

Using Feedback for Secrecy over Graphs

Shaunak Mishra*, Christina Fragouli†, Vinod Prabhakaran‡ and Suhas Diggavi*

*University of California, Los Angeles, USA

†Tata Institute of Fundamental Research, India

‡Ecole Polytechnique Federale de Lausanne, Switzerland

Abstract—We study the problem of secure message multicasting over graphs in the presence of a passive (node) adversary who tries to eavesdrop in the network. We show that use of feedback, facilitated through the existence of cycles or undirected edges, enables higher rates than possible in directed acyclic graphs of the same mincut. We demonstrate this using code constructions for canonical combination networks (CCNs). We also provide general outer bounds as well as schemes for node adversaries over CCNs.

I. INTRODUCTION

Consider a source that would like to securely multicast a message to a set of receivers in the presence of passive adversaries. It is well known that over wireless networks, if public feedback is available, we can support higher secrecy rates than if it is not [1]. We explore in this paper whether the same could be true over wired networks that are modeled as graphs.

While security against eavesdropping has been extensively examined (in a number of interesting works) in the network coding literature, the potential utility of feedback as such has not, as far as we know. Seminal works such as [2], [3] have looked both at information theoretical bounds as well as code constructions for the case of edge adversaries; works have also started examining the case of node adversaries [4], [5]. In all cases however the underlying network is modeled as a directed acyclic graph.

Yet feedback is readily available in wired networks, and could potentially help in secrecy. Many times connections between sources and receivers are undirected or bi-directional; even over directed graphs, we may have cycles, that offer a form of feedback between network nodes. The existence of such cycles could be put to good use to create for instance common randomness between intermediate network nodes, that a secrecy protocol could leverage to achieve higher rates.

We here provide a number of examples to establish that this is indeed the case. We mainly consider node adversaries, that tap a specific network node and intercept all incoming messages, but also discuss edges adversaries. We focus on a special class of (minimal) combination networks, that is often used in the network coding literature, and the simplest possible case, of a single node adversary. We derive outer bounds as well as achievability schemes for the cases where feedback is (and is not) available. We design schemes that employ feedback, which can offer rates higher than outer bounds in the case where feedback is not available. These results point

to the potential of using such feedback for network secrecy; a topic of ongoing investigation.

The paper is organized as follows. Section II introduces our notation and basic notions; Section III examines feedback over very simple abstracted examples. Section IV deals with inner and outer bounds for directed acyclic graphs, mainly developed for comparison purposes in this paper. Finally, Section V shows the benefits of feedback in undirected and bidirected graphs.

II. NOTATION AND SETUP

We model a wired network as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with unit capacity edges. A single source node has a message \mathcal{W} to send to a set of receivers $\mathcal{R} \subset \mathcal{V}$. We are interested in deriving outer bounds, as well as building N -round secure protocols with the following constraints:

- *Round* : Each edge can be used at most once in a round.
- *Decodability* : All receivers *perfectly* decode message \mathcal{W} with zero error probability.
- *Secrecy* : $H(\mathcal{W}|\mathcal{V}_A) = H(\mathcal{W})$ where \mathcal{V}_A denotes the “view” of an adversary \mathcal{A} , i.e, the information available to tapped edges or nodes during the protocol.

We say that such a protocol achieves

- *Secrecy rate* : $\frac{H(\mathcal{W})}{N}$.

We distinguish between two types of passive adversaries :

- *k-edge* adversary : the adversary has access to an arbitrary set of k edges.
- *k-node* adversary : the adversary has access to an arbitrary set of k nodes. In this paper, we mainly focus on a 1-node adversary.

We allow intermediate nodes to do operations over a finite field \mathbb{F} . We also assume that the network nodes share no prior common randomness and no side secure communication channel, they can only communicate through the network graph that is subject to eavesdropping.

Canonical combination networks (CCNs): Our results in this paper are over CCNs, that essentially are minimal¹ combination networks, see [6], [4]. Figure 1 shows a *directed* (m, h) -CCN with $m \geq h$, where m is the number of coding points and h the mincut to the receivers. It has a source S , h trivial coding nodes A_1, A_2, \dots, A_h (with indegree one), $m - h$ non-trivial coding nodes $A_{h+1}, A_{h+2}, \dots, A_m$ (with indegree h) and $\binom{m}{h}$ receivers. Each receiver is connected to h

¹Removing any edge reduces the mincut for at least one receiver.

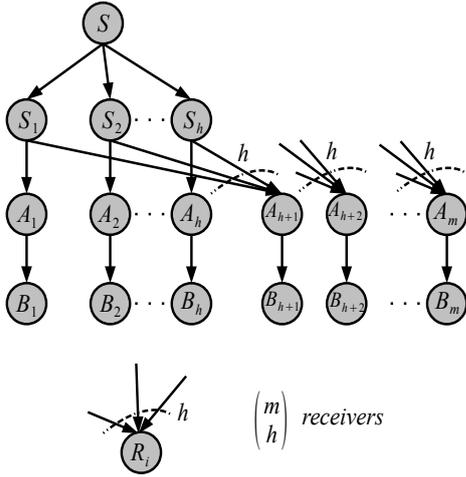


Fig. 1. Directed (m, h) -CCN.

nodes from the set $\{B_1, B_2, \dots, B_m\}$. An undirected (m, h) -CCN can be obtained by replacing all the directed edges in the directed (m, h) -CCN by undirected edges. We will also consider a bidirected (m, h) -CCN which we create by adding an edge (backward edge) in the reverse direction for every edge (forward edge) in the directed (m, h) -CCN. Note that the directed, undirected and bidirected networks all have mincut h towards each receiver.

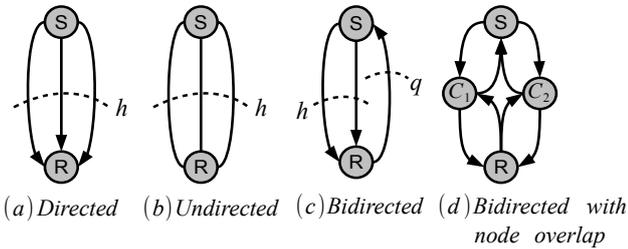


Fig. 2. Abstracted examples. The cut values are shown beside dotted lines for (a), (b) and (c).

III. ILLUSTRATING EXAMPLES

We consider the simple networks depicted in Figure 2, where a single source S wants to securely send information to a receiver R ; we think of the edges in these networks as abstracting edge-disjoint (and in the last case node-disjoint) paths in larger networks. Our goal is to build basic intuition on when feedback could be useful. For the first three examples we assume a k -edge adversary, while for the last a 1-node adversary.

Example in Figure 2(a): the outer bound on the secrecy rate is $h - k$ and is achievable. This follows trivially from [2].

Example in Figure 2(b): for this undirected graph the outer bound remains $h - k$ (proved below), and its achievability

follows from [2]. We can show the outer bound as follows. We apply the “crypto-inequality” in [7] which states that

$$I(\mathcal{W}, \mathcal{K}_S; \mathcal{K}_R | U^N) = 0 \quad (1)$$

where \mathcal{W} is the source message, \mathcal{K}_S is the private randomness of the source, \mathcal{K}_R is the private randomness of the receiver and U^N denotes the values exchanged between the source and receiver during any N -round protocol. Simply put, (1) implies that $(\mathcal{W}, \mathcal{K}_S)$ and \mathcal{K}_R , which were independent to begin with, remain independent even after conditioning on all values exchanged during the protocol. For the graph in this example,

$$H(\mathcal{W}) \stackrel{(a)}{\leq} I(\mathcal{W}; U_{1:h}^N \mathcal{K}_R) \stackrel{(b)}{=} I(\mathcal{W}; U_{1:h}^N) \stackrel{(c)}{\leq} N(h - k) \quad (2)$$

where $U_{1:h}^N$ denotes² values exchanged between the source and receiver during the N -round protocol, (a) follows from the decodability constraint, (b) follows from (1) and (c) follows from the secrecy constraint.

Example in Figure 2(c): for this cyclic graph, if we have h forward (from S to R) edges and q backward edges (from R to S), the outer bound becomes $\min\{h, h + q - k\}$ where the bound $h + q - k$ follows easily with similar steps as in (2). The outer bound is achievable as follows. To achieve it, when $k \leq q$, we can send q random packets (keys) from R to S using the backward edges, say r_1, \dots, r_q . The source creates h linear combinations of these r -packets (using for instance an MDS code), say s_1, \dots, s_h such that the r -packets and the s -packets are in general position (any selection of q of these packets are linearly independent). The source uses the s -packets as one-time pads for the forward edges. With this construction, an adversary observing any k edges will not be able to retrieve information (proved below) and thus secrecy rate h is achievable. For proof of secrecy, consider a k -edge adversary who taps l backward edges and $k - l$ forward edges. From l backward edges it infers l r -packets (say r_1, \dots, r_l). On the forward edges, the adversary observes \mathcal{V}_A (after accounting for inferred packets r_1, \dots, r_l) as shown below,

$$\mathcal{V}_A = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{k-l} \end{pmatrix} + \mathbf{A} \begin{pmatrix} r_{l+1} \\ r_{l+2} \\ \vdots \\ r_q \end{pmatrix} \quad (3)$$

where b_1, \dots, b_{k-l} are information symbols (on $k - l$ forward edges) and matrix \mathbf{A} is full rank (by construction of s -packets). Now, $H(b_1, \dots, b_{k-l} | \mathcal{V}_A) = H(b_1, \dots, b_{k-l})$ since r -packets are uniformly distributed, \mathbf{A} is full rank and $k \leq q$. We can easily extend this scheme in the case where $k > q$, by combining the previous scheme with the scheme in [2]: use again the backward edges to convey random packets to the source, have the source itself generate $k - q$ random packets, and combine these to create one-time pads to encode $h - (k - q)$ information messages to send to the receiver using secure network coding (for more details see [2]).

²We use the notation $U_{1:h}^N$ for $U_1^N, U_2^N, \dots, U_h^N$.

Example in Figure 2(d): for this bidirectional graph with two intermediate nodes C_1 and C_2 , we can achieve secrecy rate 2 even if there is a 1-node passive adversary. To do so, the receiver R can send keys k_1 and k_2 to the source S . Since C_1 and C_2 each observe only one of the keys, S can use $k_1 + k_2$ as a one-time pad for both the forward paths. If we now have a network with h overlapping forward and backward paths through intermediate nodes C_1, C_2, \dots, C_h (each forward path has a node-overlap with only one backward path), secrecy rate h is again achievable against a 1-node adversary using the same approach (this will be a technique we will use for (m, h) -bidirected CCN considered in a later section).

Intuition: These simple examples give a very intuitive message: if we can use feedback (edges in backward directions, in cyclic graphs) without affecting the mincut, then this can help to achieve higher³ secrecy rates. Essentially, we can use the feedback to create common randomness. In Section V, we show that this is indeed the case in more complex networks with multiple receivers as well, where however more elaborate schemes will be needed.

IV. DIRECTED CCN WITH 1-NODE ADVERSARY

In this section, we derive outer bounds and achievability schemes for directed CCNs where a single node is compromised and acts as a passive adversary. This is a case without feedback, which we mainly develop for comparison purposes, but we believe that these results are of independent interest. We give first an outer bound and then achievability schemes that match the outer bound in some cases.

Theorem 1: Consider a directed (m, h) -CCN with⁴ $m \geq h + 1$. An outer bound on the secrecy rate against a 1-node adversary is

$$\frac{(h-1)^2}{h} \quad (4)$$

Proof: Consider a directed (m, h) -CCN with $m = h + 1$. The high level idea of the proof will be to derive “top” and “lower” layer constraints for this layered network and then combine the two using Markovity relationships.

For an N -round protocol and $1 \leq i \leq h$, let Z_i^N, Y_i^N and L_i^N denote the values sent on edge $S \rightarrow S_i, S_i \rightarrow A_{h+1}$ and $S_i \rightarrow A_i$ respectively. For $1 \leq i \leq m$, let T_i^N denote the values sent on edge $A_i \rightarrow B_i$. Figure 3 illustrates the use of notation for a directed $(3, 2)$ -CCN.

Top layer constraints :

$$\begin{aligned} & (h-1)N - H(\mathcal{W}) \\ & \geq I(\mathcal{W}Z_1^N; Z_{2:h}^N) - H(\mathcal{W}) \\ & \stackrel{(a)}{=} I(Z_1^N; Z_{2:h}^N) + H(\mathcal{W}|Z_1^N) - H(\mathcal{W}) \stackrel{(b)}{=} I(Z_1^N; Z_{2:h}^N) \end{aligned}$$

³In addition to these examples, results for unicast in [8], [9] also show feedback leads to a relaxation in connectivity requirements for secure message transmission.

⁴We consider the case of directed CCNs with $m > h$ as in the trivial case of $m = h$ the outer bound against a 1-node adversary is $h - 1$ and is achievable using secure network coding [2].

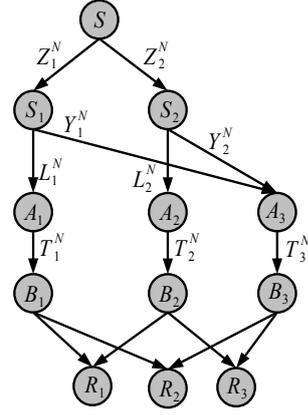


Fig. 3. A directed $(3, 2)$ -CCN illustrating notation used for proof of Theorem 1.

where (a) and (b) follow from decodability and secrecy constraints. In general, for any $i \in \{1, 2, \dots, h\}$ we have the following result,

$$I(Z_i^N; \{Z_j^N\}_{j \neq i}) \leq (h-1)N - H(\mathcal{W}) \quad (5)$$

where $\{Z_j^N\}_{j \neq i}$ denotes the set $\{Z_1^N, Z_2^N, \dots, Z_h^N\} - \{Z_i^N\}$. *Markovity :*

$$\begin{aligned} I(L_1^N Y_1^N; L_{2:h}^N Y_{2:h}^N) & \leq I(Y_1^N L_1^N Z_1^N; Y_{2:h}^N L_{2:h}^N Z_{2:h}^N) \\ & \stackrel{(a)}{=} I(Z_1^N; Z_{2:h}^N) \end{aligned} \quad (6)$$

where (a) follows from Markov chains $Y_{2:h}^N L_{2:h}^N Z_{2:h}^N \rightarrow Z_1^N \rightarrow Y_1^N L_1^N$ and $Y_1^N L_1^N Z_1^N \rightarrow Z_{2:h}^N \rightarrow Y_{2:h}^N L_{2:h}^N$. Similarly, $I(L_i^N Y_i^N; \{L_j^N Y_j^N\}_{j \neq i}) \leq I(Z_i^N; \{Z_j^N\}_{j \neq i})$.

Lower layer constraints :

$$\begin{aligned} & H(\mathcal{W}) \\ & = I(\mathcal{W}; L_{1:h-1}^N Y_{1:h}^N) \stackrel{(a)}{=} I(\mathcal{W}; L_{1:h-1}^N | Y_{1:h}^N) \\ & \leq \sum_{i=1}^{h-1} H(L_i^N | L_{1:i-1}^N Y_{1:h}^N) - H(L_i^N | L_{1:i-1}^N Y_{1:h}^N \mathcal{W} L_{i+1:h}^N) \\ & \stackrel{(b)}{=} \sum_{i=1}^{h-1} I(L_i^N; L_{i+1:h}^N | L_{1:i-1}^N Y_{1:h}^N) \\ & \leq \sum_{i=1}^{h-1} I(L_i^N Y_i^N; \{L_j^N Y_j^N\}_{j \neq i}) \\ & \stackrel{(c)}{\leq} \sum_{i=1}^{h-1} I(Z_i^N; \{Z_j^N\}_{j \neq i}) \stackrel{(d)}{\leq} (h-1)((h-1)N - H(\mathcal{W})) \end{aligned}$$

where (a) follows from secrecy constraint at node A_{h+1} , (b) follows from decodability constraint for the receiver not connected to node B_i , (c) follows from Markov chains as shown in (6) and (d) follows from the top layer constraints (5). This completes the proof for $m = h + 1$. The same outer bound holds for any directed (m, h) -CCN with $m > h$ due to the presence of receivers used in the proof for $m = h + 1$. ■

Lemma 1: Consider a directed (m, h) -CCN with a 1-node adversary. There exist achievable schemes that are tight for the cases $(m, h = 2)$, $(m = h + 1, h)$ and $(m \leq 6, h = 3)$.

Proof: See Appendix. \blacksquare

For a directed $(m, 2)$ -CCN, there exists an alternative optimal scheme (parts of which we will use in our feedback schemes in Section V). It achieves secrecy rate $\frac{h-1}{2}$ in a directed (m, h) -CCN with 1-node adversary. Described below, this scheme uses additional keys which do not reach the receivers and are *cancelled* at intermediate nodes⁵.

Key set cancellation (KSC) scheme: This is a 2-round scheme.

- 1) In the first round, the source S sends keys k_1, k_2, \dots, k_{h-1} to nodes S_1, S_2, \dots, S_{h-1} respectively and $k_h = -\sum_{i=1}^{h-1} k_i$ to S_h .
- 2) In the second round, a secure network code [2] for 1-edge adversary is used with a slight modification (described below) using keys from the first round. This delivers $h - 1$ symbols to all receivers in the second round and achieves secrecy rate $\frac{h-1}{2}$ over 2 rounds.

The modification mentioned in the second round is as follows. Consider a secure network code [2] for a directed (m, h) -CCN with 1-edge adversary. For this specific code, $\forall j > h$ let X_i^j and $\sum_{i=1}^h a_i^j X_i^j$ be the values sent on edges $S_i \rightarrow A_j$ and $A_j \rightarrow B_j$ respectively. In the second round of KSC scheme, we use this code with the modification that $\forall j > h$, S_i sends $a_i^j X_i^j + k_i$ to A_j (non-trivial coding node) instead of X_i^j . Node A_j sums up all the values received from S_1, S_2, \dots, S_h (shown in Figure 4) and sends $\sum_{i=1}^h a_i^j X_i^j + \sum_{i=1}^h k_i = \sum_{i=1}^h a_i^j X_i^j$ to B_j . Hence the *key set* $\{k_1, k_2, \dots, k_h\}$ accumulated in the first round is cancelled at all non-trivial coding nodes. The key set ensures secrecy at every non-trivial coding node and the underlying secure network code delivers $h - 1$ symbols to all receivers.

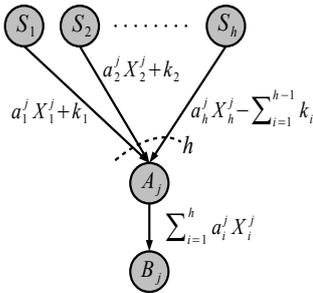


Fig. 4. Second round of KSC scheme.

V. UNDIRECTED AND BIDIRECTED CCN

In this section, we show that use of feedback can improve secrecy rates for undirected and bidirected graphs. We prove in particular the results for undirected and bidirected CCNs that are summarized in Table I.

⁵This scheme is similar to achievability schemes in [4]. In addition to [4], the approach of cancelling keys at intermediate nodes is also shown in [10].

TABLE I
SUMMARY OF RESULTS FOR 1-NODE ADVERSARY

(m, h) -CCN type	Multicast inner bound	Multicast outer bound
Directed ^a	$\frac{(h-1)^2}{h}$	$\frac{(h-1)^2}{h}$
Undirected	$(h-1) \frac{(m-h+1)}{(m-h+2)}$	$h-1$
Bidirected	$h-1$	h

^aThe multicast inner bound shown for directed CCN is for a limited number of cases described in Lemma 1.

A. Undirected CCN

An undirected CCN allows the usage of all edges in both the directions. But it is still subject to the constraint that during each round, we can use each edge only once (in any direction that we intend to).

Theorem 2: Consider an undirected (m, h) -CCN with a 1-node adversary. An outer bound for secrecy rate is $h - 1$. Moreover, when⁶ $m \geq h + 1$, there exists a scheme that achieves secrecy rate $(h - 1) \frac{m-h+1}{m-h+2}$.

For the above scheme, as $m \rightarrow \infty$, secrecy rate $\rightarrow h - 1$. This shows asymptotic optimality of the scheme. Additionally, when $m \geq 2h - 1$, the scheme achieves a secrecy rate strictly better than the outer bound $\frac{(h-1)^2}{h}$ for directed (m, h) -CCN. Thus, feedback improves secrecy rates as we transition from directed CCNs to undirected CCNs.

Proof: The outer bound proof is similar to (2) and follows from a mincut comprising of h edges between a receiver and nodes in $\{B_1, B_2, \dots, B_m\}$.

We now show a simple scheme for an undirected (m, h) -CCN which achieves secrecy rate $(h - 1) \frac{m-h+1}{m-h+2}$. The scheme operates in two phases: uplink and downlink. It begins with a single round uplink phase where keys are collected at S_1, S_2, \dots, S_h as follows.

- Source S sends keys $k_1^S, k_2^S, \dots, k_{h-1}^S$ to S_1, S_2, \dots, S_{h-1} respectively and $k_h^S = -\sum_{i=1}^{h-1} k_i^S$ to S_h . This constitutes key set \mathcal{K}^S .
- The receiver connected to B_1, B_2, \dots, B_h sends keys $k_1^R, k_2^R, \dots, k_{h-1}^R$ to S_1, S_2, \dots, S_{h-1} respectively and $k_h^R = -\sum_{i=1}^{h-1} k_i^R$ to S_h . This constitutes key set \mathcal{K}^R .
- For $i \in \{h + 1, h + 2, \dots, m\}$, A_i sends keys $k_1^{A_i}, k_2^{A_i}, \dots, k_{h-1}^{A_i}$ to S_1, S_2, \dots, S_{h-1} respectively and $k_h^{A_i} = -\sum_{i=1}^{h-1} k_i^{A_i}$ to S_h . This constitutes key set \mathcal{K}^{A_i} .

The uplink phase is followed by a downlink phase comprising of $(m - h + 1)$ downlink rounds. Each round of the downlink phase is similar to the second round of KSC scheme. The key sets collected at S_1, S_2, \dots, S_h in the uplink phase are used as part of one-time pad and cancelled at non-trivial coding nodes (before they reach the receivers) in the following manner.

- 1) Key set \mathcal{K}^S is cancelled at all non-trivial coding nodes in the first downlink round.

⁶In the trivial case of $m = h$, secrecy rate $h - 1$ is achievable using secure network coding [2].

- 2) Key set \mathcal{K}^R is cancelled at all non-trivial coding nodes in the second downlink round.
- 3) For the next $m - h - 1$ downlink rounds, a key set from $\{\mathcal{K}^{A_i}\}_{i \neq j}$ is cancelled at non-trivial coding node A_j .

Each downlink round delivers $h - 1$ symbols to all receivers. A single round uplink phase is followed by $(2 + m - h - 1)$ downlink rounds and hence, the secrecy rate is $(h - 1) \frac{m-h+1}{m-h+2}$. ■

B. Bidirected CCN

We now consider a bidirected CCN, which we create by adding for every forward edge, one parallel edge (backward edge) of the opposite directionality. Note that this does not increase the mincut to the receivers.

Lemma 2: Consider a bidirected (m, h) -CCN with a 1-node adversary. When $m \geq h + 1$, there exists a scheme which achieves secrecy rate $h - 1$.

Proof: A single round scheme achieves secrecy rate $h - 1$ as follows. The receiver connected to B_1, B_2, \dots, B_h first sends keys $k_1^R, k_2^R, \dots, k_{h-1}^R$ to S_1, S_2, \dots, S_{h-1} and $k_h^R = -\sum_{i=1}^{h-1} k_i^R$ to S_h . These keys are then cancelled at non-trivial coding nodes (similar to the second round of KSC scheme) and $h - 1$ symbols delivered to each receiver in the same round. ■

Up to now we have focused on a 1-node adversary. Interestingly, feedback using backward edges can help in the case of edge-adversaries as well.

Lemma 3: For bidirected (m, h) -CCN, secrecy rate h is achievable against a 1-edge adversary (taps only one directed edge).

Proof: For every pair of nodes sharing an edge, a key can be sent using the parallel backward edge. This key can be used as a one-time pad to secure the network code on the forward edge. ■

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APPENDIX

A. Proof of Lemma 1

We first give an intuitive outline of the achievability schemes followed by an illustrative example for directed $(4, 3)$ -CCN. The example is then extended to show schemes for the claimed cases.

Intuitive outline: The scheme is based on the following observation. If we reduce the indegree of non-trivial coding nodes to 1, a secure network code [2] against 1-edge adversary is sufficient to ensure secrecy. But reducing the indegree of non-trivial coding nodes also reduces the mincut to some receivers, hence the secrecy rate. Our approach is to have a multiple round routing strategy, using a different subset of edges in each round. We still restrict the indegree of non-trivial coding nodes to 1 in each round, but ensure that the mincut to each receiver averaged over multiple rounds is sufficient to achieve the desired secrecy rate. This has connection to the work in [11] on average throughput maximization (without any secrecy constraints) using tree packing strategies. The following example illustrates our approach for a directed $(4, 3)$ -CCN.

Example 1: Consider a directed $(4, 3)$ -CCN with 4 receivers as shown in Figure 5. Let $\{c_1^1, c_2^1, c_1^2, c_2^2, c_1^3, c_2^3\}$ be the 6 symbol codeword derived from a 4 symbol message \mathcal{W} using a rate $\frac{2}{3}$ erasure code. Let $\delta^1, \delta^2, \delta^3$ be keys generated by source S . We now describe a 3-round scheme that achieves secrecy rate $\frac{4}{3}$ (optimal for directed $(4, 3)$ -CCN). In the first round, S sends $c_1^1 + \delta^1$, $c_1^1 + c_2^1 + \delta^1$ and $c_1^1 - c_2^1 + \delta^1$ to S_1, S_2 and S_3 respectively⁷. Nodes S_1, S_2 and S_3 send these values to B_1, B_2 and B_3 via A_1, A_2 and A_3 . For A_4 , $S_1 \rightarrow A_4$ is the only incoming edge used in this round and $c_1^1 + \delta^1$ is sent from S_1 to B_4 via this edge. Each B_i now sends the received values to the receivers connected to it. At the end of this round, receivers R_1 and R_4 decode (c_1^1, c_2^1) . The other two receivers decode only c_2^1 . In the second and third round, $\{c_1^1, c_2^1, \delta^1\}$ are replaced with $\{c_1^2, c_2^2, \delta^2\}$ and $\{c_1^3, c_2^3, \delta^3\}$ respectively with the following change in routing strategy for A_4 . In the second round, $S_2 \rightarrow A_4$ is the only incoming edge used for A_4 and $c_1^2 + c_2^2 + \delta^2$ is sent along this edge. Hence, in the second round receivers R_1 and R_3 decode (c_1^2, c_2^2) and the other two receivers decode c_2^2 . In the third round, $S_3 \rightarrow A_4$ is the only incoming edge used for A_4 and $c_1^3 - c_2^3 + \delta^3$ is sent along this edge. In this round, receivers R_1 and R_2 decode (c_1^3, c_2^3) and the other two receivers decode c_2^3 . At the end of 3 rounds,

⁷These linear combinations are such that c_2^1 can be decoded from any two combinations, while (c_1^1, c_2^1) can be decoded using all three combinations.

all receivers decode at least 4 codeword symbols and hence decode the message.

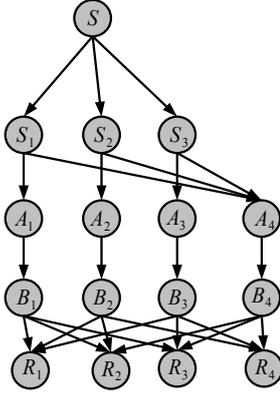


Fig. 5. Directed (4, 3)-CCN.

Extension to $(m \leq 6, h = 3)$: In Example 1, the values sent (over 3 rounds) on edges $A_i \rightarrow B_i$ for $1 \leq i \leq m$ can be listed in terms of T_i^N (defined in proof of Theorem 1, see Figure 3)

$$= \begin{pmatrix} T_1^3 & T_2^3 & T_3^3 & T_4^3 \\ c_1^1 + \delta^1 & c_1^1 + c_2^1 + \delta^1 & c_1^1 - c_2^1 + \delta^1 & c_1^1 + \delta^1 \\ c_1^2 + \delta^2 & c_1^2 + c_2^2 + \delta^2 & c_1^2 - c_2^2 + \delta^2 & c_1^2 + c_2^2 + \delta^2 \\ c_1^3 + \delta^3 & c_1^3 + c_2^3 + \delta^3 & c_1^3 - c_2^3 + \delta^3 & c_1^3 - c_2^3 + \delta^3 \end{pmatrix}$$

Since the linear combinations used in each round are similar, we denote $c_1^i + \delta^i, c_1^i + c_2^i + \delta^i, c_1^i - c_2^i + \delta^i$ in round i by routing symbols $\underline{0}, \underline{1}, \underline{2}$ respectively. In this notation,

$$\begin{pmatrix} T_1^3 & T_2^3 & T_3^3 & T_4^3 \end{pmatrix} \equiv \begin{pmatrix} \underline{0} & \underline{1} & \underline{2} & \underline{0} \\ \underline{0} & \underline{1} & \underline{2} & \underline{1} \\ \underline{0} & \underline{1} & \underline{2} & \underline{2} \end{pmatrix} = R_{m=4, h=3, N=3} \quad (7)$$

where $R_{m, h, N}$ is a routing matrix listing the routing symbols sent on edges $A_i \rightarrow B_i$ as defined above. With this notation, we are now ready to compactly describe our optimal scheme for $(m = 6, h = 3)$. We simply extend $R_{m=4, h=3, N=3}$ to $R_{m=6, h=3, N=3}$ by adding two columns as shown below.

$$R_{m=6, h=3, N=3} = \begin{pmatrix} \underline{0} & \underline{1} & \underline{2} & \underline{0} & \underline{1} & \underline{2} \\ \underline{0} & \underline{1} & \underline{2} & \underline{1} & \underline{2} & \underline{0} \\ \underline{0} & \underline{1} & \underline{2} & \underline{2} & \underline{0} & \underline{1} \end{pmatrix} \quad (8)$$

Substituting back the values of $\underline{0}, \underline{1}, \underline{2}$ for every round, one can easily check that every receiver (there are $\binom{6}{3}$ receivers) decodes at least 4 codeword symbols over 3 rounds and hence the secrecy rate is $\frac{4}{3}$ (optimal).

Extension to $(m, h = 2)$ based on Hadamard code: Let $m = 2N$ (if m is odd, simply consider a directed $(m + 1, h = 2)$ -CCN and proceed⁸). Let $\{c^1, c^2, \dots, c^N\}$ be the N symbol codeword derived from a $\frac{N}{2}$ symbol message

⁸The set of receivers in a directed $(m + 1, h)$ -CCN includes the receivers in a directed (m, h) -CCN; hence it is sufficient to show a scheme for $(m + 1, h)$.

using a rate $\frac{1}{2}$ erasure code. In addition, the source generates keys $\delta^1, \dots, \delta^N$. We will now describe an N -round scheme which achieves secrecy rate $\frac{1}{2}$. In round i , we denote $c^i + \delta^i, \delta^i$ by routing symbols $\underline{0}, \underline{1}$ respectively. In round i , the source sends $\underline{0}, \underline{1}$ to S_1, S_2 respectively and these get forwarded to B_1, B_2 . This fixes the first two columns (corresponding to T_1^N and T_2^N) of routing matrix $R_{m=2N, h=2, N}$ as the all $\underline{0}$ column vector and all $\underline{1}$ column vector respectively. The routing symbols sent on $A_3 \rightarrow B_3, \dots, A_m \rightarrow B_m$ over N rounds are derived from a Hadamard code as follows. Consider $2N$ Hadamard codewords (in terms of routing symbols $\underline{0}, \underline{1}$) of length N such that the all $\underline{0}$ and all $\underline{1}$ codewords are present in this collection (this can be easily done using Sylvester's construction). We assign these $2N$ codewords as the $2N$ columns of $R_{m=2N, h=2, N}$ such that the all $\underline{0}$ codeword is the first column and the all $\underline{1}$ codeword is the second column. The N -round schemes follows this routing matrix (the values sent on edges $A_i \rightarrow B_i$) and at the end of every round, B_1, \dots, B_{h+1} forward the received values to all the receivers connected to them. Since the Hamming distance between any two column vectors in $R_{m=2N, h=2, N}$ is at least⁹ $\frac{N}{2}$, each receiver receives both $\underline{0}, \underline{1}$ (i.e., $c^i + \delta^i, \delta^i$) in at least $\frac{N}{2}$ rounds. Hence it can decode at least $\frac{N}{2}$ codeword symbols and thus the message.

Extension to $(m = h + 1, h)$: The scheme in Example 1 can be extended for the case $(h + 1, h)$ as follows. Let $\{c_1^1, c_2^1, \dots, c_{h-1}^1, c_1^2, \dots, c_{h-1}^2, \dots, c_1^h, \dots, c_{h-1}^h\}$ be the $(h - 1)h$ symbol codeword derived from a $(h - 1)^2$ symbol message using a rate $\frac{h-1}{h}$ erasure code. The source computes $\{x_1^i, \dots, x_{h-1}^i\}$ from $\{c_1^i, \dots, c_{h-1}^i\}$ using an invertible linear transformation defined below.

$$c_j^i = \sum_{l=1}^{h-1} x_l^i - x_j^i \quad (9)$$

In addition to the above steps, the source generates keys $\delta^1, \dots, \delta^h$. We now describe an h -round scheme that achieves secrecy rate $\frac{(h-1)^2}{h}$. In round i , for $1 \leq j \leq i-1$, $x_j^i + \delta^i$ is sent from S to S_j and for $i+1 \leq j \leq h$, $x_j^i + \delta^i$ is sent to S_j . For $j = i$, $\sum_{l=1}^{h-1} x_l^i + \delta^i$ is sent to S_i . These values are forwarded by S_1, \dots, S_h to B_1, \dots, B_h respectively via A_1, \dots, A_h . Also, $S_i \rightarrow A_{h+1}$ is the only incoming edge used for A_{h+1} in round i and using this edge $\sum_{l=1}^{h-1} x_l^i + \delta^i$ is forwarded to B_{h+1} via A_{h+1} . At the end of every round, B_1, \dots, B_{h+1} forward the received values to all the receivers connected to them. In round i , only two receivers can decode $h-1$ codeword symbols, i.e., $\{c_1^i, \dots, c_{h-1}^i\}$ (they are the receivers connected to $\{B_1, \dots, B_h\}$ and $\{B_1, \dots, B_{h+1}\} - \{B_i\}$). The remaining $h-1$ receivers can decode only $h-2$ codeword symbols from $\{c_1^i, \dots, c_{h-1}^i\}$. Hence over h rounds, all receivers can decode at least $(h-1)(h-2) + (h-1) = (h-1)^2$ codeword symbols and achieve secrecy rate $\frac{(h-1)^2}{h}$.

⁹Property of Hadamard code.