Dirty Paper Coding for the MIMO Cognitive Radio Channel with Imperfect CSIT

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***Abstract*—A Dirty Paper Coding (DPC) based transmission scheme for the Gaussian multiple-input multiple-output (MIMO) cognitive radio channel (CRC) is studied when there is imperfect and perfect channel knowledge at the transmitters (CSIT) and the receivers, respectively. In particular, the problem of optimizing the sum-rate of the MIMO CRC over the transmit covariance matrices is dealt with. Such an optimization, under the DPC-based transmission strategy, needs to be performed jointly with an optimization over the inflation factor. To this end, first the problem of determination of inflation factor over the MIMO channel $Y = H_1X + H_2S + Z$ with imperfect CSIT is investigated. For this problem, two iterative algorithms, which generalize the corresponding algorithms proposed for the channel $Y = H(X + S) + Z$, are developed. Later, the necessary conditions for maximizing the sum-rate of the MIMO CRC over the transmit covariances for a given choice of inflation factor are derived. Using these necessary conditions and the algorithms for the determination of the inflation factor, an iterative, numerical algorithm for the joint optimization is proposed. Some interesting observations are made from the numerical results obtained from the algorithm. Furthermore, the high-SNR sum-rate scaling factor achievable over the CRC with imperfect CSIT is obtained.**

***Index Terms*—Cognitive radio, dirty paper coding, inflation factor, covariance optimization.**

I. Introduction

The cognitive radio channel (CRC) was introduced in [1]. A cognitive radio is a device that can sense its environment in real time and can accordingly adapt its transmission strategy. These are of current interest because of the dramatically high spectral efficiency they can achieve [1]. In [1], the authors introduced a more general cognitive protocol under which the CRC is an interference channel with degraded message sets [2].

The Gaussian multiple-input multiple-output (MIMO) interference channel consists of two transmitter-receiver pairs with each transmitter having a message for its paired receiver and the received signals are defined via equations $Y_1 = H_{11}X_1 + H_{21}X_2 + Z_1$, $Y_2 = H_{12}X_1 + H_{22}X_2 + Z_2$. Here, $\{H_{ij}\}_{i,j=1}^2$ are the fading channel matrices of dimensions $r_j \times t_i$; the transmitted signals $X_1 \sim \mathcal{CN}(0, \Sigma_1)$ and $X_2 \sim \mathcal{CN}(0, \Sigma_2)$ are subject to the power constraints[1] of $\mathrm{tr}(\Sigma_1) \leq P_1$ and $\mathrm{tr}(\Sigma_2) \leq P_2$; and $Z_1 \sim \mathcal{CN}(0, I_{r_1})$ and $Z_2 \sim \mathcal{CN}(0, I_{r_2})$ are the additive noises [2]. The Gaussian MIMO CRC is defined as the Gaussian MIMO interference channel in which the second transmitter (corresponding to signal X_2) or the cognitive transmitter (CT) knows the message (or the codeword) of the first or the primary transmitter (corresponding to signal X_1) non-causally [1].

An achievable-rate region for the Gaussian MIMO CRC has been proposed in [3]. In their coding scheme, the CT, because of its non-causal knowledge, acts as a relay to aid the primary receiver and also transmits its own message to its paired (or cognitive) receiver. It employs dirty paper coding (DPC) [4] to cancel the interference at the cognitive receiver due to the signals intended for the primary receiver. With the assumption that the required channel matrices are known at the transmitters and the receivers, it is further shown in [3] that their achievable-rate region includes points corresponding to the sum-capacity of the MIMO CRC under certain conditions. Now, to achieve the sum-capacity, an optimization over the transmit covariances is required. This problem is studied in [5] where the authors propose the so-called adaptive sum-power iterative waterfilling algorithm which computes the sum-capacity and the optimal transmit covariances.

Although the MIMO CRC is increasingly being studied under the assumption of perfect transmitter-channel-knowledge (CSIT) [6], [3], [5], [7], etc., not many papers [8] exist which deal with the practically important scenario of imperfect-CSIT CRC. We find it timely to consider the aforementioned problem of covariance optimization under imperfect CSIT. Towards this end, one needs to seek answers to the following two questions:

1) *DPC at the CT under imperfect CSIT:* Since the channel seen by the cognitive receiver is of the form $Y = H_1X + H_2S + Z$ (this will become more clear in Section III), where S is the interference known non-causally at the CT but not at its receiver, it is imperative to first study the problem of DPC over this channel when there is imperfect CSIT of H_1 and H_2. This problem is equivalent to the determination of the optimal inflation factor (see [4]) under imperfect CSIT. We studied a similar problem for the fading dirty paper channel (FDPC) $Y = H(X + S) + Z$ in [9] and developed two iterative algorithms for determination of inflation factor. These algorithms significantly improve the prior attempts mentioned therein. The same problem for the channel $Y = H_1X + H_2S + Z$, which we call the Generalized FDPC (G-FDPC), has been considered in [10], but only in the special case of (all) single-antenna terminals, and a suboptimal solution is proposed. We study this important problem in Section II of this paper.

2) *Covariance optimization under imperfect CSIT:* The problem of covariance optimization is considerably in-

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[1] Notation: For a square matrix A, $\mathrm{tr}(A)$, $|A|$, and $\mathrm{rank}(A)$ denote its trace, determinant, and rank, respectively. For any general matrix A, A^* and A^+ denote the complex-conjugate transpose and pseudo-inverse of matrix A, respectively. $\mathrm{vec}(A)$ denotes the vector obtained by stacking the columns of A. I_m is the $m \times m$ identity matrix. $\mathbb{E}_H$ denotes expectation over the random variable H.

volved, even under perfect CSIT. In [5], rather than using the achievable-rate region of [3], the authors formulate the problem in terms of an outer-bound (obtained in [3]) to the capacity region which includes points corresponding to the sum-capacity of the MIMO CRC. This is a non-convex optimization problem and is converted into an equivalent convex-concave game using the 'MAC-BC' transformations [11]. Then, for the resulting optimization, an iterative numerical algorithm is proposed. Unfortunately, the algorithm can not always guarantee the optimum solution.

The use of the outer-bound or the 'MAC-BC' transformations is not possible under imperfect CSIT. Also, unlike the perfect-CSIT case (under which the interference can be assumed to be canceled perfectly by DPC), under imperfect CSIT, an additional optimization over the inflation factor needs to be performed *jointly* with the transmit covariances. Furthermore, the problem becomes more complicated because the sum-rate optimal solution need not necessarily have the power constraints satisfied with equality (this point is detailed later). Thus the imperfect-CSIT version of this problem is also quite challenging.

A slightly different (due to a constraint on the rate of primary user) version of the problem is considered in [8], [12] for the CRC with all single-antenna terminals. The authors of [8], [12] consider the amplify-and-forward strategy for relaying at the CT, and in this sense, their scheme is less general than the one studied here.

II. DPC over the G-FDPC

Motivation to study this problem will become more clear in Section III. But, as noted before, this is an important step in the overall joint optimization. The G-FDPC is defined via equation $Y = H_1 X + H_2 S + Z$. Here, H_1 and H_2 are the channel matrices of dimensions $r \times t_x$ and $r \times t_s$, respectively; the transmitted signal $X \sim \mathcal{CN}(0, \Sigma_X)$ has a power constraint of P; the interference $S \sim \mathcal{CN}(0, \Sigma_S)$ is known non-causally at the transmitter but not at the receiver; $Z \sim \mathcal{CN}(0, \Sigma_Z)$ is the additive noise; and X, S, and Z are independent. Assume perfect receiver channel knowledge but imperfect CSIT [2]. Assume $|\Sigma_X|$, $|\Sigma_Z| > 0$; let $\mathrm{tr}(\Sigma_S) = Q$, $\mathrm{tr}(\Sigma_Z) = N$. Define SNR $= \frac{P}{N}$ [3]. Select the auxiliary random variable (see [13] for definition) as $U = X + WS$, i.e., Costa's choice [4] extended to the MIMO case, where the $t_x \times t_s$ matrix W is the inflation factor [4]. Similar to [9], we obtain the achievable rate as given by

$$R = \max_W \mathbb{E}_H \log \frac{|\Sigma_X||\Sigma_Z + H_1 \Sigma_X H_1^* + H_2 \Sigma_S H_2^*|}{|\mathrm{M}|} \quad (1)$$

with $\mathrm{M} = \begin{bmatrix} \Sigma_X + W\Sigma_S W^* & \Sigma_X H_1^* + W \Sigma_S H_2^* \\ H_1 \Sigma_X + H_2 \Sigma_S W^* & \Sigma_Z + H_1 \Sigma_X H_1^* + H_2 \Sigma_S H_2^* \end{bmatrix}$, and $H = [H_1 \; H_2]$. The above rate expression is valid only if $|\Sigma_X| > 0$. The case of $|\Sigma_X| = 0$ can be handled as in [14]. We define the no-interference upper-bound R_{noS} as the rate achievable over the G-FDPC in absence of interference (i.e., when $Q = 0$) or $R_{\mathrm{noS}} = \mathbb{E}_{H_1} \log \frac{|\Sigma_Z + H_1 \Sigma_X H_1^*|}{|\Sigma_Z|}$.

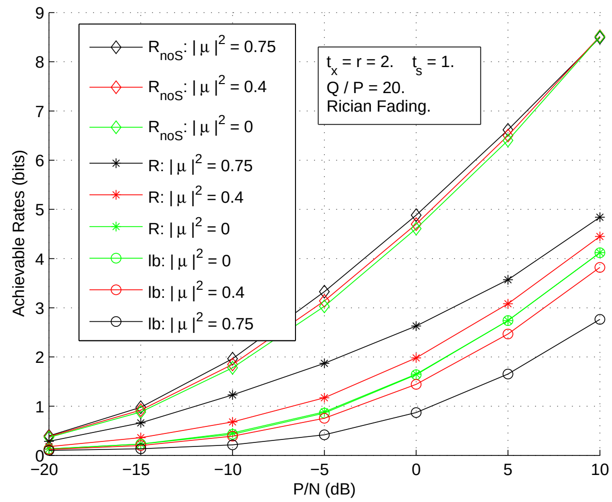

Fig. 1
Achievable Rates vs. SNR: Rician Fading.

The problem of determination of inflation factor, i.e., the maximization in (1) is equivalent to $\min_W \mathbb{E}_{[H_1 \; H_2]} \log |\mathrm{M}|$. As noted in [9], this is a non-convex optimization problem, and it seems intractable to obtain a closed-form solution. It is possible however to generalize our algorithms in [9] developed for the FDPC to the G-FDPC. Due to lack of space, we discuss here the basic idea and omit the details.

In the first algorithm, we minimize the objective function stepwise, i.e., at each step, we minimize over only one row of W, while treating all other rows as constants. Note that only the k^{th} row and the k^{th} column of matrix M depend on the k^{th} row of W. Therefore, the minimization over one row of W (while treating other rows as constants) can be done analytically if the objective function is upper-bounded by moving the expectation inside the logarithm. Thus, one iteration of the algorithm consists of successive (stepwise) minimizations over all rows of W, and these iterations are repeated until a good choice is obtained.

In the second algorithm, we solve for the stationary point of the objective function, i.e., solve an equation $\frac{d}{dW} \mathbb{E}_{[H_1 \; H_2]} \log |\mathrm{M}| = 0$. Using the obtained necessary conditions, an iterative algorithm is proposed.

Numerical Results: Here, 'lb' denotes the rate achievable using $W = 0$, i.e., by treating the interference as noise. R denotes the rate achievable using the algorithms. In Fig. 1, we take H_1 and H_2 to be independent with their elements $\sim$ i.i.d. $\mathcal{CN}(\mu, \sigma^2)$ with $|\mu|^2 + \sigma^2 = 1$ and $\mu = |\mu| \frac{(1+j)}{\sqrt{2}}$. When $\mu = 0$, the all-zero inflation factor performs almost as well as the inflation factor obtained using the algorithms. However, as $|\mu|$ increases, the algorithms outperform the simple choice of $W = 0$. This type of observation was also made in [10] in the case of SISO G-FDPC. It is generalized here to the MIMO case. In Fig. 2, the fad-

[2] We assume that the transmitter only knows the distribution H. The case of partial CSIT can be handled similarly.
[3] Note that N is total noise power.
[4] Matrix W is called the inflation factor so as to be consistent with the terminology introduced by Costa [4].

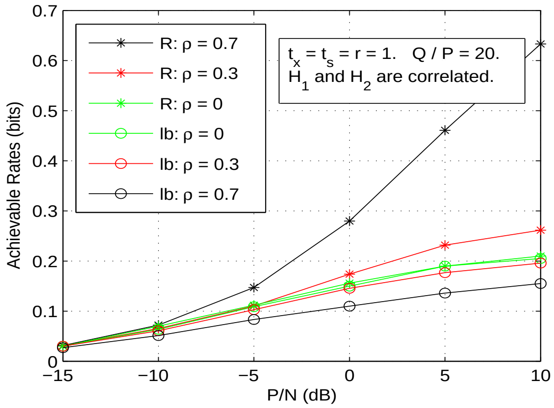

Fig. 2
ACHIEVABLE RATES VS. SNR: H_1 AND H_2 CORRELATED.

ing coefficients are correlated, i.e., H_1, $H_2 \sim \mathcal{CN}(0,1)$ with $\rho = E(H_1^* H_2)$. As ρ increases from 0 to 0.7, the algorithms perform better than simply setting $W=0$.

A considerable difference between R and $R_{\rm noS}$ is seen in Fig. 1. It should be noted that $R_{\rm noS}$ corresponds to the perfect interference-cancelation, while the curve R is for no CSIT. The gap between the two can be bridged with the availability of partial CSIT. Additionally, $R_{\rm noS}$ is loose in the high-SNR regime because of the difference in the achievable scaling factors of R and $R_{\rm noS}$ (see Theorem 1).

Loosely speaking, it appears that for DPC to perform significantly better than the naive scheme of treating the interference as noise, it is necessary to have the matrix $E(\text{vec}(H_1)^*\text{vec}(H_2))$ 'non-zero', i.e., to have H_1 and H_2 'correlated'. The 'more' non-zero the above matrix is (or the 'more highly' H_1 and H_2 are correlated), the greater is the improvement. We believe this to be the fundamental nature of DPC over the G-FDPC under imperfect CSIT. Also see the discussion following Theorem 1.

III. Optimization over the Transmit Covariances

As per the coding scheme of [3], let $X_2 = X_{21} + X_{22}$ where the signal X_{21} corresponds to relaying and is correlated with X_1 while X_{22} is the signal intended for the cognitive receiver. Let $\begin{bmatrix} X_1 \\ X_{21} \end{bmatrix} \sim \mathcal{CN}\left(0, \Sigma = \begin{bmatrix} \Sigma_1 & V \\ V^* & \Sigma_{21} \end{bmatrix}\right)$, and $X_{22} \sim \mathcal{CN}(0, \Sigma_{22})$. Also let $\Sigma = T_1 T_1^*$ and $\Sigma_{22} = T_2 T_2^*$ for some T_1 and T_2; and $X_{22} = T_2 X_{22}'$ with $X_{22}' \sim \mathcal{CN}(0, I_{t_2})$. The CT would choose the auxiliary random variable as $U = X_{22}' + W \begin{bmatrix} X_1 \\ X_{21} \end{bmatrix}$, where X_{22}' is independent of X_1 and X_{21}. Hence $\Sigma_2 = \Sigma_{22} + \Sigma_{21}$.

Now the channel between the CT-receiver pair is $Y_2 = H_{22}X_{22} + [H_{12}\ H_{22}] \begin{bmatrix} X_1 \\ X_{21} \end{bmatrix} + Z_2$ which resembles the G-FDPC. Therefore, using the algorithms of Section II, we can determine the inflation factor to be used at the CT once Σ (or T_1) and Σ_{22} (or T_2) are specified. This explains the reason to first study DPC over the G-FDPC.

Denote $\bar{H}_1 = [H_{11}\ H_{21}]$, $\bar{H}_2 = [H_{12}\ H_{22}]$, and $\bar{H} = [\bar{H}_1^*\ \bar{H}_2^*]^*$. Then the achievable sum-rate $R_{sum} = R_p + R_c$ under no CSIT and perfect receiver channel knowledge is given by equation (2) at the bottom of the page.

Since W depends on T_1 and T_2, we need to optimize R_{sum} jointly over T_1, T_2, and W, as mentioned earlier. However, since W can be determined given T_1 and T_2, let us first consider the optimization of R_{sum} over T_1 and T_2 for a given value of W; later the algorithm for the joint optimization can be formulated. Let us consider: $\max_{T_1,T_2} R_{sum}$, subject to $\text{tr}(\Sigma_1) \le P_p$ and $\text{tr}(\Sigma_{21} + \Sigma_{22}) \le P_c$. This is a non-convex optimization problem. To obtain the necessary conditions, we form the lagrangian J; and set $\frac{\partial J}{\partial T_1} = 0$ and $\frac{\partial J}{\partial T_2} = 0$. We omit the details of differentiation and directly state the necessary conditions, as given by equations (3) and (4) at the bottom of the page, where λ_p^{-1} and λ_c^{-1} are the lagrange multipliers.

Algorithm for the Joint Optimization (Alg. 1):

1. Start with some initial choices $T_1^{(0)}$ and $T_2^{(0)}$. For these choices, determine $W^{(0)}$ using the algorithms discussed in Section II.
2. At the n^{th} iteration,

- Determine the transmit covariances: to this end, we set $T_1^{(n)} = \begin{bmatrix} \lambda_p I_{t_1} & 0 \\ 0 & \lambda_c I_{t_2} \end{bmatrix} g_1(T_1^{(n-1)}, T_2^{(n-1)}, W^{(n-1)})$ and $T_2^{(n)} = \lambda_c g_2(T_1^{(n-1)}, T_2^{(n-1)}, W^{(n-1)})$. The required expectations are evaluated numerically. Find lagrange multipliers so as to meet the power con-

$$R_{sum} = \mathbb{E}_{\bar{H}}\left\{ \log \frac{|I_{r_1} + \bar{H}_1 T_1 T_1^* \bar{H}_1^* + H_{21} T_2 T_2^* H_{21}^*|}{|I_{r_1} + H_{21} T_2 T_2^* H_{21}^*|} + \log \frac{|I_{r_2} + H_{22} T_2 T_2^* H_{22}^* + \bar{H}_2 T_1 T_1^* \bar{H}_2^*|}{\begin{vmatrix} I_{t_2} + W T_1 T_1^* W^* & T_2^* H_{22}^* + W T_1 T_1^* \bar{H}_2^* \\ H_{22} T_2 + \bar{H}_2 T_1 T_1^* W^* & I_{r_2} + H_{22} T_2 T_2^* H_{22}^* + \bar{H}_2 T_1 T_1^* \bar{H}_2^* \end{vmatrix}} \right\} \quad (2)$$

$$\begin{bmatrix} \lambda_p^{-1} I_{t_1} & 0 \\ 0 & \lambda_c^{-1} I_{t_2} \end{bmatrix} T_1 = \mathbb{E}_{\bar{H}}\left\{ \bar{H}_1^* N_1^{-1} \bar{H}_1^* + \bar{H}_2^* N_2^{-1} \bar{H}_2^* - [W^*\ \bar{H}_2^*] D_2^{-1} \begin{bmatrix} W \\ \bar{H}_2 \end{bmatrix} \right\} T_1 = g_1(T_1, T_2, W), \quad (3)$$

$$\lambda_c^{-1} T_2 = \mathbb{E}_{\bar{H}}\left\{ H_{21}^* N_1^{-1} H_{21} T_2 - H_{21}^* D_1^{-1} H_{21} T_2 + H_{22}^* N_2^{-1} H_{22} T_2 - [0\ H_{22}^*] D_2^{-1} [I_{t_2}\ T_2^* H_{22}^*]^* \right\} \quad (4)$$

$$= g_2(T_1, T_2, W), \qquad \cdots \text{ where } N_1 = I_{r_1} + \bar{H}_1 T_1 T_1^* \bar{H}_1 + H_{21} T_2 T_2^* H_{21}^*,\ D_1 = I_{r_1} + H_{21} T_2 T_2^* H_{21}^*,$$

$N_2 = I_{r_2} + H_{22} T_2 T_2^* H_{22}^* + \bar{H}_2 T_1 T_1^* \bar{H}_2^*$, and D_2 is the block-partitioned matrix in equation (2).

straints.

- For $T_1^{(n)}$ and $T_2^{(n)}$ obtained above, determine $W^{(n)}$.

3. Repeat the above step until the increase in the achievable sum-rate is negligible.

The statement above regarding the determination of lagrange multipliers warrants a discussion. Note, the power transmitted by either transmitter increases with $\lambda_{p,c}$. Therefore, the feasible region for the λ's is of the form $0 < \lambda_{p,c} \leq \lambda_{p,c}^{max}$, where $(\lambda_p^{max}, \lambda_c^{max})$ is a point at which both the power constraints are satisfied with equality. One can expect the optimal point to be $(\lambda_p^{max}, \lambda_c^{max})$ at which both the transmitters operate with the maximum available power. However, since the signal intended for the primary receiver is an interference for the cognitive receiver and vice versa, the sum-rate need not necessarily be a nondecreasing function of either λ_p or λ_c. Hence, the optimal point for λ's, i.e., $\lambda_{p,c}^{opt}$ can be any interior or boundary point of the above rectangular region. Note, the choice of λ's dictates the covariance matrices, and therefore the inflation factor. Considering the fact that only an algorithmic solution is available for the inflation factor and all the required expectations need to be evaluated numerically, the problem of determination of optimal λ's looks intractable. In the numerical examples, we consider a suboptimal solution of solving the power constraints as strict equalities.

We have developed one more algorithm (Alg. 2) for the joint optimization which serves as a lower-bound on the rate achievable using Alg. 1.

1. Assume that $T_1 = 0$. Determine T_2 to maximize R_c subject to $\mathrm{tr}(\Sigma_{22}) = \frac{P_c}{2}$ (note the equality here).
2. For given T_2, determine T_1 to maximize R_p under the constraints that $\mathrm{tr}(\Sigma_1) = P_p$ and $\mathrm{tr}(\Sigma_{21}) = \frac{P_c}{2}$.
3. For given T_1, determine T_2 and W to maximize R_c under the constraint that $\mathrm{tr}(\Sigma_{22}) = \frac{P_c}{2}$.
4. Repeat Steps 2 and 3 above until the increase in the achievable rate is negligible.

Thus R_p and R_c are maximized here greedily over T_1 and (T_2, W), respectively. For these maximizations, the algorithm of joint optimization developed in [14] is used.

IV. High-SNR Analysis: Scaling Factor

Theorem 1: G-FDPC: Assume that the ratio $\frac{Q}{P}$ is constant as $P \to \infty$, and the fading processes are such that for any positive semi-definite A, $\mathrm{rank}([H_1\ H_2]A[H_1\ H_2]^*) = \min(r, \mathrm{rank}(A))$ with probability 1. The high-SNR scaling factor achievable over the no-CSIT G-FDPC using DPC is independent of the choice of W, as long as W is chosen such that the term $\log|\Sigma_X + W\Sigma_S W^*|$ scales in the high-SNR regime as $t_x \log \mathrm{SNR}$.

Thus, the naive scheme of treating the interference as noise (i.e., $W = 0$) achieves the optimal scaling factor, which is given by $\min(r, \mathrm{rank}(\Sigma_X) + \mathrm{rank}(\Sigma_S)) - \min(r, \mathrm{rank}(\Sigma_S))$. Also note that there is no loss of generality in choosing W to satisfy the condition stated in Theorem 1 because W's that do not satisfy this condition can achieve only a suboptimal scaling factor.

The intuition detailed in the paragraph just preceding Section III can explain the result of Theorem 1. Consider the FDPC with $|\Sigma_X| > 0$, or equivalently, the G-FDPC with $H_1 = H_2 = H$ and $t_x = t_s = t$. The high-SNR scaling factor of $\min(t, r)$, which is equal to that of the corresponding no-interference upper-bound, is achievable with the choice of $W = I_t$ [9]. Next consider the FDPC with $|\Sigma_X| = 0$ (let $\Sigma_X = TT^*$); this channel is then equivalent to the G-FDPC $Y = H_1X' + H_2S + Z$ with $H_1 = HT$, $H_2 = H$, and $X' \sim \mathcal{CN}(0, I)$ (so H_1 and H_2 not equal). In this case, the achievable scaling factor may not always be equal to that of the no-interference upper-bound; but in most cases, by making an appropriate choice for W (say, $W = T^+$ [14]), one can achieve a better scaling factor than that achievable with $W = 0$. Note, the G-FDPCs in the two cases above do not satisfy the assumption regarding H_1 and H_2 made in Theorem 1. Finally, consider the G-FDPC that satisfies the assumption of Theorem 1 (for example, H_1 and H_2 are independent and Rayleigh-faded). Then, as per Theorem 1, there is no advantage in optimizing over W as far as the scaling factor is concerned. Thus, the 'more highly' H_1 and H_2 are correlated, the 'larger' is the increase in the scaling factor over that achievable by treating the interference as noise.

Theorem 2: CRC: Assume that the channel matrices $\{H_{ij}\}$ are full rank and independent; and the ratio $\frac{P_p}{P_c}$ remains constant. The high-SNR($= P_p$) sum-rate scaling factor achievable over the no-CSIT CRC is given by

$$\gamma_{sum} = \max_{\mathrm{rank}(\Sigma'), \mathrm{rank}(\Sigma_{22})} \gamma_p + \gamma_c \text{ with}$$
$$\gamma_p = \min(r_1, \mathrm{rank}(\Sigma')) - \min(r_1, \mathrm{rank}(\Sigma_{22})),$$
$$\gamma_c = \min(r_2, \mathrm{rank}(\Sigma')) - \min(r_2, \mathrm{rank}(\Sigma') - \mathrm{rank}(\Sigma_{22})),$$

where Σ' is the covariance matrix of $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$; and the maximization is under the constraints of $0 < \mathrm{rank}(\Sigma') \leq t_1 + t_2$, and $0 \leq \mathrm{rank}(\Sigma_{22}) \leq \min(t_2, \mathrm{rank}(\Sigma'))$.

Proof: We present the outline here and omit the details. Given Σ and Σ_{22}, R_p achieves the scaling factor of γ_p because the primary receiver treats the interference X_{22} as noise. γ_c is achieved by the choice of $W = [0\ T_2^+]$ (see Theorem 1 above and Theorem 1 of [14]). ■

The maximization in Theorem 2 is over only finitely many values; thus, can be done via exhaustive search. Note, the power constraints are not solved as strict equalities here.

V. Numerical Results for the CRC

In figures, 'ub' denotes the sum-rate achievable by optimizing over the transmit covariances under the assumption that the interference is perfectly canceled at the cognitive receiver (i.e., the sum-rate with $R_c = E_{H_{22}} \log|I_{r_2} + H_{22}\Sigma_{22}H_{22}^*|$). We quantize each element of the fading matrices separately using an 'equally spaced level' quantizer as defined in [15]. In figures, if $B = [B_{ij}]$, then B_{ij} denotes the number of feedback bits used per element of matrix H_{ij}. Further, H_{ij} are independent. Alg. 1 is unfortunately sensitive to the initial choices. We take 4 to 5 initial choices in these examples and then select the best solution.

In Fig. 3, we consider the CRC with elements of $\{H_{ij}\} \sim$ i.i.d. $\mathcal{N}(0.6, 0.64)$. The improvement in the achievable

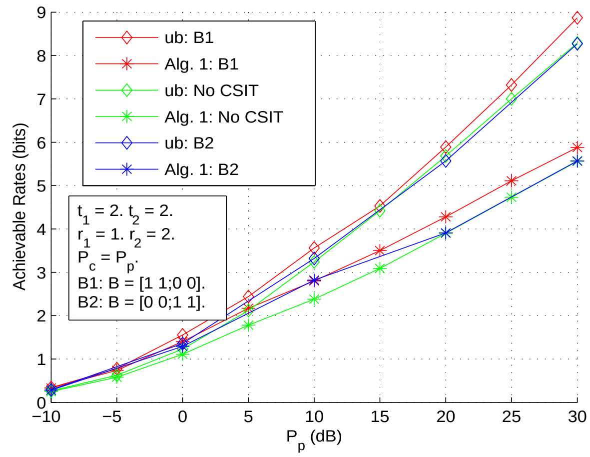

Fig. 3

ACHIEVABLE SUM-RATE VS. PP.

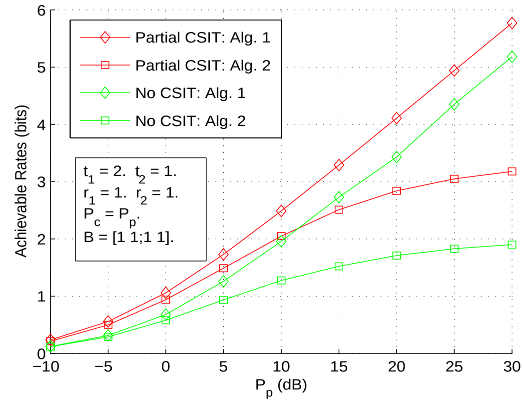

Fig. 5

COMPARISON OF ALG. 1 AND ALG. 2.

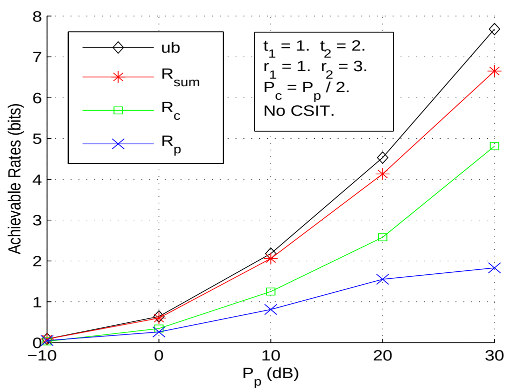

Fig. 4

ACHIEVABLE SUM-RATE VS. PP.

sum-rate with the introduction of partial CSIT is evident. Here, the scaling factor of 1 is achieved for R_{sum} by letting the CT to use its entire power for relaying. Hence, the curve corresponding to B2 merges with that corresponding to the no CSIT at high SNR. For the CRC of Fig. 4, we have the elements of $\{H_{ij}\} \sim$ i.i.d. Unif$[0,1]$. For this CRC, as per Theorem 2, the optimal solution should achieve $\gamma_{sum} = 2$ with $\gamma_p = 0$. This fact can be easily seen from the plot. In Fig. 5, we have the CRC with elements of $\{H_{ij}\} \sim$ i.i.d. $\mathcal{N}(0,1)$. It can be seen that Alg. 1 outperforms Alg. 2. However, in some cases, for example, the CRCs in Figs. 3 and 4, Alg. 2 does provide a relatively tight lower-bound. Coming back to Fig. 3 again, R_{sum} achieves the scaling factor of 1 whereas according Theorem 2, $\gamma_{sum} = 2$. This is achieved by setting $\Sigma_1 = 0$, i.e., the primary transmitter needs to turn off its power. The apparent inconsistency here is because we have considered a suboptimal solution of solving the power constraints as strict equalities. This example emphasizes the importance of the problem of determination of $\lambda's$.

VI. CONCLUSION

This paper is one of the earliest works that studies the imperfect-CSIT MIMO CRC. To the best of the authors' knowledge, it proposes for the first time a transmission strategy for the multi-antenna CRC with imperfect CSIT. En-route, brings into focus the problem of determination of λ's. Furthermore, the paper derives an achievable high-SNR sum-rate scaling factor. It would be worthwhile to obtain the highest-achievable sum-rate scaling factor. This problem can be interesting; recall its counterpart for the Gaussian MIMO broadcast channel, a problem that is open even after serious attempts. More efforts are needed to answer these two open questions.

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