Feasible alphabets for communicating the sum of sources over a network

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Abstract—We consider directed acyclic sum-networks with m sources and n terminals where the sources generate symbols from an arbitrary alphabet field F, and the terminals need to recover the sum of the sources over F. We show that for any co-finite set of primes, there is a sum-network which is solvable only over fields of characteristics belonging to that set. We further construct a sum-network where a scalar solution exists over all fields other than the binary field F_2 . We also show that a sum-network is solvable over a field if and only if its reverse network is solvable over the same field.

I. INTRODUCTION

After its introduction by the seminal work by Ahlswede et al. [1], the field of network coding has seen an explosion of interest and development. See, for instance, [2], [3], [4], [5] for some early development in the area. The work by Dougherty et al. in [6], [7] are specially relevant in the context of this paper for the nature of results and the proof techniques. They defined a network with specific demands of the terminals to be scalar linear solvable (resp. vector linear solvable) over a field F_q if there exists a scalar linear network code (resp. vector linear network code) over F_q which satisfies the demands of all the terminals. A prime p is said to be a characteristic of a network if the network is solvable over some finite field of characteristic p. They showed that for any finite or co-finite set of primes, there exists a network where the given set is the set of characteristics of the network.

In most of the past work, the terminal nodes have been considered to require the recovery of all or part of the sources' data. A more general setup is where the terminals require to recover some functions of the sources' data. Recently, the problem of communicating the sum of sources to some terminals was considered in [8], [9]. We call such a network as a sum-network. It was shown in [8] that if there are two sources or two terminals in the network, then the sum of the sources can be communicated to the terminals if and only if every source is connected to every terminal. While this condition is also necessary for any number of sources and terminals, it may not be sufficient. In [9], the authors showed that for any finite set of prime numbers, there exists a network where the sum of the sources can be communicated to the terminals using scalar or vector linear network coding if and only if the characteristic of the alphabet field belongs to the given set.

It is worth mentioning that the problem of distributed function computation in general has been considered in different contexts in the past. The work in [10], [11], [12], [13], [14] is only to mention a few.

Given a multiple unicast network, its reverse network is obtained by reversing the direction of all the links and interchanging the role of source and destination for each sourcedestination pair. It is known ([15], [16]) that a multiple unicast network is linearly solvable if and only if its reverse network is linearly solvable. However, there are multiple-unicast networks which are solvable by nonlinear network coding but whose reverse networks are not solvable ([15], [16]).

In this paper, we consider a directed acyclic network with unit-capacity links. We prove the following results.

- For every co-finite set of prime numbers, there exists a directed acyclic network of unit-capacity links with some sources and terminals so that the sum of the sources can be communicated to all the terminals using vector or scaler network coding if and only if the characteristic of the alphabet field belongs to the given set. This result complements the result in [9].
- We construct a network where the sum of the sources can be communicated to the terminals over all fields except the binary field F_2 . This shows that whether the sum of the sources can be communicated to the terminals in a network using scalar linear network coding over a field does not depend only on the characteristic of the field. It may also depend further on the order of the field.
- The sum of the sources can be communicated to the terminals in a network over some alphabet field using linear network coding if and only if the same is true for the reverse network.

Proof techniques of this paper are similar to that in [9].

In Section II, we introduce the system model. The results of this paper are presented in Section III and Section IV. We conclude the paper with a short discussion in Section V.

II. SYSTEM MODEL

A sum-network is represented by a directed acyclic graph G = (V, E) where V is a finite set denoting the vertices of the network, $E \subseteq V \times V$ is the set of edges. Among the vertices, there are m sources $s_1, s_2, \dots, s_m \in V$, and n terminals $t_1, t_2, \dots, t_n \in V$ in the network. For any edge

 $e = (i, j) \in E$, the node j will be called the head of the edge and the node i will be called the tail of the edge; and they will be denoted as head(e) and tail(e) respectively. Throughout the paper, p, possibly with subscripts, will denote a positive prime integer, and q will denote a power of a prime. Let \mathbb{F}_q denote the alphabet field. Each link in the network is assumed to be capable of carrying a symbol from \mathbb{F}_q in each use. Each link is used once in every symbol interval and this time is taken as the unit time. Each source generates one symbol from \mathbb{F}_q in every symbol interval, and each terminal requires to recover the sum of the source symbols (over \mathbb{F}_q).

For any edge $e \in E$, let $Y_e \in \mathbb{F}_q$ denote the message transmitted through e. In scalar linear network coding, each node computes a linear combination of the incoming symbols for transmission on an outgoing link. That is,

$$Y_e = \sum_{e':head(e')=tail(e)} \alpha_{e',e} Y_{e'}$$
(1)

when tail(e) is not a source node. Here $\alpha_{e',e} \in \mathbb{F}_q$ are called the local coding coefficients. A source node computes a linear combination of some data symbols generated at that source for transmission on an outgoing link, that is,

$$Y_e = \sum_{i:X_i \text{ generated at } tail(e)} \beta_{j,e} X_j \qquad (2)$$

for some $\beta_{j,e} \in \mathbb{F}_q$ if tail(e) is a source node. Since each source generates one symbol from \mathbb{F}_q per unit time, there is only one term in the summation in (2) and $\beta_{j,e}$ can be taken to be 1 without loss of generality. The decoding operation at a terminal involves taking a linear combination of the incoming messages to recover the required data.

In vector linear network coding, the data stream generated at each source node is blocked in vectors of length N. The coding operations are similar to (1) and (2) with the difference that, now $Y_e, Y_{e'}, X_j$ are vectors from \mathbb{F}_q^N , and $\alpha_{e',e}, \beta_{j,e}$ are matrices from $\mathbb{F}_q^{N \times N}$. It is known that scalar linear network coding may give better throughput in some networks than that is achievable by routing. Vector linear network coding may give further improvement over scalar linear network coding in some networks [17], [5], [18].

A sequence of nodes (v_1, v_2, \ldots, v_l) is called a path, denoted as $v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_l$, if $(v_i, v_{i+1}) \in E$ for $i = 1, 2, \ldots, l - 1$. Given a network code on the network, $\prod_{i=1}^{l-2} \alpha_{(v_i, v_{i+1}), (v_{i+1}, v_{i+2})}$ is called the path gain of the path $v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_l$.

A sum-network is said to be N-length vector linear solvable over F_q if there is a N-length vector linear network code so that each terminal recovers the sum of the N-length vectors over F_q generated at all the sources. Scalar linear solvability of a sum-network is defined similarly.

III. RESULTS

In [9], a special network S_m was defined where the sum of the sources can be communicated to the terminals using scalar or vector linear network coding only over fields of characteristics dividing m - 2. For $m \ge 3$, we now define a network $\mathcal{S}_m^* \stackrel{\triangle}{=} (V(\mathcal{S}_m^*), E(\mathcal{S}_m^*))$ which has four layers of vertices $V(\mathcal{S}_m^*) = S \cup U \cup V \cup T$. The first layer of nodes are the m-1 source nodes $S \stackrel{\triangle}{=} \{s_1, s_2, \ldots, s_{m-1}\}$. The second and third layers have m-1 nodes each, and they are denoted as $U \stackrel{\triangle}{=} \{u_1, u_2, \ldots, u_{m-1}\}$ and $V \stackrel{\triangle}{=} \{v_1, v_2, \ldots, v_{m-1}\}$ respectively. The last layer consists of the m terminal nodes $T \stackrel{\triangle}{=} \{t_1, t_2, \ldots, t_m\}$. For every $i = 1, 2, \ldots, m-1$, there is an edge from s_i to t_i , u_i to v_i , v_i to t_i , and from v_i to t_m . That is, $(s_i, t_i), (u_i, v_i), (v_i, t_i), (v_i, t_m) \in E(\mathcal{S}_m^*)$ for each $i = 1, 2, \ldots, m-1$. Also for every $i, j = 1, 2, \ldots, m-1$, $i \neq j$, there is an edge from s_i to u_j . So, the set of edges is given by

$$E(\mathcal{S}_m^*) = \bigcup_{i=1}^{m-1} \{(s_i, t_i), (u_i, v_i), (v_i, t_i), (v_i, t_m)\}$$
$$\cup \{(s_i, u_j) : i, j = 1, 2, \dots, m-1, i \neq j\}$$

The network is shown in Fig. 1.

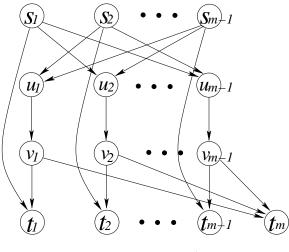


Fig. 1. The network \mathcal{S}_m^*

Now we present a lemma which will be used to prove one of the main results of this paper.

Lemma 1: For any positive integer N, the network S_m^* is N-length vector linear solvable if and only if the characteristic of the alphabet field does not divide m - 2.

Proof: First we note that every source-terminal pair in the network S_m^* is connected. This is clearly a necessary condition for being able to communicate the sum of the source messages to each terminal node over any field.

We now prove that if it is possible to communicate the sum of the source messages using vector linear network coding over \mathbb{F}_q to all the terminals in \mathcal{S}_m^* , then the characteristic of \mathbb{F}_q must not divide m - 2. As in (1) and (2), the message carried by an edge e is denoted by Y_e . For $i = 1, 2, \ldots, m$, the message vector generated by the source s_i is denoted by $X_i \in \mathbb{F}_q^N$. Each terminal t_i computes a linear combination R_i of the received vectors.

Local coding coefficients/matrices used at different layers in the network are denoted by different symbols for clarity. The message vectors carried by different edges and the corresponding local coding coefficients are as below.

$$Y_{(s_i,t_i)} = \alpha_{i,i}X_i \text{ for } 1 \le i \le m-1,$$

$$Y_{(s_i,t_i)} = \alpha_{i,i}X_i$$
(3a)

for
$$1 \le i, j \le m - 1, i \ne j$$
, (3b)

$$Y_{(u_i,v_i)} = \sum_{\substack{j=1\\j\neq i}} \beta_{j,i} Y_{(s_j,u_i)}$$

for $1 \le i \le m-1$, (3c)

$$R_{i} = \gamma_{i,1} Y_{(s_{i},t_{i})} + \gamma_{i,2} Y_{(v_{i},t_{i})}$$

for $1 < i < m - 1$, (4a)

$$R_m = \sum_{j=1}^{m-1} \gamma_{j,m} Y_{(v_j,t_m)}.$$
 (4b)

Here all the coding coefficients $\alpha_{i,j}$, $\beta_{i,j}$, $\gamma_{i,j}$ are $N \times N$ matrices over \mathbb{F}_q , and the message vectors X_i and the messages carried by the links $Y_{(.,.)}$ are length-N vectors over \mathbb{F}_q .

Without loss of generality (w.l.o.g.), we assume that $Y_{(v_i,t_i)} = Y_{(v_i,t_m)} = Y_{(u_i,v_i)}$ and $\alpha_{i,i} = \alpha_{i,j} = I$ for $1 \le i, j \le m - 1, i \ne j$, where I denotes the $N \times N$ identity matrix.

By assumption, each terminal decodes the sum of all the source messages. That is,

$$R_i = \sum_{j=1}^{m-1} X_j \text{ for } i = 1, 2, \dots, m$$
(5)

for all values of $X_1, X_2, \ldots, X_{m-1} \in \mathbb{F}_q^N$.

From equations (3) and (4), we have

$$R_i = \sum_{\substack{j=1\\j\neq i}}^{m-1} \gamma_{i,2}\beta_{j,i}X_j + \gamma_{i,1}X_i \tag{6}$$

for i = 1, 2, ..., m - 1, and

$$R_m = \sum_{i=1}^{m-1} \gamma_{i,m} \left(\sum_{\substack{j=1\\j\neq i}}^{m-1} \beta_{j,i} X_j \right)$$
$$= \sum_{j=1}^{m-1} \left(\sum_{\substack{i=1\\i\neq j}}^{m-1} \gamma_{i,m} \beta_{j,i} \right) X_j.$$
(7)

Since (5) is true for all values of $X_1, X_2, \ldots, X_m \in \mathbb{F}_q^N$, equations (6) and (7) imply

$$\gamma_{i,2}\beta_{j,i} = I \text{ for } 1 \le i, j \le m - 1, i \ne j, \qquad (8)$$

$$\gamma_{i,1} = I \text{ for } 1 \le i \le m - 1, \tag{9}$$

$$\sum_{\substack{i=1\\i\neq j}}^{m-1} \gamma_{i,m} \beta_{j,i} = I \text{ for } 1 \le j \le m-1.$$
 (10)

All the coding matrices in equations (8),(9) are invertible since the right hand side of the equations are the identity matrix. Equations (8) imply $\beta_{j,i} = \beta_{k,i}$ for $1 \le i, j, k \le m - 1$, $j \ne i \ne k$. So, let us denote all the equal co-efficients $\beta_{j,i}$; $1 \le j \le m - 1, j \ne i$ by β_i . Then (10) can be rewritten as

$$\sum_{\substack{i=1\\j\neq j}}^{m-1} \gamma_{i,m}\beta_i = I \text{ for } 1 \le j \le m-1.$$
(11)

Equation (11) implies

=

$$\gamma_{i,m}\beta_i = \gamma_{j,m}\beta_j$$
 for $1 \le i, j \le m - 1, i \ne j$

Then (11) gives

$$(m-2)\gamma_{1,m}\beta_1 = I$$

 $\Rightarrow \gamma_{1,m}\beta_1 = (m-2)^{-1}I.$ (12)

Equation (12) implies that the matrix $\gamma_{1,m}\beta_1$ is a diagonal matrix and all the diagonal elements are equal to $(m-2)^{-1}$. But the inverse of (m-2) exists over the alphabet field if and only if the characteristic of the field does not divide (m-2). So, the sum of the source messages can be communicated in \mathcal{S}_m^* by N-length vector linear network coding over F_q only if the characteristic of \mathbb{F}_q does not divide (m-2).

Now, if the characteristic of \mathbb{F}_q does not divide (m-2), then for any block length N, in particular for scalar network coding for N = 1, every coding matrix in (3a)-(3c) can be chosen to be the identity matrix. The terminals $t_1, t_2, \cdots, t_{m-1}$ then can recover the sum of the source messages by taking the sum of the incoming messages, i.e., by taking $\gamma_{i,1} = \gamma_{i,2} = I$ for $1 \le i \le m-1$ in (4a). Terminal t_m recovers the sum of the source messages by taking $\gamma_{i,m}, 1 \le i \le m-1$ in (4b) as diagonal matrices having diagonal elements as inverse of (m-2). The inverse of (m-2) exists over F_q because the characteristic of F_q does not divide (m-2).

Lemma 1 gives the following theorem.

Theorem 2: For any finite set $\mathcal{P} = \{p_1, p_2, \ldots, p_l\}$ of positive prime numbers, there exists a directed acyclic sumnetwork of unit-capacity edges where for any positive integer N, the network is N-length vector linear solvable if and only if the characteristic of the alphabet field does not belong to \mathcal{P} .

Proof: Consider the network S_m^* for $m = p_1 p_2 \dots p_l + 2$. This network satisfies the condition in the theorem by Lemma 1.

We note that the alphabet field in Theorem 2 may also be an infinite field of non-zero characteristic. In particular, the theorem also applies to the field of rationals $F_q(X)$ over F_q . So, the sum-network in Theorem 2 is also solvable using linear convolutional network code over F_q if and only if the characteristic of F_q is not in \mathcal{P} .

Now we define another sum-network G_1 with the set of vertices $V(G_1) \stackrel{\triangle}{=} \bigcup_{i=1}^3 \{s_i, u_i, v_i, t_i\}$, edges $E(G_1) \stackrel{\triangle}{=} \{(u_i, v_i) | i = 1, 2, 3\} \cup \{(s_i, u_j), (v_i, t_j) | i, j = 1, 2, 3, i \neq j\}$. The network is shown in Fig. 2. The nodes s_1, s_2, s_3 are the sources and the nodes t_1, t_2, t_3 are the terminals in the network. The symbols generated at the sources are denoted by X, Z, and W respectively.

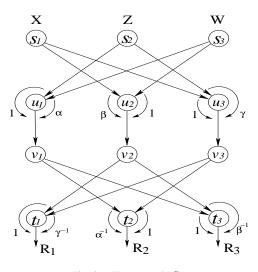


Fig. 2. The network G_1

The following lemma gives our second main result.

Lemma 3: The sum-network G_1 is scalar linear solvable over all fields other than F_2 .

Proof: The message vectors carried by different edges and the corresponding local coding coefficients are as below. Without loss of generality, we assume

$$Y_{(s_1,u_2)} = Y_{(s_1,u_3)} = X, (13a)$$

$$Y_{(s_2,u_1)} = Y_{(s_2,u_3)} = Z,$$
 (13b)

$$Y_{(s_3,u_1)} = Y_{(s_3,u_2)} = W,$$
 (13c)

and

$$Y_{(u_1,v_1)} = Y_{(s_2,u_1)} + \alpha Y_{(s_3,u_1)}, \quad (14a)$$

$$Y_{(u_2,v_2)} = Y_{(s_3,u_2)} + \beta Y_{(s_1,u_2)}, \quad (14b)$$

$$Y_{(u_3,v_3)} = Y_{(s_1,u_3)} + \gamma Y_{(s_2,u_3)}, \qquad (14c)$$

where $\alpha, \beta, \gamma \in F_q$.

Also, w.l.o.g, we assume that

$$Y_{(u_1,v_1)} = Y_{(v_1,t_2)} = Y_{(v_1,t_3)},$$
 (15a)

$$Y_{(u_2,v_2)} = Y_{(v_2,t_1)} = Y_{(v_2,t_3)},$$
 (15b)

$$Y_{(u_3,v_3)} = Y_{(v_3,t_1)} = Y_{(v_3,t_2)}.$$
 (15c)

Since there is only one path $s_2 \rightarrow u_3 \rightarrow v_3 \rightarrow t_1$ from source s_2 to terminal t_1 and also one path $s_3 \rightarrow u_2 \rightarrow v_2 \rightarrow t_1$ from source s_3 to t_1 with path gains γ and 1 respectively, the recovered symbol R_1 at t_1 must be

$$R_1 = Y_{(v_2,t_1)} + \gamma^{-1} Y_{(v_3,t_1)}.$$
 (16a)

Similarly, the recovered symbols R_1 and R_2 should be

$$R_2 = Y_{(v_3,t_2)} + \alpha^{-1} Y_{(v_1,t_2)}, \qquad (16b)$$

$$R_3 = Y_{(v_1,t_3)} + \beta^{-1} Y_{(v_2,t_3)}.$$
(16c)

The coding coefficients are depicted in Fig. 2 for clarity. From equations (13), (14), (15) and (16) it follows that

$$R_1 = (\beta + \gamma^{-1})X + Z + W,$$
(17a)

$$R_2 = X + (\gamma + \alpha^{-1})Z + W,$$
 (17b)

$$R_3 = X + Z + (\alpha + \beta^{-1})W.$$
(17a)

Note that equation (17) requires that the coding coefficients α , β and γ be non-zero. This requirement can also be seen as natural since if any of these coefficients is zero, then a particular source-terminal pair will be disconnected.

Since all the terminals must recover the sum of the source messages, i.e., $R_1 = R_2 = R_3 = X + Z + W$, we have

$$\beta + \gamma^{-1} = 1, \tag{18a}$$

$$\gamma + \alpha^{-1} = 1, \tag{18b}$$

$$\alpha + \beta^{-1} = 1. \tag{18c}$$

Now, over the binary field the values of α , β and γ must all be 1. Putting $\alpha = \beta = \gamma = 1$ in equation (18), we have 1 = 0. This gives a contradiction. So, it is not possible to communicate the sum of the sources to the terminals in this network over the binary field F_2 using scalar linear network coding.

Now, we consider any other finite field F_q ($q \neq 2$). We show that over F_q , the conditions in equation (18) are satisfied for some choice of α, β , and γ .

Since q > 2, let $\alpha \in F_q$ be any element other than 0 and 1. Also, take $\gamma = 1 - \alpha^{-1}$ and $\beta = (1 - \alpha)^{-1}$. Clearly, they satisfy (18a)-(18c). Hence, it is possible to communicate the sum of the source messages to the terminals over F_q .

It is worth noting that though the sum can not be communicated in this network by scalar network coding over F_2 , it is possible to do so by vector network coding over F_2 using any block length N > 1. This follows because it is possible to communicate the sum over the extension field F_{2^N} using scalar network coding.

IV. REVERSIBILITY OF NETWORKS

Given a sum-network (recall the definition from Sec. I) \mathcal{N} , its reverse network \mathcal{N}' is defined to be the network with the same set of vertices, the edges reversed, and the role of sources and terminals interchanged. It should be noted that since \mathcal{N} may have unequal number of sources and terminals, the number of sources (resp. terminals) in \mathcal{N} and that in \mathcal{N}' may be different. For example, the reverse network \mathcal{S}_m^* of \mathcal{S}_m^* has m sources and m-1 terminals and so the problem in $\mathcal{S}_m^{*'}$ is to communicate the sum of the source messages (say, Y_1, \ldots, Y_m) to the m-1 terminals. In this section, we show that for any sum-network \mathcal{N} and any alphabet field F_q , the sum-network \mathcal{N} is N-length vector linear solvable over F_q if and only if its reverse network \mathcal{N}' is N-length vector linear solvable over F_q .

Consider a generic sum-network \mathcal{N} depicted in Fig. 3. Consider the cuts C_1 and C_2 shown in the figure. We call these cuts, the *source-cut* and the *terminal-cut* of the sum-network respectively. The transfer function from C_1 to C_2 is defined to be the $m \times n$ matrix T over F_q which relates the vectors $\mathbf{X} = (X_1, X_2, \dots, X_m)$ and $\mathbf{R} = (R_1, R_2, \dots, R_n)$ as

$\mathbf{R}=\mathbf{X}T.$

(17b) In case of N-length vector linear coding, (17c) $X_1, X_2, \ldots, X_m, R_1, R_2, \ldots, R_n \in F_q^N, \mathbf{X} \in F_q^{mN}$, and $\mathbf{R} \in F_q^{nN}$. The transfer matrix is a $mN \times nN$ matrix which is easier viewed as an $m \times n$ matrix of $N \times N$ blocks. The (i, j)-th element ('block' for vector linear coding) of the transfer matrix is the sum of the path gains of all paths from X_i to R_j . The following lemma follows directly.

Lemma 4: A sum-network \mathcal{N} is N-length vector linear solvable if and only if there is an N-length vector linear network code so that each element/block of the transfer matrix from the source-cut to the terminal-cut is the $N \times N$ identity matrix.

Now consider the reverse network \mathcal{N}' of \mathcal{N} . Let us denote the source symbols in \mathcal{N}' as Y_1, Y_2, \ldots, Y_n and the recovered symbols at the terminals as R'_1, R'_2, \ldots, R'_m . Let us denote the network coding coefficients of \mathcal{N}' by $\beta_{e,e'}$ for any two edges $e, e' \in E(\mathcal{N}')$ so that head(e) = tail(e'). Let us denote the edge in \mathcal{N}' obtained by reversing the edge $e \in E(\mathcal{N})$ by \tilde{e} . Clearly, there is a 1-1 correspondence between the paths in \mathcal{N} and the paths in \mathcal{N}' . So, if there is a N-length vector linear network code over F_q which solves the sum-network \mathcal{N} , then \mathcal{N}' will also be N-length vector linear solvable over F_a if there is an N-length vector linear network code for \mathcal{N}' which results in the same path gain for each path in \mathcal{N}' as that for the corresponding path in \mathcal{N} . In that case, the transfer matrix from the source-cut to the terminal-cut in \mathcal{N}' for that network code will be the transpose of the transfer matrix for the network code for \mathcal{N} . Now, suppose $\{\alpha_{e,e'} \mid e, e' \in E(\mathcal{N}), head(e) =$ tail(e') is the network code which solves the sum-network $\mathcal{N}.$ Then clearly the network code $\{\beta_{\tilde{e'},\tilde{e}} \stackrel{\triangle}{=} \alpha_{e,e'} \mid e,e' \in \mathbb{R}\}$ $E(\mathcal{N}), head(e) = tail(e')$ results in a transfer matrix with all blocks as I for \mathcal{N}' , and thus solves the sum-network \mathcal{N}' . So, we have our final result:

Theorem 5: A sum-network \mathcal{N} is N-length vector linear solvable over F_q if and only if its reverse network \mathcal{N}' is also N-length vector linear solvable over F_q .

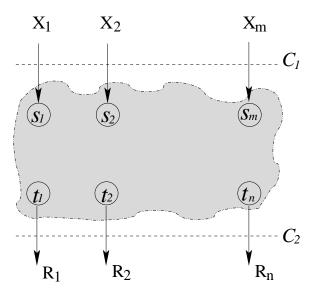


Fig. 3. A generic sum-network

V. DISCUSSION

We have presented some results on communicating the sum of source messages to a set of terminals. It was shown in [7] that there is a 1 - 1 correspondence between systems of polynomial equations and networks. This is a key result which implies existence of networks with arbitrary finite or co-finite characteristic set. Though sum-networks have very specific demands by the terminal nodes and thus are more restricted as a class, a complete characterization of the systems of polynomial equations which have equivalent sum-networks is not yet known. Investigation in this direction is in progress.

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