

Ergodic Sum Capacity of Macrodiversity MIMO Systems in Flat Rayleigh Fading

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Abstract

The prospect of base station (BS) cooperation leading to joint combining at widely separated antennas has led to increased interest in macrodiversity systems, where both sources and receive antennas are geographically distributed. In this scenario, little is known analytically about channel capacity since the channel matrices have a very general form where each path may have a different power. Hence, in this paper we consider the ergodic sum capacity of a macrodiversity MIMO system with arbitrary numbers of sources and receive antennas operating over Rayleigh fading channels. For this system, we compute the exact ergodic capacity for a two-source system and a compact approximation for the general system, which is shown to be very accurate over a wide range of cases. Finally, we develop a highly simplified upper-bound which leads to insights into the relationship between capacity and the channel powers. Results are verified by Monte Carlo simulations and the impact on capacity of various channel power profiles is investigated.

Index Terms

Macrodiversity, MIMO, MIMO-MAC, Capacity, Sum-rate, Network MIMO, CoMP, DAS, Rayleigh fading.

I. INTRODUCTION

With the advent of network multiple input multiple output (MIMO) [1], base station (BS) collaboration [2] and cooperative MIMO [3], it is becoming more common to consider MIMO links where the receive array, transmit array or both are widely separated. In these scenarios, individual antennas from a single effective array may be separated by a considerable distance. When both transmitter and receiver have distributed antennas, we refer to the link as a macrodiversity MIMO link. Little is known analytically

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about such links, despite their growing importance in research [4]-[7] and standards where coordinated multipoint transmission (CoMP) is part of 3GPP LTE Advanced.

Some analytical progress in this area has been made recently in the performance analysis of linear combining for macrodiversity systems in Rayleigh fading [8], [9]. However, there appears to be no work currently available on the capacity of general systems of this type. Similar work includes the capacity analysis of Rayleigh channels with a two-sided Kronecker correlation structure [10]. However, the Kronecker structure is much too restrictive for a macrodiversity layout and such results cannot be leveraged here. Also, there is interesting work on system capacity for particular cellular structures, including Wyner's circular cellular array model [6] and the infinite linear cell-array model [7]. Despite these contributions, the general macrodiversity model appears difficult to handle. The analytical difficulties are caused by the geographical separation of the antennas which results in different entries of the channel matrix having different powers with an arbitrary pattern. Also, these powers can vary enormously when shadowing and path loss are considered. Note that this type of channel model also occurs in the work of [11].

In this paper, we consider a macrodiversity MIMO multiple access channel (MIMO-MAC) where all sources and receive antennas are widely separated and all links experience independent Rayleigh fading. For this system, we consider the ergodic sum capacity, under the assumption of no channel state information (CSI) at the transmitters. For two sources, we derive the exact ergodic sum capacity. The result is given in closed form, but the details are complicated and for more than two sources, it would appear that an exact approach is too complex to be useful. Hence, we develop an approximation and a bound for the general case. The first technique is very accurate, but the functional form is awkward to interpret. Hence, a second, less accurate but simple bound is developed which has a familiar and appealing structure. This bound leads to insight into capacity behavior and its relationship with the channel powers. In [12], we presented a preliminary study of this problem, which focussed on the approximation for the general case. In this paper, we have extended the conference version to include the exact two source results, correlated channels, full mathematical details (see Sec. III), a motivation for the approximate analysis (see Appendix D) and a much wider range of scenarios, power profiles and discussion in the results section.

Note that, the methodology developed is for the case of arbitrary powers for the entries in the channel matrix. There is no restriction due to particular cellular structures. Hence, the results and techniques may also have applications in multivariate statistics.

The rest of the paper is laid out as follows. Section II describes the system model and Sec. III gives some

mathematical preliminaries required in the analysis. Section IV provides an exact analysis for the case of two source antennas. Sections V and VI consider the case of arbitrary numbers of sources and develop accurate approximations and bounds on capacity. Results and conclusions appear in Secs. VII and VIII.

II. SYSTEM MODEL

Consider a MIMO-MAC link with M base stations and W users operating over a Rayleigh channel where BS i has n_{R_i} receive antennas and user i has n_i antennas. The total number of receive antennas is denoted $n_R = \sum_{i=1}^M n_{R_i}$ and the total number of transmit antennas is denoted $N = \sum_{i=1}^W n_i$. An example of such a system is shown in Fig. 1, where three BSs are linked by a backhaul processing unit (BPU) and communicate with multiple, mobile users. All channels are considered to be independent since the correlated channel scenario can be transformed into the independent case as shown in Sec. II-A. The system equation is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \quad (1)$$

where \mathbf{r} is the $\mathcal{C}^{n_R \times 1}$ receive vector, \mathbf{s} is the combined $\mathcal{C}^{N \times 1}$ transmitted vector from the W users, \mathbf{n} is an additive white Gaussian noise vector, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$, and $\mathbf{H} \in \mathcal{C}^{n_R \times N}$ is the composite channel matrix containing the W channel matrices from the W users. The ergodic sum capacity of the link depends on the availability of channel state information (CSI) at the transmitter side. In particular, if no CSI at the transmitter is assumed, the corresponding ergodic sum capacity is [3, pp. 57]

$$E\{C\} = E\left\{\log_2 \left| \mathbf{I} + \frac{1}{\sigma^2} \mathbf{H}\mathbf{H}^H \right| \right\}, \quad (2)$$

where $E\{|s_i|^2\} = 1$, $i = 1, 2, \dots, N$, is the power of each transmitted symbol. It is convenient to label each column of \mathbf{H} as \mathbf{h}_i , $i = 1, 2, \dots, N$, so that $\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_N)$. The covariance matrix of \mathbf{h}_k is defined by $\mathbf{P}_k = E\{\mathbf{h}_k \mathbf{h}_k^H\}$ and $\mathbf{P}_k = \text{diag}(P_{1k}, P_{2k}, \dots, P_{n_{Rk}})$. Hence, the ik^{th} element of \mathbf{H} is $\mathcal{CN}(0, P_{ik})$. Using this notation, we can also express \mathbf{h}_k as $\mathbf{h}_k = \mathbf{P}_k^{\frac{1}{2}} \mathbf{u}_k$, where $\mathbf{u}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. Note that, for convenience, all the power information is contained in the \mathbf{P}_k matrices so that there is no normalization of the channel and, in (2), the scaling factor in the capacity equation is simply $1/\sigma^2$.

A. Correlated Channels

Consider the general scenario where sources and/or BSs have multiple co-located antennas for transmission and reception. Here, spatial correlation may be present due to the co-located antennas [13], [14].

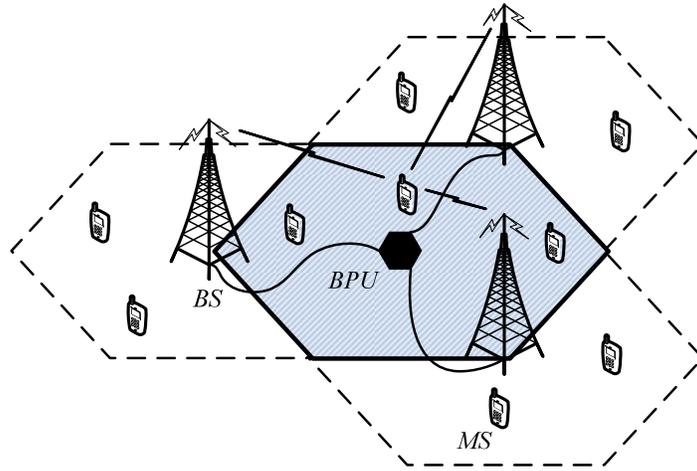


Fig. 1. A network MIMO system with a 3 sector cluster. To reduce the clutter, only paths from a single source are shown.

If a Kronecker correlation model is assumed, then the composite channel matrix is given by

$$\mathbf{H} = \begin{pmatrix} \mathbf{R}_{r1}^{\frac{1}{2}} & \mathbf{0} \\ & \ddots \\ \mathbf{0} & \mathbf{R}_{rM}^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \mathbf{H}_{w,11} & \dots & \mathbf{H}_{w,1W} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{w,M1} & \dots & \mathbf{H}_{w,MW} \end{pmatrix} \begin{pmatrix} \mathbf{R}_{t1}^{\frac{1}{2}} & \mathbf{0} \\ & \ddots \\ \mathbf{0} & \mathbf{R}_{tW}^{\frac{1}{2}} \end{pmatrix}, \quad (3)$$

where the $\mathcal{C}^{n_{R_k} \times n_i}$ matrix, $\mathbf{H}_{w,ik}$, has iid elements since all the channel powers from user k to BS i are the same. The matrix \mathbf{R}_{ri} is the receive correlation matrix at BS i and the matrix \mathbf{R}_{tk} is the transmit correlation matrix at source k as defined in [13]. Using the spectral decompositions, $\mathbf{R}_{ri} = \mathbf{\Phi}_{ri} \mathbf{\Lambda}_{ri} \mathbf{\Phi}_{ri}^H$ and $\mathbf{R}_{tk} = \mathbf{\Phi}_{tk} \mathbf{\Lambda}_{tk} \mathbf{\Phi}_{tk}^H$ and substituting (3) into (2) it is easily shown that the capacity with the channel in (3) is statistically identical to the capacity with channel

$$\mathbf{H} = \begin{pmatrix} \mathbf{\Lambda}_{r1}^{\frac{1}{2}} & \mathbf{0} \\ & \ddots \\ \mathbf{0} & \mathbf{\Lambda}_{rM}^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \mathbf{H}_{w,11} & \dots & \mathbf{H}_{w,1W} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{w,M1} & \dots & \mathbf{H}_{w,MW} \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_{t1}^{\frac{1}{2}} & \mathbf{0} \\ & \ddots \\ \mathbf{0} & \mathbf{\Lambda}_{tW}^{\frac{1}{2}} \end{pmatrix}. \quad (4)$$

Denoting (4) by $\mathbf{H} = \mathbf{\Lambda}_r^{\frac{1}{2}} \mathbf{H}_w \mathbf{\Lambda}_t^{\frac{1}{2}}$, we see that correlation is equivalent to a scaling of the channel by the relevant eigenvalues in $\mathbf{\Lambda}_r$ and $\mathbf{\Lambda}_t$. In particular, the $(u, v)^{th}$ element of \mathbf{H} has power $\mathbf{\Lambda}_{r,uu} \mathbf{\Lambda}_{t,vv} P_{uv}$, where P_{uv} is the single link power from transmit antenna v to receive antenna u . Hence, correlation can be handled by the same methodology developed in Secs. IV-VI, with suitably scaled power values ¹.

III. PRELIMINARIES

In this section we derive some useful results which will be used extensively throughout the paper.

¹Arbitrary fixed transmit power control techniques can also be handled in the same way as for the correlated scenario.

Lemma 1. Let \mathbf{X} be an $n \times n$ complex random matrix with,

$$\mathbf{A} = E \{ \mathbf{X} \circ \mathbf{X} \} \triangleq \begin{pmatrix} E \{ |X_{11}|^2 \} & E \{ |X_{12}|^2 \} & \dots \\ E \{ |X_{21}|^2 \} & E \{ |X_{22}|^2 \} & \dots \\ \dots & \dots & \dots \\ E \{ |X_{n1}|^2 \} & E \{ |X_{n2}|^2 \} & \dots \end{pmatrix}, \quad (5)$$

where \circ represents the Hadamard product. With this notation, the following identity holds.

$$E \{ |\mathbf{X}^H \mathbf{X}| \} = \text{perm}(\mathbf{A}), \quad (6)$$

where $\text{perm}(\cdot)$ is the permanent of a square matrix defined in [15].

Proof: From the definition of the determinant of a generic matrix, $\mathbf{X} = \{X_{i,k}\}_{i,k=1\dots n}$, we have

$$E \{ |\mathbf{X}^H \mathbf{X}| \} = E \left\{ \left[\sum_{\sigma} \text{sgn}(\sigma) \prod_{i=1}^n \bar{X}_{\sigma_i, i} \right] \times \left[\sum_{\mu} \text{sgn}(\mu) \prod_{i=1}^n X_{\mu_i, i} \right] \right\}, \quad (7)$$

where $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)$ is a permutation of the integers $1, \dots, n$, the sum is over all permutations, and $\text{sgn}(\sigma)$ denotes the sign of the permutation. The permutation, μ , in the second summation of (7) is defined similarly. Since all the elements of \mathbf{X} are independent, the only terms giving non-zero value expectations are $\prod_{i=1}^n \bar{X}_{a_i, i} \prod_{i=1}^n X_{b_i, i}$, where permutation $a = b$. Hence, using the permanent definition in [15] we have

$$E \{ |\mathbf{X}^H \mathbf{X}| \} = \sum_{\sigma} \prod_{i=1}^n A_{\sigma_i, i} = \text{perm}(\mathbf{A}). \quad (8)$$

■

Corollary 1. Let \mathbf{X} be an $m \times n$ random matrix with, $E \{ \mathbf{X} \circ \mathbf{X} \} = \mathbf{A}$, where \mathbf{A} is an $m \times n$ deterministic matrix and $m > n$. Then, the following identity holds.

$$E \{ |\mathbf{X}^H \mathbf{X}| \} = \text{Perm}(\mathbf{A}), \quad (9)$$

where $\text{Perm}(\mathbf{A})$ is the permanent of the rectangular matrix \mathbf{A} as defined in [15].

Proof: Using the Cauchy-Binet formula for the determinant of the product of two rectangular matrices, we can expand $|\mathbf{X}^H \mathbf{X}|$ as a sum of products of two square matrices. Each product of square matrices

can be evaluated using Lemma 1. The resulting expression is seen to be the permanent of the rectangular matrix, \mathbf{A} , which completes the proof. ■

Corollary 2. *Let \mathbf{X} be an $m \times n$ random matrix with, $E\{\mathbf{X} \circ \mathbf{X}\} = \mathbf{A}$, where \mathbf{A} is an $m \times n$ deterministic matrix and $m > n$. If the $m \times m$ deterministic matrix Σ is diagonal, then the following identity holds.*

$$E\{|\mathbf{X}^H \Sigma \mathbf{X}|\} = \text{Perm}(\Sigma \mathbf{A}). \quad (10)$$

Proof: The result follows directly from Lemma 1, and the fact that $\Sigma^{\frac{1}{2}} \circ \Sigma^{\frac{1}{2}} = \Sigma$ for any diagonal matrix. ■

Next, we give a definition for the elementary symmetric function (esf) of degree k in n variables, X_1, X_2, \dots, X_n [16]. Let $e_k(X_1, X_2, \dots, X_n)$ be the k^{th} degree esf, then

$$e_k(X_1, X_2, \dots, X_n) = \sum_{1 \leq l_1 < l_2 < \dots < l_k \leq n} X_{l_1} \dots X_{l_k}. \quad (11)$$

It is apparent from (11) that $e_0(X_1, X_2, \dots, X_n) = 1$ and $e_n(X_1, X_2, \dots, X_n) = X_1 X_2 \dots X_n$. In general, the esf of degree k in n variables, for any $k \leq n$, is formed by adding together all distinct products of k distinct variables.

Lemma 2. [16] *let \mathbf{X} be an $n \times n$ complex symmetric positive definite matrix with eigenvalues $\lambda_1, \dots, \lambda_n$. Then, the following identity holds.*

$$e_k(\lambda_1, \lambda_2, \dots, \lambda_n) = \text{Tr}_k(\mathbf{X}), \quad (12)$$

where

$$\text{Tr}_k(\mathbf{X}) = \begin{cases} \sum_{\sigma} |\mathbf{X}_{\sigma_{k,n}}| & 1 \leq k \leq n \\ 1 & k = 0 \\ 0 & k > n, \end{cases} \quad (13)$$

where $\sigma_{k,n}$ is an ordered subset of $\{n\} = \{1, \dots, n\}$ of length k and the summation is over all such subsets. $\mathbf{X}_{\sigma_{k,n}}$ denotes the principal submatrix of \mathbf{X} formed by taking only the rows and columns indexed by $\sigma_{k,n}$.

In general, $\mathbf{X}_{\sigma_{\ell,n}}^{\mu_{\ell,n}}$ denotes the submatrix of \mathbf{X} formed by taking only the rows and columns indexed by $\sigma_{\ell,n}$ and $\mu_{\ell,n}$ respectively, where $\sigma_{\ell,n}$ and $\mu_{\ell,n}$ are length ℓ subsets of $\{1, 2, \dots, n\}$. If either $\sigma_{\ell,n}$ or $\mu_{\ell,n}$ contains the complete set (i.e., $\sigma_{\ell,n} = \{1, 2, \dots, n\}$ or $\mu_{\ell,n} = \{1, 2, \dots, n\}$), the corresponding subscript/superscript may be dropped. When $\sigma_{\ell,n} = \mu_{\ell,n}$, only one subscript/superscript may be shown for brevity.

Next, we present three axiomatic identities for permanents which are required in Sec. V.

- *Axiom 1:* Let \mathbf{A} be an arbitrary $m \times n$ matrix, then

$$\sum_{\sigma} \text{Perm}((\mathbf{A})^{\mu_{0,n}}) = \sum_{\sigma} \text{Perm}((\mathbf{A})_{\sigma_{0,m}}) = 1. \quad (14)$$

- *Axiom 2:* Let \mathbf{A} be an arbitrary $m \times n$ matrix, then

$$\sum_{\sigma} \text{Perm}((\mathbf{A})_{\sigma_{k,m}}) = \sum_{\sigma} \text{Perm}((\mathbf{A})^{\sigma_{k,n}}). \quad (15)$$

- *Axiom 3:* For an empty matrix, \mathbf{A} ,

$$\text{Perm}(\mathbf{A}) = 1. \quad (16)$$

IV. EXACT SMALL SYSTEM ANALYSIS

In this section, we derive the exact ergodic sum capacity in (2) for the $N = 2$ case. This corresponds to two single antenna users or a single user with two distributed antennas. Here, the channel matrix becomes $\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2)$ and it is straightforward to write (2) as

$$\begin{aligned} E\{C\} \ln 2 &= E \left\{ \ln \left| \mathbf{I} + \frac{1}{\sigma^2} \mathbf{h}_1 \mathbf{h}_1^H \right| \right\} \\ &\quad + E \left\{ \ln \left| \mathbf{I} + \frac{1}{\sigma^2} \left(\mathbf{I} + \frac{1}{\sigma^2} \mathbf{h}_1 \mathbf{h}_1^H \right)^{-1} \mathbf{h}_2 \mathbf{h}_2^H \right| \right\} \\ &\triangleq C_1 + C_2. \end{aligned} \quad (17)$$

Both C_1 and C_2 can be expressed as scalars [17], [18, pp. 48], so the capacity analysis simply requires

$$C_1 = E \left\{ \ln \left(1 + \frac{1}{\sigma^2} \mathbf{h}_1^H \mathbf{h}_1 \right) \right\}, \quad (18)$$

$$C_2 = E \left\{ \ln \left(1 + \frac{1}{\sigma^2} \mathbf{h}_2^H \left(\mathbf{I} + \frac{1}{\sigma^2} \mathbf{h}_1 \mathbf{h}_1^H \right)^{-1} \mathbf{h}_2 \right) \right\}. \quad (19)$$

In order to facilitate our analysis, it is useful to avoid the logarithm representations in (18) and (19). We exchange logarithms for exponentials as follows. First, we note the identity,

$$\frac{1}{a} = \int_0^{\infty} e^{-at} dt, \quad \text{for } a > 0. \quad (20)$$

Now equation (20) can be used to find $\ln a$ as below:

$$\frac{\partial \ln a}{\partial a} = \int_0^{\infty} e^{-at} dt, \quad (21)$$

$$\int_0^{\ln a} d \ln a = \int_1^a \int_0^{\infty} e^{-at} dt da, \quad (22)$$

$$\ln a = \int_0^{\infty} \frac{e^{-t} - e^{-at}}{t} dt. \quad (23)$$

This manipulation is useful because there are many results which can be applied to exponentials of quadratic forms, whereas few results exist for logarithms. As an example, using (23) in (18) gives

$$C_1 = E \left\{ \int_0^{\infty} \frac{e^{-t} - e^{-\left(1 + \frac{1}{\sigma^2} \mathbf{h}_1^H \mathbf{h}_1\right)t}}{t} dt \right\}. \quad (24)$$

Note that $a = 1 + \frac{1}{\sigma^2} \mathbf{h}_1^H \mathbf{h}_1$ has been used in (23). Since $a \geq 1$, it follows that the integrand in (24) is non-negative. Also, the expected value, C_1 , is clearly finite and so, by Fubini's theorem, the order of expectation and integration in (24) may be interchanged. Using the Gaussian integral identity [9], the expectation in (24) can be computed to give

$$C_1 = \int_0^{\infty} \frac{e^{-t}}{t} - \frac{e^{-t}}{t |\boldsymbol{\Sigma}_1|} dt, \quad (25)$$

where $\boldsymbol{\Sigma}_1 = \mathbf{I} + \frac{t}{\sigma^2} \mathbf{P}_1$. Hence, the log-exponential conversion in (23) leads to a manageable integral for C_1 . Using the same approach and applying (23) in (19) gives

$$C_2 = E \left\{ \int_0^{\infty} \frac{e^{-t}}{t} - \frac{e^{-t - \frac{t}{\sigma^2} \mathbf{h}_2^H \left(\mathbf{I} + \frac{1}{\sigma^2} \mathbf{h}_1 \mathbf{h}_1^H\right)^{-1} \mathbf{h}_2}}{t} dt \right\}. \quad (26)$$

The expectation in (26) has to be calculated in two stages. First, the expectation over \mathbf{h}_2 can be solved using the Gaussian integral identity [9] and, with some simplifications, we arrive at

$$C_2 = \int_0^\infty \frac{e^{-t}}{t} - E_{\mathbf{h}_1} \left\{ \frac{e^{-t} (\sigma^2 + \mathbf{h}_1^H \mathbf{h}_1)}{t |\boldsymbol{\Sigma}_2| (\sigma^2 + \mathbf{h}_1^H \boldsymbol{\Sigma}_2^{-1} \mathbf{h}_1)} \right\} dt, \quad (27)$$

where $\boldsymbol{\Sigma}_2 = \mathbf{I} + \frac{t}{\sigma^2} \mathbf{P}_2$. Interchange of the expectation and integral in (27) follows from the same arguments used for C_1 . Equation (27) can be further simplified to give

$$C_2 = \int_0^\infty \frac{e^{-t}}{t} - \frac{e^{-t}}{t |\boldsymbol{\Sigma}_2|} dt - E_{\mathbf{h}_1} \left\{ \frac{1}{\sigma^2} \int_0^\infty \frac{e^{-t} \mathbf{h}_1^H \mathbf{P}_2 \boldsymbol{\Sigma}_2^{-1} \mathbf{h}_1}{t |\boldsymbol{\Sigma}_2| (\sigma^2 + \mathbf{h}_1^H \boldsymbol{\Sigma}_2^{-1} \mathbf{h}_1)} dt \right\}. \quad (28)$$

Defining the third term in (28) as I_b , the ergodic sum capacity, $E(C) = C_1 + C_2$, becomes

$$E\{C\} = \frac{1}{\ln 2} \left\{ \sum_{k=1}^2 I_{a_k} - I_b \right\}, \quad (29)$$

where

$$I_{a_k} = \int_0^\infty \left(\frac{e^{-t}}{t} - \frac{e^{-t}}{t |\boldsymbol{\Sigma}_k|} \right) dt. \quad (30)$$

Substituting for $\boldsymbol{\Sigma}_k$ in (30) and expanding $(t |\boldsymbol{\Sigma}_k|)^{-1}$ gives

$$I_{a_k} = \sum_{i=1}^{n_R} \eta_{ik} \int_0^\infty \frac{e^{-t}}{t + \frac{\sigma^2}{P_{ik}}} dt, \quad (31)$$

where

$$\eta_{ik} = \frac{P_{ik}^{n_R-1}}{\prod_{l \neq i} (P_{ik} - P_{lk})}. \quad (32)$$

Note that the first e^{-t}/t term in (30) cancels out with one of the terms in the partial fraction expansion leaving only the linear terms shown in the denominator of (31). The integrals in (31) can be solved in closed form [19] to give

$$I_{a_k} = \sum_{i=1}^{n_R} \eta_{ik} e^{\frac{\sigma^2}{P_{ik}}} E_1 \left(\frac{\sigma^2}{P_{ik}} \right). \quad (33)$$

In order to compute I_b we use [9, Lemma 1] to give

$$I_b = - \int_0^\infty \int_0^\infty \frac{e^{-t}}{|\boldsymbol{\Sigma}_2|} \frac{\partial E \{ e^{-\theta_1 z_1 - \theta_2 z_2} \}}{\partial \theta_1} \Big|_{\theta_1=0} d\theta_2 dt, \quad (34)$$

where $z_1 = \mathbf{h}_1^H \mathbf{P}_2 \boldsymbol{\Sigma}_2^{-1} \mathbf{h}_1$ and $z_2 = \sigma^2 + \mathbf{h}_1^H \boldsymbol{\Sigma}_2^{-1} \mathbf{h}_1$. The expectation in (34) can be solved as in [9], and with some manipulations we arrive at

$$I_b = - \int_0^\infty \int_0^\infty \frac{\partial}{\partial \theta_1} \left[\frac{e^{-\sigma^2 t - \sigma^2 \theta_2}}{|\mathbf{I} + t \mathbf{P}_2 + \theta_1 \mathbf{P}_1 \mathbf{P}_2 + \theta_2 \mathbf{P}_1|} \right]_{\theta_1=0} d\theta_2 dt. \quad (35)$$

In Appendix A, I_b in (35) is calculated in closed form and the final result is given by

$$I_b = - \frac{1}{|\mathbf{P}_1 \mathbf{P}_2|} \left\{ \sum_{i=1}^{n_R} \sum_{k \neq i}^{n_R} \sum_{l \neq i, k}^{n_R} \frac{\xi_{ikl} (\tilde{M}_{b_{ikl}} - \tilde{N}_{b_{ikl}})}{J_i} \right\}, \quad (36)$$

where $\tilde{M}_{b_{ikl}}$, $\tilde{N}_{b_{ikl}}$, J_i and ξ_{ikl} are given in (94), (95), (72) and (82) respectively. Then, the final result becomes

$$E \{C\} = \frac{1}{\ln 2} \left\{ \sum_{k=1}^2 \sum_{i=1}^{n_R} \eta_{ik} e^{\frac{\sigma^2}{P_{ik}}} E_1 \left(\frac{\sigma^2}{P_{ik}} \right) + \frac{1}{|\mathbf{P}_1 \mathbf{P}_2|} \left[\sum_{i=1}^{n_R} \sum_{k \neq i}^{n_R} \sum_{l \neq i, k}^{n_R} \frac{\xi_{ikl} (\tilde{M}_{b_{ikl}} - \tilde{N}_{b_{ikl}})}{J_i} \right] \right\}. \quad (37)$$

V. APPROXIMATE GENERAL ANALYSIS

In this section, we present an approximate ergodic sum rate capacity analysis for the case where $n_R \geq N > 2$. Extending this to $N \geq n_R$ is a simple extension of the current analysis. We use the following notation for the channel matrix,

$$\mathbf{H} = \left(\tilde{\mathbf{H}}_N, \mathbf{h}_N \right) \quad (38a)$$

$$= \left(\tilde{\mathbf{H}}_{N-1}, \mathbf{h}_{N-1}, \mathbf{h}_N \right) \quad (38b)$$

$$= \left(\tilde{\mathbf{H}}_k, \mathbf{h}_k, \dots, \mathbf{h}_{N-1}, \mathbf{h}_N \right) \quad (38c)$$

$$= \quad \vdots$$

$$= \left(\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_N \right), \quad (38d)$$

where the $n_R \times (k-1)$ matrix, $\tilde{\mathbf{H}}_k$, comprises the $k-1$ columns to the left of \mathbf{h}_k in \mathbf{H} . Using the same representation as in (17), the ergodic sum capacity is defined by [3] as,

$$E \{C\} \ln 2 \triangleq \sum_{k=1}^N C_k, \quad (39)$$

where

$$C_k = E \left\{ \ln \left| \mathbf{I} + \frac{1}{\sigma^2} \left(\mathbf{I} + \frac{1}{\sigma^2} \tilde{\mathbf{H}}_k \tilde{\mathbf{H}}_k^H \right)^{-1} \mathbf{h}_k \mathbf{h}_k^H \right| \right\} \quad (40)$$

$$= E \left\{ \ln \left(1 + \frac{1}{\sigma^2} \mathbf{h}_k^H \left(\mathbf{I} + \frac{1}{\sigma^2} \tilde{\mathbf{H}}_k \tilde{\mathbf{H}}_k^H \right)^{-1} \mathbf{h}_k \right) \right\}. \quad (41)$$

Applying (23) to (41) gives

$$C_k = \int_0^\infty \frac{e^{-t}}{t} - E \left\{ \frac{e^{-t} \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right|}{t \left| \boldsymbol{\Sigma}_k \right| \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \boldsymbol{\Sigma}_k^{-1} \tilde{\mathbf{H}}_k \right|} \right\} dt, \quad (42)$$

where $\boldsymbol{\Sigma}_k = \mathbf{I} + \frac{t}{\sigma^2} \mathbf{P}_k$. In order to calculate the second term in (42), the following expectation needs to be calculated,

$$\tilde{I}_k(t) = \frac{1}{|\boldsymbol{\Sigma}_k|} E \left\{ \frac{\left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right|}{\left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \boldsymbol{\Sigma}_k^{-1} \tilde{\mathbf{H}}_k \right|} \right\}. \quad (43)$$

Exact analysis of $\tilde{I}_k(t)$ is cumbersome, and even the $N = 2$ case (see the I_b calculation in (35)) is complicated. Hence, we employ a Laplace type approximation [20], so that $\tilde{I}_k(t)$ can be approximated by

$$\tilde{I}_k(t) \simeq \frac{1}{|\boldsymbol{\Sigma}_k|} \frac{E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right| \right\}}{E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \boldsymbol{\Sigma}_k^{-1} \tilde{\mathbf{H}}_k \right| \right\}}. \quad (44)$$

Note that the Laplace approximation is better known for ratios of scalar quadratic forms [20]. However, the identity in both the numerator and denominator of (43) can be expressed as the limit of a Wishart matrix as in [22]. This gives (43) as ratio of determinants of matrix quadratic forms which in turn can be decomposed to give a product of scalar quadratic forms as in Appendix D and [23]. Hence, the Laplace approximation for (43) has some motivation in the work of [20]. It can also be thought of as a first order delta expansion [24]. From Appendix B, the expectation in the numerator of (44) is given by

$$E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right| \right\} = \sum_{i=0}^{k-1} \sum_{\sigma} \text{Perm} \left((\mathbf{Q}_k)^{\sigma_{i,k-1}} \right) (\sigma^2)^{k-i-1}, \quad (45)$$

where \mathbf{Q}_k is defined in (100). From Appendix C, the expectation in the denominator of (44) is given by

$$|\boldsymbol{\Sigma}_k| E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \boldsymbol{\Sigma}_k^{-1} \tilde{\mathbf{H}}_k \right| \right\} = \sum_{l=0}^{n_R} t^l \varphi_{kl}, \quad (46)$$

where φ_{kl} is given in (110). Therefore, $\tilde{I}_k(t)$ becomes

$$\tilde{I}_k(t) \simeq \frac{\Theta(\mathbf{Q}_k)}{\sum_{l=0}^{n_R} t^l \varphi_{kl}} \quad (47)$$

$$= \frac{\Theta(\mathbf{Q}_k)}{\varphi_{kn_R} \sum_{l=0}^{n_R} \left(\frac{\varphi_{kl}}{\varphi_{kn_R}}\right) t^l} \quad (48)$$

$$= \frac{\Theta(\mathbf{Q}_k)}{\varphi_{kn_R} \prod_{l=1}^{n_R} (t + \omega_{kl})}, \quad (49)$$

where

$$\Theta(\mathbf{Q}_k) = \sum_{i=0}^{k-1} \sum_{\sigma} \text{Perm}((\mathbf{Q}_k)^{\sigma_{i,k-1}}) (\sigma^2)^{k-i-1}. \quad (50)$$

Note that $\omega_{kl} > 0$ for all l, k from Descartes' rule of signs as all the coefficients of the monomial in the denominator of (48) are positive. Also note that, from (110), we have $\Theta(\mathbf{Q}_k) = \varphi_{k0}$. Applying (49) in (42) gives

$$C_k \simeq \int_0^{\infty} \frac{e^{-t}}{t} - \frac{\varphi_{k0}}{\varphi_{kn_R}} \frac{e^{-t}}{t \prod_{l=1}^{n_R} (t + \omega_{kl})} dt. \quad (51)$$

Using a partial fraction expansion for the product in the denominator of the second term of (51) gives

$$\frac{1}{t \prod_{l=1}^{n_R} (t + \omega_{kl})} = \frac{\zeta_{k0}}{t} - \sum_{l=1}^{n_R} \frac{\zeta_{kl}}{t + \omega_{kl}}, \quad (52)$$

where

$$\zeta_{k0} = \frac{1}{\prod_{u=1}^{n_R} \omega_{ku}} = \frac{\varphi_{kn_R}}{\varphi_{k0}} \quad (53)$$

and

$$\zeta_{kl} = \frac{1}{\omega_{kl} \prod_{u \neq l}^{n_R} (\omega_{ku} - \omega_{kl})}. \quad (54)$$

Applying (52) in (42) gives

$$C_k \simeq \frac{\varphi_{k0}}{\varphi_{kn_R}} \sum_{l=1}^{n_R} \int_0^{\infty} \frac{\zeta_{kl}}{t + \omega_{kl}} dt \quad (55)$$

$$= \frac{\varphi_{k0}}{\varphi_{kn_R}} \sum_{l=1}^{n_R} \zeta_{kl} e^{\omega_{kl}} E_1(\omega_{kl}). \quad (56)$$

Then, applying (56) in (39) gives the final approximate ergodic sum rate capacity as

$$E\{C\} \triangleq \frac{1}{\ln 2} \sum_{k=1}^N \left(\frac{\varphi_{k0}}{\varphi_{kn_R}} \sum_{l=1}^{n_R} \zeta_{kl} e^{\omega_{kl}} E_1(\omega_{kl}) \right). \quad (57)$$

Note the simplicity of the general approximation in (57) in comparison to the two-user exact results in (37).

VI. A SIMPLE CAPACITY BOUND

In this section, we derive an extremely simple upper-bound for the ergodic capacity in (2). This provides a simpler relationship between the average link powers and ergodic sum capacity at the expense of a loss in accuracy. Using Jensen's inequality [21] and $\bar{\gamma} = \frac{1}{\sigma^2}$, equation (2) leads to

$$E\{C\} \leq \log_2 \left(E\{|\mathbf{I} + \bar{\gamma} \mathbf{H}^H \mathbf{H}|\} \right). \quad (58)$$

From Appendix B, (58) can be given as

$$E\{C\} \leq \log_2 \left(\sum_{i=0}^N \sum_{\sigma} \text{Perm}(\mathbf{P}^{\sigma_{i,N}}) \bar{\gamma}^i \right), \quad (59)$$

$$= \log_2 \left(\sum_{i=0}^N \vartheta_i \bar{\gamma}^i \right). \quad (60)$$

where $\mathbf{P} = (P_{ik})$. The simplicity of (59) is hidden by the permanent form. For small systems, expanding the permanent reveals the simple relationship between the upper bound and the channel powers. For $n_R = N = 2$ and $n_R = N = 3$, (60) gives the upper bounds in (61) and (62) respectively. These bounds show the simple pattern where cross products of L channel powers scale the $\bar{\gamma}^L$ term. Hence, at low SNR where the $\bar{\gamma}$ term is dominant, $\log_2(1 + P_T \bar{\gamma})$, where $P_T = \sum_i \sum_k P_{ik}$, is an approximation to (60). Similarly, at high SNR, the $\bar{\gamma}^N$ term is dominant and $\log_2(1 + \text{Perm}(\mathbf{P}) \bar{\gamma}^N)$ is an approximation. These approximations show that capacity is affected by the sum of the channel powers at low SNR, whereas at high SNR, the cross products of N powers becomes important.

VII. NUMERICAL AND SIMULATION RESULTS

For the numerical results, we consider three distributed BSs with either a single receive antenna or two antennas. For a two-source system, we parameterize the system by three parameters [8], [9]. The average received signal to noise ratio is defined by $\rho = P_T/\sigma^2$. In particular for a two-source system,

$$E\{C\} \leq \log_2 (1 + \bar{\gamma} (P_{11} + P_{12} + P_{21} + P_{22}) + \bar{\gamma}^2 (P_{11}P_{22} + P_{12}P_{21})). \quad (61)$$

$$\begin{aligned} E\{C\} \leq & \log_2 (1 + \bar{\gamma} (P_{11} + P_{12} + P_{13} + P_{21} + P_{22} + P_{23} + P_{31} + P_{32} + P_{33}) \\ & + \bar{\gamma}^2 (P_{11}P_{22} + P_{11}P_{32} + P_{21}P_{12} + P_{21}P_{32} + P_{31}P_{12} + P_{31}P_{22} + P_{11}P_{23} + P_{11}P_{33} + P_{21}P_{13} \\ & + P_{21}P_{33} + P_{31}P_{13} + P_{31}P_{23} + P_{12}P_{23} + P_{12}P_{33} + P_{22}P_{13} + P_{22}P_{33} + P_{32}P_{13} + P_{32}P_{23}) \\ & + \bar{\gamma}^3 (P_{11}P_{22}P_{33} + P_{11}P_{23}P_{32} + P_{12}P_{21}P_{33} + P_{12}P_{31}P_{23} + P_{13}P_{21}P_{32} + P_{13}P_{22}P_{31})). \end{aligned} \quad (62)$$

$\rho = (\text{Tr}(\mathbf{P}_1) + \text{Tr}(\mathbf{P}_2)) / \sigma^2$. The total signal to interference ratio is defined by $\varsigma = \text{Tr}(\mathbf{P}_1) / \text{Tr}(\mathbf{P}_2)$. The spread of the signal power across the three BS locations is assumed to follow an exponential profile, as in [22], so that a range of possibilities can be covered with only one parameter. The exponential profile is defined by

$$P_{ik} = K_k(\alpha) \alpha^{i-1}, \quad (63)$$

for receive location $i \in \{1, 2, 3\}$ and source k where

$$K_k(\alpha) = \text{Tr}(\mathbf{P}_k) / (1 + \alpha + \alpha^2), \quad k = 1, 2, \quad (64)$$

and $\alpha > 0$ is the parameter controlling the uniformity of the powers across the antennas. Note that as $\alpha \rightarrow 0$ the received power is dominant at the first location, as α becomes large ($\alpha \gg 1$) the third location is dominant and as $\alpha \rightarrow 1$ there is an even spread, as in the standard microdiversity scenario. Using these parameters, eight scenarios are given in Table I for the case of two single antenna users. In Fig. 2, we explore the capacity of scenarios S1-S4 where $n_R = 3$. The capacity result in (37) agrees with the simulations for all scenarios, thus verifying the exact analysis. Furthermore, the approximation in (57) is shown to be extremely accurate. All capacity results are extremely similar except for S1, where both sources have their dominant path at the first receive antenna. Here, the system is largely overloaded (two strong signals at a single antenna) and the performance is lower. The similarity of S3 and S4 is interesting as they represent very different systems. In S3, the two users are essentially separate with the dominant channels being at different antennas. In S4, both users have power equally spread over all antennas so the users are sharing all available channels. Figure 3 follows the same pattern with S6 (the overloaded case) being lower and the other scenarios almost equivalent. In Fig. 3, the overall capacity level is reduced in comparison to Fig. 2 as $\varsigma = 10$ implies a weaker second source.

Figures 4 and 5 show results for a random drop scenario with $M = n_R = 3, W = N = 3$ and $n_R =$

6, $M = 3$, $W = N = 6$ respectively. In each random drop, uniform random locations are created for the users and lognormal shadow fading and path loss are considered where $\sigma_{SF} = 8\text{dB}$ (standard deviation of shadow fading) and $\gamma = 3.5$ (path loss exponent). The transmit power of the sources is scaled so that all locations in the coverage area have a maximum received SNR greater than 3dB, at least 95% of the time. The maximum SNR is taken over the 3 BSs. Hence, each drop produces a different \mathbf{P} matrix and independent channels are assumed. The excellent agreement between the approximation in (57) and the simulations in both Fig. 4 and Fig. 5 is encouraging as this demonstrates the accuracy of (57) over different system sizes as well as over completely different sets of channel powers. Note that at high SNR, Fig. 5 gives much higher capacity values than Fig. 4 since there are 6 receive antenna rather than 3. In this high SNR region, the $\bar{\gamma}^N$ term in (60) dominates and capacity can be approximated by $\log_2(1 + \text{Perm}(\mathbf{P})\bar{\gamma}^N)$. With $n_R = N = 3$ there are 6 cross products in $\text{Perm}(\mathbf{P})$ whereas with $n_R = N = 6$ there are 720 cross products. Hence, the bound clearly demonstrates the benefits of increased antenna numbers. In practice there is a trade-off between the costs of increased collaboration between possibly distant BSs and the resulting increase in system capacity. In Figs. 2 - 5, at low SNR the capacity is controlled by P_T . Hence, since $\rho = P_T/\sigma^2$, all four drops have similar performance at low SNR and diverge at higher SNR where the channel profiles affect the results. The upper bound and associated approximations are shown in Figs. 6 and 7 both for a two user scenario (S3) and a random drop. In Fig. 6, the upper bound is shown for scenario S3 as well as the high and low SNR approximations. The results clearly show the loss in accuracy resulting from the use of the simple Jensen bound. However, the bound is quite reasonable over the whole SNR range. The low SNR approximations are quite reasonable below 0 dB and the high SNR version is as accurate as the bound above 15 dB. In Fig. 7, similar results are shown for a random drop with $M = n_R = 6$, $W = N = 6$. Here, similar patterns are observed but the low and high SNR approximations become reasonable at more widely spread SNR values. For example, the low SNR results are accurate below 0 dB and the high SNR results are poor until around 30 dB. In contrast, the upper bound is reasonable throughout. Hence, although there is a noticeable capacity error at high SNR, the cross-product coefficients in (61) and (62) are seen to explain the large majority of the ergodic capacity behavior.

VIII. CONCLUSION

In this paper, we have studied the ergodic sum capacity of a Rayleigh fading macrodiversity MIMO-MAC. The results obtained are shown to be valid for both independent channels and correlated channels,

TABLE I
PARAMETERS FOR FIGURES 2 AND 3

Sc. No.	Decay Parameter		ζ
	User 1	User 2	
S1	$\alpha = 0.1$	$\alpha = 0.1$	1
S2	$\alpha = 0.1$	$\alpha = 1$	1
S3	$\alpha = 0.1$	$\alpha = 10$	1
S4	$\alpha = 1$	$\alpha = 1$	1
S5	$\alpha = 0.1$	$\alpha = 0.1$	10
S6	$\alpha = 0.1$	$\alpha = 1$	10
S7	$\alpha = 0.1$	$\alpha = 10$	10
S8	$\alpha = 1$	$\alpha = 0.1$	10

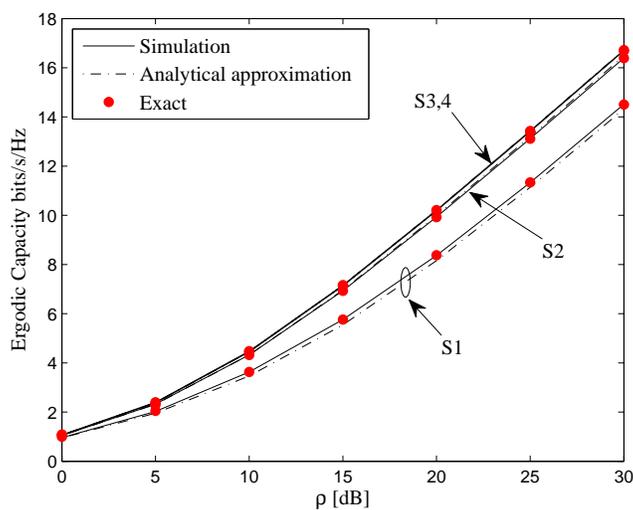


Fig. 2. Exact, approximated and simulated ergodic sum capacity in flat Rayleigh fading for scenarios S1-S4 with parameters: $n_R = 3$, $N = W = 2$ and $\zeta = 1$.

which may occur when some of the distributed transmit/receive locations have closely spaced antennas. In particular, we derive exact results for the two-source scenario and approximate results for the general case. The approximations have a simple form and are shown to be very accurate over a wide range of channel powers. In addition, a simple upper bound is presented which demonstrates the importance of various channel power cross products in determining capacity.

APPENDIX A

DERIVATION OF I_b

From (35), I_b can be written as

$$I_b = \left. \frac{\partial \tilde{I}_b}{\partial \theta_1} \right|_{\theta_1=0}, \quad (65)$$

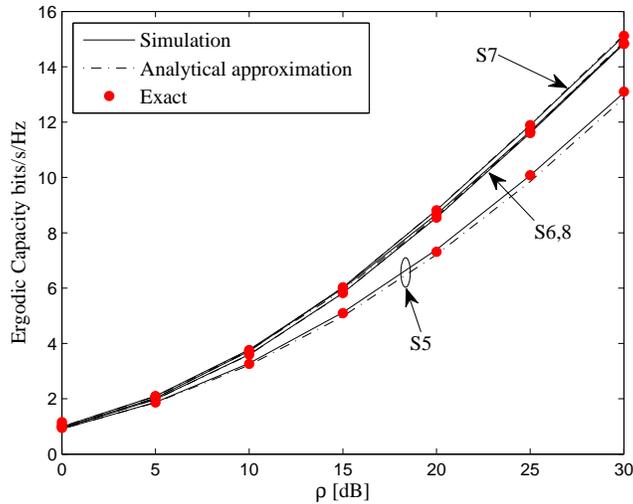


Fig. 3. Exact, approximated and simulated ergodic sum capacity in flat Rayleigh fading for scenarios S5-S8 with parameters: $n_R = 3$, $N = W = 2$ and $\zeta = 10$.

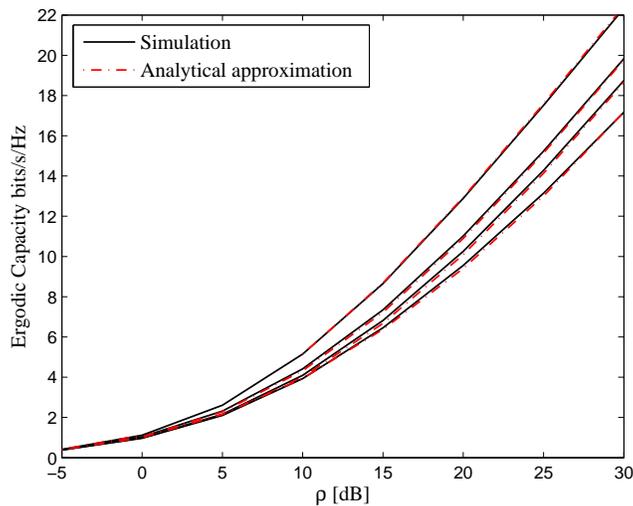


Fig. 4. Approximated and simulated ergodic sum capacity in flat Rayleigh fading for $M = n_R = 3$, $W = N = 3$ and four random drops.

where

$$\tilde{I}_b = - \int_0^\infty \int_0^\infty \frac{e^{-\sigma^2 t - \sigma^2 \theta_2}}{\prod_{i=1}^{n_R} (1 + tP_{i2} + \theta_1 P_{i1} P_{i2} + \theta_2 P_{i1})} d\theta_2 dt. \quad (66)$$

From (66), L_b becomes

$$L_b = \int_0^\infty \int_0^\infty \frac{e^{-\sigma^2 t - \sigma^2 \theta_2}}{\prod_{i=1}^{n_R} \left(\theta_1 + \frac{\theta_2}{P_{i2}} + \frac{t}{P_{i1}} + \frac{1}{P_{i1} P_{i2}} \right)} d\theta_2 dt. \quad (67)$$

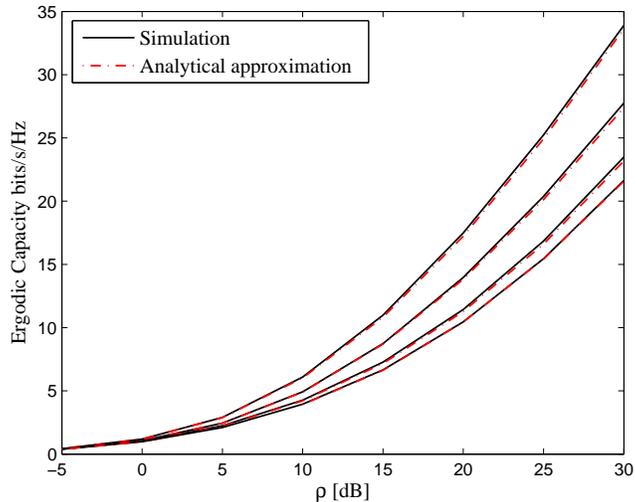


Fig. 5. Approximated and simulated ergodic sum capacity in flat Rayleigh fading for $n_R = 6$, $M = 3$, $W = N = 6$ and four random drops.

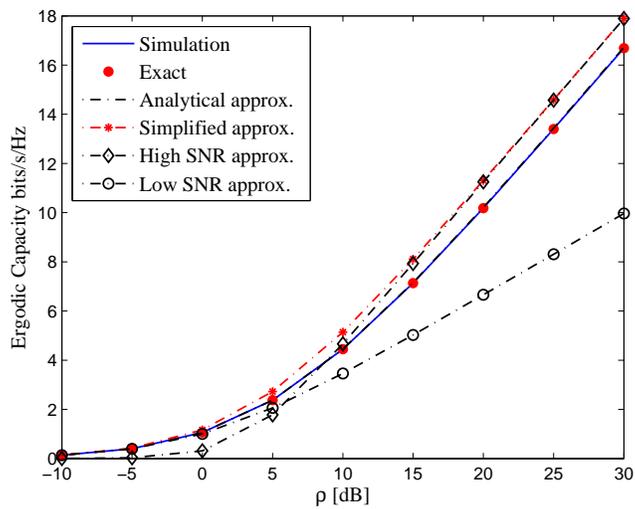


Fig. 6. Ergodic sum capacity in flat Rayleigh fading for scenario S3 with parameters: $M = n_R = 3$, $W = N = 2$ and $\zeta = 1$.

Defining

$$L_b = -|\mathbf{P}_1 \mathbf{P}_2| \tilde{I}_b, \quad (68)$$

we use a partial fraction expansion in θ_1 to give

$$L_b = \sum_{i=1}^{n_R} \int_0^\infty \int_0^\infty \frac{A_i(\theta_2, t) e^{-\sigma^2 t - \sigma^2 \theta_2}}{\left(\theta_1 + \frac{\theta_2}{P_{i2}} + \frac{t}{P_{i1}} + \frac{1}{P_{i1} P_{i2}}\right)} d\theta_2 dt, \quad (69)$$

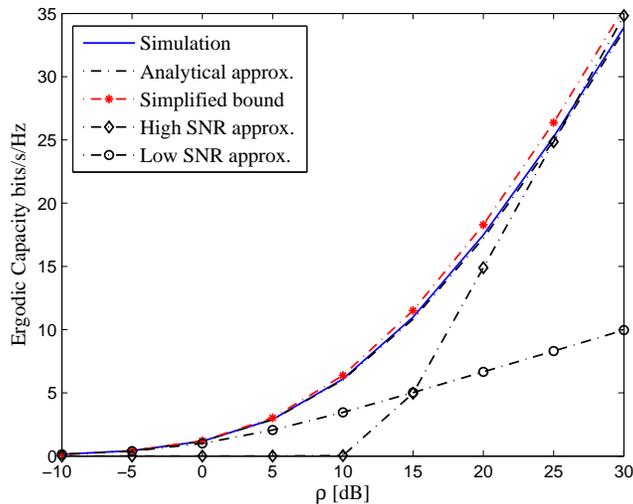


Fig. 7. Ergodic sum capacity in flat Rayleigh fading for a random drop with parameters: $n_R = 6$, $M = 3$ and $W = N = 6$.

where

$$A_i(\theta_2, t) = \frac{1}{\prod_{k \neq i}^{n_R} (\alpha_{ik}\theta_2 + \beta_{ik}t + \gamma_{ik})} \quad (70a)$$

$$\alpha_{ik} = \frac{1}{P_{k2}} - \frac{1}{P_{i2}} \quad (70b)$$

$$\beta_{ik} = \frac{1}{P_{k1}} - \frac{1}{P_{i1}} \quad (70c)$$

$$\gamma_{ik} = R_k - R_i \quad (70d)$$

$$R_i = \frac{1}{P_{i1}P_{i2}}. \quad (70e)$$

To compute (69), the following substitutions are employed

$$u = \sigma^2 t + \sigma^2 \theta_2 \quad (71a)$$

$$v_i = \frac{t}{P_{i1}} + \frac{\theta_2}{P_{i2}}. \quad (71b)$$

The Jacobian of the transformation in (71b) can be calculated as

$$J_i = \sigma^2 \left(\frac{1}{P_{i2}} - \frac{1}{P_{i1}} \right). \quad (72)$$

Substituting (71b) and (72) in (69) gives

$$L_b = \sum_{i=1}^{n_R} \int_0^\infty \int_{\frac{u}{P_{i1}\sigma^2}}^{\frac{u}{P_{i2}\sigma^2}} \frac{A_i(u, v_i) e^{-u}}{J_i(v_i + \theta_1 + R_i)} dv_i du, \quad (73)$$

where

$$A_i(u, v_i) = \frac{1}{\prod_{k \neq i}^{n_R} (a_{ik}v_i + b_{ik}u + \gamma_{ik})} \quad (74a)$$

$$a_{ik} = \frac{\sigma^2}{J_i} (\alpha_{ik} - \beta_{ik}) \quad (74b)$$

$$b_{ik} = \frac{1}{J_i} \left(\frac{\beta_{ik}}{P_{i2}} - \frac{\alpha_{ik}}{P_{i1}} \right). \quad (74c)$$

The term $A_i(u, v_i)$ in (74a) can be written as a summation using partial fractions, to give

$$A_i(u, v_i) = \sum_{k \neq i}^{n_R} \frac{B_{ik}(u)}{v_i + q_{ik}u + r_{ik}}, \quad (75)$$

where

$$B_{ik}(u) = \frac{(a_{ik})^{n_R-3}}{\prod_{l \neq i, k}^{n_R} (c_{ikl}u + d_{ikl})} \quad (76a)$$

$$c_{ikl} = b_{il}a_{ik} - a_{il}b_{ik} \quad (76b)$$

$$d_{ikl} = a_{ik}\gamma_{il} - \gamma_{ik}a_{il} \quad (76c)$$

$$q_{ik} = \frac{b_{ik}}{a_{ik}} \quad (76d)$$

$$r_{ik} = \frac{\gamma_{ik}}{a_{ik}}. \quad (76e)$$

Substituting (75) in (73) and simplifying gives

$$L_b = \sum_{i=1}^{n_R} \sum_{k \neq i}^{n_R} \int_0^\infty \int_{\frac{u}{P_{i1}\sigma^2}}^{\frac{u}{P_{i2}\sigma^2}} \frac{B_{ik}(u) e^{-u}}{J_i} \frac{dv_i du}{(v_i + \theta_1 + R_i)(v_i + q_{ik}u + r_{ik})}. \quad (77)$$

First, we integrate over v_i in (77) to give

$$L_b = \sum_{i=1}^{n_R} \sum_{k \neq i}^{n_R} \int_0^\infty \frac{C_{ik}(u, \theta_1) e^{-u}}{J_i} \ln \left[\frac{\left(\frac{u}{P_{i2}\sigma^2} + \theta_1 + R_i \right) (\lambda_{ik}u + r_{ik})}{\left(\frac{u}{P_{i1}\sigma^2} + \theta_1 + R_i \right) (\mu_{ik}u + r_{ik})} \right] du, \quad (78)$$

where

$$C_{ik}(u, \theta_1) = \frac{B_{ik}(u)}{q_{ik}u + r_{ik} - \theta_1 - R_i} \quad (79a)$$

$$\lambda_{ik} = \frac{1}{P_{i1}\sigma^2} + q_{ik} \quad (79b)$$

$$\mu_{ik} = \frac{1}{P_{i2}\sigma^2} + q_{ik}. \quad (79c)$$

Let

$$D_{ik}(u, \theta_1) = \ln \left[\frac{\left(\frac{u}{P_{i2}\sigma^2} + \theta_1 + R_i \right) (\lambda_{ik}u + r_{ik})}{\left(\frac{u}{P_{i1}\sigma^2} + \theta_1 + R_i \right) (\mu_{ik}u + r_{ik})} \right], \quad (80a)$$

then $B_{ik}(u)$ in (76a) can be rewritten as the summation

$$B_{ik}(u) = \sum_{l \neq i, k}^{n_R} \frac{\xi_{ikl}}{c_{ikl}u + d_{ikl}}, \quad (81)$$

where

$$\xi_{ikl} = \frac{(a_{ik}c_{ikl})^{n_R-3}}{\prod_{z \neq i, k, l}^{n_R} (d_{ikz}c_{ikl} - c_{ikz}d_{ikl})}. \quad (82)$$

Substituting (81) and (79c) in (78) gives

$$L_b = \sum_{i=1}^{n_R} \sum_{k \neq i}^{n_R} \sum_{l \neq i, k}^{n_R} \int_0^\infty D_{ik}(u, \theta_1) \frac{\xi_{ikl}}{J_i} \frac{du}{(c_{ikl}u + d_{ikl})(q_{ik}u + r_{ik} - \theta_1 - R_i)}. \quad (83)$$

Equation (83) can be further simplified to give

$$L_b = \sum_{i=1}^{n_R} \sum_{k \neq i}^{n_R} \sum_{l \neq i, k}^{n_R} \frac{\xi_{ikl} (M_{b_{ikl}} - N_{b_{ikl}})}{J_i}, \quad (84)$$

where

$$M_{b_{ikl}} = \int_0^\infty \frac{D_{ik}(u, \theta_1)}{f_1(\theta_1)} \frac{du}{(u + \varepsilon_{ikl})}, \quad (85)$$

$$N_{b_{ikl}} = \int_0^\infty \frac{D_{ik}(u, \theta_1)}{f_1(\theta_1)} \frac{du}{(u + f_2(\theta_1))}, \quad (86)$$

and $\varepsilon_{ikl} = d_{ikl}/c_{ikl}$. Next, we introduce the following linear functions of θ_1 :

$$f_1(\theta_1) = n_{ikl} - c_{ikl}\theta_1 \quad (87)$$

$$f_2(\theta_1) = m_{ikl} - \frac{\theta_1}{q_{ik}}, \quad (88)$$

$$\frac{\partial}{\partial \theta_1} \left[\frac{D_{ik}(u, \theta_1)}{f_1(\theta_1)} \right]_{\theta_1=0} = \frac{c_{ikl}}{n_{ikl}^2} \ln \left[\frac{\left(\frac{u}{P_{i2}\sigma^2} + R_i \right) (\lambda_{ik}u + r_{ik})}{\left(\frac{u}{P_{i1}\sigma^2} + R_i \right) (\mu_{ik}u + r_{ik})} \right] + \frac{1}{n_{ikl}} \left[\frac{P_{i2}\sigma^2}{\left(u + \frac{\sigma^2}{P_{i1}} \right)} - \frac{P_{i1}\sigma^2}{\left(u + \frac{\sigma^2}{P_{i2}} \right)} \right] \quad (91)$$

$$\tilde{M}_{b_{ikl}} = \frac{\partial M_{b_{ikl}}}{\partial \theta_1} \Big|_{\theta_1=0} = \int_0^\infty \frac{\partial}{\partial \theta_1} \left[\frac{D_{ik}(u, \theta_1)}{f_1(\theta_1)} \right]_{\theta_1=0} \frac{du}{(u + e_{ikl})} \quad (92)$$

$$\tilde{N}_{b_{ikl}} = \frac{\partial N_{b_{ikl}}}{\partial \theta_1} \Big|_{\theta_1=0} = \int_0^\infty \frac{\partial}{\partial \theta_1} \left[\frac{D_{ik}(u, \theta_1)}{f_1(\theta_1)} \right]_{\theta_1=0} \frac{du}{(u + m_{ikl})} + \int_0^\infty \left[\frac{D_{ik}(u, \theta_1)}{f_1(\theta_1)} \right]_{\theta_1=0} \frac{1/q_{ik}}{(u + m_{ikl})^2} du \quad (93)$$

$$\begin{aligned} \tilde{M}_{b_{ikl}} &= \frac{c_{ikl}}{n_{ikl}^2} \left[H_1 \left(R_i, \varepsilon_{ikl}, \frac{1}{P_{i2}\sigma^2} \right) + H_1(r_{ik}, \varepsilon_{ikl}, \lambda_{ik}) - H_1 \left(R_i, \varepsilon_{ikl}, \frac{1}{P_{i1}\sigma^2} \right) - H_1(r_{ik}, \varepsilon_{ikl}, \mu_{ik}) \right] \\ &+ \frac{\varepsilon'_{ikl}}{n_{ikl}} \left[e^{\frac{\sigma^2}{P_{i1}}} E_1 \left(\frac{\sigma^2}{P_{i1}} \right) - e^{\varepsilon_{ikl}} E_1(\varepsilon_{ikl}) \right] - \frac{\varepsilon''_{ikl}}{n_{ikl}} \left[e^{\frac{\sigma^2}{P_{i2}}} E_1 \left(\frac{\sigma^2}{P_{i2}} \right) - e^{\varepsilon_{ikl}} E_1(\varepsilon_{ikl}) \right], \end{aligned} \quad (94)$$

$$\begin{aligned} \tilde{N}_{b_{ikl}} &= \frac{c_{ikl}}{n_{ikl}^2 q_{ik}} \left[H_2 \left(R_i, m_{ikl}, \frac{1}{P_{i2}\sigma^2} \right) + H_2(r_{ik}, m_{ikl}, \lambda_{ik}) - H_2 \left(R_i, m_{ikl}, \frac{1}{P_{i1}\sigma^2} \right) - H_2(r_{ik}, m_{ikl}, \mu_{ik}) \right] \\ &+ \frac{c_{ikl}}{n_{ikl}^2} \left[H_1 \left(R_i, m_{ikl}, \frac{1}{P_{i2}\sigma^2} \right) + H_1(r_{ik}, m_{ikl}, \lambda_{ik}) - H_1 \left(R_i, m_{ikl}, \frac{1}{P_{i1}\sigma^2} \right) - H_1(r_{ik}, m_{ikl}, \mu_{ik}) \right] \\ &+ \frac{m'_{ikl}}{n_{ikl}} \left[e^{\frac{\sigma^2}{P_{i1}}} E_1 \left(\frac{\sigma^2}{P_{i1}} \right) - e^{m_{ikl}} E_1(m_{ikl}) \right] - \frac{m''_{ikl}}{n_{ikl}} \left[e^{\frac{\sigma^2}{P_{i2}}} E_1 \left(\frac{\sigma^2}{P_{i2}} \right) - e^{m_{ikl}} E_1(m_{ikl}) \right], \end{aligned} \quad (95)$$

where

$$n_{ikl} = r_{ik}c_{ikl} - d_{ikl}q_{ik} - \frac{c_{ikl}}{R_i} \quad (89)$$

$$m_{ikl} = \frac{\gamma_{ik}}{b_{ik}} - \frac{1}{q_{ik}R_i}. \quad (90)$$

Next, we can differentiate $M_{b_{ikl}}$ and $N_{b_{ikl}}$ and integrate over u to give the final result along with (65) and (68). Hence, from (87) and (80a) we get (91). Substituting (91) in (85) and (86) we get (92) and (93). (92) and (93) can be solved in closed form to give (94) and (95), where we have used the two integrals defined as follows

$$H_1(a, b, c) = \int_0^\infty \frac{e^{-t} \ln(ct + a)}{t + b} dt \quad (96)$$

$$H_2(a, b, c) = \int_0^\infty \frac{e^{-t} \ln(ct + a)}{(t + b)^2} dt \quad (97)$$

and the constants are given by

$$\begin{aligned}\varepsilon'_{ikl} &= \frac{1}{\left(\varepsilon_{ikl} - \frac{\sigma^2}{P_{i1}}\right)}, & \varepsilon''_{ikl} &= \frac{1}{\left(\varepsilon_{ikl} - \frac{\sigma^2}{P_{i2}}\right)}, \\ m'_{ikl} &= \frac{1}{\left(m_{ikl} - \frac{\sigma^2}{P_{i1}}\right)}, & m''_{ikl} &= \frac{1}{\left(m_{ikl} - \frac{\sigma^2}{P_{i2}}\right)}.\end{aligned}$$

Both H_1 and H_2 can be solved in closed form as

$$\begin{aligned}H_1(a, b, c) &= e^b \left[E_1(b) \ln c + D_1\left(\frac{a}{c} - b, b\right) \right], \\ H_2(a, b, c) &= \ln c \left[\frac{1}{b} - e^b E_1(b) \right] - 2e^b D_1\left(\frac{a}{c} - b, b\right) + \frac{1}{\left(\frac{a}{c} - b\right)} \left[e^b E_1(b) - e^{\frac{a}{c}} E_1\left(\frac{a}{c}\right) \right],\end{aligned}$$

where $D_1(a, b)$ is defined by

$$D_1(a, b) = \int_b^\infty \frac{e^{-t} \ln(t+a)}{t} dt, \quad \text{for } b \neq 0.$$

APPENDIX B

CALCULATION OF $E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right| \right\}$

Let $\lambda_1, \lambda_2, \dots, \lambda_{k-1}$ be the ordered eigenvalues of $\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k$. Since $n_R \geq (k-1)$, all eigenvalues are non zero. Then,

$$\begin{aligned}E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right| \right\} &= E \left\{ \prod_{i=1}^{k-1} (\sigma^2 + \lambda_i) \right\} \\ &= E \left\{ \sum_{i=0}^{k-1} \text{Tr}_i \left(\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right) (\sigma^2)^{k-i-1} \right\},\end{aligned}\tag{98}$$

where (98) is from (11) and Lemma 2. Therefore, the building block of this expectation is $E \left\{ \text{Tr}_i \left(\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right) \right\}$.

From Lemma 2

$$\text{Tr}_i \left(\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right) = \sum_{\sigma} \left| \left(\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right)_{\sigma_{i,k-1}} \right|.\tag{99}$$

Therefore, from Lemma 1,

$$E \left\{ \text{Tr}_i \left(\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right) \right\} = \sum_{\sigma} \text{Perm} \left((\mathbf{Q}_k)^{\sigma_{i,k-1}} \right),$$

where the $n_R \times (k-1)$ matrix, \mathbf{Q}_k , is given by

$$E \left\{ \tilde{\mathbf{H}}_k \circ \tilde{\mathbf{H}}_k \right\} = \mathbf{Q}_k. \quad (100)$$

Note that summation in (100) has $\binom{k-1}{i}$ terms. Then, the final expression becomes

$$E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \right| \right\} = \sum_{i=0}^{k-1} \sum_{\sigma} \text{Perm} \left((\mathbf{Q}_k)^{\sigma_{i,k-1}} \right) (\sigma^2)^{k-i-1}. \quad (101)$$

APPENDIX C

$$\text{CALCULATION OF } |\Sigma_k| E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \Sigma_k^{-1} \tilde{\mathbf{H}}_k \right| \right\}$$

A simple extension of (45) allows the expectation in the denominator of (44) to be calculated as

$$E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \Sigma_k^{-1} \tilde{\mathbf{H}}_k \right| \right\} = \sum_{i=0}^{k-1} \psi_{ki}(t) (\sigma^2)^{k-i-1}, \quad (102)$$

where

$$\psi_{ki}(t) = \sum_{\sigma} \text{Perm} \left((\Sigma_k^{-1} \mathbf{Q}_k)^{\sigma_{i,k-1}} \right), \quad (103)$$

and from (14)

$$\psi_{k0}(t) = 1.$$

The term in (103) can be simplified using (15) to obtain

$$\psi_{ki}(t) = \sum_{\sigma} \frac{\text{Perm} \left((\mathbf{Q}_k)^{\{\sigma_{i,k-1}\}} \right)}{\left| (\Sigma_k)_{\sigma_{i,k-1}} \right|}. \quad (104)$$

Then,

$$|\Sigma_k| E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \Sigma_k^{-1} \tilde{\mathbf{H}}_k \right| \right\} = \sum_{i=0}^{k-1} \xi_{ki}(t) (\sigma^2)^{k-i-1}, \quad (105)$$

where $\xi_{ki}(t) = |\Sigma_k| \psi_{ki}(t)$. From (104), we obtain

$$\xi_{ki}(t) = \sum_{\sigma} \left| (\Sigma_k)_{\bar{\sigma}_{n_R-i, n_R}} \right| \text{Perm} \left((\mathbf{Q}_k)_{\sigma_{i, n_R}}^{\{k-1\}} \right), \quad (106)$$

where $\bar{\sigma}_{n_R-i, n_R}$ is the compliment of σ_{i, n_R} . Therefore, it is apparent that $\xi_{ki}(t)$ is a polynomial of degree $n_R - i$. Clearly $|\Sigma_k| E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \Sigma_k^{-1} \tilde{\mathbf{H}}_k \right| \right\}$ is a polynomial of degree n_R , since $\xi_{k0}(t) = |\Sigma_k|$ is the highest degree polynomial term in t in (105). Then,

$$\left| (\Sigma_k)_{\bar{\sigma}_{n_R-i, n_R}} \right| = \sum_{l=0}^{n_R-i} \left(\frac{t}{\sigma^2} \right)^l \text{Tr}_l \left((\mathbf{P}_k)_{\bar{\sigma}_{n_R-i, n_R}} \right). \quad (107)$$

Hence, applying (107) in (106),

$$\xi_{ki}(t) = \sum_{\sigma} \sum_{l=0}^{n_R-i} \left(\frac{t}{\sigma^2} \right)^l \text{Tr}_l \left((\mathbf{P}_k)_{\bar{\sigma}_{n_R-i, n_R}} \right) \text{Perm} \left((\mathbf{Q}_k)_{\sigma_i, n_R}^{\{k-1\}} \right),$$

and $\xi_{ki}(t)$ becomes

$$\xi_{ki}(t) = \sum_{l=0}^{n_R-i} \left(\frac{t}{\sigma^2} \right)^l \hat{\varphi}_{kli} \quad (108)$$

$$= \sum_{l=0}^{n_R} \left(\frac{t}{\sigma^2} \right)^l \hat{\varphi}_{kli}, \quad (109)$$

where

$$\hat{\varphi}_{kli} = \sum_{\sigma} \text{Tr}_l \left((\mathbf{P}_k)_{\bar{\sigma}_{n_R-i, n_R}} \right) \text{Perm} \left((\mathbf{Q}_k)_{\sigma_i, n_R}^{\{k-1\}} \right),$$

and from (14), $\hat{\varphi}_{kl0}$ simplifies to give

$$\hat{\varphi}_{kl0} = \text{Tr}_l (\mathbf{P}_k).$$

Equation (109) follows from (108) due to the fact that

$$\text{Tr}_l \left((\mathbf{P}_k)_{\bar{\sigma}_{n_R-i, n_R}} \right) = 0 \quad \text{for } l > n_R - i.$$

Therefore, (102) can be written as

$$|\Sigma_k| E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \Sigma_k^{-1} \tilde{\mathbf{H}}_k \right| \right\} = \sum_{i=0}^{k-1} \sum_{l=0}^{n_R} t^l \hat{\varphi}_{kli} (\sigma^2)^{k-l-i-1},$$

which in turn can be given as

$$|\Sigma_k| E \left\{ \left| \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_k^H \Sigma_k^{-1} \tilde{\mathbf{H}}_k \right| \right\} = \sum_{l=0}^{n_R} t^l \varphi_{kl},$$

where

$$\varphi_{kl} = \sum_{i=0}^{k-1} \hat{\varphi}_{kli} (\sigma^2)^{k-l-i-1}. \quad (110)$$

APPENDIX D

EXTENDED LAPLACE TYPE APPROXIMATION

Note the well-known fact that, $\sigma^2 \mathbf{I} = E \{ \mathbf{A}^H \mathbf{A} \}$, for an iid complex Gaussian matrix ensemble, \mathbf{A} , of $\mathcal{CN} \left(0, \frac{\sigma^2}{\kappa} \right)$ random variables, where \mathbf{A} is a $\kappa \times k-1$ matrix as in [22]. This result can be rewritten in the limit to give $\sigma^2 \mathbf{I} = \lim_{\kappa \rightarrow \infty} \{ \mathbf{A}^H \mathbf{A} \}$. Using this in (43) gives

$$\tilde{I}_k(t) = \frac{1}{|\boldsymbol{\Sigma}_k|} \lim_{\kappa \rightarrow \infty} E \left\{ \frac{|\mathbf{A}^H \mathbf{A} + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k|}{|\mathbf{A}^H \mathbf{A} + \tilde{\mathbf{H}}_k^H \boldsymbol{\Sigma}_k^{-1} \tilde{\mathbf{H}}_k|} \right\}, \quad (111)$$

$$= \frac{1}{|\boldsymbol{\Sigma}_k|} \lim_{\kappa \rightarrow \infty} E \left\{ \frac{\left| \left(\mathbf{A}^H, \tilde{\mathbf{H}}_k^H \right) \begin{pmatrix} \mathbf{A} \\ \tilde{\mathbf{H}}_k \end{pmatrix} \right|}{\left| \left(\mathbf{A}^H, \tilde{\mathbf{H}}_k^H \boldsymbol{\Sigma}_k^{-\frac{1}{2}} \right) \begin{pmatrix} \mathbf{A} \\ \boldsymbol{\Sigma}_k^{-\frac{1}{2}} \tilde{\mathbf{H}}_k \end{pmatrix} \right|} \right\}, \quad (112)$$

$$= \frac{1}{|\boldsymbol{\Sigma}_k|} \lim_{\kappa \rightarrow \infty} E \left\{ \frac{|\mathbf{B}_k^H \mathbf{B}_k|}{|\mathbf{B}_k^H \bar{\boldsymbol{\Sigma}}_k \mathbf{B}_k|} \right\}, \quad (113)$$

where $\bar{\boldsymbol{\Sigma}}_k = \text{diag} \left(\mathbf{I}, \boldsymbol{\Sigma}_k^{-\frac{1}{2}} \right)$ and $\mathbf{B}_k = \begin{pmatrix} \mathbf{A} \\ \tilde{\mathbf{H}}_k \end{pmatrix}$. Using the well-known fact

$$|\mathbf{B}_k^H \mathbf{B}_k| = \prod_{i=1}^{k-1} \mathbf{b}_{ki}^H \left(\mathbf{I} - \tilde{\mathbf{B}}_{ki} \left(\tilde{\mathbf{B}}_{ki}^H \tilde{\mathbf{B}}_{ki} \right)^{-1} \tilde{\mathbf{B}}_{ki}^H \right) \mathbf{b}_{ki}, \quad (114)$$

from standard linear algebra, where \mathbf{b}_{ki} is the i^{th} column of \mathbf{B}_k , we can approximate (113) by

$$\tilde{I}_k(t) \simeq \frac{1}{|\boldsymbol{\Sigma}_k|} \prod_{i=1}^{k-1} E \left\{ \frac{\mathbf{b}_{ki}^H \left(\mathbf{I} - \tilde{\mathbf{B}}_{ki} \left(\tilde{\mathbf{B}}_{ki}^H \tilde{\mathbf{B}}_{ki} \right)^{-1} \tilde{\mathbf{B}}_{ki}^H \right) \mathbf{b}_{ki}}{\mathbf{b}_{ki}^H \left(\bar{\boldsymbol{\Sigma}}_k - \bar{\boldsymbol{\Sigma}}_k \tilde{\mathbf{B}}_{ki} \left(\tilde{\mathbf{B}}_{ki}^H \bar{\boldsymbol{\Sigma}}_k \tilde{\mathbf{B}}_{ki} \right)^{-1} \tilde{\mathbf{B}}_{ki}^H \bar{\boldsymbol{\Sigma}}_k \right) \mathbf{b}_{ki}} \right\}, \quad (115)$$

where \mathbf{b}_{ki} and \mathbf{B}_k correspond to a large but finite value of κ . Approximation (115) assumes that the terms in the product in (114) are independent. This is only true when \mathbf{b}_{ki} contains iid elements. However, in the macrodiversity case, all the elements of \mathbf{b}_{ki} are not iid. Nevertheless, part of \mathbf{b}_{ki} (the contribution from \mathbf{A}) is iid. This motivates the approximation in (115). Next, we apply the standard Laplace type approximation

[20] in (115) to give

$$\tilde{I}_k(t) \simeq \frac{1}{|\Sigma_k|} \prod_{i=1}^{k-1} \frac{E \left\{ \mathbf{b}_{ki}^H \left(\mathbf{I} - \tilde{\mathbf{B}}_{ki} \left(\tilde{\mathbf{B}}_{ki}^H \tilde{\mathbf{B}}_{ki} \right)^{-1} \tilde{\mathbf{B}}_{ki}^H \right) \mathbf{b}_{ki} \right\}}{E \left\{ \mathbf{b}_{ki}^H \left(\bar{\Sigma}_k - \bar{\Sigma}_k \tilde{\mathbf{B}}_{ki} \left(\tilde{\mathbf{B}}_{ki}^H \bar{\Sigma}_k \tilde{\mathbf{B}}_{ki} \right)^{-1} \tilde{\mathbf{B}}_{ki}^H \bar{\Sigma}_k \right) \mathbf{b}_{ki} \right\}}, \quad (116)$$

$$\simeq \frac{1}{|\Sigma_k|} \frac{E \left\{ \prod_{i=1}^{k-1} \mathbf{b}_{ki}^H \left(\mathbf{I} - \tilde{\mathbf{B}}_{ki} \left(\tilde{\mathbf{B}}_{ki}^H \tilde{\mathbf{B}}_{ki} \right)^{-1} \tilde{\mathbf{B}}_{ki}^H \right) \mathbf{b}_{ki} \right\}}{E \left\{ \prod_{i=1}^{k-1} \mathbf{b}_{ki}^H \left(\bar{\Sigma}_k - \bar{\Sigma}_k \tilde{\mathbf{B}}_{ki} \left(\tilde{\mathbf{B}}_{ki}^H \bar{\Sigma}_k \tilde{\mathbf{B}}_{ki} \right)^{-1} \tilde{\mathbf{B}}_{ki}^H \bar{\Sigma}_k \right) \mathbf{b}_{ki} \right\}}, \quad (117)$$

$$= \frac{1}{|\Sigma_k|} \frac{E \left\{ |\mathbf{B}_k^H \mathbf{B}_k| \right\}}{E \left\{ |\mathbf{B}_k^H \bar{\Sigma}_k \mathbf{B}_k| \right\}}. \quad (118)$$

Hence, a combination of approximate independence, the Laplace approximation for quadratic forms and the limiting version in (111) gives rise to the approximation used in Sec. V. The accuracy of this approach is numerically established in the simulation results in Sec. VII.

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