

Opportunistic Interference Management for Multicarrier systems

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Abstract—We study opportunistic interference management when there is bursty interference in parallel 2-user linear deterministic interference channels. A degraded message set communication problem is formulated to exploit the burstiness of interference in M subcarriers allocated to each user. We focus on symmetric rate requirements based on the number of interfered subcarriers rather than the exact set of interfered subcarriers. Inner bounds are obtained using erasure coding, signal-scale alignment and Han-Kobayashi coding strategy. Tight outer bounds for a variety of regimes are obtained using the El Gamal-Costa injective interference channel bounds and a sliding window subset entropy inequality [7]. The result demonstrates an application of techniques from multilevel diversity coding to interference channels. We also conjecture outer bounds indicating the sub-optimality of erasure coding across subcarriers in certain regimes.

I. INTRODUCTION

In multicarrier systems like OFDM, subcarriers allocated to a user may face interference due to a variety of reasons. These include the activity of other users and allocation decisions of neighbouring base stations in a cellular network. Predicting the presence or absence of interference in a particular subcarrier may not be feasible at a transmitter in such uncoordinated networks. Nevertheless, it is practical to assume that a subcarrier allocated to a user does not face interference in every channel instantiation. Thus, there is a scope for harnessing such *bursty* interference in multicarrier systems and exploring the possibility of opportunistic rate increments.

The following toy example, based on parallel linear deterministic channels, captures the intuition behind our problem formulation. Consider 2 transmitters (Tx_1 and Tx_2) and 2 receivers (Rx_1 and Rx_2). For $i \in \{1, 2\}$, Tx_i has messages for Rx_i and at discrete time index $t \in \{1, 2, \dots, N\}$, Tx_i can transmit 2 bits $[b_1^i(t) \ b_2^i(t)]$. The 2 bits correspond to 2 subcarriers (parallel channels) allocated to each transmitter-receiver pair. Depending on the interference channel realization (stays constant for $t \in \{1, 2, \dots, N\}$), Rx_i receives one of the three possibilities: $[b_1^i(t) \ b_2^i(t)]$, $[b_1^i(t) + b_1^{i'}(t) \ b_2^i(t)]$ and $[b_1^i(t) \ b_2^i(t) + b_2^{i'}(t)]$ (shown in Figure 1), where $i, i' \in \{1, 2\}$ and $i' \neq i$. The first possibility corresponds to the interference free case (for Rx_i) and the remaining two possibilities correspond to interference from $Tx_{i'}$ (only one of the subcarriers of Rx_i gets interfered). Hence, there are $3 \times 3 = 9$ distinct possibilities for the pair of received values at Rx_1 and Rx_2 over time duration N . The crucial constraint in this setup is

$$Tx_i \text{ sends } [b_1^i(t) \ b_2^i(t)] \xrightarrow{3 \text{ possibilities for } Rx_i} \begin{matrix} [b_1^i(t) \ b_2^i(t)] \\ [b_1^i(t) + b_1^{i'}(t) \ b_2^i(t)] \\ [b_1^i(t) \ b_2^i(t) + b_2^{i'}(t)] \end{matrix}$$

Fig. 1. Channel realizations for Rx_i in the toy example. The “+” operator denotes modulo 2 addition and indicates the presence of interference. As shown above, interference is not present in all channel realizations for Rx_i (hence bursty); but whenever it is present, it is limited to just 1 out of the 2 transmitted bits.

that the transmitters do not know *a priori* the interference channel realization. The channel is used N times (time index $t \in \{1, 2, \dots, N\}$) and we have the following (symmetric) rate requirement: ensure base rate R_1 at a receiver when *any* one of the subcarriers (of the receiver) gets interfered and ensure rate $R_0 + R_1$ at a receiver when both subcarriers (of the receiver) are interference free (*i.e.*, opportunistically deliver incremental rate R_0 , in addition to R_1 , whenever a receiver is interference free). In this setup, we are interested in characterizing the rate region (R_1, R_0) as the performance metric. Clearly, $R_0 \leq 2$ (a maximum of 2 bits per time index can be sent by a transmitter) and corner point $(R_1, R_0) = (0, 2)$ is easily achievable. Also, the corner point $(R_1, R_0) = (1, 0)$ can be easily achieved by using a repetition code across the 2 subcarriers (*i.e.*, $b_1^1(t) = b_2^1(t)$ and $b_2^2(t) = b_1^2(t)$). The repetition code ensures decodability of the message (of rate R_1) irrespective of which subcarrier gets interfered. Using time sharing between corner points $(0, 2)$ and $(1, 0)$, we can achieve $2R_1 + R_0 \leq 2$. Intuitively this looks like the best we can do, and indeed it can be shown to be tight using entropy inequalities. The problem pursued in this paper is a generalization of this example through parallel linear deterministic interference channels (leading to a rate region with more than two non-trivial corner points in most cases).

In [1] and [2], the problem of harnessing bursty interference was studied for a single carrier scenario using a degraded message set approach. This approach guarantees a base rate when the carrier faces interference. In addition to the base rate, an incremental rate is provided whenever the carrier is interference free. In the multicarrier version considered in this paper, every user (receiver) is allocated M subcarriers (parallel channels) and we extend the degraded message set approach for a rate tuple (R_0, R_L, R_M) as follows: (a) when all M

subcarriers of a user get interfered, the user achieves rate R_M (b) when any L out of M subcarriers get interfered, the user achieves rate $R_M + R_L$ and (c) when all M subcarriers are interference free, the user achieves rate $R_M + R_L + R_0$. Thus, the user experiences opportunistic rate increments as the number of interfered subcarriers decreases. Maintaining low message complexity is the practical idea behind considering the number of interfered subcarriers rather than the specific set of subcarriers interfered. The problem formulation has some similarity with symmetric multilevel diversity coding [3] and our results demonstrate that similar tools (subset entropy inequalities) as in [7] can be used in this context.

Our main contributions in this paper are:

- Inner bounds for $(R_0, R_L, R_M = 0)$ and $(R_0 = 0, R_L, R_M)$ setups using erasure coding across subcarriers (employed for specific interfered *levels* in a subcarrier), signal-scale alignment [1], [5] and Han-Kobayashi scheme.
- Develop outer bounds using techniques inspired by multilevel diversity codes.
- The inner and outer bounds coincide for several regimes.

The remainder of this paper is organized as follows. Section II formalizes the setup and rate requirements. Section III states the main results. Inner bounds and outer bounds are discussed in Sections IV and V respectively. We conclude the paper with a short discussion in Section VI.

II. NOTATION AND SETUP

We consider a system with two base stations (transmitters) Tx_1 and Tx_2 and two users (receivers) Rx_1 and Rx_2 . For $i \in \{1, 2\}$, user Rx_i is allocated M subcarriers $s_1^i, s_2^i, \dots, s_M^i$ by the base station Tx_i . The transmit signals of base stations Tx_1 and Tx_2 are assumed to be independent.

A. Channel Model

The channel is modeled by a 2-user multicarrier (parallel) linear deterministic interference channel [4] where, similar to [1], interfering links in each subcarrier may or may not be active (unknown to the transmitters). At discrete time index $t \in \{1, 2, \dots, N\}$, the transmit signal on subcarrier s_j^i is $\mathbf{x}_j^i(t) \in \mathbb{F}^q$ where \mathbb{F} is a finite field. The received signals on subcarrier s_j^i of Rx_i when s_j^i faces interference from $s_j^{i'}$ (corresponding to user $i' \neq i$) and when it is interference free are described below as (1) and (2) respectively,

$$\mathbf{y}_j^i(t) = \mathbf{G}^{q-n} \mathbf{x}_j^i(t) + \mathbf{G}^{q-k} \mathbf{x}_j^{i'}(t) \quad (1)$$

$$\mathbf{y}_j^i(t) = \mathbf{G}^{q-n} \mathbf{x}_j^i(t) \quad (2)$$

where \mathbf{G} is a $q \times q$ shift matrix in the terminology of deterministic channel models [4] and $\mathbf{x}_j^{i'}(t)$ denotes the transmit signal on subcarrier $s_j^{i'}$ for user i' . All operations above are in \mathbb{F}^q . Similar to [1], the transmitters are assumed to have prior knowledge of parameters n and k (direct and interfering channel strengths), and the presence (or absence) of interference in a subcarrier is assumed to be constant throughout the channel usage duration. Without loss of generality, we assume

$q = \max(n, k)$. Let $\alpha = \frac{k}{n}$ denote the normalized strength of the interfering signal. Since interference free capacity for a single carrier can be achieved when $\alpha \geq 2$ [8], we focus on $0 \leq \alpha \leq 2$. For every time instant, it is convenient to consider a subcarrier as indexed levels of bit pipes. Each bit pipe can carry a symbol from \mathbb{F} .

Let $\mathbf{v}_j^i(t) = \mathbf{G}^{q-k} \mathbf{x}_j^{i'}(t)$ denote the interfering signal for Rx_i on subcarrier s_j^i . We use $\mathbf{X}_j^i = [\mathbf{x}_j^i(1) \mathbf{x}_j^i(2) \dots \mathbf{x}_j^i(N)]$ to denote the transmit signals sent during N time slots on s_j^i and \mathbf{V}_j^i is defined similarly from $\mathbf{v}_j^i(t)$. Also, we define $\mathbf{X}_{j_1:j_2}^i = [\mathbf{X}_{j_1}^i \mathbf{X}_{j_1+1}^i, \dots, \mathbf{X}_{j_2}^i]$.

B. Rate Requirements

The rate requirements for both the users are constrained to be symmetric. For Rx_i , messages (W_0^i, W_L^i, W_M^i) corresponding to rate tuple $(R_0^i, R_L^i, R_M^i) = (R_0, R_L, R_M)$ are encoded in $\mathbf{X}_{1:M}^i$. Based on the number of interfered subcarriers for Rx_i , we have the following requirements for the desired messages:

- 1) Rx_i decodes W_M^i when all M subcarriers of Rx_i get interfered.
- 2) Rx_i decodes (W_L^i, W_M^i) when any L out of M subcarriers of Rx_i get interfered.
- 3) Rx_i decodes (W_0^i, W_L^i, W_M^i) when all M subcarriers of Rx_i are interference free.

A rate tuple is considered achievable if the probability of decoding error is vanishingly small as $N \rightarrow \infty$. To simplify our analysis, we consider two setups: $(R_0, R_L, 0)$ -setup and $(0, R_L, R_M)$ -setup. In the $(R_0, R_L, 0)$ -setup, R_M is assumed to be zero and in the $(0, R_L, R_M)$ -setup R_0 is assumed to be zero. The rate regions for these two setups are analyzed separately in this paper.

III. MAIN RESULTS

Depending on whether $L \leq \frac{M}{2}$ or $L \geq \frac{M}{2}$, we have different results for $(R_0, R_L, 0)$ -setup and $(0, R_L, R_M)$ -setup.

A. Results for $(R_0, R_L, 0)$ -setup

1) $L \leq \frac{M}{2}$: We have a tight characterization of capacity in this case.

Theorem 1: For $L \leq \frac{M}{2}$, the capacity region for $(R_0, R_L, 0)$ -setup is as follows.

$$MR_L + (M - L)R_0 \leq M((M - 2L) + L(\max(1, \alpha) + \max(1 - \alpha, 0)))n \quad (3)$$

$$R_L + R_0 \leq Mn \quad (4)$$

2) $L \geq \frac{M}{2}$: In this case, we have a tight characterization in certain regimes.

Theorem 2: For $L \geq \frac{M}{2}$, consider the following rate inequalities:

$$\begin{aligned} MR_L + (M - L)R_0 &\leq M((M - L)(\max(1 - \alpha, 0) + \max(1, \alpha)) \\ &\quad + (2L - M)\max(\alpha, 1 - \alpha))n \end{aligned} \quad (5)$$

$$R_L + R_0 \leq Mn \quad (6)$$

$$2R_L + R_0 \leq M(\max(1, \alpha) + \max(1 - \alpha, 0))n \quad (7)$$

Inequalities (5), (6) and (7) are inner bounds; (5) and (6) are outer bounds.

Corollary 1: We have a tight characterization for the regime $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$ in the $(R_0, R_L, 0)$ -setup. This follows from the observation that (7) is not active in presence of (5) and (6) for $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$ (see Appendix for detailed proof).

Conjecture 1: For the $(R_0, R_L, 0)$ -setup with $L \geq \frac{M}{2}$, (7) is an outer bound.

If Conjecture 1 holds, we have a tight characterization for $(R_0, R_L, 0)$ -setup when $L \geq \frac{M}{2}$.

B. Results for $(0, R_L, R_M)$ -setup

1) $L \leq \frac{M}{2}$: In this case, we have a tight characterization in certain regimes.

Theorem 3: For $L \leq \frac{M}{2}$, consider the following rate inequalities:

$$R_L + R_M \leq ((M - 2L) + L(\max(1, \alpha) + \max(1 - \alpha, 0)))n \quad (8)$$

$$R_M \leq M \max(1 - \alpha, \alpha)n \quad (9)$$

$$MR_L + 2(M - L)R_M \leq M(M - L)(\max(1, \alpha) + \max(1 - \alpha, 0))n \quad (10)$$

Inequalities (8), (9) and (10) are inner bounds; (8) and (9) are outer bounds.

Corollary 2: We have a tight characterization for the regime $\{L \leq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$ in the $(0, R_L, R_M)$ -setup. This follows from the observation that (10) is not active in presence of (8) and (9) for $\{L \leq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$ (see Appendix for detailed proof).

Conjecture 2: For the $(0, R_L, R_M)$ -setup with $L \leq \frac{M}{2}$, (10) is an outer bound.

If Conjecture 2 holds, we have a tight characterization for $(0, R_L, R_M)$ -setup when $L \leq \frac{M}{2}$.

2) $L \geq \frac{M}{2}$: In this case, we have a tight characterization in certain regimes.

Theorem 4: For $L \geq \frac{M}{2}$, consider the following rate inequalities:

$$R_L + R_M \leq ((M - L)(\max(1, \alpha) + \max(1 - \alpha, 0)) + (2L - M) \max(1 - \alpha, \alpha))n \quad (11)$$

$$R_M \leq M \max(1 - \alpha, \alpha)n \quad (12)$$

$$R_L + R_M \leq \frac{M}{2}(\max(1, \alpha) + \max(1 - \alpha, 0))n \quad (13)$$

Inequalities (11), (12) and (13) are inner bounds; (11) and (12) are outer bounds.

Corollary 3: We have a tight characterization for the regime $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{2}{3}\}$ in the $(0, R_L, R_M)$ -setup. This follows from the observation that (13) is not active in presence of (11) and (12) for $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{2}{3}\}$ (see Appendix for detailed proof).

Conjecture 3: We conjecture that (13) is an outer bound for $(0, R_L, R_M)$ -setup when $L \geq \frac{M}{2}$.

If Conjecture 3 holds, we have a tight characterization for $(0, R_L, R_M)$ -setup when $L \geq \frac{M}{2}$.

IV. INNER BOUNDS

Figure 3 summarizes the inner bounds for different regimes depending on values of α , M and L . The inner bound rate region is obtained from achievable corner points (shown in Figure 3) using time-sharing. Achievability schemes for corner points shown in Figure 3 can be described as follows.

A. Achievable corner points (R_L, R_0) in $(R_0, R_L, 0)$ -setup

- $(0, Mn)$: This appears in cases (1)-(5) in Figure 3. It can be achieved by using the top n levels in all the M subcarriers for message W_0^i .
- $(M(1 - \alpha)n, M\alpha n)$: This corner point is achievable for $\alpha \leq 1$ and appears in cases (1)-(3) in Figure 3. To achieve this, the top $(1 - \alpha)n$ levels of each subcarrier are used for W_L^i and the bottom αn levels are used for W_0^i . Since the top $(1 - \alpha)n$ levels of a subcarrier are always interference free, using M subcarriers we achieve $(M(1 - \alpha)n, M\alpha n)$.
- $((M - L\alpha)n, 0)$: This corner point is achievable for $\alpha \leq 1$ and appears in case (1) in Figure 3. Since any L out of M subcarriers get interfered, an erasure code¹ (across M subcarriers) can recover symbols at rate $(M - L)\alpha n$ from the bottom αn levels of M subcarriers. Also, an additive rate of $M(1 - \alpha)n$ can be obtained by using the top $(1 - \alpha)n$ levels of M subcarriers. Adding the contributions from the bottom αn levels and top $(1 - \alpha)n$ levels of all M subcarriers, we achieve $R_L = (M - L)\alpha n + M(1 - \alpha)n = (M - L\alpha)n$.
- $(M\alpha n, M(2 - 3\alpha)n)$ and $((M\alpha + (M - L)(2 - 3\alpha))n, 0)$: These appear in case (2) in Figure 3 and are achievable for $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$ using the following signal-scale alignment technique [5], [1]. The n levels in a subcarrier s_j^i are divided into 4 bands L_1, L_2, L_3 and L_4 as shown in Figure 2. For $i \neq i'$, when subcarrier s_j^i faces interference, only L_1 of $s_j^{i'}$ interferes with L_2 and L_3 of s_j^i . Also, only L_2 of $s_j^{i'}$ interferes with L_4 of s_j^i . Given this structure, the trick will be to not transmit any information in band L_2 . This keeps L_4 interference free as shown in Figure 2. Using L_1 and L_4 of M subcarriers for W_L^i , we achieve $R_L = M\alpha n$. Using only L_3 of M subcarriers for W_0^i we achieve $R_0 = M(2 - 3\alpha)n$. Hence $(M\alpha n, M(2 - 3\alpha)n)$ is achievable. For $((M\alpha + (M - L)(2 - 3\alpha))n, 0)$, the same signal-scale alignment trick is used in addition to a rate $\frac{M-L}{M}$ erasure code across M subcarriers for L_3 .
- $(M(1 - \frac{\alpha}{2})n, 0)$: This appears in case (3) in Figure 3 and is achievable for $\frac{2}{3} \leq \alpha \leq 1$. Han-Kobayashi scheme [6] can achieve rate $(1 - \frac{\alpha}{2})n$ for a single interfered subcarrier when $\frac{2}{3} \leq \alpha \leq 1$. This scheme is used for each of the M subcarriers to achieve this corner point.
- $(M(\alpha - 1)n, M(2 - \alpha)n)$ and $((M - L(2 - \alpha))n, 0)$: These are achievable for $1 \leq \alpha \leq 2$. The corner point $(M(\alpha - 1)n, M(2 - \alpha)n)$ appears in cases (4) and (5)

¹Interfered levels in the interfered subcarriers are treated as erasures.

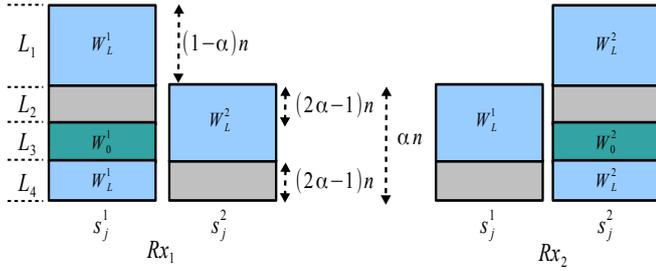


Fig. 2. Signal-scale alignment technique to achieve $(M\alpha n, M(2-3\alpha)n)$

in Figure 3 and is achievable using the following signal-scale alignment strategy. The top $(2-\alpha)n$ levels of a subcarrier are used for W_0^i . The next $(\alpha-1)n$ levels are used for W_L^i . This ensures that the levels used for W_L^i are always interference free. Using M subcarriers we achieve, $(M(\alpha-1)n, M(2-\alpha)n)$. To achieve $((M-L(2-\alpha))n, 0)$ (which appears in case (4) in Figure 3), a similar scheme is used with a rate $\frac{M-L}{M}$ erasure code (across M subcarriers) for the top $(2-\alpha)n$ levels of a subcarrier.

- $(\frac{M\alpha}{2}n, 0)$: This appears in case (5) in Figure 3 and is achievable for $1 \leq \alpha \leq 2$. For the classical two user interference channel (single carrier) with $1 \leq \alpha \leq 2$, rate $\frac{\alpha}{2}n$ is easily achievable. Using this single carrier scheme for M subcarriers, we achieve $(\frac{M\alpha}{2}n, 0)$.

B. Achievable corner points (R_M, R_L) in $(0, R_L, R_M)$ -setup

- $(M(1-\alpha)n, (M-L)\alpha n)$: This appears in cases (6), (7) and (9) in Figure 3 and is achievable for $\alpha \leq 1$. Using the top $(1-\alpha)n$ levels of M subcarriers for W_M^i , we achieve $R_M = M(1-\alpha)n$. For W_L^i , a rate $\frac{M-L}{M}$ erasure code is used for the bottom αn levels across M subcarriers to obtain $R_L = (M-L)\alpha n$.
- $(0, (M-L\alpha)n)$: This appears in cases (6), (7) and (9) in Figure 3 and is achievable for $0 \leq \alpha \leq 1$. The achievability is same as that of $(R_L, R_0) = ((M-L\alpha)n, 0)$ in the $(R_0, R_L, 0)$ -setup.
- $(M\alpha n, (M-L)(2-3\alpha)n)$: This appears in cases (7)-(8) in Figure 3 and is achievable for $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$. A signal-scale alignment technique similar to the one in Figure 2 is used to achieve $R_M = M\alpha n$. Additionally, a rate $\frac{M-L}{M}$ erasure code across M subcarriers for L_3 is used to achieve $R_L = (M-L)(2-3\alpha)n$.
- $(0, (M\alpha + (M-L)(2-3\alpha))n)$: This appears in case (8) in Figure 3 and is achievable for $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$. The achievability is same as that of $(R_L, R_0) = ((M\alpha + (M-L)(2-3\alpha))n, 0)$ in the $(R_0, R_L, 0)$ -setup.
- $(0, M(1-\frac{\alpha}{2})n)$ and $(M(1-\frac{\alpha}{2})n, 0)$: The corner point $(M(1-\frac{\alpha}{2})n, 0)$ appears in cases (9) and (10) while $(0, M(1-\frac{\alpha}{2})n)$ appears in case (10) in Figure 3. Both corner points are achievable for $\frac{2}{3} \leq \alpha \leq 1$. To achieve $(0, M(1-\frac{\alpha}{2})n)$, we use the scheme for achieving $(R_L, R_0) = (M(1-\frac{\alpha}{2})n, 0)$ in the $(R_0, R_L, 0)$ -setup

(i.e., Han-Kobayashi scheme is used for all the M subcarriers). Also, by using W_M^i instead of W_L^i , the above scheme achieves corner point $(M(1-\frac{\alpha}{2})n, 0)$ in the $(0, R_L, R_M)$ -setup.

- $(M(\alpha-1)n, (M-L)(2-\alpha)n)$ and $(0, (M-L(2-\alpha))n)$: These are achievable for $1 \leq \alpha \leq 2$. The corner point $(M(\alpha-1)n, (M-L)(2-\alpha)n)$ appears in case (11) in Figure 3 and is achievable using the following signal-scale alignment strategy. The top $(2-\alpha)n$ levels of a subcarrier are used for W_L^i with a rate $\frac{M-L}{M}$ erasure code across M subcarriers. The next $(\alpha-1)n$ levels are used for W_M^i . This ensures that the levels used for W_M^i are always interference free. Using M subcarriers we achieve, $(M(\alpha-1)n, (M-L)(2-\alpha)n)$. To achieve $(0, M-L(2-\alpha))n, 0)$ (which appears in case (11) in Figure 3), we use the same scheme as that for $(R_L, R_0) = ((M-L(2-\alpha))n, 0)$ in the $(R_0, R_L, 0)$ -setup.
- $(0, \frac{M\alpha}{2}n)$ and $(\frac{M\alpha}{2}n, 0)$: The corner point $(\frac{M\alpha}{2}n, 0)$ appears in cases (11) and (12) while $(0, \frac{M\alpha}{2}n)$ appears in case (12) in Figure 3. Both corner points are achievable for $1 \leq \alpha \leq 2$. To achieve $(0, \frac{M\alpha}{2}n)$, we use the scheme for achieving $(R_L, R_0) = (\frac{M\alpha}{2}n, 0)$ in the $(R_0, R_L, 0)$ -setup (case(5) in Figure 3). Also, by using W_M^i instead of W_L^i , the above scheme achieves the corner point $(\frac{M\alpha}{2}n, 0)$ in the $(0, R_L, R_M)$ -setup.

V. OUTER BOUNDS

In this section, we first define additional notation for outer bound proofs. This is followed by outer bound proofs for $(R_0, R_L, 0)$ -setup (which use techniques [7] from multilevel diversity coding) and outer bound proofs for $(0, R_L, R_M)$ -setup.

A. Receiver Configurations

There are $\binom{M}{L}$ ways in which any L out of M subcarriers get interfered. Every such choice is a receiver configuration for a user. We use additional notation for a special set of receiver configurations described below. Consider a circulant matrix $\mathbf{C}_{M,L}$ of dimension M with the first row consisting of $M-L$ consecutive ones followed by L zeros. The other rows are cyclic right shifts of the first row. As an example, $\mathbf{C}_{3,1}$ is shown below.

$$\mathbf{C}_{3,1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

We use $\mathbf{C}_{M,L}$ to list a specific set of receiver configurations in the following manner. Each row corresponds to a receiver configuration with M subcarriers indexed by the columns. In each row, 1 denotes an interference free subcarrier and 0 denotes an interfered subcarrier. Hence, out of $\binom{M}{L}$ choices, $\mathbf{C}_{M,L}$ lists only M receiver configurations. For example, the third row in $\mathbf{C}_{3,1}$ shown above indicates a situation for R_{x_i} where only subcarrier s_2^i gets interfered. The structure of $\mathbf{C}_{M,L}$ corresponds to the choice of receiver configurations we use in

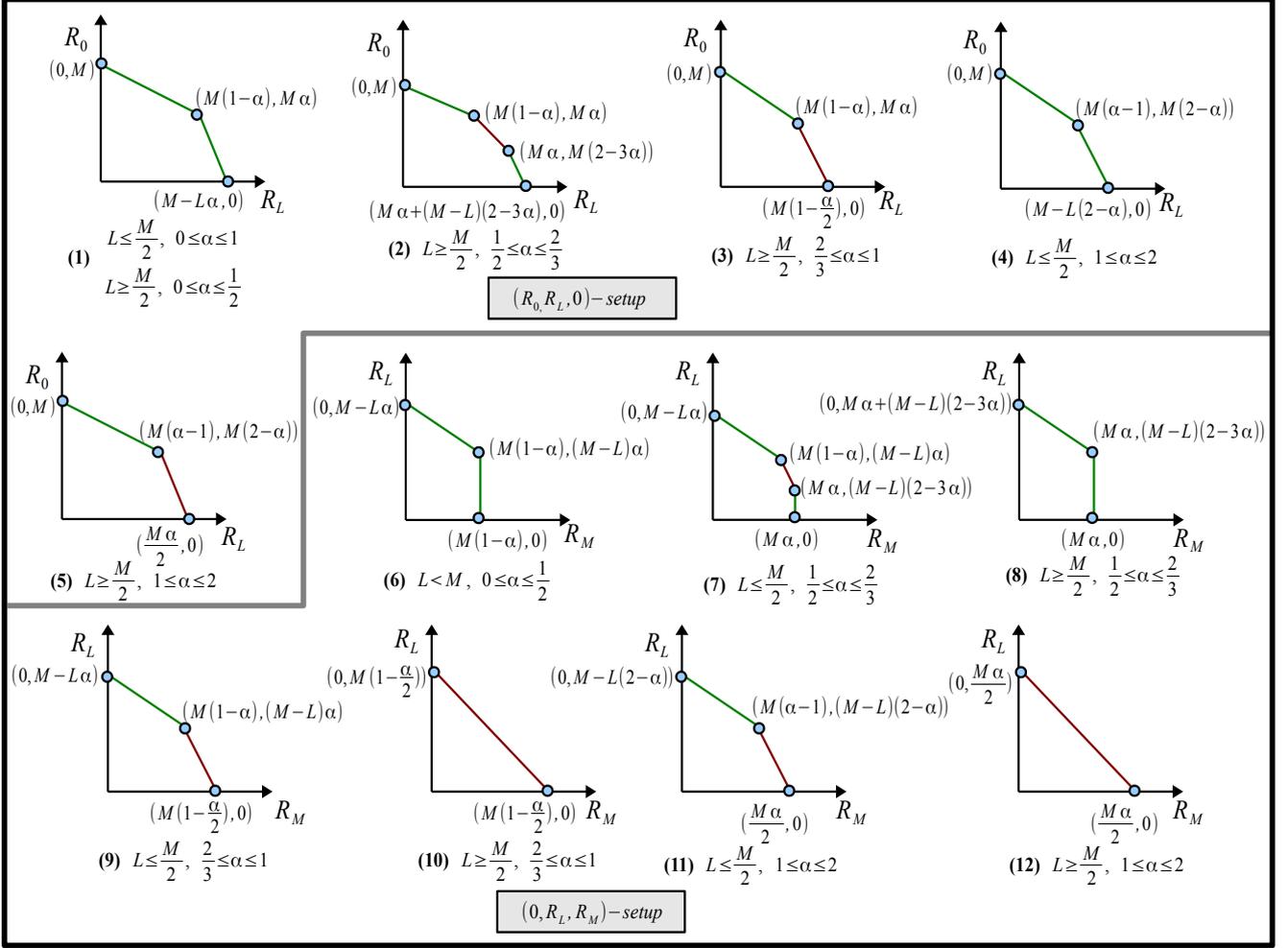


Fig. 3. Inner bound rate regions for $(0, R_L, R_M)$ -setup and $(R_0, R_L, 0)$ -setup in different regimes. The achievable corner points have been normalized with respect to n and are indicated by blue dots. Lines corresponding to tight outer bounds are colored green and the conjectured outer bounds are colored red.

some of our outer bound proofs. This structure enables the use of sliding window subset inequality [7] in such proofs.

We now describe additional notation related to receiver configurations of a user. When Rx_i is in receiver configuration indicated by row j of $\mathbf{C}_{M,L}$, we use $\mathcal{Y}_{M,L,j}^i$ to denote the received signal on M subcarriers (over N time slots). In the same spirit, we define $\mathcal{V}_{M,L,j}^i$ as the interfering signal over all M subcarriers for Rx_i in this receiver configuration. The received signal in interference free subcarriers in $\mathcal{Y}_{M,L,j}^i$ is denoted by $\mathcal{X}_{M,L,j}^i$ and the received signal in interfered subcarriers in $\mathcal{Y}_{M,L,j}^i$ is denoted by $\tilde{\mathcal{Y}}_{M,L,j}^i$. When all M subcarriers of Rx_i are interference free, the received signal is denoted by $\mathcal{X}_{M,0}^i = \mathcal{X}_{M,0,j}^i$.

Now, a direct consequence of the sliding window subset

inequality [7] in our setting can be stated as follows.

$$\begin{aligned} \sum_{j=1}^M H(\mathcal{X}_{M,M-1,j}^i) &\geq \frac{1}{2} \sum_{j=1}^M H(\mathcal{X}_{M,M-2,j}^i) \dots \\ &\dots \geq \frac{1}{M} \sum_{j=1}^M H(\mathcal{X}_{M,0,j}^i) \end{aligned} \quad (14)$$

B. Outer bounds for $(R_0, R_L, 0)$ -setup

1) *Proof of outer bound (3):* We prove outer bound (3) using a careful choice of receiver configurations represented by rows of $\mathbf{C}_{M,L}$. The high level idea is to divide the received signal into interfered and interference free terms followed by the use of (14) on the interference free terms. The proof can be described as follows.

Using Fano's inequality for Rx_i $i \in \{1, 2\}$, for any $\epsilon > 0$

there exists a large enough N such that,

$$\begin{aligned}
& N(MR_L + (M - L)R_0 - (2M - L)\epsilon) \\
\leq & (M - L)I(W_0^i; \mathcal{X}_{M,0}^i | W_L^i) \\
& + \sum_{j=1}^M I(W_L^i; \mathcal{X}_{M,L,j}^i | \tilde{\mathcal{Y}}_{M,L,j}^i) \\
= & (M - L)H(\mathcal{X}_{M,0}^i | W_L^i) - \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^i | W_L^i) \\
& + \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^i) + \sum_{j=1}^M I(W_L^i; \tilde{\mathcal{Y}}_{M,L,j}^i | \mathcal{X}_{M,L,j}^i) \\
\stackrel{(a)}{\leq} & \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^i) + \sum_{j=1}^M I(W_L^i; \tilde{\mathcal{Y}}_{M,L,j}^i | \mathcal{X}_{M,L,j}^i) \quad (15) \\
\leq & \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^i) + \sum_{j=1}^M H(\tilde{\mathcal{Y}}_{M,L,j}^i) \\
& - \sum_{j=1}^M H(\tilde{\mathcal{Y}}_{M,L,j}^i | \mathcal{X}_{M,L,j}^i | W_L^i | W_0^i) \\
= & \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^i) \\
& + \sum_{j=1}^M H(\tilde{\mathcal{Y}}_{M,L,j}^i) - \sum_{j=1}^M H(\mathcal{V}_{M,L,j}^i) \quad (16) \\
\stackrel{(b)}{\leq} & \frac{M - L}{L} \sum_{j=1}^M H(\mathcal{X}_{M,M-L,j}^i) \\
& + \sum_{j=1}^M H(\tilde{\mathcal{Y}}_{M,L,j}^i) - \sum_{j=1}^M H(\mathcal{V}_{M,L,j}^i) \quad (17)
\end{aligned}$$

(a) follows from (14) and (b) follows from $M - L \geq L$ and (14). Substituting $i = 1$ and $i = 2$ in (17), we can obtain two inequalities corresponding to different users. On adding these two inequalities,

$$\begin{aligned}
& 2N(MR_L + (M - L)R_0 - (2M - L)\epsilon) \\
\leq & \frac{(M - 2L)}{L} \sum_{j=1}^M H(\mathcal{X}_{M,M-L,j}^1) + \sum_{j=1}^M H(\tilde{\mathcal{Y}}_{M,L,j}^1) \\
& + \frac{(M - 2L)}{L} \sum_{j=1}^M H(\mathcal{X}_{M,M-L,j}^2) + \sum_{j=1}^M H(\tilde{\mathcal{Y}}_{M,L,j}^2) \\
& + \left(\sum_{j=1}^M H(\mathcal{X}_{M,M-L,j}^1) - \sum_{j=1}^M H(\mathcal{V}_{M,L,j}^2) \right) \\
& + \left(\sum_{j=1}^M H(\mathcal{X}_{M,M-L,j}^2) - \sum_{j=1}^M H(\mathcal{V}_{M,L,j}^1) \right) \\
\stackrel{(a)}{\leq} & 2N(M(M - 2L) \\
& + ML \max(1, \alpha) + ML \max(1 - \alpha, 0))n \quad (18)
\end{aligned}$$

where (a) follows from the structure of $\mathbf{C}_{M,L}$.

2) *Proof of outer bound (5)*: The inequality (15) in the proof of outer bound (3) also holds for $L \geq \frac{M}{2}$. Hence,

$$\begin{aligned}
& N(MR_L + (M - L)R_0 - (2M - L)\epsilon) \\
\leq & \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^i) + \sum_{j=1}^M I(W_L^i; \tilde{\mathcal{Y}}_{M,L,j}^i | \mathcal{X}_{M,L,j}^i) \\
\leq & \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^i | \tilde{\mathcal{Y}}_{M,L,j}^i) - \sum_{j=1}^M H(\mathcal{V}_{M,L,j}^i) \quad (19)
\end{aligned}$$

Substituting $i = 1$ and $i = 2$ in (19), we can obtain two inequalities corresponding to different users. On adding these two inequalities,

$$\begin{aligned}
& 2N(MR_L + (M - L)R_0 - (2M - L)\epsilon) \\
\leq & \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^1 | \tilde{\mathcal{Y}}_{M,L,j}^1) - \sum_{j=1}^M H(\mathcal{V}_{M,L,j}^2) \\
& + \sum_{j=1}^M H(\mathcal{X}_{M,L,j}^2 | \tilde{\mathcal{Y}}_{M,L,j}^2) - \sum_{j=1}^M H(\mathcal{V}_{M,L,j}^1) \\
\stackrel{(a)}{\leq} & 2N(M(M - L)(\max(1 - \alpha, 0) + \max(1, \alpha)) \\
& + M(2L - M) \max(\alpha, 1 - \alpha))n \quad (20)
\end{aligned}$$

where (a) follows from $L \geq \frac{M}{2}$ and the structure of $\mathbf{C}_{M,L}$.

C. Outer bounds for $(0, R_L, R_M)$ -setup

Outer bounds (8), (9), (11) and (12) can be shown by using the El Gamal-Costa injective interference channel bounds [6] as follows.

1) *Proof of outer bound (8)*: For this outer bound proof, we consider two receiver configurations with no interfered subcarriers in common and apply the injective channel bound [6] as shown below.

For any $\epsilon > 0$ there exists a large enough N such that,

$$\begin{aligned}
& N(2(R_M + R_L) - 2\epsilon) \\
\stackrel{(a)}{\leq} & H(\mathcal{Y}_{M,L,1}^1 | \mathcal{V}_{M,L,L+1}^2) + H(\mathcal{Y}_{M,L,L+1}^2 | \mathcal{V}_{M,L,1}^1) \\
\leq & 2N(L \max(1, \alpha) \\
& + \max(1 - \alpha, 0)) + M - 2L)n \quad (21)
\end{aligned}$$

where (a) follows from the injective channel bound [6].

2) *Proof of outer bounds (9) and (12)*: Outer bounds (9) and (12) have the same proof. For the proof, we consider the receiver configuration with all M subcarriers interfered and apply the injective channel bound [6] as shown below.

For any $\epsilon > 0$ there exists a large enough N such that,

$$\begin{aligned}
& N(2R_M - 2\epsilon) \\
\stackrel{(a)}{\leq} & H(\mathcal{Y}_{M,M,j}^1 | \mathcal{V}_{M,M,j}^2) + H(\mathcal{Y}_{M,M,j}^2 | \mathcal{V}_{M,M,j}^1) \\
\leq & 2NM \max(1 - \alpha, \alpha)n \quad (22)
\end{aligned}$$

where $\mathcal{Y}_{M,M,j}^i$ corresponds to the receiver configuration with all M subcarriers interfered and (a) follows from the injective channel bound [6].

3) *Proof of outer bound (11)*: For this outer bound proof, we consider two receiver configurations with minimum number of interfered subcarriers (*i.e.*, $2L - M$) in common and apply the injective channel bound [6] as shown below.

For any $\epsilon > 0$ there exists a large enough N such that,

$$\begin{aligned} & N(2(R_M + R_L) - 2\epsilon) \\ & \stackrel{(a)}{\leq} H(\mathcal{Y}_{M,L,1}^1 | \mathcal{V}_{M,L,L+1}^2) + H(\mathcal{Y}_{M,L,L+1}^2 | \mathcal{V}_{M,L,1}^1) \\ & \leq 2N((M-L)(\max(1, \alpha) + \max(1 - \alpha, 0)) \\ & \quad + (2L - M) \max(1 - \alpha, \alpha))n \end{aligned} \quad (23)$$

where (a) follows from the injective channel bound [6].

VI. DISCUSSION

It is optimal to treat interference as noise in the regimes where erasure coding across subcarriers leads to tight inner bounds. However, outer bound conjectures on (7) and (13) (*i.e.*, Conjectures 1 and 3) suggest that this may not be the case for all regimes. For $\alpha = 1$ (and $L > \frac{M}{2}$), both imply $R_L \leq \frac{M}{2}n$; this can be simply achieved by dividing the M subcarriers between the two users. An erasure coding scheme in this case will lead to $R_L = (M - L)n < \frac{M}{2}n$. Hence, erasure coding across subcarriers may not be optimal in all regimes.

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APPENDIX

A. Proof of Corollary 1

For $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$, inequality (7) is not active in presence of inequalities (5) and (6). This can be proved as follows.

In this regime, inequalities (5), (6) and (7) can be rewritten (shown below) as (24), (25) and (26) respectively.

$$MR_L + (M - L)R_0 \leq M(M - L\alpha)n \quad (24)$$

$$R_L + R_0 \leq Mn \quad (25)$$

$$2R_L + R_0 \leq M(2 - \alpha)n \quad (26)$$

Figure 4 shows the situation in this regime²; it is clear that (26) (dashed red line in Figure 4) is not active in presence of (24) and (25) (solid green lines in Figure 4). Since inequalities (24) and (25) are inner bounds as well as outer bounds in this regime, we have a tight characterization.

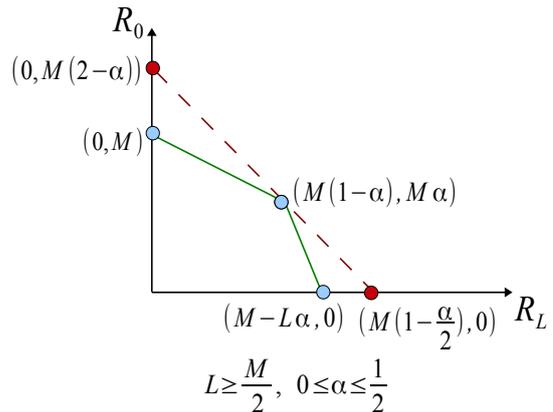


Fig. 4. Rate inequalities (normalized with respect to n) for the regime $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$ in the $(R_0, R_L, 0)$ -setup. Inequality (26) (dashed red line) is not active in presence of (24) and (25) (solid green lines).

B. Proof of Corollary 2

For $\{L \leq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$, inequality (10) is not active in presence of inequalities (8) and (9). This can be proved as follows.

In this regime, inequalities (8), (9) and (10) can be rewritten (shown below) as (27), (28) and (29) respectively.

$$R_L + R_M \leq (M - L\alpha)n \quad (27)$$

$$R_M \leq M(1 - \alpha)n \quad (28)$$

$$MR_L + 2(M - L)R_M \leq M(M - L)(2 - \alpha)n \quad (29)$$

Figure 5 shows the situation in this regime; it is clear that (29) (dashed red line in Figure 5) is not active in presence of (27) and (28) (solid green lines in Figure 5). Since inequalities (27) and (28) are inner bounds as well as outer bounds in this regime, we have a tight characterization.

C. Proof of Corollary 3

In the regime $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{2}{3}\}$, inequality (13) is not active in presence of inequalities (11) and (12). This can be shown as follows.

²For $L \geq \frac{M}{2}$, $M - L\alpha = M(1 - \frac{\alpha}{2}) + (\frac{M}{2} - L)\alpha \leq M(1 - \frac{\alpha}{2})$

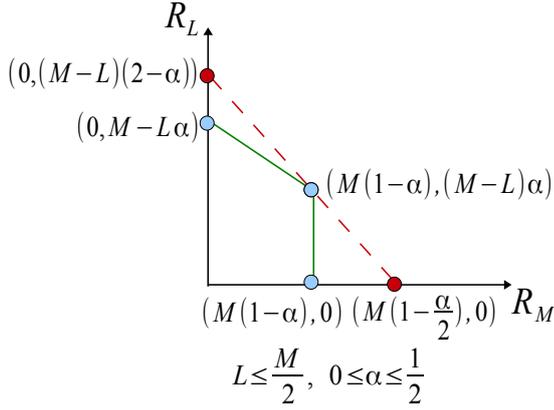


Fig. 5. Rate inequalities (normalized with respect to n) for the regime $\{L \leq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$ in the $(0, R_L, R_M)$ -setup. Inequality (29) (dashed red line) is not active in presence of (27) and (28) (solid green lines).

For this regime, inequalities (11), (12) and (13) can be rewritten (shown below) as (30), (31) and (32) respectively.

$$R_L + R_M \leq ((M-L)(2-\alpha) + (2L-M)\max(1-\alpha, \alpha))n \quad (30)$$

$$R_M \leq M\max(1-\alpha, \alpha)n \quad (31)$$

$$R_L + R_M \leq \frac{M}{2}(2-\alpha)n \quad (32)$$

To show (32) is not active in presence of (30) and (31), it is sufficient to prove (30) *dominates*³ (32) in this regime. We prove this in two steps as shown below (analysis for $0 \leq \alpha \leq \frac{1}{2}$ followed by analysis for $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$).

For $0 \leq \alpha \leq \frac{1}{2}$, (30) can be simplified to

$$R_L + R_M \leq (M - L\alpha)n$$

Since $L \geq \frac{M}{2}$; $(M - L\alpha) \leq \frac{M}{2}(2 - \alpha)$. Thus, (30) dominates (32) for $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$.

For $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$, (30) can be simplified to

$$\begin{aligned} R_L + R_M &\leq (M(2-2\alpha) - L(2-3\alpha))n \\ &= \left(\frac{M}{2}(2-\alpha) + \left(\frac{M}{2} - L\right)(2-3\alpha)\right)n \end{aligned}$$

Since $\alpha \leq \frac{2}{3}$ and $\frac{M}{2} \leq L$, $\frac{M}{2}(2-\alpha) + (\frac{M}{2} - L)(2-3\alpha) \leq \frac{M}{2}(2-\alpha)$. Thus, (30) dominates (32) for $\{L \geq \frac{M}{2}, \frac{1}{2} \leq \alpha \leq \frac{2}{3}\}$.

As shown above, (30) dominates (32) for both $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{1}{2}\}$ and $\{L \geq \frac{M}{2}, \frac{1}{2} \leq \alpha \leq \frac{2}{3}\}$. Since inequalities (30) and (31) are inner bounds as well as outer bounds, we have a tight characterization in the regime $\{L \geq \frac{M}{2}, 0 \leq \alpha \leq \frac{2}{3}\}$.

³gives a smaller bound for $R_L + R_M$