FLEXIBLE BACKHAUL DESIGN WITH COOPERATIVE TRANSMISSION IN CELLULAR INTERFERENCE NETWORKS

BY

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THESIS

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ABSTRACT

Interference is an important factor that limits the rates that can be achieved by mobile users in a cellular network. Interference management through cooperation has emerged as a major consideration for next-generation cellular networks. In this thesis, we focus on the downlink of a sectored hexagonal cellular network, under the assumption of local interference i.e., the interference at each user is only due to transmitters in neighboring sectors. We explore the potential degrees of freedom (DoF) gain in this network under constraints on the cooperation between base-stations. The constraints that we consider are the cooperation order M, and the average backhaul load B, which denote the maximum and the average number of transmitters, respectively, that jointly transmit any message. We first study the DoF gains in a scenario where mobile receivers can be associated to any neighboring cell but no cooperative transmission is allowed, and derive bounds on the maximum achievable per user DoF for orthogonal schemes. We then show that by combining cooperative transmission with flexible message assignment to the transmitters, it is possible to achieve a per user DoF strictly greater than that without cooperation. The proposed cooperative transmission scheme does not require extra backhaul capacity, as it uses a smart assignment of messages to transmitters to meet an average backhaul load constraint of one message per transmitter. The schemes presented are simple zero-forcing beamforming schemes that require linear precoding over a single time/frequency slot (one-shot). Similar schemes are proposed which achieve a per user DoF greater than half with a minimal increase in the backhaul load. These results are derived for networks with intra-cell interference and networks without intra-cell interference.

To my parents, for their love and support

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CHAPTER 1

INTRODUCTION

There has been a rapid growth in the usage of wireless networks in the past few years. In a cellular network, interference due to neighboring cells is a major factor that limits the rates of users. Cooperation among base stations or mobile users in the cellular network has emerged as one of the important technologies for managing interference [1].

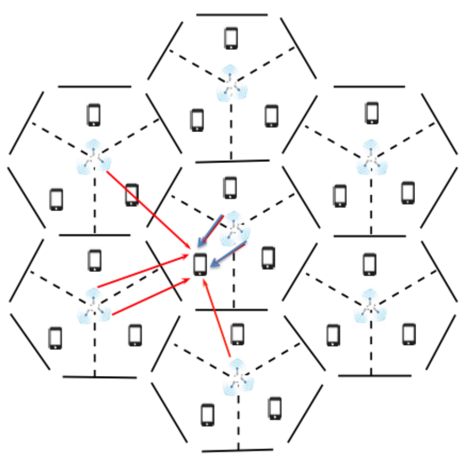


Figure 1.1: A sectored cellular network with three sectors per cell. Each sector is surrounded by six neighboring sectors, out of which two belong to the same cell.

In this thesis, we focus on the downlink of a two-dimensional hexagonal sectored cellular network shown in Figure 1.1. It is assumed that the interference is local, i.e., interference at each receiver (user) in a sector is only due to neighboring sectors. The network can be modeled as a two-dimensional locally connected interference network as shown in [2]. We consider two scenarios, networks without intra-cell interference, i.e., sectors that belong to the same cell do not interfere, and networks with intra-cell interference. The proposed schemes for interference management are evaluated using the degrees of freedom (DoF) criterion. The degrees of freedom criterion provides a preliminary way of characterizing the capacity of a network, as the number of interference-free channels in the network at a high signal-to-noise ratio (SNR).

In interference management without cooperation, a per user DoF (PUDoF) of half can be achieved, using the asymptotic interference alignment scheme in [3]. Interference alignment is a precoding scheme that achieves this PUDoF by aligning the interference at each receiver into approximately half the dimensions, leaving the remaining dimensions for transmission of the desired signal. However, this PUDoF value is approached by coding over an impractically large number of symbol extensions (time/frequency slots).

In this model, we look at potential DoF gains using cooperative transmission. The transmitters (base stations) in the downlink cooperate by means of sharing messages through a backhaul network. Complete cooperation among all the transmitters can be used to achieve a PUDoF of one, but at the cost of overloading the backhaul network. Hence, we are interested in understanding the gains in the PUDoF that are achievable under partial cooperation. The finite capacity of the backhaul network is modeled by two cooperation constraints M and B, that reflect the maximum and average transmit set size, respectively, where the transmit set size refers to the number of transmitters at which a message is available. Coding schemes under the cooperation constraint B are relevant in practical scenarios, as the transmit set size is allowed to vary across messages while maintaining an average backhaul load. The proposed downlink interference management schemes rely on a flexible design of the cellular backhaul, that takes into account the topology of the network, to make decisions about associating mobile receivers to cells. The flexible backhaul design is augmented to include cooperative transmission, and to show that DoF gains can be achieved through a minimal increase in the backhaul capacity. All of our proposed schemes are single-shot schemes and do not require any symbol extensions. The notion of topological robustness is discussed for the proposed schemes, to guarantee a minimum DoF, irrespective of the strength of the interference links.

The DoF gain achieved by cooperative transmission using local message sharing was studied and characterized for Wyner's linear interference networks in [4], [5], [6] under the maximum transmit set size cooperation constraint. This was extended to linear networks with possible link erasures in [7]. Flexible backhaul design for linear interference networks was considered in [8], where it was shown that DoF gains can be achieved in large linear networks without requiring an increase in the average transmit set size. In this thesis, we extend the work of [8] to hexagonal cellular interference networks.

Cooperation between base station receivers in the *uplink* was considered in [2]. The cooperation between base station receivers is through the exchange of decoded messages over the backhaul network. It was shown that practical interference alignment schemes can achieve the optimal degrees of freedom in the uplink using cooperation between receivers. The advantage of the message passing framework of [2] is that it does not require analog signal sharing over the backhaul unlike traditional approaches for cooperative uplink reception.

Another approach for managing interference in the cellular downlink through cooperative transmission was introduced in [9], where transmitting base stations cooperate by exchanging quantized dirty paper coded signals. However, implementing this approach can face practical challenges as each transmitter only gets its message after a series of preceding transmitters have encoded their messages; this will either require significant delay requirements or require coding over multiple time slots. Further, under this setting, the only way for messages to be delivered to transmitters through a centralized controller is for the controller to be aware of channel state information.

It is finally worth mentioning that while the schemes of [9] and [2] are designed for multiple antenna systems and require extra backhaul capacity to support the sharing of messages, the proposed scheme in this thesis is for single antenna systems and employs an assignment of messages to transmitters that attains a better DoF gain for similar backhaul loads. We envision a novel paradigm where the message passing framework of [2] for the uplink is complemented by our proposed approach for the downlink.

CHAPTER 2

SYSTEM MODEL AND NOTATION

Consider a network with K users. Assume that each transmitter and receiver has a single antenna. The signal Y_i at receiver i is given by

$$Y_i = H_{ii}X_i + \sum_{j \in N_i} H_{ij}X_j + Z_i, \tag{2.1}$$

where X_j denotes the signal transmitted by transmitter j under an average transmit power constraint P, Z_i denotes the additive white Gaussian noise at receiver i, H_{ij} denotes the channel gain coefficient from transmitter j to receiver i, and N_i denotes the set of interferers at receiver i. Channel coefficients that are not identically zero are assumed to be drawn from a continuous joint distribution. Channel state information is assumed to be available at all transmitters.

2.1 Linear Interference Network

Consider the downlink of the linear cellular model presented by Wyner. In this model, the cells are located on an infinite linear equi-spaced grid and each transmitter is associated with a single user. In the asymmetric Wyner model, each user observes interference from a preceding transmitter, while in the symmetric Wyner model, each receiver observes interference from a preceding and a succeeding transmitter. This setting may be extended to define a locally connected linear interference network with connectivity parameter L. Here, L denotes the number of dominant interferers per user, where each user observes interference from $\lceil \frac{L}{2} \rceil$ preceding transmitters and $\lfloor \frac{L}{2} \rfloor$ succeeding transmitters. An asymmetric Wyner model corresponding to $L = 1$ and a symmetric Wyner model corresponding to $L = 2$ are shown in Figure 2.1. The channel coefficients corresponding to a locally connected linear channel model are as follows:

$$H_{i,j} \text{ is not identically 0 iff } i \in [j - \lfloor \frac{L}{2} \rfloor, j + \lceil \frac{L}{2} \rceil].$$

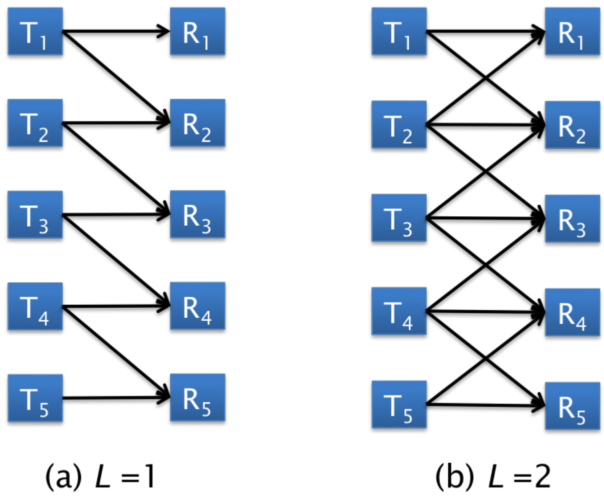
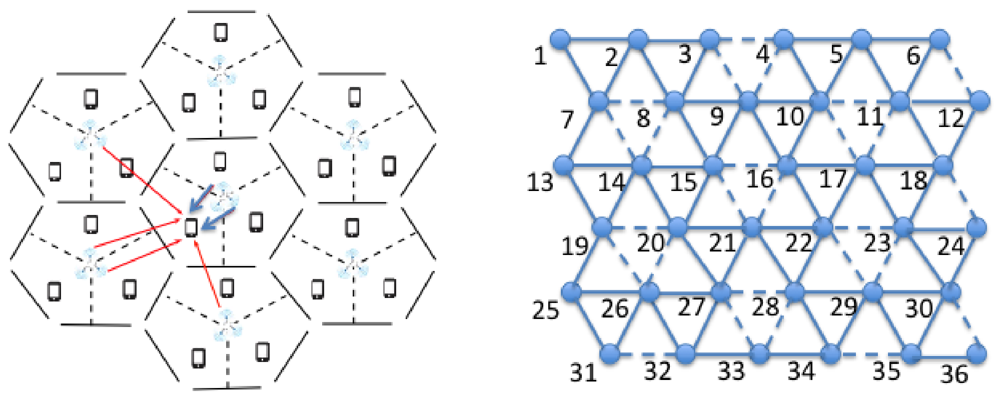


Figure 2.1: Locally connected linear interference networks are shown with the number of users $K = 5$. The transmitter corresponding to cell i is denoted by T_i and the user (receiver) by R_i. In (a), a Wyner model with $L = 1$ is shown. In (b), a symmetric Wyner model with $L = 2$ is shown.

2.2 Hexagonal Cellular Network

Consider a hexagonal cellular network with three sectors per cell as shown in Figure 2.2(a). Each user is associated with a single sector. It is assumed that the intended message to a user is transmitted by a single transmitter. A local interference channel model is assumed, where the interference at each receiver is only due to the base stations in the neighboring sectors. We first consider the scenario where it is assumed that sectors belonging to the same cell do not interfere with each other. This is motivated by the fact that the interference power due to sectors in the same cell is usually far lower than the interference from out-of-cell users located in the sector's line of sight. We then relax this assumption and consider both intra-cell and out-of-cell interference at each receiver.



(a) (b)

Figure 2.2: (a) Cellular network and (b) interference graph. A sectored cellular network with three sectors per cell. Each sector is surrounded by six neighboring sectors, out of which two belong to the same cell. Interference due to sectors of the same cell are denoted by blue arrows and the red arrows indicate out-of-cell interference. In (b), each vertex denotes a transmitter-receiver pair belonging to a sector. An edge between vertices denotes an interfering link. The dotted lines in (b) represent interference between sectors belonging to the same cell.

2.2.1 Interference Graph

The cellular model is represented by an undirected interference graph $G(V, E)$ shown in Figure 2.2(b) where each vertex $u \in V$ corresponds to a transmitter-receiver pair corresponding to a particular sector. An edge $e \in E$ between two vertices $u, v \in V$ corresponds to interference between the transmit-receiver pairs i.e., the transmitter at u causes interference at the receiver corresponding to v and vice versa. We note that each sector is surrounded by six neighboring sectors, out of which two belong to the same cell. The dotted lines denote interference between sectors that belong to the same cell. Hence we see in the interference graph that each vertex is connected by an edge to six other vertices, out of which two belong to the same cell (indicated by dotted lines). For any node a, the transmitter, receiver and intended message corresponding to the node are denoted by T_a, R_a and W_a. We consider K-user networks where $\sqrt{K}$ is an integer, and nodes are numbered as in Figure 2.2(b). In the figure, $\sqrt{K} = 6$. We study the performance in the asymptotic limit of the number of users and hence we make this assumption on the value of $\sqrt{K}$ can be made to simplify the analysis.

2.3 Message Assignment

Let $[K]$ denote the set $\{1, 2, ..., K\}$. For any user $i \in [K]$, W_i denotes the message associated with user i and $\mathcal{T}_i$ denotes the transmit set of W_i i.e., $\mathcal{T}_i$ contains the indices of all transmitters at which W_i is available. A particular message assignment is denoted by $\{\mathcal{T}_i\}_{i\in[K]}$. We use message assignment strategies to define the transmit sets for a sequence of K-user channels. For a particular message assignment, M denotes the maximum transmit set size and is also referred to as cooperation order, while B denotes the average transmit set size,

$$M = \max_i |\mathcal{T}_i|, \tag{2.2}$$

$$B = \frac{\sum |\mathcal{T}_i|}{K}. \tag{2.3}$$

We use $\mathcal{R}_i$ to denote the set of indices of received signals that are connected to transmitter T_i.

2.4 Zero-Forcing Transmit Beamforming (Interference Avoidance)

In this thesis, we consider only zero-forcing (ZF) transmit beamforming schemes. In these schemes, the transmit signal X_i at transmitter T_i is given by

$$X_i = \sum_{\ell : i \in \mathcal{T}_\ell} X_{i,\ell}, \tag{2.4}$$

where $X_{i,\ell}$ only depends on message W_ℓ. Further, it is assumed that each message is either not transmitted or allocated one degree of freedom. For every user $j \in [K]$, let $\tilde{Y}_j = Y_j - Z_j$, where Y_j is given in (2.1). For the case where W_j is not transmitted $I(\tilde{Y}_j; W_j) = 0$ and for the case where W_j is transmitted interference-free, $\tilde{Y}_j$ is completely determined from W_j. In any ZF scheme, a receiver R_j is active iff $I(\tilde{Y}_j; W_j) > 0$. Note that if R_j is active, $I(\tilde{Y}_j; W_i) = 0, \forall i \neq j$.

2.5 Capacity and Degrees of Freedom

Let P be the average transmit power constraint at any transmitter, and let $\mathcal{W}_i$ denote the alphabet for W_i. The rates $R_i(P) = \frac{\log|\mathcal{W}_i|}{n}$ are achievable if and only if the error probabilities of all messages can simultaneously be arbitrarily small for large n, using an interference avoidance scheme. A DoF tuple $(d_1, d_2, \ldots, d_K)$ is achievable using an interference avoidance scheme, if for every $P > 0$, there exists an achievable rate tuple $(R_1(P), R_2(P), \ldots, R_K(P))$ such that

$$d_i = \lim_{P\to\infty} \frac{R_i(P)}{\log P}, \forall i \in [K]. \tag{2.5}$$

Sum DoF corresponding to an achievable DoF tuple is given by $\sum\limits_{i\in[K]} d_i$. Sum DoF denotes the total number of interference-free sessions that can be supported in a multi-user channel at a high signal-to-noise ratio (SNR). If $\mathcal{D}$ denotes the closure of the set of all achievable DoF tuples, then η_L for a locally connected channel with connectivity L is defined as

$$\eta_L = \max_{\mathcal{D}} \sum_{i\in[K]} d_i. \tag{2.6}$$

For a K-user channel, we define $\eta_L(K, M)$ and $\eta_L^{\text{avg}}(K, B)$ as the maximum achievable η_L over all possible message assignments satisfying the constraints (2.2) and (2.3) respectively. We define the following asymptotic quantities which capture how η_L scales with K.

$$\tau_L(M) = \lim_{K\to\infty} \frac{\eta_L(K, M)}{K} \tag{2.7}$$

$$\tau_L^{\text{avg}}(B) = \lim_{K\to\infty} \frac{\eta^{\text{avg}}(K, B)}{K} \tag{2.8}$$

CHAPTER 3

LINEAR INTERFERENCE NETWORKS

Most of the prior work on characterizing DoF for cellular networks focuses on linear interference networks, and in particular, the Wyner model described in Section 2.1. Recall that for a locally connected K-user channel with connectivity L, each transmitter is connected to $\lfloor \frac{L}{2} \rfloor$ preceding and $\lceil \frac{L}{2} \rceil$ succeeding receivers. We define an equivalent $(K-x)$-user channel that is used in the rest of this chapter, where $x = \lfloor \frac{L}{2} \rfloor$. We silence the first x transmitters, deactivate the last x receivers, and relabel the transmit signals to obtain the $(K-x)$-user channel, where transmitter j is connected to receivers in the set $\{\mathrm{R}_i : i \in \{j, j+1, ..., j+L\}\}$ as shown in Figure 3.1. Note that the new channel model gives the same value of $\tau_L(M)$ as the original one, since $x = o(K)$. More precisely, we consider the following channel model:

$$H_{i,j} \text{ is not identically 0 iff } i \in [j, j+1, \ldots, j+L].$$

3.1 Traditional Interference Management

We start by exploring the limits of a traditional approach for interference management for the case of no cooperation, without the restriction to the considered class of interference avoidance schemes. It is assumed that each message $W_i, i \in [K]$ is assigned to the transmitter T_i with the same index. We restate the following lemma from [6],

Lemma 1. *For the case of no cooperation, if $\mathcal{T}_i = \{\mathrm{T}_j\}$, then $d_i + d_s \leq 1$, $\forall s \in R_j, s \neq i$.*

The linear L connected channel as well as the hexagonal model introduced in Chapter 2, can be divided into groups of two transmitter-receiver pairs

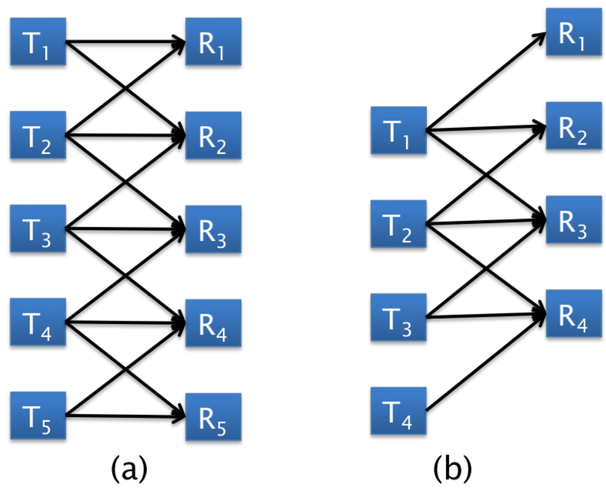


Figure 3.1: In (a), a locally connected channel model with number of users $K = 5$ and connectivity parameter $L = 2$ is shown. In (b), the new equivalent model for the network in (a) is shown.

each, such that the transmitter of one user is connected to the receiver of the other. Note that each message $W_i, i \in [K]$ is assigned to the transmitter T_i with the same index. Applying Lemma 1 to each group, it is easy to see that the PUDoF of the network cannot exceed $\frac{1}{2}$ in this scenario.

Hence, using a traditional approach for interference management, the asymptotic per user DoF is at most $\frac{1}{2}$. Further, it can be shown that this DoF value can only be approached in the limit of the length of symbol extension, i.e., by coding over a large number of time/frequency slots as in the asymptotic interference alignment scheme [3].

3.2 Flexible Message Assignment

Unlike the scheme described in Section 3.1, where each message has to be assigned to the transmitter with the same index, we explore a flexible cell association approach in this section. We assume that a message need not be assigned to the transmitter with the same index. An example is first discussed to demonstrate that this leads to DoF gains.

3.2.1 Example

Consider the asymmetric Wyner model ($L = 1$) and consider the case of no cooperation ($M = 1$). Let W_1 be available at the first transmitter, W_3 be available at the second transmitter, and deactivate both the second receiver and the third transmitter. Then it is easily seen that messages W_1 and W_3 can be received interference-free at their corresponding receivers. Moreover, the deactivation of T_3 splits this part of the network from the rest of the network and the same scheme can be repeated by assigning W_4, W_6 to the transmitters T_4,T_5, respectively, and so on. Thus, 2 degrees of freedom can be achieved for each set of three users, thereby, achieving an asymptotic PUDoF of $\frac{2}{3}$. The described message assignment is depicted in Figure 3.2.

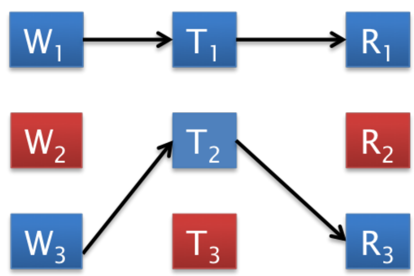


Figure 3.2: Flexible association of W_3 to T_2 achieves a PUDoF of $\frac{2}{3}$, without any cooperation in an asymmetric Wyner network. The red boxes represent deactivated nodes.

3.3 Achievable Schemes Using Cooperation

We present schemes that use cooperative transmission with flexible message assignment. Even though the average backhaul load constraint B is more relevant to practical applications, the cooperation order M simplifies the combinatorial aspect of the problem, and solutions under constraint M can be used to provide solutions under constraint B. The following theorem from [6] gives a simple message assignment strategy under the cooperation order M, which is later proved to be optimal under interference avoidance schemes.

Theorem 1. *The following lower bound holds for the asymptotic PUDoF of* *a locally connected interference network with connectivity parameter L,*

$$\tau_L(M) \geq \frac{2M}{2M+L}.$$

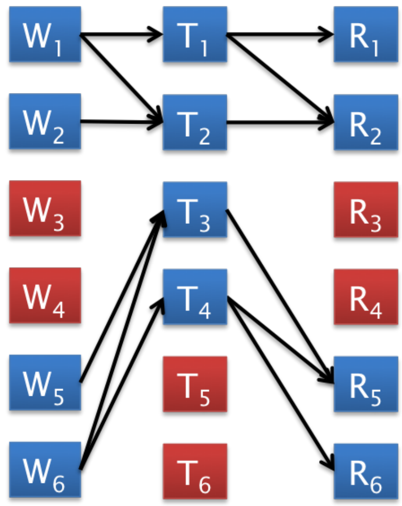


Figure 3.3: The message assignment is shown for the network with $L = 2$. A PUDoF of $\frac{2}{3}$ is attained using cooperation constraint $M = 2$. The red boxes represent deactivated nodes.

Proof. Let the network be divided into subnetworks consisting of $2M + L$ consecutive transceivers. The last L transmitters in each subnetwork are deactivated to eliminate inter subnetwork interference. It suffices to show that sum DoF of $2M$ can be achieved in each subnetwork. Consider a scheme in which the messages $W_{M+1}, W_{M+2}, ..., W_{M+L}$ are not transmitted and the remaining messages are sent interference-free. The users in the subnetwork are further divided into two clusters. Cluster A_1 consists of the first M users and cluster A_2 consists of the last M users, denoted by the following subsets of $[2M + L]$,

$$A_1 = [M],$$
$$A_2 = \{L + M + 1, L + M + 2, ..., L + 2M\}.$$

Messages are assigned so there is no interference between sets A_1 and A_2 as follows,

$$\mathcal{T}_i = \begin{cases} \{i, i+1, ..., M\}, & \forall i \in A_1, \\ \{i - L, i - L - 1, ..., M + 1\}, & \forall i \in A_2. \end{cases}$$

The message assignment for $L = 2$ and $M = 2$ is depicted in Figure 3.3.

The strategy for transmitting messages of users in A_1 interference-free is demonstrated for W_1. The first receiver does not observe interference from any transmitter and hence the message W_1 is encoded into $X_{1,1}$. The interference seen at receiver R_2 due to W_1 needs to be canceled, and $X_{2,1}$ is designed as

$$X_{2,1} = \frac{-H_{2,1}}{H_{2,2}} X_{1,1}.$$

The beams $X_{i,1}$ where $i \in \{3, \dots, M\}$ are designed in a similar fashion to cancel the interference caused by W_1 at receivers R_i, where $i \in \{3, \dots, M\}$. Thus, we achieve a sum DoF of $2M$ among every consecutive $2M + 1$ users in the network. $\square$

The proposed schemes are zero-forcing beamforming schemes, which do not require precoding over more than one time/frequency slot (one shot). Note that in the coding scheme of Theorem 1, some messages are not transmitted, so that the remaining messages can be sent interference-free. Fairness can be maintained in the allocation of the available DoF over all users, through fractional reuse in the system by deactivating different sets of receivers in different sessions, e.g., in different time or frequency slots.

The maximum transmit set size constraint M is not met tightly for all messages in the message assignment scheme presented in Theorem 1. We therefore consider the average backhaul constraint in which the transmit set size is allowed to vary across the messages, while maintaining an average transmit set size of B. We present a theorem from [8], for the asymmetric Wyner model, which uses the achievable schemes in Theorem 1.

Theorem 2. *The following lower bound holds for the asymptotic PUDoF for a Wyner network over all achievable schemes*

$$\tau^{avg}(B) = \frac{4B - 1}{4B}.$$

The achievable scheme involves dividing the network into subnetworks of consecutive $4B$ users. The last transmitter in each subnetwork is silenced to eliminate inter subnetwork interference. The message W_{2B+1} is not transmitted. The first $2B$ messages are transmitted similar to the scheme for messages in A_1 in the proof of Theorem 1 with $M = 2B$, while the last $2B - 1$ messages are sent similar to the scheme for messages in A_2 with $M = 2B - 1$. It can be verified that this strategy satisfies the average backhaul load constraint of B and achieves a PUDoF of $\frac{4B-1}{4B}$. This scheme was shown to be optimal over all achievable schemes in [8].

It can be verified that under the average backhaul load constraint $B = 1$, i.e., without requiring extra backhaul capacity, a PUDoF of greater than half can be achieved for $L \leq 6$ in a similar fashion, using combinations of schemes designed for different values of M.

Although we assume availability of all channel coefficients at every transmitter in the network, the schemes presented in this section require only local channel state information, i.e., each node need only be aware of the channel coefficients between itself and its neighbors.

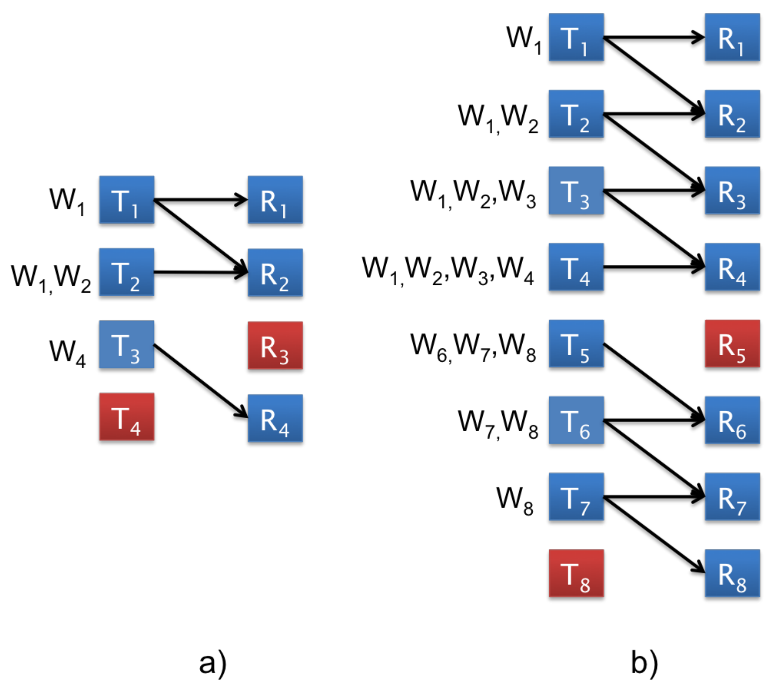


Figure 3.4: Message assignments are shown for the asymmetric Wyner network. In (a), a PUDoF of $\frac{3}{4}$ is attained with $B = 1$ and in (b), a PUDoF of $\frac{7}{8}$ is attained with $B = 2$.

3.4 Irreducible Message Assignments

In order to find an upper bound on $\tau_L(M)$, we need to consider all possible message assignments for interference avoidance schemes. In this section, we discuss the notion of irreducible message assignments. A message assignment is irreducible if no element can be removed from any transmit set without decreasing the sum rate.

For a message W_i and a fixed transmit set $\mathcal{T}_i$, construct a graph $G_{W_i,\mathcal{T}_i}$ in which an edge exists between vertices x, y iff

$$x, y \in \mathcal{T}_i,$$
$$|x - y| \leq L.$$

Vertices corresponding to transmitters that are connected to receiver R_i are called marked vertices. We restate the following lemma and its corollary from [6].

Lemma 2. *For any $k \in \mathcal{T}_i$ such that the vertex k in $G_{W_i,\mathcal{T}_i}$ is not connected to a marked vertex, removing k from $\mathcal{T}_i$ does not decrease the sum rate.*

Corollary 1. *Let $\mathcal{T}_i$ be an irreducible message assignment and $|T_i| \leq M$, then $\forall k, k \in \mathcal{T}_i$ only if the vertex k in $G_{W_i,\mathcal{T}_i}$ lies at a distance that is less than or equal to $M-1$ from a marked vertex.*

3.5 DoF Upper Bounds

In this section, we discuss upper bounds that show the optimality of the achievable schemes from Theorem 1. For each transmitter in $\mathcal{T}_i$, we assume that W_i contributes to the transmit signal of transmitter i, i.e., $\forall j \in \mathcal{T}_i, I(X_j, W_i) > 0$; otherwise transmitter T_j can be removed from $\mathcal{T}_i$. For a set S, let V_S denote the set of indices for active receivers connected to transmitters with indices in S. We restate the following lemma from [6] for interference avoidance schemes.

Lemma 3. *For any message W_i, the number of active receivers connected to at least one transmitter carrying the message is no greater than the number of transmitters carrying the message,*

$$|V_{\mathcal{T}_i}| \leq |\mathcal{T}_i|.$$

Among the $|\mathcal{T}_i|$ transmit signals carrying W_i, one transmit signal is designed to send message W_i to the intended receiver R_i, and the remaining

$|\mathcal{T}_i|-1$ signals cannot be designed to cancel W_i at more than $|\mathcal{T}_i|-1$ receivers.

The following theorem from [6] provides an upper bound on PUDoF using Lemma 3 and the concept of irreducible message assignments, that shows the optimality of the achievable schemes discussed in Theorem 1.

Theorem 3. *The following lower bound holds for the asymptotic PUDoF of a locally connected interference network with connectivity parameter L, under restriction to zero-forcing transmit beamforming schemes,*

$$\tau_L(M) \geq \frac{2M}{2M+L}.$$

Proof. We show that in any set S of $2M+L$ consecutive users, a DoF of only $2M$ can be attained. For a user $i \in [S]$, let U_i be the set of active users in S with an index $j > i$. Similarly, let D_i be the set of active users in S with an index $j < i$. Assume that S has at least $2M+1$ active users, then there is an active user in S that lies in the middle of a subset of $2M+1$ active users in S. Let this middle user have index i for the rest of the proof. Let s_{min} and s_{max} be the users in S with minimum and maximum indices, respectively, i.e., $s_{min} = \min\{s : s \in S\}$ and $s_{max} = \max\{s : s \in S\}$, we then consider the following cases.

Case 1: W_i is transmitted from a transmitter that is connected to the receiver with index s_{min}. It follows from Lemma 2 that $V_{\mathcal{T}_i} \supseteq D_i \cup i$, and hence, $|V_{\mathcal{T}_i}| \geq M+1$, which contradicts Lemma 3, as $|\mathcal{T}_i| \leq M$.

Case 2: W_i is being transmitted from a transmitter that is connected to the receiver with index s_{max}. It follows from Lemma 2 that $V_{\mathcal{T}_i} \supseteq U_i \cup i$, and hence, $|V_{\mathcal{T}_i}| \geq M+1$, which again contradicts Lemma 3.

Case 3: There is no transmitter in $\mathcal{T}_i$ that is connected to any of the receivers with indices s_{min} and s_{max}. It follows from Lemma 2 that all the receivers connected to transmitters carrying W_i belong to S. At least $L+|\mathcal{T}_i|$ receivers in S are connected to one or more transmitters in $\mathcal{T}_i$, and since S has at least $2M+1$ active receivers, any subset of $L+|\mathcal{T}_i|$ receivers in S has at least $|\mathcal{T}_i|+1$ active receivers, which is a contradiction to Lemma 3. $\square$

CHAPTER 4

NETWORK WITH NO INTRA-CELL INTERFERENCE

We extend the results of linear interference networks to hexagonal sectored cellular networks. In this chapter, we focus on the case with no interference between sectors belonging to the same cell. The interference graph is shown in Figure 4.1. The connectivity is fixed for this network, and we drop L in the notation τ_L, and use τ to denote the maximum PUDoF under appropriate constraints.

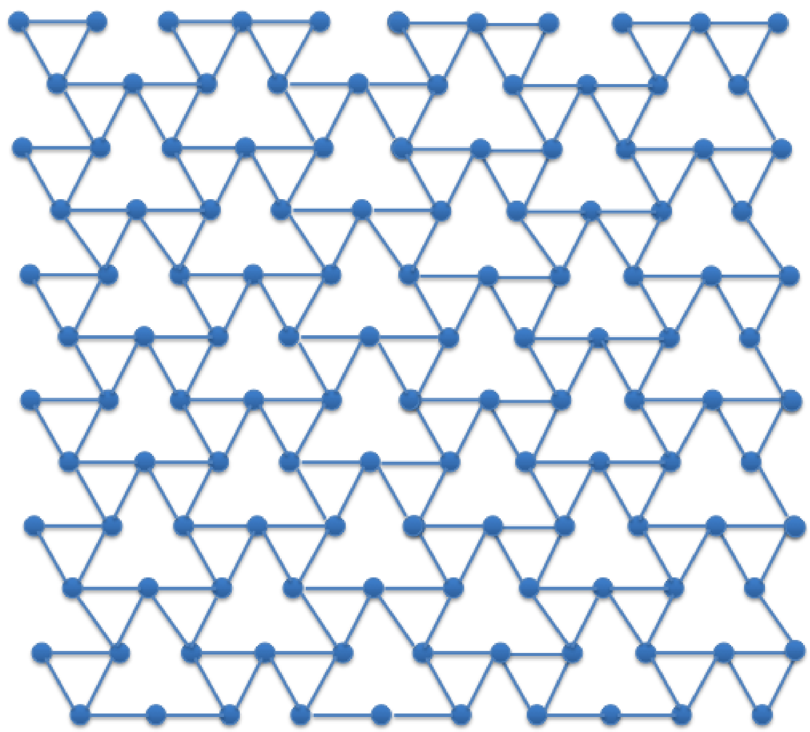


Figure 4.1: Interference graph for the case of no intracell interference. Each sector is surrounded by four neighboring out-of-cell sectors.

4.1 Flexible Cell Association

In this section, we explore a flexible cell association approach in which each message can be assigned to only one transmitter, but this transmitter can be chosen as any transmitter connected to the message's desired destination. We formally study this scenario by imposing the maximum transmit size constraint $M = 1$. We characterize lower and upper bounds for the maximum achievable per user DoF. Note that when $M = 1$, an interference avoidance scheme is just an orthogonal scheme (e.g., TDMA).

Theorem 4. *The following bounds hold under restriction to orthogonal schemes for the PUDoF, as the number of users goes to infinity, with flexible cell association and no cooperation,*

$$\frac{1}{3} \leq \tau(M = 1) \leq \frac{3}{7}.$$

Proof. Lower Bound: The network can be divided as in Figure 4.2 into disjoint, fully connected triangles. In each triangle, by deactivating nodes 1 and 2 as shown in Figure 4.2, it is easy to see that a PUDoF of $\frac{1}{3}$ is achieved.

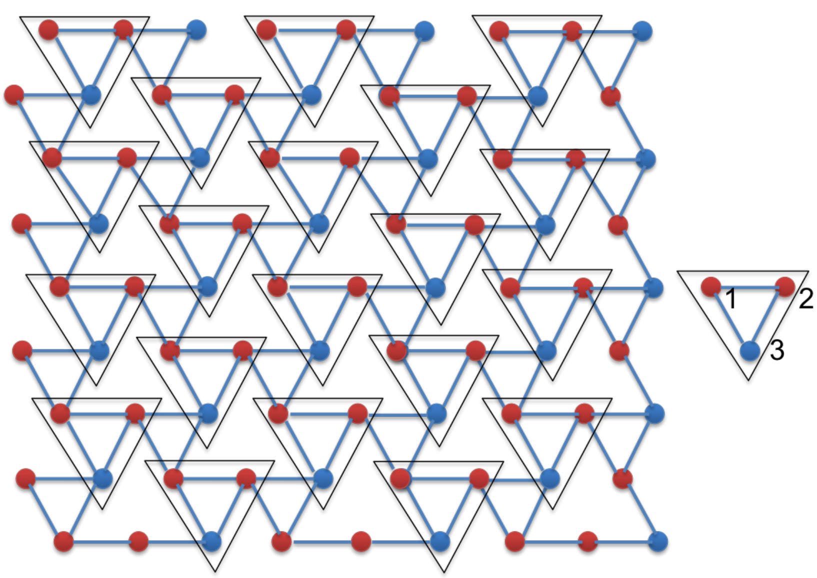


Figure 4.2: Division of network into triangular subnetworks. In each triangle, we note that by deactivating nodes 1 and 2 (in red), we obtain non-interfering triangles and a per user DoF of $\frac{1}{3}$ is achieved.

Upper Bound: Consider the division of the network into disjoint fully connected triangles as shown in Figure 4.3. For any orthogonal coding scheme, we note that any fully connected triangle in the network is in one of the following states:

State 0 (inactive triangle): All transmitters and receivers in the triangle are inactive.

State 1 (self-serving triangle): Exactly one transmitter in the triangle sends a message to exactly one receiver within the triangle. None of the other transmitters or receivers can be active in this triangle.

State 2 (serving triangle): At least one transmitter in the triangle serves a receiver in another triangle and there are no active receivers in the triangle.

State 3 (served triangle): At least one receiver in the triangle is served by a transmitter in another triangle and there are no active transmitters.

For the triangles in state 1 and state 0, the number of active receivers is bounded by the number of triangles, i.e., one-third of the number of users.

For every transmitter c that is active in a triangle S_c in state 2, there exists a neighboring receiver b in a different triangle S_b in state 3 that is being served by it and a neighboring node a in another different triangle S_a, whose transmitter and receiver are both inactive.

We now consider the following cases:

Case 1: S_a is in state 2 or 3. The remaining neighbors of a, b, c are the nodes in their own triangles. We now know that $d_a + d_b + d_c \leq 1$, because receivers R_a and R_c are inactive. Further, because none of the nodes a, b and c have other neighbors except in their own triangles, there is no overcounting when we repeat this procedure to obtain DoF bounds on other similar sets of users.

Case 2: S_a is in state 1. Suppose in S_a, there is a node a_2 which serves itself. Then there is another inactive node in S_a which may form a group similar to a, b, c with its neighbors from different triangles, say b_1, c_1. We note that these two groups are disjoint. Therefore, among the seven nodes $(S_a \cup \{b, c, b_1, c_1\})$, there are at most three active receivers. Suppose S_a does not contain a self-serving node. Then a is the only node with inactive transmitter and receiver in S_a, and among the five nodes $(S_a \cup \{b, c\})$, we attain a sum DoF of at most two.

Case 3: S_a is in state 0. Then in the set of the five nodes $(S_a \cup \{b, c\})$, we attain a sum DoF of at most one.

For any scheme, the network can be rearranged into a combination of disjoint groups of three, five and seven users, and PUDoF for each group is at most $\frac{3}{7}$. It follows that $\tau \leq \frac{3}{7}$ holds asymptotically for any choice of cell associations and interference avoidance schemes. $\square$

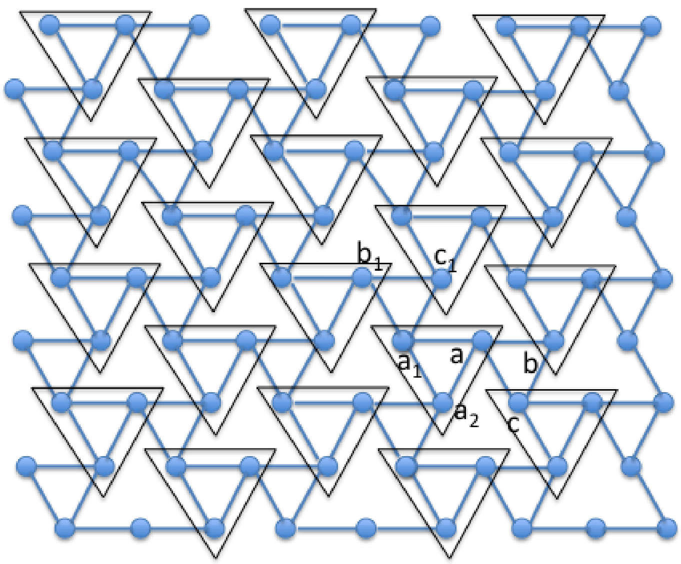


Figure 4.3: Division of network into triangular subnetworks. Transmitter c in triangle S_c in state 2 (serving triangle) serves a neighboring receiver b in a different triangle S_b in state 3 (served triangle). The neighboring node a in another different triangle S_a is inactive. If S_a is in state 1 (self-serving) and a_2 serves itself, then a_1, b_1, c_1 may form a group similar to a, b, c. Among the seven nodes ($S_a \cup \{b, c, b_1, c_1\}$), at most three receivers are active.

4.2 Flexible Message Assignment with Cooperation

We now consider cooperative transmission with flexible message assignment, first under the cooperation order M, and then use these results to propose schemes that achieve per user DoF gains under constraints on the average backhaul load B.

4.2.1 Cooperative Transmission Lower Bounds

We first present a lower bound on the PUDoF under cooperation order $M = 2$ and then for particular values of M, where $M = 5\ell + 6, \forall \ell \in \mathbb{N} \cup \{0\}$.

Theorem 5. *Under the cooperation order $M = 2$, the following lower bound holds on the PUDoF, as the number of users goes to infinity,*

$$\tau(M = 2) \geq \frac{4}{9}.$$

Proof. Consider division of the network into subnetworks of nine nodes as shown in Figure 4.4. In each subnetwork, we have two blocks, one consisting of six nodes (nodes 1, 2, 3, 4, 5, 6 in Figure 4.4) and the other consisting of three nodes (nodes 7, 8, 9 in Figure 4.4). Note that deactivating the nodes 7, 8, 9 eliminates interference between subnetworks. Hence we present

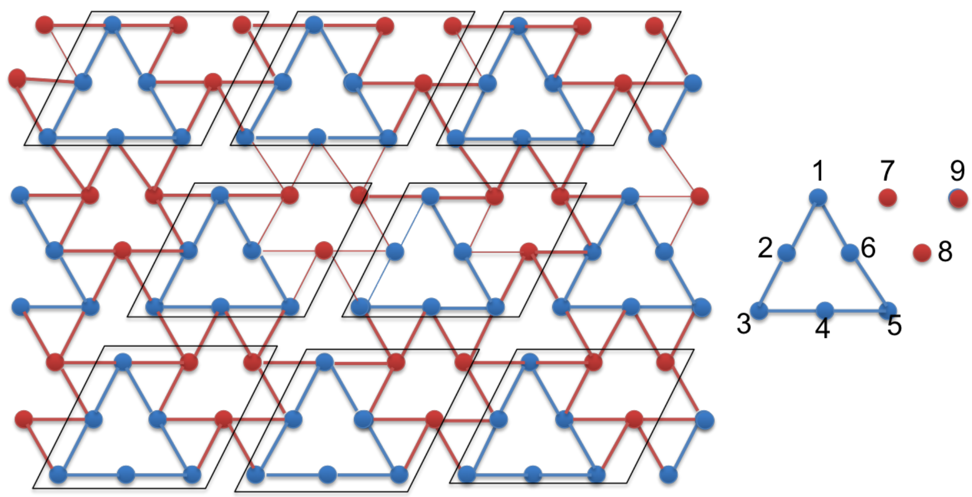


Figure 4.4: Division of cellular network into subnetworks consisting of nine nodes each. By deactivating three sectors belonging to a cell (red nodes) in each subnetwork, interference between subnetworks is eliminated. Each subnetwork consists of six active nodes that form a triangle.

a coding scheme for each subnetwork separately. We treat the triangular block of six nodes as a linear network shown in Figure 4.5, $\mathcal{T}_2 = \{1\}, \mathcal{T}_3 = \{1, 2\}, \mathcal{T}_4 = \{5\}, \mathcal{T}_5 = \{5, 6\}$ and messages W_1, W_6 are not sent. Transmitters $\mathrm{T}_3, \mathrm{T}_4$ and receivers $\mathrm{R}_1, \mathrm{R}_6$ are deactivated. Since the transmitters are aware of the channel state information, the messages W_2, W_3, W_4, W_5 can be sent without any interference through zero-forcing linear beamforming, similar to the scheme in Theorem 1. Thus $\eta \geq 4$ and $\tau(M = 2) \geq \frac{4}{9}$ and we note that for this scheme, the average backhaul load $B = \frac{2}{3}$. □

Theorem 6. *Under the cooperation order M, where $M = 5\ell + 6, \forall \ell \in \mathbb{N} \cup \{0\}$, the following lower bound holds on the PUDoF, as the number of users goes*

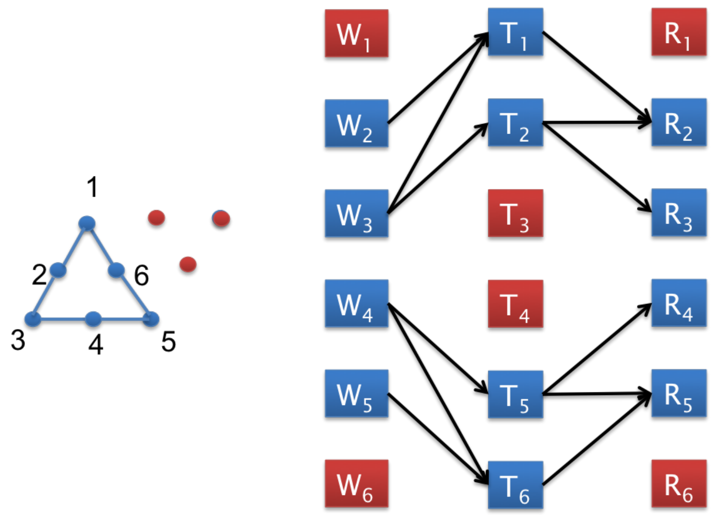


Figure 4.5: The subnetwork is shown as a linear network and the message assignment in the linear network is shown for $M = 2$. $\mathrm{T}_i, \mathrm{R}_i$ denote the transmitter and receiver associated with the sector containing user i, and W_i denotes the intended message for user i. The red boxes indicate that the corresponding transmitter/receiver is deactivated.

to infinity,

$$\tau(M) = \frac{5M}{6M+9}.$$

Proof. We first prove the result for $\ell = 0$ and $M = 6$ and then extend this scheme to higher values of ℓ. We consider the network division shown in Figure 4.4 and consider each subnetwork separately. In the block of six nodes, if each message is available at all the transmitters, then by the use of simple linear transmit beamforming, we obtain a sum DoF of 6 thus giving $\tau(M = 6) \geq \frac{2}{3}$. Note that for this scheme, the average backhaul load on the network $B = 4$.

For a general case with $\ell \geq 1$, consider subnetworks of size $9 + 6\ell$. The case $\ell = 2$ is shown in the Figure 4.6. We have two kinds of units here - one block of nine nodes as in the previous case and ℓ blocks containing six nodes each. Note that by deactivating the transmitters of three nodes corresponding to the block containing nine nodes (as before) and one transmitter each corresponding to each block of six nodes, the interference between subnetworks can be eliminated. Hence in each block, by simple linear transmit beamforming, $\tau(M) = \frac{M}{6\ell+9} = \frac{5M}{6M+9}$ can be attained. We note that for this scheme, $B = \frac{5M^2}{6M+9}$ which equals 4 for $M = 6$. Note that for the case $5\ell + 6 < M < 5(\ell + 1) + 6$, $\tau = \max\{\frac{5\ell+6}{6\ell+9}, \frac{M}{6\ell+9}\}$. □

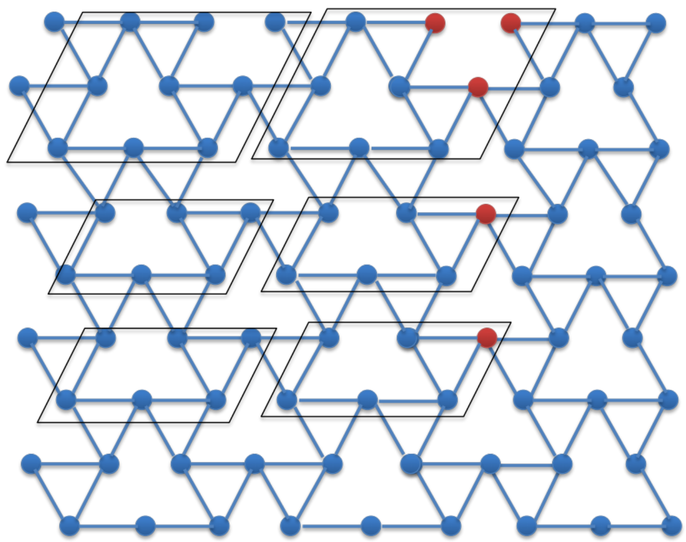


Figure 4.6: Division of cellular network into subnetworks for $M = 11$. Here $\ell = 2$ and each subnetwork consists of a block containing nine nodes and two blocks containing six nodes each. The red nodes indicate deactivated nodes in a block.

We now show, through the result in Theorem 7, how a smart choice for assigning messages to transmitters, aided by cooperative transmission, can achieve scalable DoF gains through an interference avoidance coding scheme. In particular, we show a lower bound on the achievable PUDoF greater than the $\frac{3}{7}$ upper bound of the case without cooperation, without the need for an extra backhaul load.

Theorem 7. *Under the average backhaul constraint $B = 1$, the following lower bound holds on the PUDoF, as the number of users goes to infinity,*

$$\tau^{avg}(B = 1) \geq \frac{7}{15}.$$

Proof. Consider the message assignment strategy from Theorem 6 with $B = 4$ and $\tau \geq \frac{2}{3}$, and the message assignment strategy from Theorem 5 with $B = \frac{2}{3}$ and $\tau \geq \frac{4}{9}$. A convex combination in the ratio $9 : 1$ of these schemes gives us $B = 1$ and a PUDoF of $\frac{7}{15}$ for the entire network. $\square$

In the achievable scheme in Theorem 5, we have $B = \frac{2}{3}$ and a PUDoF of $\frac{4}{9}$. Since $B < 1$, we can have a scheme which achieves a greater PUDoF by overloading the network, and combining these schemes gives us $\tau(B = 1) \geq \frac{7}{15}$. By combining a flexible backhaul design and cooperative transmission, we achieve a PUDoF very close to $\frac{1}{2}$ through simple interference avoidance.

Theorem 8. *Under the average backhaul constraint $B = 2$, the following lower bound holds for the asymptotic PUDoF,*

$$\tau^{avg}(B = 2) \geq \frac{8}{15}.$$

Proof. Consider the message assignment strategy from Theorem 6 with $B = 4$ and $\tau \geq \frac{2}{3}$, and the message assignment strategy from Theorem 5 with $B = \frac{2}{3}$ and $\tau \geq \frac{4}{9}$. A convex combination in the ratio $2 : 3$ of these schemes gives us $B = 1$ and a PUDoF of $\frac{8}{15}$ for the entire network. □

Theorem 9. *Under the average backhaul constraint $B = 3$, the following lower bound holds for the asymptotic per user DoF,*

$$\tau^{avg}(B = 3) \geq \frac{3}{5}.$$

Proof. Consider the message assignment strategy from Theorem 6 with $B = 4$ and $\tau \geq \frac{2}{3}$, and the message assignment strategy from Theorem 5 with $B = \frac{2}{3}$ and $\tau \geq \frac{4}{9}$. A convex combination in the ratio $3 : 7$ of these schemes gives us $B = 1$ and a per user DoF of $\frac{8}{15}$ for the entire network. □

The schemes discussed in this section are one-shot zero-forcing beamforming schemes that require only local channel state information, i.e., each node only needs to be aware of the channel coefficients between itself and its neighbors. Also, some messages are being sent interference-free while a few messages are not being transmitted. A sense of fairness is maintained among all users through fractional reuse in the system, by deactivating different sets of receivers in different sessions, e.g., in different time or frequency slots.

The insights obtained from the result of Theorem 7 are expected to hold even when some of the users in the considered cellular network are absent. As we are considering the average transmit set size constraint, allowing for a flexible backhaul design with cooperative transmission can still achieve significant DoF gains. This is because the backhaul resources corresponding to missing users can be used to facilitate the interference management for the remaining users.

4.2.2 Cooperative Transmission Upper Bound

In order to obtain a converse for $\tau^{\text{avg}}(B)$, we need upper bounds on $\tau(M)$ for higher values of M. As a first step, we derive a bound on $\tau(M = 2)$.

Theorem 10. *Under the maximum transmit set size constraint $M = 2$, the following upper bound holds for the asymptotic PUDoF, under restriction to interference avoidance schemes,*

$$\tau(M = 2) \leq \frac{3}{4}.$$

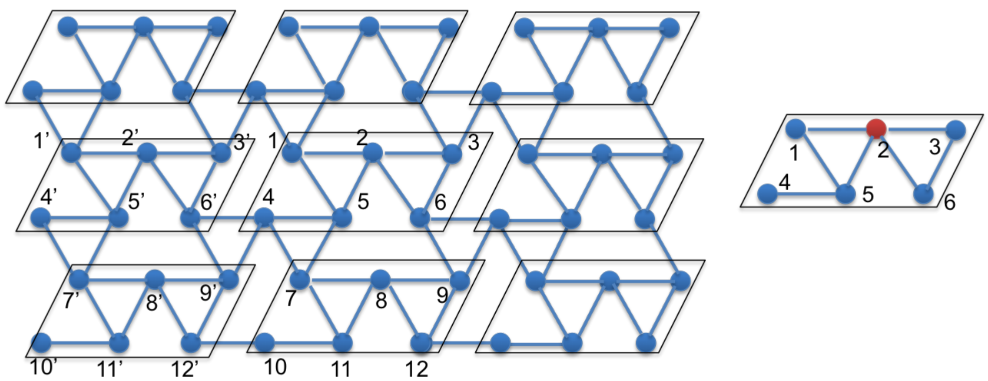


Figure 4.7: Division of the network into subnetworks of six nodes. Subnetworks of 12 nodes (nodes numbered 1 through 12) are also considered, which consist of two blocks.

Proof. The interference graph can be divided into blocks containing six nodes as shown in Figure 4.7. We first show that among nodes 1, 2, 3, 4, 5 (in the Figure 4.7) in each subnetwork, at most 4 degrees of freedom can be achieved.

Each block is in one of the following states:

State A1: At least one of the transmitters $\{\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3, \mathrm{T}_5, \mathrm{T}_6\}$ is active. Each transmitter in the set is connected to at least three receivers and for Lemma 3 to hold, one of the receivers must be deactivated.

State A2: None of the transmitters $\{\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3, \mathrm{T}_5, \mathrm{T}_6\}$ is active. Then the message W_2 cannot be transmitted to R_2. In both the states, DoF in each subnetwork is at most 5.

Now consider the division of the network into subnetworks, where each subnetwork consists of twelve nodes consisting of two blocks discussed before, as shown in Figure 4.7. Let C and D denote the subnetworks containing nodes numbered 1 through 6 and 7 through 12, respectively.

Case 1: Both C and D are in state A2. Then messages W_2, W_8 cannot be transmitted. Note that the messages W_5, W_7 in this scenario can be carried only by T_4 and hence at most one receiver among $\mathrm{R}_5, \mathrm{R}_7$ can be active, from Lemma 3. In this scenario, a DoF greater than 9 cannot be attained.

Case 2: Block C is in state A1 and block D in state A2. Then message W_8 cannot be transmitted. If any of the transmitters $\{\mathrm{T}_5,\ \mathrm{T}_2,\ \mathrm{T}_6\}$ are active, then at least two receivers in block C must not be active and a DoF of at most 9 is attained. Suppose that none of the transmitters $\{\mathrm{T}_5,\ \mathrm{T}_2,\ \mathrm{T}_6\}$ is active. Note that message W_7 can be transmitted only by T_4 or T_5 and in any case, at most one receiver among $\mathrm{R}_5, \mathrm{R}_7$ can be active. Since we have assumed that $\mathrm{T}_2, \mathrm{T}_6$ are inactive, for $\mathrm{R}_6, \mathrm{R}_9$ to be simultaneously active, T_3 must be active, in which case one among $\mathrm{R}_3, \mathrm{R}_2$ is not active. Hence a DoF greater than 9 cannot be attained in this scenario as well.

Case 3: Block C is in state A2 and block D in state A1. Then message W_2 cannot be transmitted. If any of $\{\mathrm{T}_7,\ \mathrm{T}_8,\ \mathrm{T}_9, \mathrm{T}_{11}\}$ are active, then at least two receivers in block D cannot be active and a DoF of at most 9 is attained. Suppose none of the transmitters in the set $\{\mathrm{T}_7,\ \mathrm{T}_8,\ \mathrm{T}_9,\ \mathrm{T}_{11}\}$ is active. Then T_{12} has to be active. Then at least one receiver among $\mathrm{R}_8, \mathrm{R}_9, \mathrm{R}_{12}$ is not active and similar to the previous case, at least one receiver among $\mathrm{R}_5, \mathrm{R}_7$ must not be active. Hence, a DoF greater than 9 cannot be attained in this scenario.

Case 4: Both blocks are in state A1. If any transmitters among $\{\mathrm{T}_2,\ \mathrm{T}_5,\ \mathrm{T}_7,\ \mathrm{T}_8\}$ is active, then a DoF of at most 9 is attained. Also if T_{11} is active, then for any transmitter active in block C, a DoF greater than 9 cannot be achieved. Suppose T_9 is active in D and T_1 in C, a DoF of at most 9 is attained. Note that only T_4 can transmit W_7 and hence one among the receivers $\mathrm{R}_4, \mathrm{R}_5, \mathrm{R}_7$ must be inactive. If T_6 or T_9 is active, two additional receivers must be inactive and hence a DoF greater than 9 cannot be attained. If T_3 and T_{12} are active simultaneously then we cannot achieve a DoF greater than 9. So the only possibility of attaining DoF greater than 9 is when $\mathrm{T}_1, \mathrm{T}_{12}$ are active. This is possible only when R_5 and R_9 are the only inactive receivers in C and D, and T_1 transmits W_2, T_{12} transmits W_8 and T_4 transmits W_7. In this case note that either $\mathrm{T}_{6'}$ or $\mathrm{T}_{9'}$ transmits W_4 and in the subnetwork containing the 12 nodes numbered $1'$ through $12'$, three receivers among $\{\mathrm{R}_{2'}, \mathrm{R}_{3'}, \mathrm{R}_{6'}, \mathrm{R}_{8'}, \mathrm{R}_{9'}, \mathrm{R}_{12'}\}$ must be inactive. If any transmitter among $\{\mathrm{T}_{2'},\ \mathrm{T}_{5'},\ \mathrm{T}_{7'},\ \mathrm{T}_{8'},\ \mathrm{T}_{11'}\}$ is active, we have a DoF of at most 8. Otherwise at least one among $\mathrm{R}_{4'}, \mathrm{R}_{5'}, \mathrm{R}_{7'}$ is inactive leading to a DoF of at most 8 in this subnetwork. Hence, in the two combined subnetworks, $\tau \leq \frac{3}{4}$.

□

4.3 Robustness Analysis

We now discuss the notion of topological robustness defined in [2]. An achievable scheme is robust with respect to a particular network topology, if its performance does not depend on the existence of interference. Most of the present cellular networks are designed such that the interfering links are weak. In some of our proposed schemes, we use the interfering crosslinks to transmit messages. We want to ensure that the same PUDoF can be achieved, even if some of the crosslinks are missing. We want our proposed schemes to be robust to the topology of the network i.e., under any configuration of interfering links.

We show that the results from Section 4.2 hold even when some of the crosslinks are missing.

Theorem 11. *Under the average backhaul constraint $B = 4$, the following lower bound holds for the asymptotic PUDoF, under any configuration of interfering crosslinks,*

$$\tau^{avg}(B = 4) \geq \frac{2}{3}.$$

Proof. We consider the network division in Figure 4.4 and show that in each subnetwork containing nine nodes, if some of the crosslinks are missing, we can still achieve a PUDoF of $\frac{2}{3}$, for $B = 4$, as in Theorem 6. In the achievable scheme used in Theorem 6, each transmitter in the block of six nodes knows all the other messages. If any of the crosslinks are missing, we modify the transmit beams accordingly to cancel interference due to the existing crosslinks, for any possible configuration of the missing crosslinks. □

Theorem 12. *Under the average backhaul constraint $B = \frac{2}{3}$, the following lower bound holds for the asymptotic PUDoF, under any configuration of interfering crosslinks,*

$$\tau^{avg}(B = \frac{2}{3}) \geq \frac{4}{9}.$$

Proof. Consider the network division and the achievable scheme used in Theorem 5. We show that in each subnetwork consisting of nine nodes, if any of the crosslinks are missing, we can still achieve a PUDoF of $\frac{4}{9}$, for the case $B = \frac{2}{3}$, as before. In each subnetwork, only the crosslinks in the block of six nodes can affect the DoF.

In the interference graph, each edge represents interference in both directions. We consider the two crosslinks that represent an edge, separately. Let the links be numbered in the following way. From a node i, the crosslink to node $(i-1)\text{mod}6$ is ia, and the crosslink to node $i+1\text{mod}6$ is ib, as shown in Figure 4.8. We use i' to represent the edge between nodes i and i+1mod6 i.e., i'=$\{ib$,(i+1mod6)$a\}$. We refer to the achievable scheme of Theorem 5 with $M = 2$, as scheme 1. Starting with node 1, the links $1b, 2b, 5a, 6a$ should not be missing for the scheme to achieve a PUDoF of $\frac{4}{9}$. Similarly, starting with node 2, scheme 1 can be used as long as links $2b, 3b, 6a, 1a$ are not missing. Note that any node can start with the number 1. Let scheme 1.i represent scheme 1 starting from node i, such that the links ib, (i+1mod6)b, (i+4mod6)a, (i+5mod6)a are not be missing.

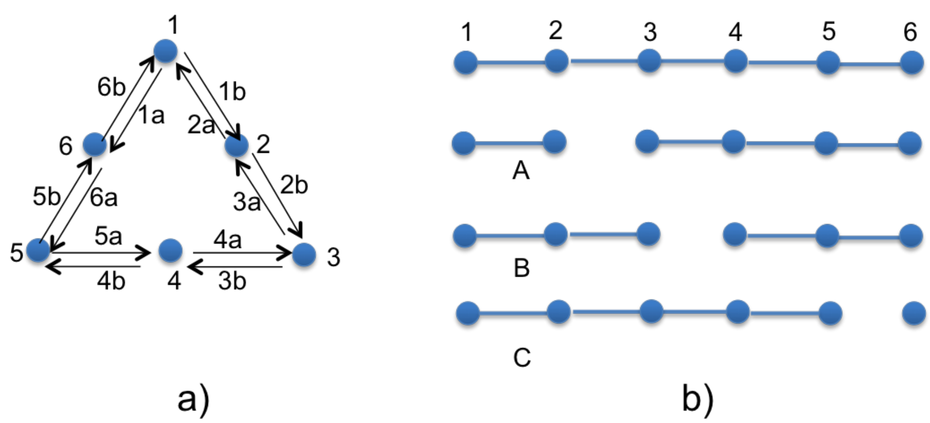


Figure 4.8: In (a), we consider the two-directional edges between every two nodes in the triangular subnetwork. From each node i, the crosslink to node $(i-1)$mod 6 is ia and to node $i+1$mod 6 is ib. In (b), we consider separation of the subnetwork into subnetworks when few crosslinks are missing.

If any of the links are missing, then the block of six nodes exists in any of the following three states:

State S1: The interference graph of the block can be split into two or more components.

State S2: The interference graph of the block cannot be split into smaller components and does not contain a cycle.

State S3: The interference graph of the block contains a cycle.

Consider the block in state S1. Suppose it contains two components. The possible cases are as shown in Figure 4.8(b). In case A, we may deactivate the nodes $4, 5$ in the second block. In case B, we may deactivate nodes 2 and 4 to achieve the DoF. In case C, we may deactivate nodes $2, 4$ to achieve the required DoF. There are no crosslinks used to transmit intended messages in the schemes of cases B and C, and therefore if any other crosslinks are missing, there is no effect on the DoF. In case A, crosslinks are present only in the block containing two nodes, and if any of them are missing, the same DoF can be achieved with a smaller average transmit set size. Note that more components are obtained by removing crosslinks from the two component case.

Consider a block in state S2, and suppose that we deactivate nodes 2 and 4. Among nodes $5, 6$, if there are crosslinks present, the required message is made available at the other transmitter, and if any of the crosslinks is missing, the same DoF can be achieved with a smaller average transmit set size.

Let the block be in state S3. Note that none of $\{1', 2', 3', 4', 5', 6'\}$ is missing. If four or more links are missing, there exist at most two elements in $\{1', 2', 3', 4', 5', 6'\}$ which have both crosslinks present. If they are adjacent to each other say $1', 2'$, we may deactivate the common node 2 and the central node 5 in the remaining part. This results in two sets containing two nodes such that each set has only one crosslink each, so that we can attain a DoF of 2 in each set, with $B = \frac{2}{3}$ in the subnetwork. If they are at a distance of one, say $1', 3'$, then consider the set in between, say $2'$ which contains $2b$ and $3a$. Consider the set of nodes $2, 3$ and the network after deactivating nodes 1 and 4. We again have two sets containing two nodes such that each set has only one crosslink each as in the previous case.

If three links are missing, there exist three elements in $\{1', 2', 3', 4', 5', 6'\}$ that have both crosslinks present. Suppose these elements are $\{1', 3', 5'\}$ or $\{2', 4', 6'\}$ i.e., every other element in the set $\{1', 2', 3', 4', 5', 6'\}$. Without loss of generality, assume they are $\{1', 3', 5'\}$ and the corresponding missing links belong to one each among $\{2b, 3a\}$, $\{4b, 5a\}$ and $\{6b, 1a\}$. If the missing links are $\{3a, 5a, 1a\}$ or $\{2b, 4b, 6b\}$, we may use the schemes 2 and 3 (see

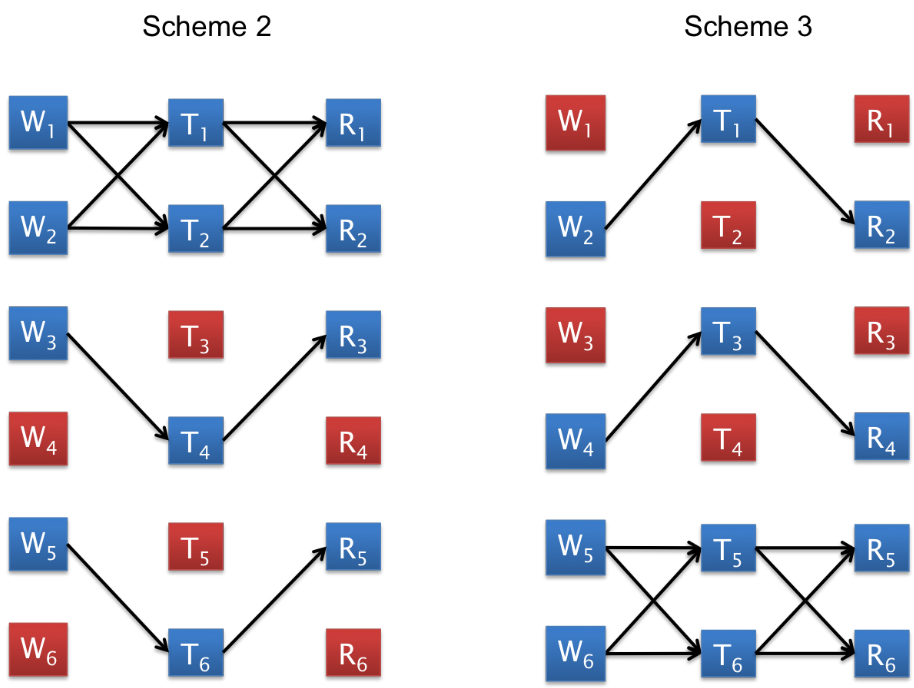


Figure 4.9: Scheme 2 is used when crosslinks from T_3 to R_2, T_5 to R_4, T_1 to R_6 are missing. Scheme 3 is used when crosslinks from T_2 to R_3, T_4 to R_5, T_6 to R_1.

Figure 4.9). When the missing links are $\{2b, 4b, 6a\}$, $\{2b, 5a, 6b\}$, $\{2b, 5a, 6a\}$, $\{3a, 4b, 6b\}$, $\{3a, 4b, 6a\}$, $\{3a, 5a, 6b\}$, we may use the schemes 1.5, 1.3, 1.3, 1.2, 1.6, 1.2 respectively. If the elements of the set $\{1', 2', 3', 4', 5', 6'\}$ are consecutive, assume they are $4', 5', 6'$ and the corresponding missing links belong to one each among $\{1b, 2a\}$, $\{2b, 3a\}$ and $\{3b, 4a\}$. We have eight possibilities among which $\{1b, 2b, 3b\}$, $\{1b, 2b, 4a\}$, $\{1b, 3a, 4a\}$, $\{2a, 2b, 3b\}$, $\{2a, 3a, 3b\}$, $\{2a, 3a, 4a\}$ use scheme 1.5, 1.3, 1.3, 1.6, 1.6, 1.1 and $\{1b, 3a, 3b\}$, $\{2a, 2b, 4a\}$ use scheme 4 and scheme 5 respectively (see Figure 4.10). If only two of them are consecutive, without loss of generality let the consecutive ones be $1', 2'$ and the other can be either $4'$ or $5'$. We may deactivate the common node 2 and the central node among the remaining, which is node 5. This results in two sets containing two nodes such that each set has only one crosslink, so that we can attain a DoF of 2 in each set, with $B = \frac{2}{3}$ in the subnetwork.

If two links are missing, there exist two elements among $\{1', 2', 3', 4', 5', 6'\}$ that have only one crosslink. Suppose they are at a distance two from each other. Let them be $1', 4'$ and deactivate nodes 3, 6, so that we end up with two sets containing two nodes, such that each set has only one crosslink, so that we can attain a DoF of 2 in each set, with $B = \frac{2}{3}$ in the subnetwork. If they have a distance one, let them be $1', 3'$. Then one crosslink each from the sets $\{1b, 2a\}$, $\{3b, 4a\}$ is missing. If $2a$ is missing, scheme 1.1 may be used and if $1b$ is missing then scheme 1.4 may be used. If they are adjacent, let them be $2', 3'$. Then one crosslink each from the sets $\{2b, 3a\}$, $\{3b, 4a\}$ is missing. If $3a$ is missing then we may use scheme 1.1, and if $2b$ is missing, we may use scheme 1.4 to achieve the required DoF under the given constraint.

If one of the links is removed, let the node at transmitter connected to the link be T_4, then using scheme 1.4 we are done. $\square$

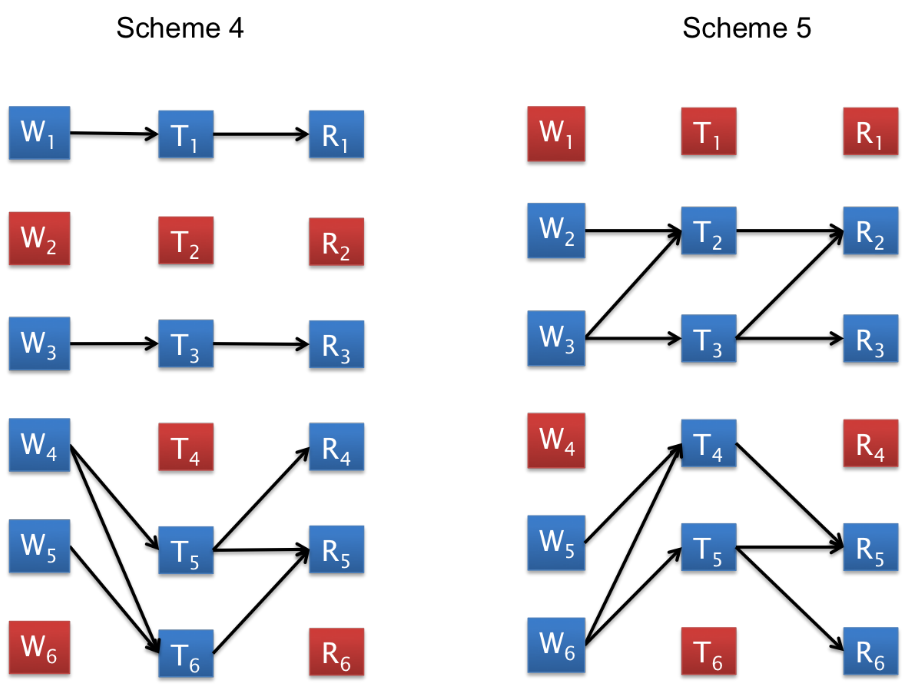


Figure 4.10: Scheme 4 is used when crosslinks from T_1 to R_2, T_3 to R_2, T_3 to R_4 are missing. Scheme 5 is used when crosslinks from T_2 to R_1, T_2 to R_3, T_4 to R_3.

CHAPTER 5

NETWORK WITH INTRA-CELL INTERFERENCE

In this chapter we consider interference between the sectors belonging to the same cell. The interference graph is as shown in Figure 5.1

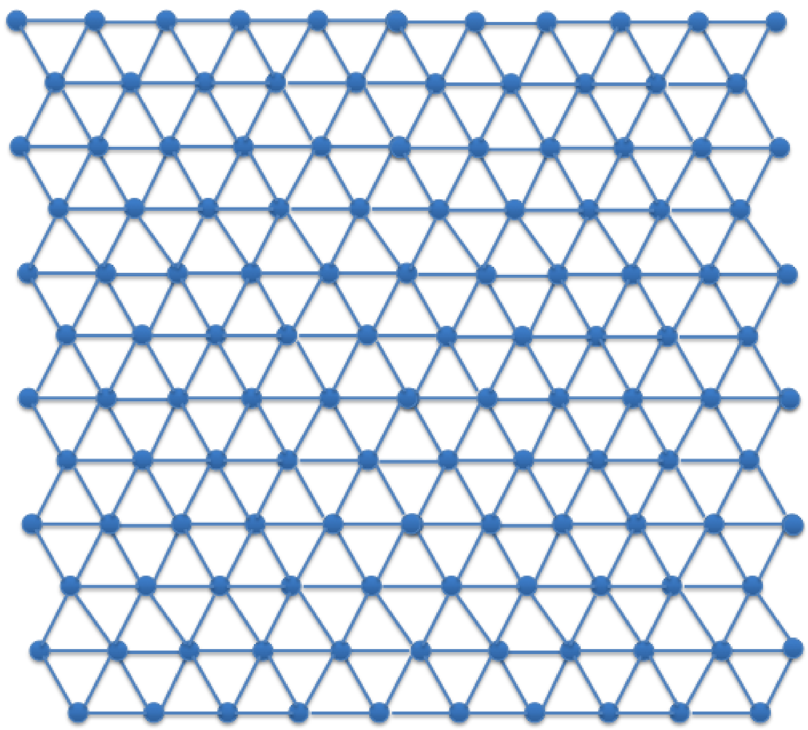


Figure 5.1: Interference graph for the case of intra-cell interference. Note that each sector sees interference from six neighboring sectors.

5.1 Flexible Cell Association

Similar to the case of no intra-cell interference, we first study the case of flexible cell association, by imposing the cooperation order $M = 1$ and characterizing lower and upper bounds for the maximum achievable PUDoF.

Theorem 13. *The following bounds hold for the asymptotic PUDoF, under restriction to orthogonal schemes, with flexible cell association and no cooperation, as the number of users goes to infinity,*

$$\frac{1}{3} \leq \tau(M = 1) \leq \frac{2}{5}.$$

Proof. Lower Bound: Considering a division of the network into triangles, and by deactivating two nodes in each triangle as shown in Figure 5.2, it is easy to see that a PUDoF of $\frac{1}{3}$ is achieved, similar to the case with no intra-cell interference.

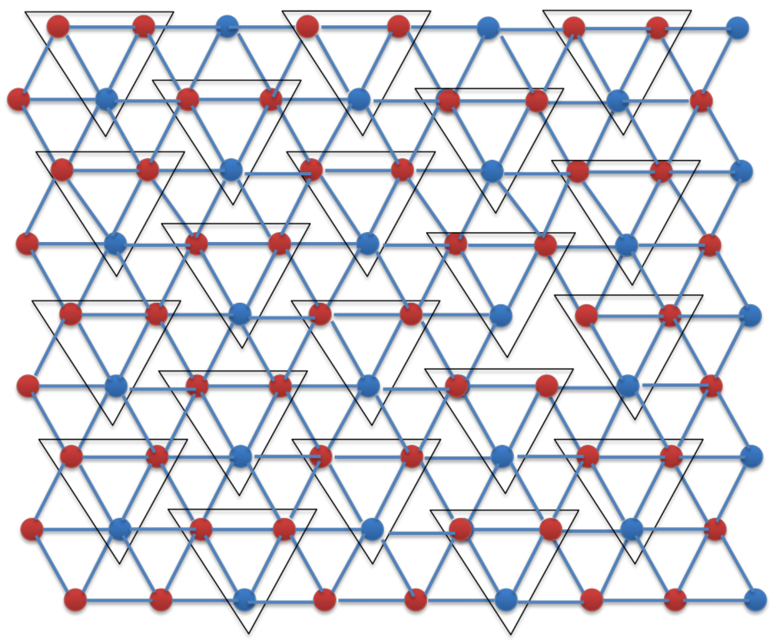


Figure 5.2: Division of network into triangular subnetworks. We note that by deactivating the upper nodes in each triangle, a per user DoF of $\frac{1}{3}$ is achieved.

Upper Bound: Consider the division of the network into disjoint fully connected triangles, as shown in Figure 4.3. For any orthogonal coding scheme, we note that any fully connected triangle in the network is in one of the following states:

State 0 (inactive triangle): All transmitters and receivers in the triangle are inactive.

State 1 (self-serving triangle): Exactly one transmitter in the triangle sends a message to exactly one receiver within the triangle. None of the other transmitters or receivers can be active in this triangle.

State 2 (serving triangle): At least one transmitter in the triangle serves a receiver in another triangle, and there are no active receivers in the triangle.

State 3 (served triangle): At least one receiver in the triangle is being served by a transmitter in another triangle, and there are no active transmitters.

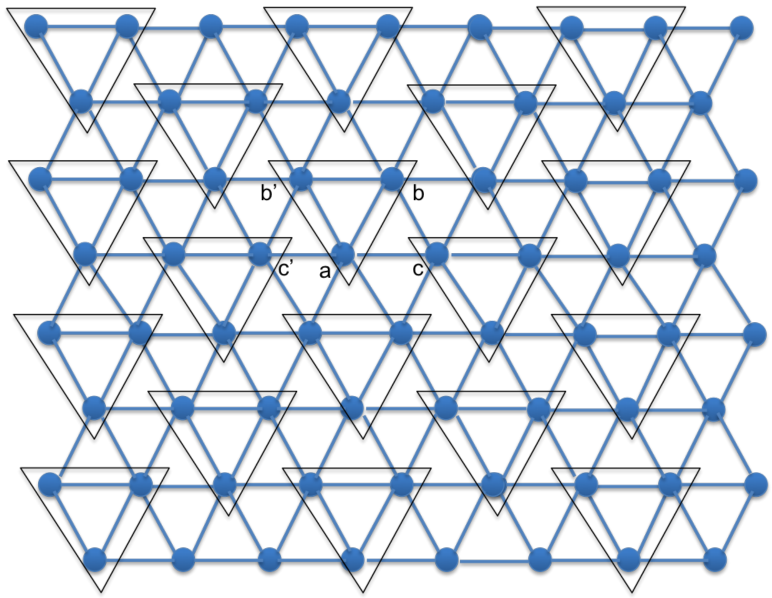


Figure 5.3: Division of the network into triangular subnetworks. Node c in triangle S_c in state 2, serves neighboring receiver b in a different triangle S_b in state 3. One neighboring node a in S_b is inactive due to this transmission, which results in PUDoF of $\frac{2}{5}$.

For the triangles in state 1 and state 0, the number of active receivers is bounded by the number of triangles, i.e., one-third of the number of users.

For every transmitter c that is active in a triangle S_c in state 2, there exists a neighboring receiver b in a different triangle S_b in state 3 that is being served by it. Furthermore, there exists exactly one neighboring node a in either S_b or S_c, whose transmitter and receiver are rendered inactive owing to this transmission.

We now consider the following cases:

Case 1: There is another neighboring transmission (from c' to b') between a triangle containing node a and a neighboring triangle, which requires node a to be inactive. In this case, we have two transmissions among five nodes, giving a PUDoF of $\frac{2}{5}$.

Case 2: No other transmission between a triangle containing a and another triangle results in node a being deactivated. Then we group a, b, c into a triangle of type state 1 with PUDoF of $\frac{1}{3}$.

For any scheme, the network can be rearranged into a combination of disjoint groups of three and five users, such that the PUDoF for each group is at most $\frac{2}{5}$. It follows that $\tau \leq \frac{2}{5}$ holds asymptotically for any choice of cell associations and interference avoidance schemes. □

5.2 Flexible Message Assignment with Cooperation

In this section, we propose interference avoidance schemes that use flexible assignment with cooperative transmission, under a cooperation order M, and use these schemes in an appropriate combination to obtain a scheme that satisfies an average backhaul load B.

5.2.1 Lower Bounds

Theorem 14. *Under the maximum transmit set size constraint $M = 3$, the following lower bound holds for the PUDoF, as the number of users goes to infinity,*

$$\tau(M = 3) \geq \frac{5}{12}.$$

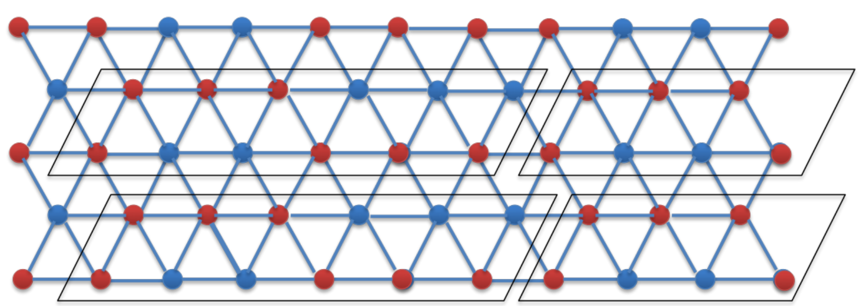


Figure 5.4: Division of cellular network into subnetworks containing twelve nodes each. In each subnetwork, seven nodes are deactivated (red nodes), and using linear beamforming, the remaining seven messages are sent interference-free.

Proof. Consider the division of cellular network into non-interfering subnetworks containing twelve nodes each as shown in Figure 5.4. Within each subnetwork, seven nodes are deactivated in a way that leads to further subdivision into non-interfering blocks of two and three nodes, within each subnetwork. Using zero-forcing linear beamforming, with $M = 2$ in the block of two nodes and $M = 3$ in the block of three nodes, five of the messages are sent interference-free, leading to a PUDoF of $\frac{5}{12}$ with $B = \frac{13}{12}$. $\square$

In Theorem 15, we present an interference avoidance coding scheme that uses a smart message assignment, aided by cooperative transmission, to achieve scalable DoF gains. Note that the lower bound on the achievable PUDoF is greater than the $\frac{2}{5}$ upper bound of the case without cooperation, without the need for extra backhaul load.

Theorem 15. *Under the average backhaul constraint $B = 1$, the following lower bound holds for the PUDoF, as the number of users goes to infinity,*

$$\tau^{avg}(B = 1) \geq \frac{11}{27} > \frac{2}{5}.$$

Proof. Consider the message assignment strategy from Theorem 13 with $B = \frac{1}{3}$ and $\tau \geq \frac{1}{3}$, and the message assignment strategy from Theorem 14 with $B = \frac{13}{12}$ and $\tau \geq \frac{5}{12}$. A convex combination in the ratio $1 : 8$ of these schemes gives $B = 1$ and a PUDoF of $\frac{11}{27}$ for the entire network. $\square$

In the achievable scheme in Theorem 13, we have $B = \frac{1}{3}$ and a PUDoF of $\frac{1}{3}$. Since $B < 1$, we can have a scheme that achieves a greater PUDoF by overloading the network and combining these schemes gives us $\tau(B = 1) > \frac{2}{5}$.

Theorem 16. *Under the average backhaul constraint B, the following lower bound holds for the PUDoF, as the number of users goes to infinity,*

$$\tau^{avg}(B) \geq \frac{2B}{3(B+1)}.$$

Proof. Consider the scenario where we deactivate every third row of nodes in the graph as shown in Figure 5.5. Then we are left with blocks of two rows out of which we deactivate a column (set of two nodes) periodically after B nodes. Considering a repeating block of $2B$ nodes, we see that by linear transmit beamforming with $M = 2B$, a PUDoF of $\frac{2}{3}\frac{B}{B+1}$ can be achieved. $\square$

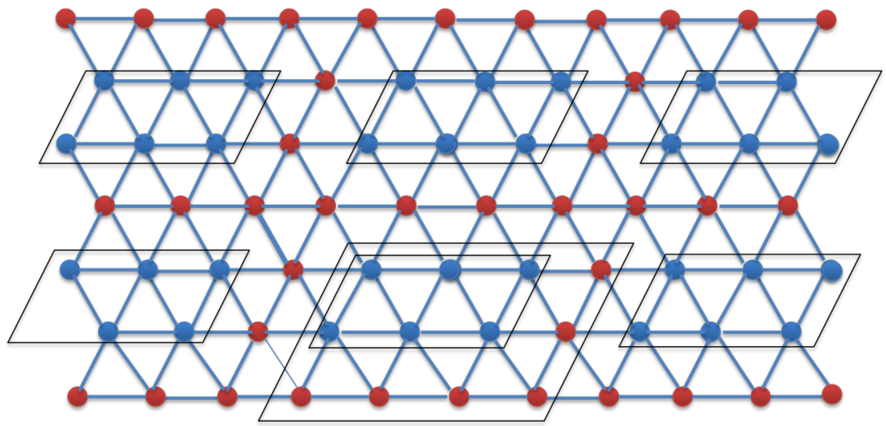


Figure 5.5: Division of the network into subnetworks.

As in the previous chapters, the proposed schemes are one-shot zero-forcing beamforming schemes. In these schemes, some messages are being sent interference-free while few messages not being transmitted. Fairness is maintained by deactivating different sets of receivers in different sessions, e.g., in different time or frequency slots.

Robustness: All the achievable schemes discussed in this chapter involve dividing the network into non-interfering subnetworks, in which all the messages are known at all the transmitters. If some of the interfering cross links are missing, the transmit beams can be designed taking the missing interference links into account, so that the same PUDoF can be achieved, while satisfying the same average backhaul load constraint. Hence, all the achievable schemes discussed in this chapter are topologically robust.

5.2.2 Upper Bound

In order to obtain a converse for $\tau^{\text{avg}}(B)$, we need upper bounds on $\tau(M)$ for higher values of M. As a first step, we derive a bound on $\tau(M = 2)$.

Theorem 17. *Under the maximum transmit set size constraint M, the following lower bound holds for the asymptotic PUDoF, under restriction to interference avoidance schemes,*

$$\tau(M = 2) \leq \frac{14}{19} = 0.73.$$

Proof. Consider two concentric hexagons with a total of 19 nodes as shown in Figure 5.6. The inner hexagon consists of seven nodes, out of which we refer to the one at the center of the hexagon as the central node. For Lemma 3 to hold, $V_{T_i} \leq 2$. We show that a sum DoF greater than 14 cannot be attained among the 19 nodes. If any of the inner hexagon transmitters is active, then at least five receivers must be deactivated and our claim holds. Assume none of the inner hexagon transmitters are active. Then the central receiver cannot be active. If none of the outer hexagon transmitters are active, then the inner hexagon receivers cannot be active and our claim holds. For every outer transmitter that is active, and transmitting to an inner receiver, at least one receiver per transmitter is deactivated, and the sum DoF cannot be larger than 14. In all cases, the sum DoF does not exceed 14 in the concentric hexagons. $\square$

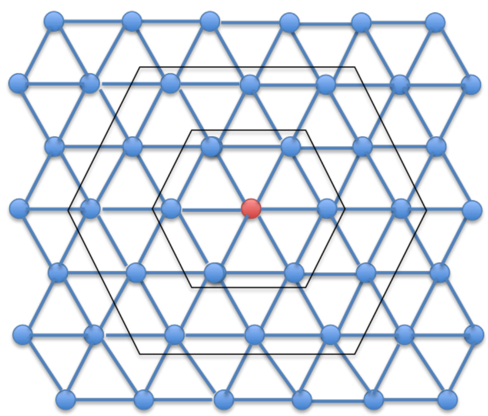


Figure 5.6: Division of the network into hexagonal subnetworks. The central node is colored by red.

CHAPTER 6

CONCLUSION AND FUTURE WORK

Interference management is crucial for meeting the increasing demand for data rates in a cellular network. In this thesis, we focused on interference management using cooperative transmission in the downlink of a cellular network. Using the DoF criterion, we analyzed the capacity gain achieved through cooperative transmission in cellular networks of practical significance.

We studied a two-dimensional hexagonal sectored cellular network under both the assumptions of intra-cell interference and no intra-cell interference. We showed that one-shot linear beamforming schemes can be used to achieve DoF gains in large networks. The proposed schemes rely on a flexible design of the cellular backhaul, which takes into account the topology of the network to make decisions about associating mobile users to transmitters. The schemes use cooperative transmission to cancel interference, through an assignment of messages to transmitters that requires minimal or no extra backhaul capacity. We also discussed the robustness of the schemes, as some of the schemes rely on the interfering links for communication.

In [2], the DoF gains that can be achieved in the uplink was studied using interference alignment and cooperation between base station receivers through the exchange of decoded messages over the backhaul link. We believe that the insights drawn from the results in this thesis can lead to a novel design for interference management schemes for the cellular downlink, while the message passing framework of [2] is used in the uplink.

The results derived in this thesis can be extended to other two-dimensional networks with higher connectivity and also to networks where the nodes have multiple antennas. We need upper bounds on the PUDoF under constraints on the cooperation order M, in order to find upper bounds on the PUDoF under constraints on the average backhaul load B, for all interference avoidance schemes.

Our thesis assumes the availability of perfect channel state information (CSI) at the base station transmitters. The performance of the proposed schemes can be analyzed when perfect CSI is not available at the transmitters and also for practical values of signal-to-noise ratio (SNR). We believe that for practical SNR values, the message assignment of our proposed schemes can be used in combination with precoding schemes that maximize the signal-to-interference noise ratio (SINR) at the receiver, to approach the rates promised by the DoF analysis in this thesis.

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