

Analysis of Alternative Metrics for the PAPR Problem in OFDM Transmission

Gerhard Wunder
 Fraunhofer Heinrich Hertz Institut
 Berlin, Germany
 Email: gerhard.wunder@hhi.fraunhofer.de

Abstract—The effective PAPR of the transmit signal is the standard metric to capture the effect of nonlinear distortion in OFDM transmission. A common rule of thumb is the $\log(N)$ barrier where N is the number of subcarriers which has been theoretically analyzed by many authors. Recently, new alternative metrics have been proposed in practice leading potentially to different system design rules which are theoretically analyzed in this paper. One of the main findings is that, most surprisingly, the $\log(N)$ barrier turns out to be much too conservative: e.g. for the so-called amplifier-oriented metric the scaling is rather $\log[\log(N)]$. To prove this result, new upper bounds on the PAPR distribution for coded systems are presented as well as a theorem relating PAPR results to these alternative metrics.

I. INTRODUCTION

The peak-to-average power ratio (PAPR) problem is a well-established problem in OFDM literature and has entailed numerous research papers since the mid nineties [1]. Nowadays, even though OFDM has become the predominant wireless technology in the downlink, there are still many concerns about the application of OFDM in the uplink. This is mainly due to the fact that the PAPR reduction capabilities of state-of-the-art algorithms and their respective impact on relevant performance measures such as power efficiency, error probability, and spectral regrowth are not easy to track and mostly presented in terms of simulations. This situation is indeed dissatisfactory for system design, where provable performance limits are required. Another important driving factor within the context of *Green Information Technology* is the growing energy cost of network operation setting standards beyond capabilities of current PAPR reduction algorithms [2]. Hence, it becomes more and more apparent that the problem can not be considered as solved yet and that the PAPR metric itself has to be carefully reviewed overthrowing some of the common understanding and results particularly in the context of MIMO [3], [4], [5].

This paper revisits the PAPR problem and analyzes new performance metrics in terms of their effective behaviour. Standard results suggest that power amplifier backoff is to be adjusted along the $\log(N)$ rule of thumb where N is the number of subcarriers [6], [7]. However, high but very narrow peaks obviously cause spectral regrowth but the effect on e.g. symbol error probability might be negligible which suggests that amplifier backoff and PAPR of the transmit signal can indeed fall apart while, still, zero symbol error probability can be achieved. This motivates the analysis of alternative

performance measures recently proposed in practice [8]. The detailed contributions are as follows:

Contributions: First, we provide a new analytical upper bound on the PAPR distribution for coded OFDM systems generalizing some known results in the literature. The theorems are used to bound some given alternative performance metric introducing the so-called balancing method. In this context we prove that even though PAPR is of order $\log(N)$ with high probability, the amplifier backoff can be adjusted according to a much lower value. Specifically, for the so-called amplifier oriented metric the scaling turns out to be of order $\log[\log(N)]$ which is almost a constant in practical terms and suggests new system design rules.

II. THE COMMUNICATION MODEL

Let us introduce coded OFDM systems. We adopt the system model introduced by [9]: Let \mathcal{C} be a code that maps k_b input bits into blocks of N constellation symbols c_0, \dots, c_{N-1} , from a complex constellation \mathcal{Q} forming the codeword \mathbf{c} . We assume here $\mathcal{Q} := \{-1, 1\} = \text{BPSK}$. The rate R of this code is defined to be $R = k_b/N$ such that \mathcal{C} has $M_1 = 2^{RN}$ codewords.

Given a codeword \mathbf{c} , a single OFDM baseband symbol can be described by

$$S_{\mathbf{c}}(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k e^{2\pi j k \Delta f t}, \quad 0 \leq t \leq T_s, j = \sqrt{-1} \quad (1)$$

where N is the number of subcarriers, $\Delta f = 1/T_s$ is the subcarrier frequency offset and T_s is the symbol duration. For mathematical convenience the time axis can be normalized by T_s , i.e. we substitute $\theta(t) = 2\pi t/T_s$ and write $S_{\mathbf{c}}(\theta)$, $0 \leq \theta \leq 2\pi$. Furthermore, for later reference define $S_{\mathbf{c}}(\theta, \alpha) := \Re e(S_{\mathbf{c}}(\theta) e^{j\alpha})$ with sampling points

$$\theta_{l,L} := \frac{2\pi l}{2LN}, \alpha_{l,K} := \frac{l2\pi}{K},$$

which are collected in the two-dimensional lattice

$$\Omega_{L,K} := \{(\theta_{l_1,L}, \alpha_{l_2,K}), 0 \leq l_1 < LN, 0 \leq l_2 < K\},$$

of the square $[0, 2\pi) \times [0, 2\pi)$. Here, $L > 1$ is the oversampling factor and $K > 2$ is some auxiliary variable. The Nyquist-rate samples are $\theta_l := \theta_{l,1}$. In the baseband model the OFDM symbols undergo a nonlinear transformation denoted as

$$\Phi : S_{\mathbf{c}}(\theta) \mapsto \Phi[S_{\mathbf{c}}(\theta)]$$

representing some high power amplifier (HPA) model. In the sequel, we assume for simplicity that the nonlinearity acts solely on the samples obtained with some oversampling factor L .

III. HPA MODELS

A. Soft envelope limiter model

The soft envelope limiter (SEL) model is given by

$$\Phi_{sel}(S_c(\theta)) = \begin{cases} S_c(\theta), & |S_c(\theta)| \leq \lambda \\ \lambda e^{j \arg(S_c(\theta))}, & |S_c(\theta)| > \lambda \end{cases},$$

where λ is the saturation level of the non-linearity and the event $\{|S_c(\theta)| > \lambda\}$ is commonly described as clipping. The samples after the SEL nonlinearity can be decomposed as

$$\Phi(S_c(\theta_{l,L})) = S_c(\theta_{l,L}) + D_c(\theta_{l,L})$$

and obviously $D_c(\theta_{l,L}) = 0 \forall l$ when no clipping occurs. The SEL model is a standard model when there are additional predistortion techniques.

B. Cubic polynomial model

Particular in the 3GPP context [8], [4] the cubic model has become popular and is given by

$$\Phi_{cu}(S_c(\theta)) = a \cdot S_c(\theta) + b \cdot S_c(\theta) |S_c(\theta)|^2, \quad a, b > 0.$$

The advantage is in most cases a simpler analytical treatment.

IV. FIGURES OF MERIT

A. Crest-factor

The crest-factor (CF)¹ of eqn. (1) is defined by

$$CF_L(S_c) := \max_{0 \leq l < LN} |S_c(\theta_{l,L})| \quad (2)$$

with $1 \leq CF_L(S_c) \leq \sqrt{N}$. A first choice to assess the impact of a nonlinearity in the transmitter path would be the maximum CF taken over all codewords (that we call the CF of a code) defined by

$$CF_L(\mathcal{C}) := \max_{c \in \mathcal{C}} CF_L(S_c).$$

It was shown in [10], [11] that for spherical codes the CF of a code can be computed with arbitrary accuracy provided that the code supports minimum-distance decoding. On the other hand, the occurrence of this „worst-case“ codeword may be extremely unlikely. In this case, the distribution of the CF must be taken into account. The complementary cumulative distribution function (CCDF) of (2) is defined by

$$B_L(x) := \Pr(\{CF_L(S_c) > x; c \in \mathcal{C}\}).$$

(Pr denotes probability). Using the CCDF a more appropriate measure can be defined such as the „effective“ CF defined by the CF of which the probability of occurrence may be considered negligible in practice, i.e.

$$B_L(CF_{eff}(\mathcal{C})) = \epsilon$$

where $CF_{eff}(\mathcal{C})$ is the effective CF and ϵ is some small number, say $10^{-3} \dots 10^{-8}$ (outage probability). Our aim is to bound this term for codes.

¹We consider CF instead of PAPR.

B. Amplifier-oriented metric

The definition of $D_c(\theta)$ suggest the following metric

$$AOM_L(S_c) := \frac{1}{LN} \sum_{l=0}^{LN-1} |D_c(\theta_{l,L})|^2$$

In the following we will see how we can relate this metric to B_L .

V. FUNDAMENTALS ON CF DISTRIBUTION

The general approach is as follows: suppose the probabilities

$$\Pr(S_c(\theta, \alpha) > x)$$

are given where the tuple (θ, α) runs through $\Omega_{L,K}$. By the method of projections and union bound we have

$$B_L(x) \leq \min_{K > 2} \sum_{(\theta, \alpha) \in \Omega_{L,K}} \Pr\left(S_c(\theta, \alpha) > \frac{x}{C_K}\right)$$

where $C_K := \cos^{-1}\left(\frac{\pi}{K}\right)$, $K \geq 3$. Thus, all we have to do is to bound the distribution of the instantaneous envelope for the different modulation schemes. Thus, we replace the probability terms with the Chernoff (or any other Marcov style) bound, i.e.

$$\Pr(S_c(\theta, \alpha) > x) \leq \mathbb{E}\left(e^{\varrho(S_c(\theta, \alpha) - x)}\right) \quad (3)$$

for any (real) $\varrho > 0$ ($\mathbb{E}(\cdot)$ is the expectation operator). We call the bound (3) a union bound on the CCDF of the CF.

We also need some elements from coding theory. Let $d(c', c'')$ be the *Hamming distance* between codewords c', c'' , i.e. the number of positions where the codewords differ. The distance distribution is then given by

$$W_k^\circ := \frac{|\{c', c'' \in \mathcal{C} : d(c', c'') = k\}|}{M_1}$$

Note that since for a linear code it does not matter which individual codeword we pick when calculating the distance to another codeword, the distance distribution coincides with the weight distribution W_k for linear codes, i.e.

$$W_k := |\{c : w(c) = k, c \in \mathcal{C}\}|$$

where $w(\cdot)$ denotes the weight of a codeword. Furthermore, if the code contains the all-one codeword (i.e. all components are negative under our identification of constellation symbols) we have $W_k = W_{N-k}$ and the weight distribution becomes *symmetric*.

The main purpose of the following derivations is to prove an interesting connection between CF distributions and distance distributions. Since many results require symmetric weight and distance distributions we start with the following lemma. It says that the CF distribution can be estimated by its symmetrized version.

Lemma 1: Suppose \mathcal{C}_A is a binary code. Then, for any set $\mathcal{A} := \{c \in \mathcal{C}_A : CF(S_c) > x\}$ the probability that this occurs is upperbounded by

$$\frac{|\mathcal{A}|}{|\mathcal{C}_A|} \leq 2 \frac{|\mathcal{A} \cup \mathcal{B}|}{|\mathcal{C}_A \cup \mathcal{C}_B|} \leq 4 \frac{|\mathcal{A}|}{|\mathcal{C}_A|},$$

where \mathcal{C}_B is the binary code constructed by adding the all-one codeword to any codeword of \mathcal{C}_A and $\mathcal{B} := \{\mathbf{c} \in \mathcal{C}_B : CF(S_{\mathbf{c}}) > x\}$.

Proof: First, observe that $|\mathcal{C}_A| = |\mathcal{C}_B|$ and $|\mathcal{A}| = |\mathcal{B}|$. Furthermore we have

$$|\mathcal{C}_A \cup \mathcal{C}_B| \leq 2|\mathcal{C}_A|$$

so that we get

$$|\mathcal{C}_A| \geq \frac{|\mathcal{C}_A \cup \mathcal{C}_B|}{2}.$$

Hence, we have

$$\frac{|\mathcal{A}|}{|\mathcal{C}_A|} \leq \frac{|\mathcal{A} \cup \mathcal{B}|}{|\mathcal{C}_A|} \leq 2 \frac{|\mathcal{A} \cup \mathcal{B}|}{|\mathcal{C}_A \cup \mathcal{C}_B|}.$$

The converse is

$$\frac{|\mathcal{A} \cup \mathcal{B}|}{|\mathcal{C}_A \cup \mathcal{C}_B|} \leq \frac{2|\mathcal{A}|}{|\mathcal{C}_A \cup \mathcal{C}_B|} \leq \frac{2|\mathcal{A}|}{|\mathcal{C}_A|}$$

proving the claim. \blacksquare

Now, we are ready for our first theorem which generalizes [1, Thm. 6.16].

Theorem 1: For any binary code \mathcal{C} the CCDF of the CF is upperbounded by

$$B_L(x) \leq \min_{\varrho > 0} \sqrt{\sum_{k=0}^N \frac{f^*(\varrho, x) W_k^\circ \cosh(\varrho N^{\frac{1}{4}}(N-2k))}{M_1}}$$

where

$$f^*(\varrho, x) := \min_{K > 2} 2LNK \exp\left(-\frac{\varrho\sqrt{Nx}}{C_K}\right) \sqrt{\cosh(\varrho N^{\frac{3}{4}})}.$$

Proof: We first assume codes with symmetric weight distributions. For ease of presentation let us define

$$b_{\mathbf{i}}^{(j)} := \frac{j!}{i_0! i_1! \dots i_{N-1}!}$$

and

$$k_{\mathbf{i}}(\theta, \alpha) := (\cos^{i_0}(\alpha), \dots, \cos^{i_{N-1}}((N-1)\theta + \alpha)),$$

as well as the code moments

$$M_{\mathbf{i}} := \mathbb{E}(c_0^{i_0} c_1^{i_1} \dots c_{N-1}^{i_{N-1}}).$$

Fixing $\varrho > 0, L > 1, K > 2, N_1 > 1$, and expanding the exponential function in the Chernoff bound yields

$$B_L(x) \lesssim \sum_{(\theta, \alpha) \in \Omega_{L, K}} e^{-\frac{\varrho\sqrt{Nx}}{C_K}} \sum_{j=0}^{N_1} \frac{\varrho^j}{j!} \sum_{\mathbf{i} \in \mathcal{I}_j} b_{\mathbf{i}}^{(j)} k_{\mathbf{i}}(\theta, \alpha) M_{\mathbf{i}}$$

where we have omitted the error term (indicated by \lesssim) on the right hand side which depends on the natural number $N_1 > 0$ and is given by Taylor's theorem. Applying the Cauchy-Schwartz's inequality yields

$$B_L(x) \lesssim \sum_{(\theta, \alpha) \in \Omega_{L, K}} e^{-\frac{\varrho\sqrt{Nx}}{C_K}} \sum_{j=0}^{N_1} \frac{\varrho^j}{j!} \left(\sum_{\mathbf{i} \in \mathcal{I}_j} b_{\mathbf{i}}^{(j)} k_{\mathbf{i}}^2(\theta, \alpha) \right)^{\frac{1}{2}} \left(\sum_{\mathbf{i} \in \mathcal{I}_j} b_{\mathbf{i}}^{(j)} M_{\mathbf{i}}^2 \right)^{\frac{1}{2}}.$$

Observing that

$$\begin{aligned} \sum_{\mathbf{i} \in \mathcal{I}_j} b_{\mathbf{i}}^{(j)} k_{\mathbf{i}}^2(\theta, \alpha) &= \left(\sum_{k=0}^{N-1} \cos^2(k\theta + \alpha) \right)^j \\ &=: \Psi_N^j(\theta, \alpha) \end{aligned}$$

we have

$$B_L(x) \lesssim \sum_{(\theta, \alpha) \in \Omega_{L, K}} e^{-\frac{\varrho\sqrt{Nx}}{C_K}} \sum_{j=0}^{N_1} \frac{\varrho^j \Psi_N^{\frac{j}{2}}(\theta, \alpha)}{j!} \left(\sum_{\mathbf{i} \in \mathcal{I}_j} b_{\mathbf{i}}^{(j)} M_{\mathbf{i}}^2 \right)^{\frac{1}{2}}.$$

Next, we can replace the squared moments in terms of the distance distribution of the (in general nonlinear) code [12, [1, Proof of Thm. 6.16], i.e.

$$\sum_{\mathbf{i} \in \mathcal{I}_j} b_{\mathbf{i}}^{(j)} \left(\sum_{\mathbf{c} \in \mathcal{C}} \prod_{k=0}^{N-1} c_k^{i_k} \right)^2 = M_1 \sum_{k=0}^N (N-2k)^j W_k^\circ,$$

and therefore

$$B_L(x) \lesssim \sum_{(\theta, \alpha) \in \Omega_{L, K}} e^{-\frac{\varrho\sqrt{Nx}}{C_K}} \sum_{j=0}^{N_1} \frac{\varrho^{2j} \Psi_N^j(\theta, \alpha)}{(2j)!} \left(\frac{1}{M_1} \sum_{k=0}^N (N-2k)^{2j} W_k^\circ \right)^{\frac{1}{2}}$$

since the distance distribution is symmetric. Again applying Cauchy-Schwartz's inequality yields

$$\begin{aligned} &\sum_{j=0}^{N_1} \frac{\varrho^{2j} [\Psi_N(\theta, \alpha)]^{\frac{3j}{4}}}{(2j)!} \left(\frac{[\Psi_N(\theta, \alpha)]^{\frac{j}{2}}}{M_1} \sum_{k=0}^N (N-2k)^{2j} W_k^\circ \right)^{\frac{1}{2}} \\ &\leq \left(\sum_{j=0}^{N_1} \frac{\varrho^{2j} ([\Psi_N(\theta, \alpha)]^{\frac{3}{4}})^{2j}}{(2j)!} \right)^{\frac{1}{2}} \\ &\left(\sum_{j=0}^{N_1} \frac{\varrho^{2j}}{M_1 (2j)!} \sum_{k=0}^N ([\Psi_N(\theta, \alpha)]^{\frac{1}{4}})^{2j} (N-2k)^{2j} W_k^\circ \right)^{\frac{1}{2}}. \end{aligned}$$

Since N_1 is arbitrary, we have

$$\sum_{j=0}^{N_1} \frac{\varrho^{2j} ([\Psi_N(\theta, \alpha)]^{\frac{3}{4}})^{2j}}{(2j)!} \xrightarrow{N_1 \rightarrow \infty} \cosh(\varrho [\Psi_N(\theta, \alpha)]^{\frac{3}{4}}),$$

and

$$\begin{aligned} &\sum_{j=0}^{N_1} \frac{\varrho^{2j}}{M_1 (2j)!} \sum_{k=0}^N ([\Psi_N(\theta, \alpha)]^{\frac{1}{4}})^{2j} (N-2k)^{2j} W_k^\circ \xrightarrow{N_1 \rightarrow \infty} \\ &\frac{1}{M_1} \sum_{k=0}^N \cosh(\varrho [\Psi_N(\theta, \alpha)]^{\frac{1}{4}} (N-2k)) W_k^\circ \end{aligned}$$

for any $\varrho > 0, L > 1, K > 2$. The final result follows from $\Psi_N^j(\theta, \alpha) \leq N^j$ and invoking Lemma 1 to lift the proof to

the non-symmetric case. The additional factor $\frac{1}{2}$ is due to the real BPSK symbols where $|S_c(\theta)| = |S_c(2\pi - \theta)|$. ■

We can improve on the result by assuming linearity of the code. The following theorem relies on the fact that moments of linear codes are non-negative which generalizes [12], [1, Thm. 6.13].

Theorem 2: Let \mathcal{C} be a linear, binary code. Then the CCDF of the CF is upperbounded by

$$B_L(x) \leq \min_{\varrho > 0} \sum_{k=0}^N \frac{f^{**}(\varrho, x) W_k \cosh(\varrho(N-2k))}{M_1}$$

where

$$f^{**}(\varrho, x) := \min_{K > 2} 2LNK \exp\left(-\frac{\varrho\sqrt{Nx}}{C_K}\right).$$

Proof: The proof uses Lemma 1 and invokes the same proof steps as in [12] which are omitted. ■

The applicability of the latter theorems is ensured by the following result.

Theorem 3: Suppose that for all W_k° there is a constant C_w independent of k so that

$$W_k^\circ \leq (1 + C_w) \frac{1}{2^{N-k_b}} \binom{N}{k}. \quad (4)$$

Then the CF is upperbounded by:

$$\begin{aligned} B_L(x) &\leq 2(1 + C_w) LKN \exp\left(-\frac{x^2}{2C_K^2\sqrt{N}}\right) && \text{nonlinear } \mathcal{C} \\ B_L(x) &\leq 2(1 + C_w) LKN \exp\left(-\frac{x^2}{2C_K^2}\right) && \text{linear } \mathcal{C} \end{aligned}$$

Proof: We omit the proof for the linear part which follows directly from [12]. In the nonlinear case by Theorem 1 and by virtue of

$$\begin{aligned} \sqrt{\sum_{k=0}^N \binom{N}{k} \frac{\cosh\left(\varrho N^{\frac{1}{4}}(N-2k)\right)}{2^N}} &\leq \left(\cosh\left(N^{\frac{1}{4}}\varrho\right)\right)^{\frac{N}{2}} \\ &\leq \exp\left(\frac{\varrho^2 N^{\frac{3}{2}}}{4}\right), \end{aligned}$$

and since

$$\sqrt{\cosh\left(\varrho N^{\frac{3}{4}}\right)} \leq \exp\left(\frac{\varrho^2 N^{\frac{3}{2}}}{4}\right),$$

we obtain

$$B_L(x) \leq 2(1 + C_w) LNK \exp\left(-\frac{\varrho x \sqrt{N}}{C_K}\right) \exp\left(\frac{\varrho^2 N^{\frac{3}{2}}}{4}\right).$$

Setting $\varrho = \frac{2x}{C_K N}$ yields

$$\bar{F}_{cf}^{\mathcal{C}}(x) \leq 2(1 + C_w) LNK \exp\left(-\frac{x^2}{C_K^2\sqrt{N}}\right).$$

The asymptotics follow immediately then. ■

VI. ALTERNATIVE METRICS: UPPER BOUNDS

In this section we use Theorems 1, 2 to obtain upper bounds for the alternative metrics. Let us define the random variable

$$N_c^{(L)}(\lambda) := |\{l : |S_c(\theta_{l,L})| > \lambda, l = 0, \dots, LN-1\}| \quad (5)$$

counting the number of samples which exceed a level λ . In the following theorem we apply a balancing technique between the CCDF of the CF and the number of samples that exceed a given level. For ease of presentation we set $L = 1$.

We start with the uncoded BPSK case where a tighter bound can be obtained. Note that BPSK is a linear code with $C_w = 0$ in Theorem 3.

Theorem 4: Assume $\mathcal{Q} := \{-1, 1\} = \text{BPSK}$ and suppose S_c is clipped at the level λ . For any given performance metric h which is increasing in its argument, the average distortion is upperbounded by

$$\begin{aligned} \Pr\left(\sum_{l=0}^{N-1} h(|D_c(\theta_l)|) > x\right) \\ \leq \min_{\mu > \lambda} \left[B_1(\mu) + [B_1(\lambda) + B_1^2(\lambda)] \frac{h^2(\mu - \lambda)}{x^2} \right]. \end{aligned}$$

Proof: Define the event

$$\mathcal{A} := \left\{ \sum_{l=0}^{LN-1} h(|D_c(\theta_l)|) > x \right\}.$$

\mathcal{A} can be partitioned into disjoint events $\mathcal{A} \cap \{CF_L(S_c) > \mu\}$ or $\mathcal{A} \cap \{CF_L(S_c) \leq \mu\}$. Thus

$$\Pr(\mathcal{A}) \leq B_L^*(\mu) + \Pr(\mathcal{A} \cap \{CF_L(S_c) \leq \mu\}). \quad (6)$$

Next we need to calculate the term $\Pr(\mathcal{A} \cap \{CF_L(S_c) \leq \mu\})$. Clearly the event $\mathcal{A} \cap \{CF_L(S_c) \leq \mu\}$ is contained in the event

$$\left\{ \left\{ N_c^{(L)}(\lambda) \cdot \max_{0 \leq l < LN} h(|D_c(\theta_{l,L})|) \geq x \right\} \cap \{CF_L(S_c) \leq \mu\} \right\}$$

which itself is within the event

$$\left\{ N_c^{(L)}(\lambda) \cdot h(\mu - \lambda) \geq x \right\}.$$

Hence, we take the unconstrained number of points exceeding λ but lift up the level that is needed for a countable event. Writing

$$\begin{aligned} N_c(\lambda, L) &= \sum_{l=0}^{LN-1} \mathbb{I}\{|S_c(\theta_{l,L})| > \lambda\} \\ &\leq \sum_{(\theta, \alpha) \in \Omega_{L,K}} \mathbb{I}\left\{S_c(\theta, \alpha) > \frac{\lambda}{C_K}\right\} \end{aligned}$$

and by Markov's inequality applied to the squared term and evaluating the exponential moments thereby using the inherent structure of $S_c(\theta, \alpha)$ [13] yields

$$\begin{aligned} \Pr(N_c^2(\lambda, L) > x^2) \\ \leq \frac{NK e^{\frac{\varrho^2 N}{2} - \frac{\varrho \lambda \sqrt{N}}{C_K}} + N(N-1)K^2 e^{\frac{2\varrho^2 N}{2} - \frac{2\varrho \lambda \sqrt{N}}{C_K}}}{x^2}. \end{aligned}$$

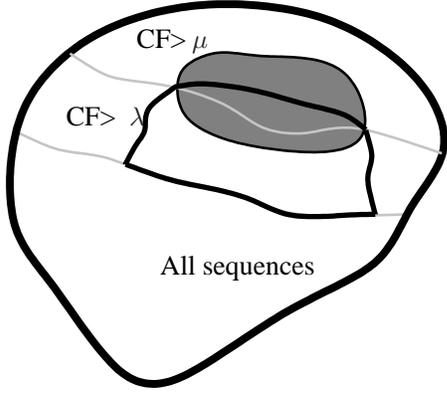


Fig. 1. Illustration of bounding method: the shaded area is the set of which the probability measure is to be bounded. The upper bound is by adjusting the μ level. The bold encircled area is the set where a sufficient number of points crosses the λ level.

Due to lack of space we omit the details. ■

The following theorem holds for any binary code.

Theorem 5: Suppose S_c is clipped at the level λ . For any given performance metric h which is increasing in its argument, the average distortion is upperbounded by

$$\Pr \left(\sum_{l=0}^{N-1} h(|D_c(\theta_l)|) > x \right) \leq \min_{\mu > \lambda} \left[B_1(\mu) + B_1(\lambda) \frac{h(\mu - \lambda)}{x} \right].$$

VII. APPLICATIONS

In this section we relate the theoretical results obtained so far to get scaling result for alternative metrics. Suppose that we use a binary code and that condition (4) is satisfied. Note that the condition holds for uncoded transmission as well as many families of codes (such as the BCH family [1, pp. 158 ff.]). Suppose further we apply the AOM metric in case of the SEL amplifier model. Setting

$$h(|D_c(\theta_l)|) := \frac{|D_c(\theta_l)|^2}{N}$$

then by Theorem 5 we have (omitting constants which have no effect on the asymptotic results)

$$\begin{aligned} & \Pr \left(\sum_{l=0}^{LN-1} h(|D_c(\theta_{l,L})|) > x \right) \\ & \leq \min_{\mu > \lambda} \left[B_1(\mu) + B_1(\lambda) \frac{(\mu - \lambda)^2}{N \cdot x} \right] \\ & \leq \min_{\mu > \lambda} \left[NK e^{-\frac{\mu^2}{2c_K^2}} + NK e^{-\frac{\lambda^2}{2c_K^2}} \frac{(\mu - \lambda)^2}{N \cdot x} \right], \end{aligned}$$

and setting

$$\begin{aligned} \lambda &= \lambda_N = \sqrt{(1 + \varepsilon) \log[\log(N)]} \\ \mu &= \mu_N = \sqrt{(1 + \varepsilon) \log(N)} \end{aligned}$$

where $\varepsilon > 0$ is an arbitrarily small constant yields

$$\Pr \left(\sum_{l=0}^{LN-1} \frac{|D_c(\theta_{l,L})|^2}{LN} > x \right) \rightarrow 0, \quad N \rightarrow \infty$$

for any fixed $x > 0$. Hence we have the remarkable result that the clipping level of the amplifier can be almost set to a constant and still the AOM metric can be made arbitrarily small.

VIII. CONCLUSIONS

In this paper we show that the standard design rule for the power amplifier in OFDM transmission might be too conservative if alternative metrics are considered. It is worth emphasizing that we do not claim that this is the ultimate scaling as in practice other metrics might be important (e.g. spectral regrowth). We provided an example and considered the amplifier oriented metric recently proposed in practice. We have not yet considered new algorithms (e.g. based on derandomization) for these metrics which is an interesting extension of the results in this paper.

REFERENCES

- [1] S. Litsyn, *Peak Power Control in Multicarrier Communications*, Cambridge University Press, 2007.
- [2] L. M. Correia, D. Zeller, O. Blume, D. Ferling, Y. Jading, I. Gdor, G. Auer, L. Van der Perre, "Challenges and enabling technologies for energy aware mobile radio networks," *IEEE Signal Processing Magazine*, vol. 48, no. 11, pp. 66–72, November 2010.
- [3] R. F. H. Fischer and C. Siegl, "Reed-solomon and simplex codes for PAPR reduction in OFDM," *IEEE Trans. Inform. Theory*, vol. 55, no. 4, pp. 1519–1528, April 2009.
- [4] C. Siegl, *Peak-to-average Power Ratio Reduction in Multi-antenna OFDM Via Multiple Signal Representation*, vol. 29, Dissertation, Erlanger Berichte aus Informations- und Kommunikationstechnik, Shaker Verlag, Aachen, 2010.
- [5] H. Boche and V. Pohl, "Signal representation and approximation - fundamental limits," *European Trans. on Telecomm. (ETT)*, vol. 18, no. 5, pp. 445–456, August 2007.
- [6] N. Dinur and D. Wulich, "Peak-to average power ratio in high-order OFDM," *IEEE Trans. on Communications*, vol. 49, no. 6, pp. 1063–1072, June 2001.
- [7] H. Ochiai and H. Imai, "On the distribution of the peak-to-average power ratio in OFDM signals," *IEEE Trans. on Communications*, vol. 49, no. 2, pp. 282–289, February 2001.
- [8] 3GPP TSG RAN WG1 37, "Comparison of PAR and cubic metric for power de-rating," in *Motorola Tdoc R1-040642*, May 2004.
- [9] K. G. Paterson and V. Tarokh, "On the existence and construction of good codes with low peak-to-average power ratios," *IEEE Trans. Inform. Theory*, vol. 46, no. 6, pp. 1974–1986, September 2000.
- [10] G. Wunder and H. Boche, "A baseband model for computing the PAPR in OFDM systems," in *4th International ITG Conference on Source and Channel Coding*, Berlin, January 2002, VDE, pp. 273–280, VDE-Verlag GmbH.
- [11] V. Tarokh and H. Jafarkhani, "On the computation and reduction of the peak-to average power ratio in multicarrier communications," *IEEE Trans. on Comm.*, vol. 48, no. 1, pp. 37–44, January 2000.
- [12] S. Litsyn and G. Wunder, "Generalized bounds on the CF distribution of OFDM signals with application to code design," *IEEE Trans. Inform. Theory*, vol. 52, no. 3, pp. 992–1006, March 2006.
- [13] G. Wunder and H. Boche, "New results on the statistical distribution of the crest-factor of OFDM signals," *IEEE Trans. Inform. Theory*, vol. 49, no. 2, pp. 488–494, February 2003.