

A Construction of Quantum Codes via A Class of Classical Polynomial Codes

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Abstract—There have been various constructions of classical codes from polynomial valuations in literature [2], [7], [8], [10], [11]. In this paper, we present a construction of classical codes based on polynomial construction again. One of the features of this construction is that not only the classical codes arisen from the construction have good parameters, but also quantum codes with reasonably good parameters can be produced from these classical codes. In particular, some new quantum codes are constructed (see Examples V.5 and V.6).

Index Terms—Cyclotomic cosets, Polynomials, Hermitian self-orthogonal, Quantum distance.

I. INTRODUCTION

One way to produce good quantum codes is to make use of Hermitian self-orthogonal classical codes [1]. To get ℓ -ary quantum codes, one needs Hermitian self-orthogonal classical codes over \mathbb{F}_{ℓ^2} with good minimum distance of dual codes. Due to the fact that the Hermitian inner product involves power ℓ (see (IV.2)), the parameters of quantum codes derived from Hermitian self-orthogonal classical codes are usually constrained. For instance, in [6] (also see [5]), quantum MDS codes produced by using Hermitian self-orthogonal classical codes have relatively small dimension.

In this paper, we first go to a field of larger size to obtain classical codes over \mathbb{F}_{ℓ^2} and then we select Hermitian self-orthogonal codes from these classical codes over \mathbb{F}_{ℓ^2} . In this way, we can produce good quantum codes. Our idea to produce classical codes over \mathbb{F}_{ℓ^2} from a field of large size has already been studied in the previous papers [2], [7], [8], [10], [11] where polynomial codes were considered. The main idea of this paper is to convert some of these codes into Hermitian self-orthogonal in order to construct quantum codes. It turns out that some new quantum codes can be produced (see Examples V.5 and V.6).

The paper is organized as follows. In Section II, we introduce some basic notations and results about cyclotomic cosets and corresponding polynomials. In Section III, we show how classical codes can be constructed from these cosets and polynomials. To construct quantum codes, we study dual codes of these classical codes in Section IV. In the last section, we apply the results in the previous sections to construction of quantum codes.

II. CYCLOTOMIC COSETS AND CORRESPONDING POLYNOMIALS

Let q be a prime power and let $n > 1$ be a positive integer with $\gcd(q, n) = 1$. Let m be the order of q modulo n , i.e., m is the smallest positive integer such that n divides $q^m - 1$.

For any $a \in \mathbb{Z}_n$, we define a q -cyclotomic coset modulo n

$$S_a := \{a \cdot q^i \bmod n : i = 0, 1, 2, \dots\}.$$

It is a well-know fact that all q -cyclotomic cosets partition the set \mathbb{Z}_n . Let $S_{a_1}, S_{a_2}, \dots, S_{a_t}$ stand for all distinct q -cyclotomic cosets modulo n . Then, we have that $\mathbb{Z}_n = \cup_{i=1}^t S_{a_i}$ and $n = \sum_{i=1}^t |S_{a_i}|$. We denote by s_a the size of the q -cyclotomic coset S_a .

The following fact can be easily derived.

Lemma II.1. *For every $a \in \mathbb{Z}_n$, the size s_a of S_a divides m which is the order of q modulo n .*

Proof: It is clear that s_a is the smallest positive integer such that $a \equiv aq^{s_a} \bmod n$, i.e., s_a is the smallest positive integer such that $n/\gcd(n, a)$ divides $q^{s_a} - 1$. Since $n/\gcd(n, a)$ also divides $q^m - 1$, we have $m \equiv 0 \bmod s_a$ by applying the long division. ■

Now for each S_a , we form s_a polynomials in the following way. Let $\alpha_1, \dots, \alpha_{s_a}$ be an \mathbb{F}_q -basis of $\mathbb{F}_{q^{s_a}}$ (note that $\mathbb{F}_{q^{s_a}}$ is a subfield of \mathbb{F}_{q^m}). Consider the polynomials $f_{a,j}(x) := \sum_{i=0}^{s_a-1} (\alpha_j x^a)^{q^i}$ for $j = 1, 2, \dots, s_a$.

Lemma II.2. *For every $a \in \mathbb{Z}_n$, we have the following facts.*

- (i) *The polynomials $f_{a,j}(x)$ for $j = 1, 2, \dots, s_a$ are linearly independent over \mathbb{F}_q .*
- (ii) *$f_{a,j}(\beta)$ belongs to \mathbb{F}_q for all $\beta \in U_n \cup \{0\}$, where U_n is the subgroup of n -th roots of unity in $\mathbb{F}_{q^m}^*$, i.e., $U_n := \{\beta \in \mathbb{F}_{q^m}^* : \beta^n = 1\}$.*

Proof: (i) is clear since the coefficients of x^a in $f_{a,j}(x)$ are α_j and $\alpha_1, \alpha_2, \dots, \alpha_{s_a}$ form an \mathbb{F}_q -basis of $\mathbb{F}_{q^{s_a}}$.

To prove (ii), it is sufficient to prove that $(f_{a,j}(\beta))^q = f_{a,j}(\beta)$ for every $\beta \in U_n \cup \{0\}$. Consider

$$\begin{aligned} (f_{a,j}(\beta))^q &= \left(\sum_{i=0}^{s_a-1} (\alpha_j \beta^a)^{q^i} \right)^q \\ &= \sum_{i=0}^{s_a-1} (\alpha_j \beta^a)^{q^{i+1}} = \sum_{i=1}^{s_a-1} (\alpha_j \beta^a)^{q^i} + \alpha_j^{q^{s_a}} \beta^{aq^{s_a}} \\ &= \sum_{i=1}^{s_a-1} (\alpha_j \beta^a)^{q^i} + \alpha_j \beta^a = f_{a,j}(\beta). \end{aligned}$$

This completes the proof. ■

III. CONSTRUCTION OF CLASSICAL CODES

In this section, we give a construction of classical codes basing on the facts from Section 2. For a positive integer r

with $1 \leq r \leq n-1$, consider the set of polynomials

$$P_r := \{f_{a,j}(x) : 0 \leq a \leq r, j = 1, 2, \dots, s_a\}.$$

Denote the size of P_K by k_r . From Lemma II.2, it is clear that the polynomial space V_r spanned by P_r over \mathbb{F}_q has dimension k_r .

The code C_r is defined by

$$\{(f(\beta))_{\beta \in U_n \cup \{0\}} : f \in V_r\}. \quad (\text{III.1})$$

Proposition III.1. *The code C_r defined in (III.1) is a q -ary linear code with parameters $[n+1, k_r, \geq n+1-r]$.*

Proof: As the degree of every polynomial $f(x)$ in V_r is at most $r \leq n-1$, it has at most r roots. Thus, $(f(\beta))_{\beta \in U_n \cup \{0\}}$ has the Hamming weight at least $n+1-r$ as long as f is a nonzero polynomial. Hence, the dimension of C_r is the same as the one of V_r , i.e., $\dim(C_r) = k_r$. Moreover, the minimum distance of C_r is at least $n+1-r$. ■

Example III.2. Let $q = 4$ and $n = 51$. Then the order of 4 modulo 51 is $m = 4$. All 4-cyclotomic cosets modulo 51 are

$\{0\}$	$\{1, 4, 13, 16\}$	$\{2, 8, 26, 32\}$
$\{3, 12, 39, 48\}$	$\{5, 14, 20, 29\}$	$\{6, 24, 27, 45\}$
$\{7, 10, 28, 40\}$	$\{9, 15, 36, 42\}$	$\{11, 23, 41, 44\}$
$\{17\}$	$\{18, 21, 30, 33\}$	$\{19, 25, 43, 49\}$
$\{22, 31, 37, 46\}$	$\{34\}$	$\{35, 38, 47, 50\}$

For instance, for $r = 16$, we obtain a 4-ary $[52, 5, \geq 36]$ -linear code. This is an optimal code in the sense that for given length and dimension, the minimum distance can not be improved. For $r = 17$, we obtain a 4-ary $[52, 6, \geq 35]$ -linear code which is best known based on the online table [4].

Example III.3. Let $q = 4$ and $n = 63$. Then the order of 4 modulo 63 is $m = 3$. All 4-cyclotomic cosets modulo 63 are

$\{0\}$	$\{1, 4, 16\}$	$\{2, 8, 32\}$
$\{3, 12, 48\}$	$\{5, 17, 20\}$	$\{6, 24, 33\}$
$\{7, 28, 49\}$	$\{9, 18, 36\}$	$\{10, 34, 40\}$
$\{11, 44, 50\}$	$\{13, 19, 52\}$	$\{14, 35, 56\}$
$\{15, 51, 60\}$	$\{21\}$	$\{22, 25, 37\}$
$\{23, 29, 53\}$	$\{26, 38, 41\}$	$\{27, 45, 54\}$
$\{30, 39, 57\}$	$\{31, 55, 61\}$	$\{47, 59, 62\}$
$\{43, 46, 58\}$	$\{42\}$	

For instance, for $r = 16$, we get a 4-ary $[64, 4, \geq 48]$ -linear code. This is an optimal code in the sense that for given length and dimension, the minimum distance can not be improved. For $r = 20$, again we get an optimal 4-ary $[64, 7, \geq 44]$ -linear code. For $r = 21$, an optimal 4-ary $[64, 8, \geq 43]$ -linear code can be derived as well.

IV. DUAL CODES

In this section, we study dual codes for those codes arisen from cyclotomic cosets. From now on, we assume that q is even. Then n is always odd (as $\gcd(n, q) = 1$) and hence $n+1$ is even.

Two q -cyclotomic cosets S_a and S_b are called *dual* if there exists $c \in S_b$ such that $a+c$ is divisible by n . For instance,

in Example III.2, $\{1, 4, 13, 16\}$ and $\{35, 38, 47, 50\}$ are dual to each other. It is clear that the dual of a given cyclotomic coset is unique. Moreover, we have the following facts.

Lemma IV.1. *Let S_a be the dual of a cyclotomic coset S_b . Then we have*

- (i) $|S_a| = |S_b|$
- (ii) *For every $x \in S_a$, there exists $y \in S_b$ such that $x+y$ is divisible by n .*

Proof: We may assume that $a+b$ is divisible by n . By definition, s_b is the smallest positive integer such that n divides $b(q^{s_b} - 1)$. Thus, s_b is the smallest positive integer such that n divides $-b(q^{s_b} - 1)$. As $-b(q^{s_b} - 1) \equiv a(q^{s_b} - 1) \pmod{n}$, the desired result of part (i) follows.

Let $x \equiv aq^i \pmod{n}$ for some integer i . By definition, there exists $c \in S_b$ such that $a \equiv -c \pmod{n}$. Thus, $x \equiv aq^i \equiv -cq^i \pmod{n}$. Put $y = cq^i \pmod{n} \in S_b$. We obtain the desired result of part (ii). ■

Consider a set \mathcal{S} of cyclotomic cosets such that $\{0\} \in \mathcal{S}$. Let \mathcal{S}^* denote the collection of duals of cyclotomic cosets in \mathcal{S} . We denote by $P_{\mathcal{S}}$ the polynomial set

$$\{f_{a,j}(x) : S_a \in \mathcal{S}; j = 1, 2, \dots, s_a\}.$$

Let $V_{\mathcal{S}}$ be the \mathbb{F}_q -space spanned by all polynomials in $P_{\mathcal{S}}$. Define the \mathbb{F}_q -linear code by

$$C_{\mathcal{S}} := \{(f(\beta))_{\beta \in U_n \cup \{0\}} : f \in V_{\mathcal{S}}\} \quad (\text{IV.1})$$

Then we have the following result.

Proposition IV.2. *Let $\mathcal{A} = \cup_{i=1}^t S_{a_i}$ be the set of all q -cyclotomic cosets modulo n . Then the Euclidean dual of $C_{\mathcal{S}}$ is $C_{\mathcal{R}}$, where $\mathcal{R} = \{\{0\}\} \cup (\mathcal{A} - \mathcal{S}^*)$.*

Proof: First of all, the dimension of the code $C_{\mathcal{S}}$ is $\sum_{S \in \mathcal{S}} |S|$. Thus, the dimension of $C_{\mathcal{R}}$ is $1 + \sum_{S \in \mathcal{A}} |S| - \sum_{T \in \mathcal{S}} |T| = n+1 - \dim(C_{\mathcal{S}})$ (note the fact that $\sum_{S \in \mathcal{A}} |S| = |\mathbb{Z}_n| = n$). To prove our lemma, it is sufficient to show that every codeword in $C_{\mathcal{S}}$ is orthogonal to all codewords of $C_{\mathcal{R}}$ under the dot product.

For a polynomial $u(x)$ in $P_{\mathcal{A}}$, we denote by \mathbf{c}_u the codeword $(u(\beta))_{\beta \in U_n \cup \{0\}}$. Let $f(x), g(x)$ be polynomials in $P_{\mathcal{S}}$ and $P_{\mathcal{R}}$, respectively. If both $f(x)$ and $g(x)$ are equal to 1. Then $\mathbf{c}_f = \mathbf{c}_g$ is the all-one vector $\mathbf{1}$. It is clear that in this case \mathbf{c}_f and \mathbf{c}_g are orthogonal under the dot product. Now assume that at least one of $f(x), g(x)$ is not equal to 1. Then for any terms x^i in $f(x)$ and terms x^j in $g(x)$, we have $i+j \not\equiv 0 \pmod{n}$. Thus, the product $f(x)g(x)$ contains only terms x^k with $k \not\equiv 0 \pmod{n}$. For such k we have

$$\sum_{\beta \in U_n \cup \{0\}} \beta^k = \frac{\alpha^{kn} - 1}{\alpha^k - 1} = 0,$$

where α is an n -th primitive root of unity in U_n . This implies that \mathbf{c}_f and \mathbf{c}_g are orthogonal under the dot product. The desired result follows. ■

Example IV.3. Let $q = 4$ and $n = 51$. Let $\mathcal{S} = \{\{0\}, \{1, 4, 13, 16\}\}$. By Example III.2, we know that $\mathcal{R} = \mathcal{A} - \{\{35, 38, 47, 50\}\}$.

In order to apply our results to quantum codes, we want to discuss the Hermitian dual of $C_{\mathcal{S}}$ as well. Let us assume that q is equal to ℓ^2 . The Hermitian inner product of the two vectors $(u_1, u_2, \dots, u_{n+1})$ and $(v_1, v_2, \dots, v_{n+1})$ in $\mathbb{F}_{\ell^2}^n$ is defined by

$$\sum_{i=1}^{n+1} u_i^\ell v_i. \quad (\text{IV.2})$$

By abuse of notations, for a set $\mathcal{S} = \{S_a\}_{a \in I}$ of cyclotomic cosets, we denote by $\ell\mathcal{S}$ the set $\{S_{a\ell}\}_{a \in I}$ of the cyclotomic cosets.

Proposition IV.4. *Under the inner product (IV.2), the Hermitian dual of $C_{\mathcal{S}}$ is $C_{\mathcal{T}}$, where $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (\ell\mathcal{S})^*)$.*

Proof: It is clear that the Hermitian dual of $C_{\mathcal{S}}$ is the Euclidean dual of $C_{\ell\mathcal{S}}$. Now the desired result follows from Proposition IV.2. ■

Example IV.5. Let $q = 4$ and $n = 51$. Let $\mathcal{S} = \{\{0\}, \{1, 4, 13, 16\}\}$. By Example III.2, we know that $\mathcal{T} = \mathcal{A} - \{\{19, 25, 43, 49\}\}$.

V. APPLICATION TO QUANTUM CODES

In this section, we show how to apply the results from the previous sections to obtain quantum codes.

Instead of giving several complicated results with detailed formula, we give a general result in this section. Then we use examples to illustrate our result.

Theorem V.1. *Let \mathcal{S} be a set of q -cyclotomic cosets modulo n and let $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (\ell\mathcal{S})^*)$ such that $(\ell\mathcal{S})^*$ contains all cyclotomic cosets $\{S_a : n + 2 - d \leq a \leq n - 1\}$. If \mathcal{S} is a subset of \mathcal{T} , then there exists an ℓ -ary quantum code $[[n + 1, n + 1 - 2k, \geq d]]$, where k is the \mathbb{F}_q -dimension of $C_{\mathcal{S}}$.*

Proof: By Proposition IV.4, the Hermitian dual of $C_{\mathcal{S}}$ is $C_{\mathcal{T}}$. Under our assumption, $C_{\mathcal{S}}$ is Hermitian self-orthogonal under the inner product (IV.2). Thus, we obtain an ℓ -ary quantum code $[[n + 1, n + 1 - 2k]]$ with minimum distance at least the Hamming distance of $C_{\mathcal{T}}$ (see [1]). As $P_{\mathcal{T}}$ contains polynomials of degree at most $n + 1 - d$, the Hamming distance of $C_{\mathcal{T}}$ is at least d . This completes the proof. ■

Example V.2. Let $q = 4$ and $n = 21$. Then the order of 4 modulo 21 is $m = 3$. All 4-cycloyomic cosets modulo 21 are

$\{0\}$	$\{1, 4, 16\}$	$\{2, 8, 11\}$
$\{3, 6, 12\}$	$\{5, 17, 20\}$	$\{7\}$
$\{9, 15, 18\}$	$\{10, 13, 19\}$	$\{14\}$

Let $\mathcal{S} = \{\{0\}, \{1, 4, 16\}, \{2, 8, 11\}, \{3, 6, 12\}\}$. Then $2\mathcal{S} = \mathcal{S}$ and $(2\mathcal{S})^* = \{\{0\}, \{5, 17, 20\}, \{10, 13, 19\}, \{9, 15, 18\}\}$. Moreover, \mathcal{S} is contained in $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (2\mathcal{S})^*)$. As S_{17}, S_{18}, S_{19} and S_{20} belong to $(2\mathcal{S})^*$, we obtain a binary quantum $[[22, 2, 6]]$ code which achieves the best-known parameters [4].

Example V.3. Let $q = 4$ and $n = 51$. Then the order of 4 modulo 51 is $m = 4$. Let $\mathcal{S} = \{\{0\}, \{1, 4, 13, 16\}, \{2, 8, 26, 32\}, \{6, 24, 27, 45\}\}$. Then $2\mathcal{S} = \{\{0\}, \{1, 4, 13, 16\}, \{2, 8, 26, 32\}, \{3, 12, 39, 48\}\}$ and $(2\mathcal{S})^* = \{\{0\}, \{35, 38, 47, 50\}, \{19, 25, 43, 49\}, \{3, 12, 39, 48\}\}$. Moreover, \mathcal{S} is contained in $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (2\mathcal{S})^*)$. As S_{47}, S_{48}, S_{49} and S_{50} belong to $(2\mathcal{S})^*$, we obtain a binary quantum $[[52, 26, 6]]$ code which meets the best-known one in the online table [4].

In the similar way, we obtain binary quantum codes with parameters $[[52, 24, 7]]$ and $[[52, 8, 10]]$. Both codes meet the parameters of the best-known ones in [4].

Example V.4. Let $q = 4$ and $n = 63$. Then the order of 4 modulo 63 is $m = 3$.

- (i) $\mathcal{S} = \{\{0\}, \{1, 4, 16\}, \{2, 8, 32\}\}$. Then $2\mathcal{S} = \mathcal{S}$ and $(2\mathcal{S})^* = \{\{0\}, \{31, 55, 61\}, \{47, 59, 62\}\}$. Moreover, \mathcal{S} is contained in $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (2\mathcal{S})^*)$. As S_{61} and S_{50} belong to $(2\mathcal{S})^*$, we obtain a binary quantum $[[64, 50, 4]]$ code which is optimal [4].
- (ii) $\mathcal{S} = \{\{0\}\}, \{1, 4, 16\}, \{2, 8, 32\}, \{6, 24, 33\}\}$. Then $2\mathcal{S} = \{\{0\}\}, \{1, 4, 16\}, \{2, 8, 32\}, \{3, 12, 48\}\}$ and $(2\mathcal{S})^* = \{\{0\}, \{15, 51, 60\}, \{31, 55, 61\}, \{47, 59, 62\}\}$. Moreover, \mathcal{S} is contained in $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (2\mathcal{S})^*)$. As S_{59}, S_{60}, S_{61} and S_{62} belong to $(2\mathcal{S})^*$, we obtain a binary quantum $[[64, 44, 6]]$ code which is optimal again [4].

Analogously, binary quantum codes with parameters $[[64, 38, 7]]$ and $[[64, 32, 8]]$ can be derived. Both codes meet the parameters of the best-known ones in [4].

Example V.5. Let $q = 16$ and $n = 51$. Then the order of 16 modulo 51 is $m = 2$. Let $\mathcal{S} = \{\{0\}, \{12, 39\}, \{8, 26\}, \{4, 13\}\}$. Then $4\mathcal{S} = \{\{0\}, \{1, 16\}, \{2, 32\}, \{3, 48\}\}$ and $(4\mathcal{S})^* = \{\{0\}, \{3, 48\}, \{19, 49\}, \{35, 50\}\}$. Moreover, \mathcal{S} is contained in $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (2\mathcal{S})^*)$. As S_{50}, S_{49} and S_{48} belong to $(4\mathcal{S})^*$, we obtain a 4-ary quantum $[[52, 38, 5]]$ -code.

Likewise, we obtain 4-ary quantum codes with parameters $[[52, 34, 6]]$, $[[52, 30, 7]]$, $[[52, 26, 8]]$, $[[52, 22, 9]]$, $[[52, 18, 10]]$ and $[[52, 14, 12]]$. The last one meets the parameters of the best-known ones in [3] and the rest are new to the online table [3].

Example V.6. Let $q = 64$ and $n = 585$. Then the order of 64 modulo 585 is $m = 2$. Let $\mathcal{S} = \{\{0\}, \{8, 512\}, \{16, 439\}\}$. Then $8\mathcal{S} = \{\{0\}, \{1, 64\}, \{2, 128\}\}$ and $(8\mathcal{S})^* = \{\{0\}, \{457, 583\}, \{521, 584\}\}$. Moreover, \mathcal{S} is contained in $\mathcal{T} = \{\{0\}\} \cup (\mathcal{A} - (2\mathcal{S})^*)$. As S_{584} and S_{583} belong to $(8\mathcal{S})^*$, we obtain a 8-ary quantum $[[586, 576, 4]]$ -code.

In the similar way, we draw 8-ary quantum codes with parameters $[[586, 572, 5]]$, $[[586, 568, 6]]$, $[[586, 564, 7]]$, $[[586, 560, 8]]$, $[[586, 556, 9]]$, $[[586, 552, 10]]$, $[[586, 548, 11]]$, $[[586, 544, 12]]$, $[[586, 540, 13]]$, $[[586, 536, 14]]$, $[[586, 532, 15]]$ and so on. Now, compared with the online table [3], these codes have better parameters. For instance, 8-quantum codes with the parameters $[[589, 553, 4]]$,

[[589, 513, 6]], [[627, 561, 5]], [[627, 531, 6]], [[627, 501, 7]], [[629, 557, 6]], [[629, 533, 7]], [[629, 521, 8]] are given in [3]. We can see that with the same distances our codes have bigger dimensions, but smaller lengths.

REFERENCES

- [1] A. Ashikhmin and E. Knill, "Nonbinary quantum stabilizer codes," *IEEE Trans. Inf. Theory*, vol. 47, no. 7, pp. 3065–3072, Nov. 2001.
- [2] N. Aydin and D. K. Ray-Chaudhuri, "On some classes of optimal and near-optimal polynomial codes," *Finite Fields and Their Applications*, 10(1004), pp.24-35.
- [3] J. Bierbrauer, "Some good quantum twisted codes," <http://www.mathi.uni-heidelberg.de/yves/Matritzen/QT BCH/QT BCHIndex.html>, January, 2012.
- [4] M. Grassl, "Code Tables: Bounds on the parameters of various types of codes," <http://www.codetables.de/>, January, 2012.
- [5] M. Grassl, T. Beth, M. Roetteler, "On optimal quantum codes," *International Journal of Quantum Information*, Vol. 2, No. 1, pp. 55-64, 2004.
- [6] L. Jin, L. San, J. Luo and C. Xing, "Application of classical Hermitian self-orthogonal MDS codes to quantum MDS codes," *IEEE Trans. Inform. Theory*, vol. 56, no. 8, pp. 4735–4740, Sep. 2010.
- [7] S. Ling, H. Niederreiter and C. Xing, "Symmetric polynomials and some good codes," *Finite Fields and Their Applications*, 7 (2001), pp.142C148.
- [8] S. Ling and C. P. Xing, *Coding Theory – A first course*, Cambridge, 2004.
- [9] F. J. MacWilliams and N. J. A. Sloane, *The theory of error-correcting codes*, Amsterdam: North-Holland, 1977.
- [10] C. P. Xing and Y. Fang, "A class of polynomial codes," *IEEE Trans. on Inform. Theory*, Vol.50(5)(2004), pp. 884-887.
- [11] C. P. Xing and S. Ling, "A class of linear codes with good parameters," *IEEE Trans. on Inform. Theory*, Vol.46(6)(2000), pp.1527-1532.