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A Greedy Search for Improved QC LDPC Codes with Good Girth Profile and Degree Distribution

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***Abstract*—The girth profile is introduced and search algorithms for regular and irregular quasi-cyclic LDPC block codes with both good girth profile and good degree distribution are presented. New QC LDPC block codes of various code rates are obtained and their bit error rate performance is compared with that of the corresponding LDPC block codes defined in the IEEE 802.16 WiMAX standard of the same block length and code rate.**

I. INTRODUCTION

Due to their low decoding complexity and good bit error rate (BER) performance close to the theoretical limit, low-density parity-check (LDPC) codes are a suitable choice for modern communication standards [1]–[3]. For example, the IEEE 802.16 WiMAX standard defines LDPC block codes with code rates from $1/2$ up to $5/6$ and block lengths between 576 and 2304.

In this paper, we shall focus on quasi-cyclic (QC) LDPC block codes, which can be efficiently represented in the form of tailbitten convolutional codes, and, hence, support searching for new low-complexity codes. QC LDPC block codes can be characterized by being either *regular* or *irregular*. If each column and each row of its parity-check matrix H contains exactly J and K ones, respectively, the corresponding QC LDPC block code is said to be (J, K)-regular; and irregular otherwise. While the symbol node and constraint node *degree distribution* of the parity-check matrix H is constant for regular codes (equal to J and K, respectively), it varies for irregular codes, which significantly affects the BER performance. Hereinafter we shall focus solely on the degree distribution of symbol nodes and will refer to it simply as the degree distribution.

Interpreting the parity-check matrix of an LDPC block code as the *biadjacency matrix* of a bipartite graph yields its corresponding Tanner graph [4]. The length of the shortest cycle in a Tanner graph is called the *girth* of the LDPC block code and is commonly considered to be one of the most important code parameter, as it determines the number of independent iterations in belief-propagation (BP) decoding. Typically, new QC LDPC block codes are constructed by assigning suitable labels to the edges of its *base graph*, that is, replacing the nonzero entries of its binary *base matrix* with monomial entries, such that the girth of the corresponding QC LDPC block code after tailbiting to length M is larger than or equal to a certain target value (see, for example, [5]–[7] and references therein). When constructing QC LDPC block codes in such a way, it is intuitively clear that their BER performance depends on the underlying structure of the base matrix. Moreover, there exists a common conjecture [5]–[7] that the structure of the base matrix has a larger influence on its BER performance than its girth. For example, a QC LDPC block code with "only" girth 6, but good degree distribution, which, for example, satisfies approximately the criterion on the degree proportions given in [8], can achieve a BER performance close to its theoretical limit. A set of such irregular QC LDPC block codes are defined in the IEEE 802.16 WiMAX standard [1], and, to the best of our knowledge, nobody reported QC LDPC block codes with improved BER performance, especially for block lengths as short as $n \approx 576$.

In this paper, we shall study the influence of the degree distribution and the girth profile of QC LDPC block codes, as defined in Section II, on the BER performances of BP decoding. Two greedy algorithms are presented in Section III: one for constructing a base matrix for an irregular QC LDPC block code with a given *degree distribution* and another for obtaining a set of labels for (regular and irregular) QC LDPC block codes yielding a good *girth profile*. Note that in case of regular QC LDPC block codes which have a constant degree distribution, base matrices constructed from all-one matrices (AO) and Steiner triple systems (STS) will be used. In Section IV, the BER performance of these QC LDPC block codes is compared with the corresponding codes defined in the IEEE 802.16 standard of the same code rate and block length. In particular, we shall show that for a fixed degree distribution, the BER performance can be further improved by choosing edge labels yielding a good girth profile. The paper is concluded with some final remarks in Section V.

II. REGULAR AND IRREGULAR QC LDPC CODES

A rate $R = b/c$ binary convolutional code $\mathcal{C}$ is determined by its polynomial parity-check matrix of memory m

$$H(D) = \begin{pmatrix} h_{11}(D) & h_{12}(D) & \dots & h_{1c}(D) \\ h_{21}(D) & h_{22}(D) & \dots & h_{2c}(D) \\ \vdots & \vdots & \ddots & \\ h_{(c-b)1}(D) & h_{(c-b)2}(D) & \dots & h_{(c-b)c}(D) \end{pmatrix} \quad (1)$$

where the parity-check polynomials $h_{ij}(D)$ are either zero or monomial entries D_{ij}^{w} of degree w_{ij}, where w_{ij} are nonnegative integers. Such a parity-check matrix $H(D)$ can be represented by its *degree matrix* $W=(w_{ij})$, $i=1,2,\ldots,c-b$, and $j=1,2,\ldots,c$, where $w_{ij}=-1$ if $h_{ij}(D)=0$. If every row and column of $H(D)$ contains K and J nonzero entries, respectively, we call $\mathcal{C}$ a (J,K)-regular LDPC convolutional code; and irregular otherwise.

Next we express the $(c-b)\times c$ parity-check matrix $H(D)$ in terms of its $m+1$ binary matrices H_i, $i=0,1,\ldots,m$,

$$H(D)=H_0+H_1D+\cdots+H_mD^m$$

Tailbiting the convolutional code $\mathcal{C}$ to length M c-tuples, where $M>m$, yields the $M(c-b)\times Mc$ parity-check matrix of a quasi-cyclic (QC) block code $\mathcal{B}$ as

$$H_{\text{TB}}^{\text{T}}=\begin{pmatrix} H_0^{\text{T}} & H_1^{\text{T}} & \cdots & H_{m-1}^{\text{T}} & H_m^{\text{T}} & \mathbf{0} \\ \mathbf{0} & H_0^{\text{T}} & H_1^{\text{T}} & \cdots & H_{m-1}^{\text{T}} & H_m^{\text{T}} \\ H_m^{\text{T}} & \mathbf{0} & H_0^{\text{T}} & H_1^{\text{T}} & \cdots & H_{m-1}^{\text{T}} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ H_1^{\text{T}} & \cdots & H_{m-1}^{\text{T}} & H_m^{\text{T}} & \mathbf{0} & H_0^{\text{T}} \end{pmatrix} \quad (2)$$

where T denotes transpose. Interpreting H_{TB}^{T} as a biadjacency matrix [9] yields the corresponding Tanner graph $\mathcal{G}$ of this QC LDPC block code. The length of the shortest cycle in this graph is called the girth g.

On the other hand, if we let M tend to infinity we return to the convolutional parity-check matrix $H(D)$ (1) of the parent convolutional code $\mathcal{C}$. In terms of Tanner graph representations, this corresponds to unwrapping the underlying graph and extending it in the time domain towards infinity. The shortest cycle in such an infinite Tanner graph shall be called the *free girth* g_{free}, where $g_{\text{free}}\geq g$.

Moreover, let the *base matrix* of a tailbiting LDPC block code, whose parent convolutional code with parity-check matrix $H(D)$ (1) has only zero or monomial entries, be defined as

$$B=H(D)\big|_{D=1} \quad (3)$$

that is, all nonzero entries in $H(D)$ are replaced by 1. Its *base graph* follows as the bipartite graph, whose biadjacency matrix is given by the base matrix B. Denote the girth of such a base graph by g_{B}.

The problem of finding new QC LDPC block codes can be separated into two different tasks: finding a suitable base matrix (base graph) and determining a set of suitable labels (monomial degrees). In particular, it has been shown that for every such base matrix B (with base graph $\mathcal{G}_{\text{B}}$) there exists a tailbiting length M and a set of labels, such that the girth g of the Tanner graph after tailbiting to length M c-tuples satisfies the inequality [7]

$$g\geq 2\max\left\{g_{\text{B}}+\lceil g_{\text{B}}/2\rceil, d_2\right\}\geq 3g_{\text{B}} \quad (4)$$

where d_2 is the minimum second generalized Hamming distance of the linear $(Jc,(J-2)c+b)$ block code whose parity-check matrix is equal to the incidence matrix of the base graph $\mathcal{G}_{\text{B}}$.

Let $H_i(D)$, $i=1,2,\ldots,c$, denote the submatrix formed by the i columns of $H(D)$ with the smallest column weight and let $g_{\text{free},i}$ be the free girth of the corresponding infinite Tanner graph with biadjacency matrix $H_i(D)$. Then the c-tuple

$$\boldsymbol{g}_{\text{free}}^{\text{p}}=(g_{\text{free},1}\, g_{\text{free},2}\cdots g_{\text{free},c}) \quad (5)$$

is called the *free girth profile* (GP) of the parent infinite Tanner graph, where $g_{\text{free},i}\geq g_{\text{free},i+1}$, $i=1,2,\ldots,c-1$, and $g_{\text{free},c}=g_{\text{free}}$. It can be efficiently represented as the ordered set

$$\mathcal{S}_g=\left\{g_{\text{free}}^{(1)}\big(n^{(1)}\big), g_{\text{free}}^{(2)}\big(n^{(2)}\big),\ldots,g_{\text{free}}^{(l)}\big(n^{(l)}\big)\right\} \quad (6)$$

where $n^{(j)}<n^{(j+1)}$, $n^{(l)}=c$, and $g_{\text{free},i}=g_{\text{free}}^{(j)}(n^{(j)})$ for $n^{(j-1)}<i\leq n^{(j)}$ with $n^{(0)}=0$. In other words, the submatrix, consisting of up to $n^{(j)}$ columns with smallest column weights, has a free girth which is greater than or equal to $g_{\text{free}}^{(j)}$. Note that a girth profile $\boldsymbol{g}_{\text{free}}^{\text{p}}$ is said to be better ($>$) than another profile $\boldsymbol{g}_{\text{free}}^{\text{p}\prime}$ if there exists a positive integer ℓ such that

$$g_{\text{free}}^{(j)}\begin{cases}=g_{\text{free}}^{(j)\prime} & j=1,2,\ldots,\ell-1\\ >g_{\text{free}}^{(j)\prime} & j\geq\ell\end{cases} \quad (7)$$

Similarly, let $J_{\max}$ be the maximum number of nonzero entries in any column of the parity-check matrix $H(D)$ of the parent convolutional code $\mathcal{C}$. Moreover, denote by n_d, $d=1,2,\ldots,J_{\max}$, the number of columns (symbols) of the parity-check matrix $H(D)$ with d nonzero entries (of degree d). Then the $J_{\max}$-tuple

$$\boldsymbol{d}_s^{\text{p}}=(n_1\, n_2\ldots n_{J_{\max}}) \quad (8)$$

is called the (symbol node) *degree distribution* of the parent infinite Tanner graph. Since $\boldsymbol{d}_s^{\text{p}}$ contains mostly zero entries, it can be efficiently represent as the set

$$\lambda_{\text{s}}=\left\{d^{(1)}\big(n^{(1)}\big), d^{(2)}\big(n^{(2)}\big),\ldots,d^{(l)}\big(n^{(l)}\big)\right\} \quad (9)$$

where the $n_{d^{(i)}}=n^{(i)}$ and, clearly, $\sum_{i=1}^{l}n^{(i)}=c$. Note that the degree distribution of the convolutional parity-check matrix $H(D)$ and its base matrix B coincide. Moreover, its degree distribution is invariant among column permutations.

III. Two greedy algorithms for obtaining QC LDPC block codes with good degree distribution and free girth profile

First, we shall determine a base matrix B with a given degree distribution λ_{s}, having at most P_{c} and P_{r} common entires among its columns and rows, respectively. Then, suitable labels (monomial degrees w_{ij}) will be obtained, such that the infinite Tanner graph corresponding to the constructed polynomial parity-check matrix $H(D)$ has a good free girth profile. Finally, tailbiting $H(D)$ to length M yields the corresponding parity-check matrix of an (Mc,Mb) LDPC block code with girth $g\leq g_{\text{free}}$ and similar girth profile. By taking the tailbiting length M into account when determining the labels, $g=g_{\text{free}}$ can be guaranteed.

Assume that a base matrix B of size $(c-b)\times c$ shall be constructed, and let the set

$$\lambda_{\text{s}}^{\text{t}}=\left\{\overline{d}^{(1)}\big(\overline{n}^{(1)}\big),\overline{d}^{(2)}\big(\overline{n}^{(2)}\big),\ldots,\overline{d}^{(l)}\big(\overline{n}^{(l)}\big)\right\} \quad (10)$$

TABLE I
PARAMETERS OF RATE $R=1/2$ AND $R=2/3$ QC LDPC BLOCK CODES OF LENGTH $n=576$ (*$n=572$)

Description	g	g_{free}	distribution
		$R=1/2$	
IEEE 802.16	4	6	$\lambda_s=\{2(11),3(8),6(5)\}$
Irregular	8	8	$\lambda_s=\{2(12),3(4),4(4),6(4)\}$
Irregular (good GP & BD)	6	6	$\lambda_s=\{2(12),3(4),4(4),6(4)\}$ $\mathcal{S}_g=\{12(18),10(20),8(23),6(24)\}$
$(3,6)$-reg. STS*	6	10	$\lambda_s=\{3(26)\}$
$(3,6)$-reg. STS* (good GP)	8	8	$\lambda_s=\{3(26)\}$ $\mathcal{S}_g=\{10(22),8(26)\}$
$(3,6)$-reg. AO (good GP)	8	8	$\lambda_s=\{3(6)\}$ $\mathcal{S}_g=\{12(4),10(5),8(6)\}$
$(3,6)$-reg. AO (random)	4	4	$\lambda_s=\{3(6)\}$
		$R=2/3$	
IEEE 802.16 (A)	6	6	$\lambda_s=\{2(7),3(12),6(5)\}$
IEEE 802.16 (B)	4	4	$\lambda_s=\{2(7),3(1),4(16)\}$
IEEE 802.16 (A) (good GP)	6	6	$\lambda_s=\{2(7),3(12),6(5)\}$ $\mathcal{S}_g=\{14(11),12(12),10(17),8(20),6(24)\}$
Irregular (good GP & BD)	8	8	$\lambda_s=\{2(1),3(5),4(6)\}$ $\mathcal{S}_g=\{12(2),8(12)\}$
$(3,9)$-reg. AO (good GP)	8	8	$\lambda_s=\{3(9)\}$ $\mathcal{S}_g=\{12(3),8(9)\}$
$(3,9)$-reg. AO (random)	4	4	$\lambda_s=\{3(9)\}$

TABLE II
PARAMETERS OF RATE $R=3/4$ AND $R=5/6$ QC LDPC BLOCK CODES OF LENGTH $n=576$

Description	g	g_{free}	profiles
		$R=3/4$	
IEEE 802.16 (A)	4	4	$\lambda_s=\{2(5),3(1),4(18)\}$
IEEE 802.16 (B)	4	6	$\lambda_s=\{2(5),3(12),6(7)\}$
IEEE 802.16 (A) (good GP)	6	6	$\lambda_s=\{2(5),3(1),4(18)\}$ $\mathcal{S}_g=\{14(6),12(7),8(13),6(24)\}$
$(3,12)$-reg. AO (good GP)	6	6	$\lambda_s=\{3(12)\}$ $\mathcal{S}_g=\{12(3),10(4),8(11),6(12)\}$
$(3,12)$-reg. AO (random)	4	4	$\lambda_s=\{3(12)\}$
		$R=5/6$	
IEEE 802.16	4	6	$\lambda_s=\{2(3),3(10),4(11)\}$
IEEE 802.16 (good GP)	6	6	$\lambda_s=\{2(3),3(10),4(11)\}$ $\mathcal{S}_g=\{14(4),12(5),10(6),8(13),6(24)\}$
$(3,18)$-reg. AO (good GP)	6	6	$\lambda_s=\{3(18)\}$ $\mathcal{S}_g=\{12(3),8(9),6(18)\}$
$(3,18)$-reg. AO (random)	4	4	$\lambda_s=\{3(18)\}$

be a given target degree distribution with l entries, such that their proportions approximately satisfy, for example, the criterion given in [8].

Algorithm BD: *Determine randomly a base matrix B for a degree distribution λ_s^t and the numbers of overlapping positions P_c and P_r.*

1) Let B be an empty matrix, $\boldsymbol{v}$ a column vector of length $c-b$, set $i=1$, $j=n=0$, and denote by S the maximum number of trials.
2) While the number of columns of B is less than c:
   (a) If $n=0$, then $j \leftarrow j+1$ and $n \leftarrow \overline{n}^{(j)}$.
   (b) Set $t \leftarrow \overline{d}^{(j)}$ and $s \leftarrow 1$.
   (c) Set t randomly chosen entries of the vector $\boldsymbol{v}$ to 1; and all other entries to 0.
   (d) If the vector $\boldsymbol{v}$ and none of the columns and rows of B coincide in less than or equal to P_c and P_r entries, respectively, set $B \leftarrow [B\,;\boldsymbol{v}]$, increase $i \leftarrow i+1$, decrease $n \leftarrow n-1$, and go to step 2).
   (e) If $s=S$, the algorithm fails and a base matrix B with such a degree distribution can not be found within S number of trials. Restart at step 1.
   (f) Otherwise, increase $s \leftarrow s+1$ and go to step 2(c).

The numbers of overlapping positions P_c and P_r are predetermined constants which directly influence the girth of the base graph g_B and its second generalized Hamming distance d_2, and, hence, the lower bound (4) on the girth g, achievable by a proper labeling of B. From the point of maximizing the girth value g, choosing $P_c=1$ is most suitable but is only feasible when constructing rather large base matrices. On the other hand, $P_c=2$ yields the girth of the base graph as $g_B=4$. However, for suitable structured base matrices, it can be shown that their second generalized Hamming distance is $d_2=7$ and, hence, it is possible to find a labeling yielding an LDPC block code with girth $g=14$. This motivated the choice of $P_c=2$ in our search procedures, while P_r is iteratively increased until such a base matrix can be found.

Next, denote by B_i, $i=1,2,\dots,c$, the submatrix formed by i columns of the base matrix B with the smallest column weight, while the set $\boldsymbol{p}_i^z$ describes the indices of all rows with zero entries in column i. Moreover, let the ordered set

$$\mathcal{S}_g^t=\left\{\overline{g}_{\text{free}}^{(1)}(\overline{n}^{(1)}),\overline{g}_{\text{free}}^{(2)}(\overline{n}^{(2)}),\dots,\overline{g}_{\text{free}}^{(l)}(\overline{n}^{(l)})\right\} \tag{11}$$

be an upper bound on the desired target free girth profile with l entries, where $\overline{g}_{\text{free}}^{(j)}>\overline{g}_{\text{free}}^{(j+1)}$, $n^{(j)}<n^{(j+1)}$, $j=1,2,\dots,l-1$, and $n^{(l)}=c$. Optionally, denote by M the desired tailbiting length.

Algorithm GP: *Determine a set of labels for a given base matrix B such that the corresponding infinite Tanner graph has a good girth profile $\mathcal{S}_g$.*

1) Set $i=2$, $j=1$, $n=0$, initialize W with a binary column vector of length $c-b$, and set all of its entries determined by $\boldsymbol{p}_1^z$ to -1; and to 0 otherwise. Moreover, let $\boldsymbol{v}$ be a column vector of length $c-b$ and choose the maximum number of trials to be S.
2) While the number of columns of W is less than c:
   (a) If $n=0$, then $j \leftarrow j+1$ and $n \leftarrow \overline{n}^{(j)}$.
   (b) Set $t \leftarrow \overline{g}_{\text{free}}^{(j)}$, $s \leftarrow 1$, and $u \leftarrow c-b-|\boldsymbol{p}_i^z|$, where $|\cdot|$ denotes the cardinality.
   (c) Set all entries in the vector $\boldsymbol{v}$ specified by the set $\boldsymbol{p}_i^z$ to -1. That is, $\boldsymbol{v}$ is a column vector with u unknowns.
   (d) Construct a system of girth inequalities for $[W\,;\boldsymbol{v}]$ with target girth t using algorithm A or B from [7]

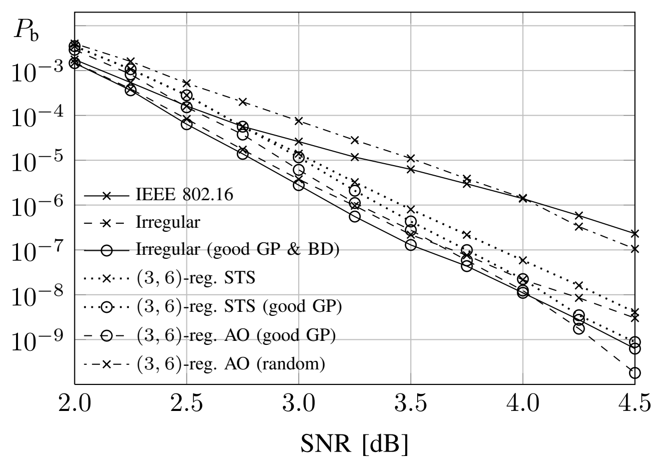

Fig. 1. Bit error rate performance for $R = 1/2$ LDPC block codes.

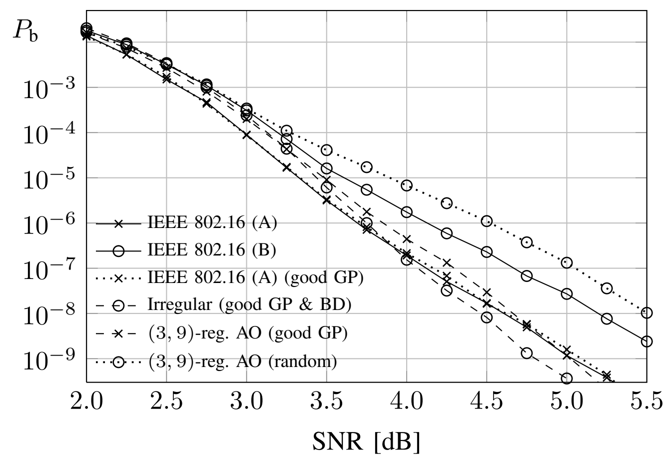

Fig. 2. Bit error rate performance for $R = 2/3$ LDPC block codes.

containing u unknowns, where the tailbiting length M can be optionally taken into account.

(e) Choose randomly these u unknowns.

(f) If all inequalities are satisfied, let $W \leftarrow [W\,;\boldsymbol{v}]$, set $i \leftarrow i+1$, $n \leftarrow n-1$, and go to step 2).

(g) If $s = S$, then decrease $t \leftarrow t-2$ (since the girth is even), set $s \leftarrow 1$, and go to step 2(d).

(h) Otherwise, increase $s \leftarrow s+1$ and go to step 2(e).

The final degree matrix W of size $(c-b)\times c$ specifies a convolutional parity-check matrix $H(D)$, whose infinite Tanner graph has free girth t. Moreover, if the tailbiting length M has been taken into account in step 2(c), the corresponding Tanner graph, whose biadjacency matrix is obtained by tailbiting $H(D)$ to length M, follows as $g = t$.

IV. Simulation Results and Comparisons

In the following a small selection of regular and irregular QC LDPC block codes of rates $1/2$, $2/3$, $3/4$, and $5/6$, with block length $n \approx 576$ shall be presented and compared with the corresponding LDPC block codes as defined in the IEEE 802.16 WiMAX standard [1]. The corresponding codes are given in Table I and Table II together with the free girth g_{free} of the infinite Tanner graph, the girth g of the Tanner graph after tailbiting to length M, as well as their degree distribution λ_{s} and, if applicable, their girth profile $\mathcal{S}_g$ in compact form. The corresponding degree matrices are omitted due to space restriction, but are available online at [10]. Note, that the girth profile is only included if the GP algorithm has been applied, as it depends on the column order of the parity-check matrix $H(D)$ and would otherwise lead to an improper comparison.

In particular, the following QC LDPC block codes shall be compared: (The applied algorithms are given in parenthesis)

- Irregular codes as defined in the IEEE 802.16 standard
- Irregular codes with identical degree distribution as above, but good girth profile (GP) for rates $R \geq 3/4$
- Irregular codes with both good degree distribution [8] and good girth profile (BD & GP) for rates $R \leq 2/3$
- Regular codes with base-matrices constructed from Steiner triple systems (STS) and good girth profile (GP) for rate $R = 1/2$
- Regular codes with all-one (AO) base matrices with good girth profile (GP)
- Regular codes with all-one (AO) base matrices and randomly chosen monomial entries

The corresponding BER performance for those codes is given in Figs. 1–4 using 60 BP decoding iterations. Generally, codes with a good girth profile and/or good degree distribution yield an improved BER performance, typically around $0.5 - 1\,$dB for small and medium signal-to-noise (SNR) values. In the following, the influence of these parameters shall be discussed in more detail.

A. Monomial labels yielding a good girth profile

Comparing for example the two irregular QC LDPC block codes of $R = 1/2$ in Fig. 1 with identical degree distribution, free girth g_{free} and girth g, we see that QC LDPC block codes with a good girth profile usually yield a better BER performance. Note, however, that our search for rate $R = 2/3$ QC LDPC block codes with good girth profiles did not yield any code with an improved BER performance compared to the rate $R = 2/3$ IEEE 802.16 (A) code in Fig. 2, since the girth profile for this code, after reordering its columns, is already $\lambda_{\text{s}} = \{14(11), 12(12), 10(18)\}$, and, hence, better than the best result that we have obtained from the GP algorithm for this code rate. However, when starting from a base matrix with a good degree distribution, and applying our GP algorithm, we were able to improve the BER performance for SNR values above $> 4.0\,$dB.

B. Base matrices with a good degree distribution

Determining a base matrix for a given target degree distribution $\lambda_{\text{s}}^{\text{t}}$ and the numbers of overlapping positions P_{c} and P_{r} imposes a lot of restrictions on the entries of the base matrix, which often can be satisfied only for rather large matrices. For rate $R = 1/2$ LDPC block codes we were able to find such an irregular base matrix of size 12×24 and potential girth 14. Its BER performance with random labelings as well as after improving its girth profile is illustrated in Fig. 1, where the latter case yields the best BER performance for this rate and block length.

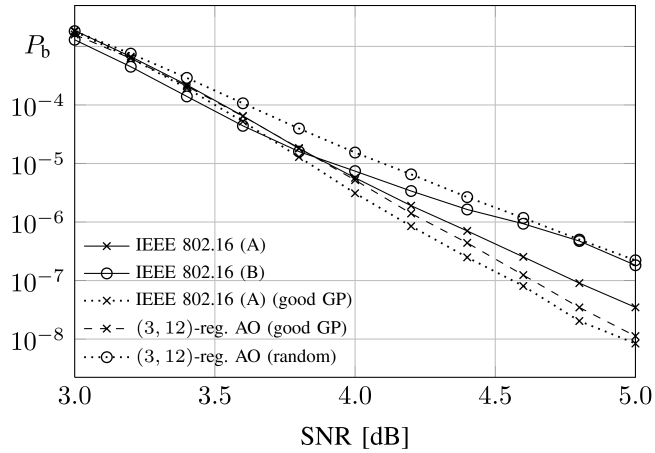

Fig. 3. Bit error rate performance for $R = 3/4$ LDPC block codes.

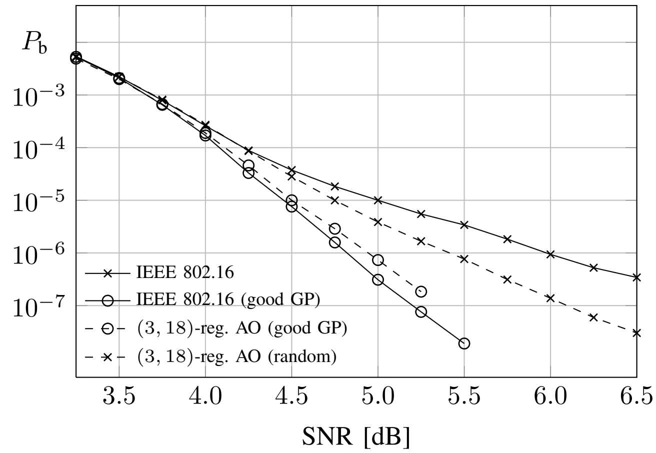

Fig. 4. Bit error rate performance for $R = 5/6$ LDPC block codes.

For rate $R = 2/3$ we were only able to determine a rather small irregular base matrix of size 4×12 with potential girth 12, which yielded the best BER performance for high SNR values. For higher rates, we were so far unable to construct examples of base matrices for good degree distributions due to the additional restrictions imposed by the reduced row per column ratio of the corresponding parity-check matrices. However, using larger computational resources it should be possible to determine similar base matrices of larger sizes and for higher code rates.

C. Regular versus irregular

While irregular LDPC block codes yield the best BER performance among all considered rates, the difference between short regular LDPC block codes with good girth profile and the best irregular LDPC block codes is surprisingly small, in particular for higher rates. For example, the rate $R = 5/6$ $(3, 18)$-regular QC LDPC block code has a better BER performance than the irregular code defined within IEEE 802.16 with same rate and block length.

Since $(3, K)$-regular LDPC block codes can be described by a small set of parameters, it should be possible to further improve such high-rate codes by combining an extensive search with a set of suitable rejection rules.

D. Error floor

In our simulations of the BER performance using BP decoding, we were so far unable to observe any error floor behavior for QC LDPC block codes with good girth profile.

E. Search complexity

Determining suitable labels for a given base matrix is the most complex step in the presented algorithms. We used the algorithm as presented in [7] and generated between 10^3 and 10^6 different labelings per column, taking in average between one and three hours per code. Among the final set of QC LDPC block codes with good girth profile, we randomly selected 10 codes and based our final decision on their SNR values which lead to a bit error probability around 10^{-5}.

Typically the BER performances of all randomly chosen codes were very close to each other, where the BER was improved by a factor of two for the best code, while it increased by a factor of two for the worst code.

V. Summary

Two greedy algorithms for constructing base matrices with a given degree distribution and for finding monomial labels yielding a good girth profile have been presented. Newly found QC LDPC block codes of rates between $R = 1/2$ and $R = 5/6$ have been compared with the corresponding codes defined in the IEEE 802.16 standard. QC LDPC block codes with a good girth profile yielded significantly better BER performances for the rates $R = 1/2$ and $R = 5/6$ compared to the IEEE 802.16 standard, while for the rates $R = 2/3$ and $R = 3/4$, only moderate improvement, especially in the high SNR region, could be achieved. By starting from base matrices with a good degree distribution for rates $R = 1/2$ and $R = 2/3$, these results have been further improved.

Acknowledgements

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