lemma 1 in [11]): Let X_1, X_2, \dots, X_k be arbitrarily distributed zero-mean random variables with covariance matrix K. Let S be any subset of $\{1, 2, ..., k\}$ and S be its complement. Then

$$h(X_S|X_{\widetilde{S}}) \le h(X_S^*|X_{\widetilde{S}}^*),\tag{8}$$

where $X_{1}^{*}, X_{2}^{*}, ..., X_{k}^{*} \sim N(0, K)$.

Lemma 3: Let X_1, X_2, V be an arbitrarily distributed zeromean random variables with covariance matrix K, where X_2 and V is independent of each other. Let X_1^*, X_2^*, V^* be the zero mean Gaussian distributed random variables with the same covariance matrix as X_1, X_2, V . Then,

$$\mathbb{E}[X_1 X_2] = \mathbb{E}[X_1^* X_2^* | V^*]$$
(9)

Proof: Without loss of generality X_1^* can be written as $X_1^* =$ $W^* + cV^*$, where W^* is the zero mean Gaussian random variable independent of V^* . Then

$$\begin{split} \mathbb{E}[X_1 X_2] &= \mathbb{E}[X_1^* X_2^*] \\ &= \mathbb{E}[\mathbb{E}[X_1^* X_2^* | V^*]] \\ &= \mathbb{E}[\mathbb{E}[(W^* + cV^*)X_2^* | V^*]] \\ &= \mathbb{E}[\mathbb{E}[(W^* X_2^* | V^*]] + c\mathbb{E}[\mathbb{E}[V^* X_2^* | V^*]] \\ &\stackrel{(a)}{=} \mathbb{E}[X_1^* X_2^* | V^*] + c\mathbb{E}[V^* \mathbb{E}[X_2^*]] \\ &\stackrel{(b)}{=} \mathbb{E}[X_1^* X_2^* | V^*] \end{split}$$

where (a) results from the independence of X_2^* and V^* . And, (b) results from the fact that X_2^* is zero mean.

Lemma 4:

$$\mathbb{E}[X_1^*X_2^*|V^*] \le (\mathbb{E}[(X_1^*)^2|V^*])^{\frac{1}{2}} (\mathbb{E}[(\mathbb{E}[X_2^*|X_1^*])^2])^{\frac{1}{2}}$$
Proof: Note that

$$\begin{split} \mathbb{E}[X_1^*X_2^*|V^*] &\cong \mathbb{E}[\mathbb{E}[X_1^*X_2^*|V^*, X_1^*]] \\ \stackrel{(b)}{=} \mathbb{E}[X_1^*\mathbb{E}[X_2^*|V^*, X_1^*]|V^*] \\ \stackrel{(c)}{\leq} (\mathbb{E}[(X_1^*)^2|V^*])^{\frac{1}{2}} (\mathbb{E}[(\mathbb{E}[X_2^*|V^*, X_1^*])^2])^{\frac{1}{2}} \\ \stackrel{(d)}{\leq} (\mathbb{E}[(X_1^*)^2|V^*])^{\frac{1}{2}} (\mathbb{E}[(\mathbb{E}[X_2^*|X_1^*])^2])^{\frac{1}{2}} \end{split}$$

where (a) comes from the law of iterated expectations, (b) from the independence of X_2^* and V^* , (c) from the Cauchy-Schwartz inequality, and (d) from the fact that entropy can only be reduced by conditioning.

Definition 1: Define the rate region $\mathcal{R}_{out}^{\alpha,\beta}$ to be the convex hull of all rate pairs (R_0, R_1, R_2) satisfying

$$R_{0} \leq \frac{1}{2} \log \left(1 + \frac{\beta P_{1} + b^{2}(1-\alpha)P_{2} + 2b\sqrt{(\beta(1-\alpha)P_{1}P_{2})}}{(1+b^{2}\alpha P_{2})} \right)$$

$$R_{0} + R_{1} \leq \frac{1}{2} \log \left(1 + \frac{P_{1} + b^{2}(1-\alpha)P_{2} + 2b\sqrt{(\beta(1-\alpha)P_{1}P_{2})}}{(1+b^{2}\alpha P_{2})} \right)$$

$$R_{2} \leq \log(\alpha P_{2} + 1)$$

$$R_{1} \geq \mu R_{0}$$
(10)

for some $\alpha \in [0,1]$ and $\beta \in [0,1]$

Definition 2: Define the rate region \mathcal{R}_{out} to be convex hull of the union of rate region $\mathcal{R}_{out}^{\alpha,\beta}$:

$$\mathcal{R}_{out} \triangleq \overline{\bigcup_{0 < \alpha, \beta < 1} \mathcal{R}_{out}^{\alpha, \beta}}.$$
 (11)

We denote C to be the capacity region of the Gaussian weak partially cognitive radio channel. An outer bound for C is obtained as follows.

Theorem 2: \mathcal{R}_{out} is an outer bound of the capacity region for the Gaussian weak partially cognitive radio channel:

$$\mathcal{C} \subset \mathcal{R}_{out}.$$

Proof: We start from the rate region in Theorem 1.

$$R_0 \le I(U, X_1; Y_1|V) = h(Y_1|V) - h(Y_1|V, U, X_1)$$

= $h(Y_1|V) - h(Y_1|U, X_1)$ (12)

$$R_0 + R_1 \le I(U, X_1; Y_1) = h(Y_1) - h(Y_1|U, X_1)$$
(13)

$$R_2 \le I(X_2; Y_2 | U, X_1) = h(Y_2 | U, X_1) - h(N_2) \quad (14)$$

(12) follows from the Markov chain, $V \Rightarrow (U, X_1) \Rightarrow Y_1$. First, we set

$$h(Y_2|U, X_1) = \frac{1}{2}\log(2\pi e(1+\alpha P_2))$$
(15)

without loss of generality for some $\alpha \in [0, 1]$. Note that

$$Y_1 = b(X_2 + Z_1) + X_1 + Z'$$

$$h(Y_1|U, X_1) = h(b(X_2 + Z_1) + Z'|U, X_1), \qquad (16)$$

where b < 1 because legitimate receiver faces a weak interference, and Z' is a Gaussian distributed random variable with variance $1 - b^2$. By entropy power inequality (EPI)[14], we have,

$$2^{2h(Y_1|U,X_1)} \ge 2^{2h(bY_2|U,X_1)} + 2^{2h(Z')}.$$

= $b^2 2^{2h(Y_2|U,X_1)} + 2\pi e(1-b^2)$
= $2\pi e(1+b^2\alpha P_2),$

which yields

$$h(Y_1|U, X_1) \ge \frac{1}{2}\log(2\pi e(1+b^2\alpha P_2)).$$
 (17)

Next, we need to bound $h(Y_1)$ and $h(Y_1|V)$. Note that by setting $h(Y_2|U, X_1) = \frac{1}{2}\log(2\pi e(1 + \alpha P_2))$ we have the following result.

$$h(Y_2|U, X_1) \le h(X_2 + Z_2|X_1)$$

$$\le h(X_2^* + Z_2|X_1^*)$$

$$= \frac{1}{2} \log(2\pi e(1 + \operatorname{Var}(X_2^*|X_1^*))), \quad (18)$$

where $Var(\cdot|\cdot)$ denotes the conditional covariance. Combining (15) with (18), we obtain the bound

$$\operatorname{Var}(X_2^*|X_1^*) \ge \alpha P_2. \tag{19}$$

Also,

$$\operatorname{Var}(X_2^*|X_1^*) = \mathbb{E}[(X_2^*)^2] - \mathbb{E}[(\mathbb{E}[X_2^*|X_1^*])^2].$$
(20)

From (19) and (20), we obtain,

$$\mathbb{E}[(\mathbb{E}[X_2^*|X_1^*])^2] \le (1-\alpha)P_2.$$
(21)

Again, we set $\mathbb{E}[(X_1^*)^2|V^*] = \beta P_1$ for some $\beta \in [0, 1]$ without loss of generality. Now combining Lemma 3, Lemma 4, and the above results,

$$\mathbb{E}[X_1 X_2] \le \sqrt{(\beta P_1)} \sqrt{(1-\alpha)P_2}.$$
(22)

Therefore, we obtain the bound for $h(Y_1)$ as

$$h(Y_{1}) \leq \frac{1}{2} \log \left(2\pi e \left(\begin{array}{c} 1 + \operatorname{Var}(X_{1}) + b^{2} \operatorname{Var}(X_{2}) \\ +2b\mathbb{E}[X_{1}X_{2}] \end{array} \right) \right) \\ \leq \frac{1}{2} \log \left(2\pi e \left(\begin{array}{c} 1 + P_{1} + b^{2}P_{2} \\ +2b\sqrt{\beta(1-\alpha)}P_{1}P_{2} \end{array} \right) \right)$$
(23)

For $h(Y_1|V)$, note that (Y_1^*, V^*) has the same covariance matrix as (Y_1, V) if $Y_1 = X_1^* + bX_2^*$. Also, Y_1 is a mean zero Gaussian distributed random variable. Thus,

$$h(Y_{1}|V) \leq h(Y_{1}^{*}|V^{*})$$

$$=h(X_{1}^{*}+bX_{2}^{*}+Z_{1}|V^{*})$$

$$=\frac{1}{2}\log\left(2\pi e\left(\begin{array}{c}1+\operatorname{Var}(X_{1}^{*}|V^{*})\\+b^{2}\operatorname{Var}(X_{2}^{*}|V^{*})\\+2b\mathbb{E}[X_{1}^{*}X_{2}^{*}|V^{*}]\end{array}\right)\right)$$

$$\leq\frac{1}{2}\log\left(2\pi e\left(\begin{array}{c}1+\beta P_{1}+b^{2}P_{2}\\+2b\sqrt{(\beta(1-\alpha)P_{1}P_{2})}\end{array}\right)\right), (24)$$

which gives the desired outer bound for the capacity region.

IV. ACHIEVABLE REGION FOR THE GAUSSIAN CHANNEL

In this section, we describe an achievable region for the Gaussian channel model described in (1). In deriving the achievable region, we combine superposition coding and dirty paper coding [13]. The legitimate transmitter encodes messages W_0 and W_1 using Gaussian codebooks and superimposes them to form its final codeword. The cognitive transmitter allocates a portion of the power in communicating message W_0 to the legitimate receiver. The remaining power is used in encoding its own message W_2 using dirty paper coding treating the codewords (from W_0) as non-causally known interference. Then the following two definitions and theorem present the achievable region for the Gaussian partially cognitive radio channel.

Definition 3: Define the rate region $\mathcal{R}_i^{\alpha,\beta}$ to be the convex hull of all rate pairs (R_0, R_1, R_2) satisfying

$$R_{0} \leq \frac{1}{2} \log \left(1 + \frac{\beta P_{1} + b^{2}(1-\alpha)P_{2} + 2b\sqrt{\beta(1-\alpha)P_{1}P_{2}}}{1+b^{2}\alpha P_{2}} \right)$$

$$R_{1} \leq \frac{1}{2} \log \left(1 + \frac{(1-\beta)P_{1}}{1+\beta P_{1} + b^{2}P_{2} + 2b\sqrt{\beta(1-\alpha)P_{1}P_{2}}} \right)$$

$$R_{1} \leq \frac{1}{2} \log \left(1 + \frac{a^{2}(1-\beta)P_{1}}{1+a^{2}\beta P_{1} + P_{2} + 2a\sqrt{\beta(1-\alpha)P_{1}P_{2}}} \right)$$

$$R_{2} \leq \frac{1}{2} \log(1+\alpha P_{2})$$

$$(25)$$

for some $\alpha \in [0,1]$ and $\beta \in [0,1]$.

Definition 4: Define the rate region \mathcal{R}_i to be convex hull of the union of rate region $\mathcal{R}_i^{\alpha,\beta}$:

$$\mathcal{R}_i \triangleq \overline{\bigcup_{0 \le \alpha, \beta \le 1} \mathcal{R}_i^{\alpha, \beta}}.$$
 (26)

Theorem 3: For the Gaussian channel with partially cognitive radio as described in (1), the region described by

$$\mathcal{R}_{in} = \{ (R_0, R_1, R_2) \in \mathcal{R}_i : R_1 \ge \mu R_0 \}$$
(27)

is achievable.

Proof: In proving the theorem, we use an encoding strategy that combines superposition coding and dirty paper coding. We first describe the encoding strategy at the two transmitters.

Encoding Strategy at legitimate transmitter: For every message $W_0 \in \{1, \ldots, M_0\}$, the legitimate transmitter generates a codeword $X_{10}^n(W_0)$ from the distribution $p(X_{10}^n) = \prod_{i=1}^n p(X_{10}(i))$ and $X_{10}(i) \sim \mathcal{N}(0, \beta P_1)$ for some $0 \leq \beta \leq 1$. For every message $W_1 \in \{1, \ldots, M_1\}$, the legitimate transmitter generates a codeword $X_{11}^n(W_1)$ from the distribution $p(X_{11}^n) = \prod_{i=1}^n p(X_{11}(i))$ and $X_{11}(i) \sim \mathcal{N}(0, (1 - \beta)P_1)$. The legitimate transmitter then superimposes these codewords to form the net codeword X_1^n as

$$X_1^n = X_{10}^n + X_{11}^n.$$

Encoding strategy at cognitive transmitter: The cognitive transmitter allocates a portion of its power in communicating the message W_0 to the legitimate receiver. For message W_0 , the cognitive transmitter generates a codeword $X_{20}^n(W_0)$ as follows:

$$X_{20}^{n}(W_{0}) = \sqrt{\frac{(1-\alpha)P_{2}}{\beta P_{1}}} X_{10}^{n}(W_{0}).$$

That is, the cognitive transmitter uses the same codeword for encoding message W_0 as used by the legitimate transmitter except that it is scaled to power $(1-\alpha)P_2$ for some $0 \le \alpha \le 1$. Next, the cognitive transmitter encodes message W_2 to codeword X_{22}^n . The codeword is generated using dirty paper coding treating $aX_{10}^n + X_{20}^n$ as non-causally known interference. A characteristic feature of Costa's dirty paper coding is that the codeword X_{22}^n is independent of the interference $X_{20}^n + aX_{10}^n$, and is distributed as $p(X_{22}^n) = \prod_{i=1}^n p(X_{22}(i))$ and $X_{22}(i) \sim \mathcal{N}(0, \alpha P_2)$. The cognitive transmitter superimposes the two codewords X_{20}^n and X_{22}^n to form its net codeword X_2^n . That is,

$$X_2^n = X_{20}^n + X_{22}^n.$$

Next, we describe the decoding strategy and the rate constraints associated at the two receivers.

Decoding strategy at legitimate receiver: The legitimate receiver obtains the signal

$$Y_1^n = X_{10}^n + X_{11}^n + bX_{20}^n + bX_{22}^n + Z_1^n$$

The receiver first decodes message W_1 treating X_{10}^n, X_{20}^n and X_{22}^n as Gaussian noise. After decoding message W_1 , the receiver decodes message W_0 by treating X_{22}^n as Gaussian noise after canceling out X_{11}^n . In the first stage, the receiver can decode message W_1 successfully if

$$R_{1} \leq \frac{1}{2} \log \left(1 + \frac{(1-\beta)P_{1}}{1+\beta P_{1}+b^{2}P_{2}+2b\sqrt{\beta(1-\alpha)P_{1}P_{2}}} \right).$$
(28)

Similarly, the receiver can decode message W_0 successfully if

$$R_0 \le \frac{1}{2} \log \left(1 + \frac{\beta P_1 + b^2 (1 - \alpha) P_2 + 2b \sqrt{\beta (1 - \alpha) P_1 P_2}}{1 + b^2 \alpha P_2} \right)$$
(29)

Decoding strategy at cognitive receiver: The cognitive receiver obtains the signal

$$Y_2^n = aX_{10}^n + aX_{11}^n + X_{20}^n + X_{22}^n + Z_2^n$$

Similar to the legitimate receiver, the cognitive receiver first decodes message W_1 treating X_{10}^n, X_{20}^n and X_{22}^n as Gaussian

noise. The receiver can decode message W_1 successfully if

$$R_{1} \leq \frac{1}{2} \log \left(1 + \frac{a^{2}(1-\beta)P_{1}}{1 + a^{2}\beta P_{1} + P_{2} + 2a\sqrt{\beta(1-\alpha)P_{1}P_{2}}} \right).$$
(30)

After decoding message W_1 , the cognitive receiver decodes message W_2 using Costa's dirty paper decoding. In decoding message W_2 , the cognitive receiver sees only Z_2^n as Gaussian noise. X_{10}^n and X_{20}^n do not appear as noise as they were canceled out at the encoder side using Costa's dirty paper coding. Hence, the receiver can decode message W_2 successfully if

$$R_2 \le \frac{1}{2}\log(1+\alpha P_2).$$
 (31)

Hence, the region described by \mathcal{R}_{in} in (27) is achievable in the Gaussian partially cognitive radio channel. This completes the proof of Theorem 3.

Remark 1: It should be noted here that the cognitive receiver first cancels the interference due to message W_1 before decoding message W_2 . This places a constraint on rate R_1 given by (30). Ideally, we would want the constraint on R_1 given by (28) to be more binding than the constraint on R_1 given by (30). This is possible if

$$\frac{a^{2}(1-\beta)P_{1}}{1+a^{2}\beta P_{1}+P_{2}+2a\sqrt{\beta(1-\alpha)P_{1}P_{2}}} \geq \frac{(1-\beta)P_{1}}{1+\beta P_{1}+b^{2}P_{2}+2b\sqrt{\beta(1-\alpha)P_{1}P_{2}}}.$$
(32)

V. CONDITIONS OF OPTIMALITY OF ACHIEVABLE REGION

In this section, we compare the achievable region and the outer bound and derive conditions when the two meet. We say that the achievable region described in Section IV is (μ_0, μ_1, μ_2) optimal if

$$\max_{\substack{(R_0,R_1,R_2)\in\mathcal{R}_{in}}} \mu_0 R_0 + \mu_1 R_1 + \mu_2 R_2$$

=
$$\max_{\substack{(R_0,R_1,R_2)\in\mathcal{R}_{out}}} \mu_0 R_0 + \mu_1 R_1 + \mu_2 R_2$$
(33)

Let (R_0^o, R_1^o, R_2^o) be (μ_0, μ_1, μ_2) optimal with respect to the outer bound. That is,

$$(R_0^o, R_1^o, R_2^o) = \operatorname*{arg\,max}_{(R_0, R_1, R_2) \in \mathcal{R}_{out}} \mu_0 R_0 + \mu_1 R_1 + \mu_2 R_2.$$
(34)

Let (α^o, β^o) be the optimal power splits at the two transmitters that maximizes the (μ_0, μ_1, μ_2) sum rate with respect to the outer bound. That is,

$$(\alpha^{o}, \beta^{o}) = \underset{0 \le \alpha, \beta \le 1}{\arg \max} \max_{(R_{0}, R_{1}, R_{2}) \in \mathcal{R}_{out}^{\alpha, \beta}} \mu_{0} R_{0} + \mu_{1} R_{1} + \mu_{2} R_{2}.$$
(35)

Then, we have the following lemma.

Lemma 5: $\beta^{o} = 1$ for all $(\mu_{0}, \mu_{1}, \mu_{2})$.

The proof of the lemma follows from the observation that $\mathcal{R}_{out}^{\alpha,1} \supseteq \mathcal{R}_{out}^{\alpha,\beta}$ for all $0 \leq \beta \leq 1$. We next look at the conditions when the achievable region meets the outer bound.

We first consider the case $\mu_0 \ge \mu_1$. Then, we have

$$\begin{aligned} R_0^o + R_1^o &= \frac{1}{2} \log \left(1 + \frac{P_1 + b^2 (1 - \alpha^o) P_2 + 2b \sqrt{\beta^o (1 - \alpha^o) P_1 P_2}}{1 + b^2 \alpha^o P_2} \right), \\ R_1^o &= \mu R_0^o \\ R_2^o &= \frac{1}{2} \log(1 + \alpha^o P_2). \end{aligned}$$
(36)

The conditions for optimality are then given by the following lemma.

Lemma 6: If the following two conditions are satisfied

$$\geq \frac{\frac{a^2}{1+a^2\beta^o P_1 + P_2 + 2a\sqrt{(1-\alpha^o)P_1P_2}}}{\frac{1}{1+\beta^o P_1 + b^2P_2 + 2b\sqrt{\beta^o(1-\alpha^o)P_1P_2}}},$$
(37)

$$\log\left(1 + \frac{P_1}{1 + b^2 P_2}\right) \ge \mu \log\left(1 + \frac{b^2(1 - \alpha^\circ)P_2}{1 + b^2\alpha^\circ P_2}\right) , \qquad (38)$$

then the achievable region is (μ_0, μ_1, μ_2) sum optimal for $\mu_0 \ge \mu_1$.

Proof: The proof of the lemma is fairly simple and we briefly explain the two conditions.

The first condition comes from ensuring that constraint on R_1 in the achievable region due to decoding message m_1 at the legitimate receiver is more binding than the constraint due to decoding message m_1 at the cognitive receiver.

The second condition comes in ensuring that the point which maximizes the (μ_0, μ_1, μ_2) sum in $\mathcal{R}_{out}^{\alpha^0, 1}$ is also achievable. The main details of the proof are omitted here.

Next, we consider the case $\mu_0 < \mu_1$. In this case, R_0^o, R_1^o and R_2^o are given by

$$\begin{aligned} R_0^o &= 0\\ R_1^o &= \frac{1}{2} \log \left(1 + \frac{P_1 + b^2 (1 - \alpha^o) P_2 + 2b \sqrt{(1 - \alpha^o) P_1 P_2}}{1 + b^2 \alpha^o P_2} \right) \quad (39)\\ R_2^o &= \frac{1}{2} \log (1 + \alpha^o P_2). \end{aligned}$$

The condition of optimality when $\mu_0 < \mu_1$ is given by the following lemma.

Lemma 7: When $\mu_0 < \mu_1$, if we have $\alpha^o = 1$, then the achievable region is (μ_0, μ_1, μ_2) sum optimal.

The proof of the lemma follows from the argument that if $\alpha^o = 1$, then the corresponding point (R_0^o, R_1^o, R_2^o) is also achievable by substituting $\alpha = 1$ and $\beta = 0$.

VI. CONCLUSIONS

In this paper, we investigated the capacity region of interference channel with partially cognitive radios. For the general discrete memoryless IFC setting, we obtained the outer bound for the capacity region when the legitimate receiver observes the weak interference. And, for a mixed interference Gaussian channel, we showed that the portions of the outer bound can be achieved.

VII. ACKNOWLEDGMENT

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