On D2D Caching with Uncoded Cache Placement

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Abstract

We consider a cache-aided wireless device-to-device (D2D) network under the constraint of *one-shot delivery*, where the placement phase is orchestrated by a central server. We assume that the devices' caches are filled with uncoded data, and the whole file database at the server is made available in the collection of caches. Following this phase, the files requested by the users are serviced by inter-device multicast communication. For such a system setting, we provide the exact characterization of load-memory trade-off, by deriving both the minimum average and the minimum peak sum-loads of links between devices, for a given individual memory size at disposal of each user.

I. INTRODUCTION

The killer application of wireless networks has evolved from real-time voice communication to on-demand multimedia content delivery (e.g., video), which requires a nearly 100-fold increase in the per-user throughput, from tens of kb/s to 1 Mb/s. Luckily, the pre-availability of such content allows for leveraging storage opportunities at users in a proactive manner, thereby reducing the amount of necessary data transmission during periods of high network utilization.

A caching scheme is composed of two phases. The *placement phase* refers to the operation during low network utilization, when users are not requesting any content. During this phase, the

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Rafael F. Schaefer was with the Information Theory and Applications Chair, Technische Universität Berlin, Germany. Currently, he is with the Chair of Communications Engineering and Security, University of Siegen, Germany (email: rafael.schaefer@unisiegen.de). The work of R. F. Schaefer was funded by the German Ministry of Education and Research (BMBF) within the national initiative for "Post Shannon Communication (NewCom)" under Grant 16KIS1004. cache memories of users are filled by a central server proactively. When each user directly stores a subset of bits, the placement phase is uncoded. The transmission stage when users request their desired content is termed *delivery phase*. By utilizing the content stored in their caches during the placement phase, users aim to reconstruct their desired content from the signals they receive. The sources of such signals may differ depending on the context and network topology. In this work, we focus on the device-to-device (D2D) caching scenario, in which the signals available during the delivery phase are generated merely by the users themselves, whereas the central server remains inactive.

A coded caching strategy was proposed by Maddah-Ali and Niesen (MAN) [1]. Their model consists of users with caches and of a server which is in charge of the distribution of content to users through an error-free shared-link, during both the placement and delivery phases. This seminal work showed that a *global caching gain* is possible by utilizing multicasting linear combinations during the delivery phase. By observing that some MAN linear combinations are redundant, the authors [2] proposed an improved scheme, which is optimal under the constraint of uncoded cache placement. It was proved in [3] that the uncoded caching scheme is optimal generally within a factor of 2, e.g. even when more involved (coded) cache placement schemes are allowed.

D2D caching problem was originally considered in [4]–[6], where users are allowed to communicate with each other. By extending the caching scheme in [1] to the D2D scenario, we can also have the global caching gain. It was proved in [4], [5] that the proposed D2D caching scheme is order optimal within a constant when the memory size is not small.

Particularly, the D2D caching setting with uncoded placement considered in this work is closely related to the distributed computing problem originally proposed [7] and distributed data-shuffling problem [8]. The coded distributed computing setting can be interpreted as a symmetric D2D caching setting with multiple requests, whereas the coded data shuffling problem can be viewed as a D2D caching problem with additional constraints on the placement.

Contributions: Based on the D2D achievable caching scheme in [5], with K the number of users and N the number of files, for $N \ge K$ and the shared-link caching scheme in [2] for N < K, we propose a novel achievable scheme for D2D caching problem, which is shown to be order optimal within a factor of 2 under the constraint of uncoded placement, in terms of the average transmitted load for uniform probability of file requests and the worst-case transmitted load among all possible demands.

For each user, if any bit of its demanded file not already in its cache can be recovered from its cache content and a transmitted packet of a single other user, we say that the delivery phase is *one-shot*. Under the constraint of uncoded placement and one-shot delivery, we can divide the D2D caching problem into K shared-link models. Under the above constraints, we then use the index coding acyclic converse bound in [9, Corollary 1] to lower bound the total load transmitted in the K shared-link models. By leveraging the connection among the K shared-link models, we propose a novel way to use the index coding acyclic converse bound compared to the method used for single shared-link model in [2], [10], [11]. With this converse bound, we prove that the proposed achievable scheme is exactly optimal under the constraint of uncoded placement and one-shot delivery, in terms of the average transmitted load and the worst-case transmitted load among all possible demands.

In the longer version of this work [12], we also consider random user inactivity where the identities of inactive users are unknown. The one-shot delivery property allows for an extension of the proposed scheme that provides robustness against outage that user inactivity may lead to.

II. PROBLEM SETTING

A. Notations

 $|\cdot|$ is used to represent the cardinality of a set or the length of a file in bits; we let $\mathcal{A} \setminus \mathcal{B} := \{x \in \mathcal{A} | x \notin \mathcal{B}\}, [a:b:c] := \{a, a+b, a+2b, ..., c\}, [a:c] = [a:1:c] \text{ and } [n] = [1:n];$ the bit-wise XOR operation between binary vectors is indicated by \oplus ; for two integers x and y, we let $\binom{x}{y} = 0$ if x < y or $x \le 0$.

B. D2D Caching Problem Setting

We consider a D2D network composed of K users, which are able to receive all the other users' transmissions. Users make requests from a database of N files $W = (W_1, ..., W_N)$, each with a length of F bits. Every user has a memory of MF bits, where $M \in [N/K, N)$.

The system operation can be divided into the *placement* and *delivery* phases. During the placement phase users have access to a central server. In this work, we only consider the caching problem with uncoded cache placement, where each user k directly stores MF bits of N files in its memory. For the sake of simplicity, we do not repeat this constraint in the rest of paper. Since the placement is uncoded, we can divide each file into subfiles, $W_q = \{W_{q,\mathcal{V}} : \mathcal{V} \subseteq [K]\}$, where $W_{q,\mathcal{V}}$ represents the set of bits exclusively cached by users in \mathcal{V} . We denote the indices

of the stored bits at user k by \mathcal{M}_k . For convenience, we denote the cache placement of the whole system by $\mathcal{M} := (\mathcal{M}_1, \ldots, \mathcal{M}_K)$. During the delivery phase, each user demands one file. We define *demand* vector $d := (d_1, \ldots, d_K)$, with $d_k \in [N]$ denoting user k's requested file index. The set of all possible demands is denoted by \mathcal{D} , so that $\mathcal{D} = [N]^K$. Given the demand information, each user k generates a codeword X_k of length $R_k F$ bits and broadcasts it to other users, where R_k indicates the load of user k. For a given subset of users $S \subseteq [K]$, we let X_S denote the ensemble of codewords broadcasted by these users. From \mathcal{M}_k and $X_{[K]\setminus k}$, each user k recovers its desired file.

In this work we concentrate on the special case of *one-shot delivery*, which we formally define in the following.

Definition 1 (**One-shot delivery**) If each user $k \in [K]$ can decode any bit of its requested file not already in its own cache from its cache and the transmission of a single other user, we say that the delivery phase is one-shot. Mathematically, we indicate by $W_{d_k}^{k,i}$ the block of bits needed by user k and recovered from the transmission of user i, i.e., $H(W_{d_k}^{k,i}|X_i, \mathcal{M}_k) = 0$, indicating that $W_{d_k}^{k,i}$ is a deterministic function of X_i and \mathcal{M}_k . Then, a one-shot scheme implies that $(W_{d_k} \setminus \mathcal{M}_k) \subseteq \bigcup_{i \in [K] \setminus \{k\}} W_{d_k}^{k,i}$. In addition, we also define $W_{d_k,\mathcal{V}}^{k,i}$ as the block of bits needed by user k and recovered from the transmission of user i, which are exclusively cached by users in \mathcal{V} . Hence, we have for each user $k \in [K] \bigcup_{\mathcal{V} \subseteq ([K] \setminus \{k\}):i \in \mathcal{V}} W_{d_k,\mathcal{V}}^{k,i} = W_{d_k}^{k,i}, \forall i \in [K] \setminus \{i\}$.

Letting $R = \sum_{k=1}^{K} R_k$, we say that a communication load R is *achievable* for a demand dand placement \mathcal{M} , with $|\mathcal{M}_k| = \mathcal{M}$, $\forall k \in [K]$, if and only if there exists an ensemble of codewords $X_{[K]}$ of size RF such that each user k can reconstruct its requested file W_{d_k} . We let $R^*(d, \mathcal{M})$ indicate the minimum achievable load given d and \mathcal{M} . We also define $R_0^*(d, \mathcal{M})$ as the minimum achievable load given d and \mathcal{M} under the constraint of one-shot delivery. We consider uniform demand distribution and aim to minimize the average and worst-case loads, $R_{ave}^* = \min_{\mathcal{M}} \mathbb{E}_d[R^*(d, \mathcal{M})]$ and $R_{worst}^* = \min_{\mathcal{M}} \max_d R^*(d, \mathcal{M})$. Similarly, we define $R_{ave, o}^*$ and $R_{worst, o}^*$ as the minimum average and worst-case loads under the constraint of one-shot delivery, respectively.

Further, for a demand d, we let $N_e(d)$ denote the number of distinct indices in d. In addition, we let $d_{\backslash\{k\}}$ and $N_e(d_{\backslash\{k\}})$ stand for the demand vector of users $[K]\backslash\{k\}$ and the number of distinct files requested by all users but user k, respectively. As in [2], [13], we group the demand vectors in \mathcal{D} according to the frequency of common entries that they have. Towards this end,

for a demand d, we stack in a vector of length N the number of appearances of each request in descending order, and denote it by s(d). We refer to this vector as *composition* of d. Clearly, $\sum_{n=1}^{N} s_n(d) = K$. By \mathscr{S} we denote the set of all possible compositions. We denote the set of demand vectors with the same composition $s \in \mathscr{S}$ by \mathcal{D}_s . We refer to these subsets as *types*. Obviously, they are disjoint and $\bigcup_{s \in \mathscr{S}} \mathcal{D}_s = \mathcal{D}$.

III. MAIN RESULTS

In the following theorem, we characterize the exact memory-average load trade-off under the constraint of one-shot delivery. The achievable scheme is introduced in Section IV and the converse bound is proved in Section V.

Theorem 1 (Average load): For a D2D caching scenario with a database of N files and K users each with a cache of size M, the following average load under the constraint of uncoded placement and one-shot delivery with uniform demand distribution, is optimal

$$R_{\text{ave, o}}^{*} = \mathbb{E}_{d} \left[\frac{\binom{K-1}{t} - \frac{1}{K} \sum_{k=1}^{K} \binom{K-1 - N_{e}(d_{\backslash \{k\}})}{t}}{\binom{K-1}{t-1}} \right]$$
(1)

with $t = \frac{KM}{N} \in [K]$, where d is uniformly distributed over $\mathcal{D} = \{1, ..., N\}^K$. Additionally, $R^*_{\text{ave, o}}$ corresponds to the lower convex envelope of its values at $t \in [K]$, when $t \notin [K]$.

We can extend the above results to worst-case load in the following theorem, whose proof can be found in [12].

Theorem 2 (Worst-case load): For a D2D caching scenario with a database of N files and K users each with a cache of size M, the following peak load $R^*_{\text{worst, o}}$ under the constraint of uncoded placement and one-shot delivery, is optimal

$$R_{\text{worst, o}}^{*} = \begin{cases} \frac{\binom{K-1}{t}}{\binom{K-1}{t-1}} & K \leq N \\ \frac{\binom{K-1}{t} - \frac{2N-K}{K} \binom{K-N}{t} - \frac{2(K-N)}{K} \binom{K-1-N}{t}}{K} & \text{otherwise} \\ \frac{\binom{K-1}{t} - \binom{K-1-N}{t}}{\binom{K-1}{t-1}} & K \geq 2N \end{cases}$$
(2)

with $t = \frac{KM}{N} \in [K]$. Additionally, $R^*_{\text{worst, o}}$ corresponds to the lower convex envelope of its values at $t \in [K]$, when $t \notin [K]$.

Remark 1: As we will present in Section IV and discuss in Remark 2, our achievable scheme is in fact composed of K shared-link sub-systems, where each ith sub-system includes $N_e(d) = N_e(d_{\{i\}})$ demanded files. The scheme is symmetric in the file-splitting step in the delivery phase. It is interesting to observe that, even if the system is asymmetric in the sense that each K sharedlink sub-system may not have the same $N_e(\mathbf{d}_{\setminus \{i\}})$, the symmetric file-splitting is nevertheless optimal.

By comparing the achievable load by our proposed scheme and the minimum achievable load for shared-link model, we obtain the following order optimality result (see the longer version [12] of this paper for the proof).

Theorem 3 (Order optimality): For a D2D caching scenario with a database of N files and K users each with a cache size of M, the proposed achievable average and worst-case transmitted loads in (1) and (2), is order optimal within a factor of 2.

IV. A NOVEL ACHIEVABLE D2D CODED CACHING SCHEME

In this section, we present a caching scheme that achieves the loads stated in Theorem 1 and Theorem 2. To this end, we show that for any demand vector d the proposed scheme achieves the load

$$R^{*}(\boldsymbol{d}, \mathcal{M}_{\text{MAN}}) = \frac{\binom{K-1}{t} - \frac{1}{K} \sum_{i=1}^{K} \binom{K-1-N_{e}(\boldsymbol{d}_{\setminus\{i\}})}{t}}{\binom{K-1}{t-1}},$$
(3)

where \mathcal{M}_{MAN} refers to the symmetric placement which was originally presented in [1]. This immediately proves the achievability of the average and worst case loads given in Theorem 1 and Theorem 2, respectively. In Subsection IV-A, we will present our achievable scheme and provide a simple example, illustrating how the idea of exploiting common demands [2] is incorporated in the D2D setting. In Remark 2, we will discuss our approach of decomposing the D2D model into *K* shared-link models.

A. Achievability of $R^*(\boldsymbol{d}, \boldsymbol{\mathcal{M}}_{MAN})$

In the following, we present the proposed caching scheme for integer values of $t \in [K]$. For non-integer values of t, resource sharing schemes [1], [5], [14] can be used to achieve the lower convex envelope of the achievable points.

1) Placement phase: Our placement phase is based on the MAN placement [1], where each file W_q is divided into $\binom{K}{t}$ disjoint sub-files denoted by $W_{q,\mathcal{V}}$ where $\mathcal{V} \subseteq [K]$ and $|\mathcal{V}| = t$. During the placement phase, each user k caches all bits in each sub-file $W_{q,\mathcal{V}}$ if $k \in \mathcal{V}$. As there are $\binom{K-1}{t-1}$ sub-files for each file where $k \in \mathcal{V}$ and each sub-file is composed of $F/\binom{K}{t}$ bits, each user caches NFt/K = MF bits.

2) Delivery phase: The delivery phase starts with the file-splitting step: Each sub-file is divided into t equal length disjoint sub-pieces of $F/t{K \choose t}$ bits which are denoted by $W_{q,\mathcal{V},i}$, where $i \in \mathcal{V}$. Subsequently, each user i selects any subset of $N_e(\mathbf{d}_{\backslash \{i\}})$ users from $[K]\backslash\{i\}$, denoted by $\mathcal{U}^i = \{u_1^i, ..., u_{N_e(\mathbf{d}_{\backslash \{i\}})}^i\}$, which request $N_e(\mathbf{d}_{\backslash \{i\}})$ distinct files. Extending the nomenclature in [2], we refer to these users as *leading demanders of user i*.

Let us now fix a user *i* and consider an arbitrary subset $\mathcal{A}^i \subseteq [K] \setminus \{i\}$ of *t* users. Each user $k \in \mathcal{A}^i$ needs the sub-piece $W_{d_k,\{\mathcal{A}^i \cup \{i\}\} \setminus \{k\},i}$, which is cached by all the other users in \mathcal{A}^i and the user *i*. Precisely, all users in a set \mathcal{A}^i wants to exchange these sub-pieces $W_{d_k,\{\mathcal{A}^i \cup \{i\}\} \setminus \{k\},i}$ from the transmissions of user *i*. By letting user *i* broadcast

$$Y^{i}_{\mathcal{A}^{i}} := \bigoplus_{k \in \mathcal{A}^{i}} W_{d_{k}, \{\mathcal{A}^{i} \cup \{i\}\} \setminus \{k\}, i},$$

$$(4)$$

this sub-piece exchanging can be accomplished, as each user $k \in \mathcal{R}^i$ has all the sub-pieces on the RHS of (4), except for $W_{d_k,\{\mathcal{R}^i \cup \{i\}\} \setminus \{k\},i}$.

We let each user *i* broadcast the binary sums that are useful for at least one of its leading demanders. That is, each user *i* broadcasts all $Y_{\mathcal{A}^i}^i$ for all subsets \mathcal{A}^i that satisfy $\mathcal{A}^i \cap \mathcal{U}^i \neq \emptyset$, i.e. $X_i = \{Y_{\mathcal{A}^i}^i\}_{\mathcal{A}^i \cap \mathcal{U}^i \neq \emptyset}$. For each user $i \in [K]$, the size of the broadcasted codeword amounts to $\binom{K-1}{t} - \binom{K-1-N_e(d_{\{i\}})}{t}$ times the size of a sub-piece, summing which for all $i \in [K]$ results in the load stated in (3).

We now show that each user $k \in [K]$ is able to recover its desired sub-pieces. When k is a leading demander of a user i, i.e., $k \in \mathcal{U}^i$, it can decode any sub-piece $W_{d_k,\mathcal{B}^k \cup \{i\},i}$, for any $\mathcal{B}^k \subseteq \mathcal{R}^i \setminus \{k\}, |\mathcal{B}^k| = t - 1$, from $Y^i_{\mathcal{B}^k \cup \{k\}}$ which is broadcasted from user i, by performing

$$W_{d_k,\mathcal{B}^k\cup\{i\},i} = \left(\bigoplus_{x\in\mathcal{B}^k} W_{d_k,\{\mathcal{B}^k\cup\{i,k\}\}\setminus\{x\},i}\right) \bigoplus Y^i_{\mathcal{B}^k\cup\{k\}}$$
(5)

as can be seen from (4).

However, when $k \notin \mathcal{U}^i$, not all of the corresponding codewords $Y^i_{\mathcal{B}^k \cup \{k\}}$ for its required sub-pieces $W_{d_k, \mathcal{B}^k \cup \{i\}, i}$ are directly broadcasted from user *i*. User *k* can still decode its desired sub-piece by generating the missing codewords based on its received codewords from user *i* (see [12] for the proof).

In the following, we provide a short demonstration of the above presented ideas.

An example: Let us consider the case when N = 2, K = 4, M = 1, t = KM/N = 2 and d = (1, 2, 1, 1). Notice that $N_e(d_{\{2\}}) = 1$ and $N_e(d_{\{i\}}) = 2$ for $i \in \{1, 3, 4\}$. Each file is divided into $\binom{4}{2} = 6$ sub-files and users cache the following sub-files for each $i \in \{1, 2\}$:

$$\mathcal{M}_{1} = \{W_{i,\{1,2\}}, W_{i,\{1,3\}}, W_{i,\{1,4\}}\},\$$

$$\mathcal{M}_{2} = \{W_{i,\{1,2\}}, W_{i,\{2,3\}}, W_{i,\{2,4\}}\},\$$

$$\mathcal{M}_{3} = \{W_{i,\{1,3\}}, W_{i,\{2,3\}}, W_{i,\{3,4\}}\},\$$

$$\mathcal{M}_{4} = \{W_{i,\{1,4\}}, W_{i,\{2,4\}}, W_{i,\{3,4\}}\}.$$

After splitting the sub-files into 2 equal length sub-pieces, users 1, 3, 4 transmit the codewords $X_1 = \{Y_{\{2,3\}}^1, Y_{\{2,4\}}^1, Y_{\{3,4\}}^1\}, X_3 = \{Y_{\{1,2\}}^3, Y_{\{1,4\}}^3, Y_{\{2,4\}}^3\}, X_4 = \{Y_{\{1,2\}}^4, Y_{\{1,3\}}^4, Y_{\{2,3\}}^4\},$ where $Y_{\mathcal{R}^i}^i$ is given by (4).

Notice that for these users, there exists no subset \mathcal{A}^i s.t. $\mathcal{A}^i \subseteq [K] \setminus \{i\}, |\mathcal{A}^i| = t = 2$ which satisfies $\mathcal{U}^i \cap \mathcal{A}^i \neq \emptyset$. However, depending on the choice of \mathcal{U}^2 , user 2 can find $\binom{K-1-N_e(d_{\{2\}})}{t} = 1$ subset \mathcal{A}^2 with $\mathcal{U}^2 \cap \mathcal{A}^2 \neq \emptyset$. Such an \mathcal{A}^2 can be determined as $\{3,4\}, \{1,4\}, \{1,3\}$ for the cases of $\mathcal{U}^2 = \{1\}, \mathcal{U}^2 = \{3\}, \mathcal{U}^2 = \{4\}$, respectively.

Picking user 1 as its leading demander, i.e., $\mathcal{U}^2 = \{1\}$, user 2 only transmits $X_2 = \{Y_{\{1,3\}}^2, Y_{\{1,4\}}^2\}$ sparing the codeword $Y_{\{3,4\}}^2 = W_{1,\{2,3\},2} \oplus W_{1,\{2,4\},2}$. As mentioned before, the choice of the leading demanders is arbitrary and any one of the $Y_{\{1,3\}}^2, Y_{\{1,4\}}^2, Y_{\{3,4\}}^2$ can be determined as the superfluous codeword. In fact, any one of these codewords can be attained by summing the other two, since $Y_{\{1,3\}}^2 \oplus Y_{\{1,4\}}^2 \oplus Y_{\{3,4\}}^2 = 0$.

From the broadcasted codewords, all users can decode all their missing sub-pieces by using the sub-pieces in their caches as side-information, by performing (5). As each sub-piece is composed of $F/t{K \choose t} = F/12$ bits and as $3 \times 3 + 1 \times 2 = 11$ codewords of such size are broadcasted, our scheme achieves a load of 11/12, which could be directly calculated by (3).

Remark 2: Notice that the proposed scheme is in fact composed of K shared-link models each with N files of size F' = F/K bits and K' = K - 1 users with caches of size $M' = \frac{N(t-1)}{(K-1)}$ units each. The corresponding parameter for each model is found to be $t' = \frac{K'M'}{N} = t - 1$. Summing the loads of each $i \in [K]$ shared-link sub-systems ((3) in [2]) with parameters F = F', K = $K', M = M', t = t', N_e(d) = N_e(d_{\backslash \{i\}})$, yields (3).

Remark 3: When each user requests a distinct file $(N_e(d) = K)$, our proposed scheme corresponds to the one presented in [5]. The potential improvement of our scheme when

V. CONVERSE BOUND UNDER THE CONSTRAINT OF ONE-SHOT DELIVERY

In this section we propose the converse bound under the constraint of one-shot delivery given in Theorem 1. Under the constraint of one-shot delivery, we can divide each sub-file $W_{i,\mathcal{V}}$ into sub-pieces. Recall that $W_{d_k,\mathcal{V}}^{k,i}$ represents the bits of W_{d_k} decoded by user k from X_i . Under the constraint of one-shot delivery, we can divide the D2D caching problem into K shared-link models. In the *i*th shared-link model where $i \in [K]$, user *i* transmits X_i such that each user $k \in [K] \setminus \{i\}$ can recover $W_{d_k,\mathcal{V}}^{k,i}$ for all $\mathcal{V} \subseteq ([K] \setminus \{k\})$ where $i \in \mathcal{V}$.

A. Converse Bound for $R_o^*(\mathbf{d}, \mathbf{M})$

Fix a demand vector **d** and a cache placement \mathcal{M} . We first focus on the shared-link model where user $i \in [K]$ broadcasts.

Consider a permutation of $[K] \setminus \{i\}$, denoted by $\mathbf{u} = (u_1, u_2, ..., u_{K-1})$, where user u_1 is in position 1 of \mathbf{u} , user u_2 is in position 2 of \mathbf{u} , etc. We define a function f which maps the vectors \mathbf{u} into another vector $f(\mathbf{u}, \mathbf{d})$ based on the demand vector \mathbf{d} ,

$$f(\mathbf{u}, \mathbf{d}) := (u_j : j \in [K-1] \text{ and } \{j' \in [1 : j-1] : d_{u_{j'}} = d_{u_j}\} = \emptyset).$$

In other words, to obtain $f(\mathbf{u}, \mathbf{d})$, for each demanded file, from the vector \mathbf{u} we remove all the users in \mathbf{u} demanding this file except the user in the lowest position demanding this file. For example, if $\mathbf{u} = (2, 3, 5, 4)$, $\mathbf{d} = (1, 2, 2, 3, 3)$, we have $d_{u_1} = d_{u_2} = 2$ and $d_{u_3} = d_{u_4} = 3$, and thus $f(\mathbf{u}, \mathbf{d}) = (u_1, u_3) = (2, 5)$. It can be seen that $f(\mathbf{u}, \mathbf{d})$ contains $N_e(\mathbf{d}_{\backslash \{k\}})$ elements. Furthermore, we denote the j^{th} element of $f(\mathbf{u}, \mathbf{d})$ by $f_j(\mathbf{u}, \mathbf{d})$. For the permutation \mathbf{u} , we can choose a set of sub-pieces, $(W_{d_{f_j}(\mathbf{u},\mathbf{d})}^{f_j(\mathbf{u},\mathbf{d}),i}: \mathcal{V}_j \subseteq [K] \setminus \{f_1(\mathbf{u}, \mathbf{d}), \dots, f_j(\mathbf{u}, \mathbf{d})\}, i \in \mathcal{V}_j, j \in [N_e(\mathbf{d}_{\backslash \{i\}})]$). By a similar proof as [10, Lemma 1], we have the following lemma.

Lemma 1: For each permutation of $[K] \setminus \{i\}$, denoted by $\mathbf{u} = (u_1, u_2, ..., u_{K-1})$, we have

$$H(X_i) \ge \sum_{j \in [N_{\mathbf{e}}(\boldsymbol{d}_{\backslash \{i\}})]} \sum_{\mathcal{V}_j \subseteq [K] \setminus \{f_1(\mathbf{u}, \mathbf{d}), \dots, f_j(\mathbf{u}, \mathbf{d})\}: i \in \mathcal{V}_j} |W_{d_{f_i}(\mathbf{u}, \mathbf{d}), \mathcal{V}_j}^{f_j(\mathbf{u}, \mathbf{d}), i}|.$$
(6)

Considering all the permutations of $[K] \setminus \{i\}$ and all $i \in [K]$, we sum the inequalities in form of (6) to obtain,

$$(K-1)!(H(X_1)+\ldots+H(X_K)) \ge \sum_{k\in[K]} \sum_{\mathcal{V}\subseteq[K]\setminus\{k\}} \sum_{i\in\mathcal{V}} a_{\mathcal{V}}^{k,i} |W_{d_k,\mathcal{V}}^{k,i}|,$$
(7)

where $a_{\mathcal{V}}^{k,i}$ represents the coefficient of $|W_{d_k,\mathcal{V}}^{k,i}|$ in the sum. In [12], we prove the following lemma.

Lemma 2: $a_{\mathcal{V}}^{k,i_1} = a_{\mathcal{V}}^{k,i_2}$, for each $i_1, i_2 \in \mathcal{V}$. From Lemma 2, we define $a_{\mathcal{V}}^k = \frac{a_{\mathcal{V}}^{k,i}}{(K-1)!}$ for all $i \in \mathcal{V}$. Hence, from (7) we have

$$R_{o}^{*}(\mathbf{d}, \mathcal{M})F \ge \left(H(X_{1}) + \ldots + H(X_{K})\right)$$
(8)

$$\geq \sum_{k \in [K]} \sum_{\mathcal{V} \subseteq [K] \setminus \{k\}} a_{\mathcal{V}}^{k} |W_{d_{k},\mathcal{V}}|$$
(9)

where in (9) we used

$$\sum_{i \in \mathcal{V}} |W_{d_k, \mathcal{V}}^{k, i}| \ge |W_{d_k, \mathcal{V}}|.$$

$$\tag{10}$$

Remark 4: To derive the converse bound under the constraint of uncoded cache placement in [2], [11], the authors consider all the demands and all the permutations and sum the inequalities together. By the symmetry, it can be easily checked that in the summation expression, the coefficient of subfiles known by the same number of users is the same. However, in our problem, notice that (8) and (10) only hold for one demand. So each time we should consider one demand and let the coefficients of $H(X_k)$ where $k \in [K]$ be the same. Meanwhile, for each demand, we also should let the cofficients in Lemma 2 be the same. However, for each demand, the K shared-link models are not the symmetric. If we use the choice of the acyclic sets in [2], [11] for each of the K shared-link models, we cannot ensure that for one demand, the coefficients are the symmetric.

B. Converse Bound for $R^*_{ave, o}$

We focus on a type of demands s. For each demand vector $\mathbf{d} \in \mathcal{D}_s$, we lower bound $R_o^*(\mathbf{d}, \mathcal{M})$ as (9). Considering all the demands in \mathcal{D}_s , we then sum the inequalities in form of (9),

$$\sum_{\mathbf{d}\in\mathcal{D}_{s}}R_{o}^{*}(\mathbf{d},\mathcal{M})F \geq \sum_{q\in[N]}\sum_{\mathcal{V}\subseteq[K]}b_{q,\mathcal{V}}|W_{q,\mathcal{V}}|$$
(11)

where $b_{q,\mathcal{V}}$ represents the coefficient of $|W_{q,\mathcal{V}}|$. By the symmetry, it can be seen that $b_{q_1,\mathcal{V}_1} = b_{q_2,\mathcal{V}_2}$ if $|\mathcal{V}_1| = |\mathcal{V}_2|$. So we let $b_t := b_{q,\mathcal{V}}$ for each $q \in [N]$ and $\mathcal{V} \subseteq [K]$ where $|\mathcal{V}| = t$. Hence, from (11) we get

$$|\mathcal{D}_{s}|F\mathbb{E}_{\boldsymbol{d}\in\mathcal{D}_{s}}[R_{o}^{*}(\boldsymbol{d},\boldsymbol{\mathcal{M}})] = \sum_{\boldsymbol{d}\in\mathcal{D}_{s}}R_{o}^{*}(\boldsymbol{d},\boldsymbol{\mathcal{M}})F \geq \sum_{t\in[0:K]}b_{t}x_{t}$$
(12)

where we define $x_t := \sum_{q \in [N]} \sum_{\mathcal{V} \subseteq [K]: |\mathcal{V}|=t} |W_{q,\mathcal{V}}|$. The value of b_t is found as (see [12])

$$b_{t} = \frac{|\mathcal{D}_{s}| \left(\sum_{i \in [K]} {\binom{K-1}{t}} - {\binom{K-N_{e}(d_{\backslash \{i\}})-1}{t}}\right)}{tN\binom{K}{t}}.$$
(13)

We take (13) into (12) to obtain

$$\mathbb{E}_{\boldsymbol{d}\in\mathcal{D}_{\boldsymbol{s}}}[R_{\boldsymbol{o}}^{*}(\boldsymbol{d},\boldsymbol{\mathcal{M}})] \geq \sum_{t\in[0:K]} \frac{\binom{K-1}{t} - \frac{1}{K}\sum_{i\in[K]}\binom{K-N_{\boldsymbol{e}}(\boldsymbol{d}_{\backslash\{i\}})-1}{t}}{\binom{K-1}{t-1}NF} x_{t}.$$
(14)

We also have the constraint of file size $\sum_{t \in [0:K]} x_t = NF$, and the constraint of cache size $\sum_{t \in [1:K]} tx_t \leq KMF$.

We let $r_{t,s} := \frac{\binom{K-1}{t} - \frac{1}{K} \sum_{i \in [K]} \binom{K-N_e(d \setminus \{i\})^{-1}}{t}}{\binom{K-1}{t-1} NF}$. Similar to [2], we can lower bound (14) using Jensen's inequality and the monotonicity of $\text{Conv}(r_{t,s})$,

$$\mathbb{E}_{\boldsymbol{d}\in\mathcal{D}_{\boldsymbol{s}}}[R_{\boldsymbol{o}}^{*}(\boldsymbol{d},\boldsymbol{\mathcal{M}})] \geq \operatorname{Conv}(r_{t,\boldsymbol{s}}).$$
(15)

So we have

$$\min_{\mathcal{M}} \mathbb{E}_{\boldsymbol{d} \in \mathcal{D}_{\boldsymbol{s}}} [R_{\boldsymbol{o}}^*(\boldsymbol{d}, \mathcal{M})] \ge \min_{\mathcal{M}} \operatorname{Conv}(r_{t, \boldsymbol{s}}) = \operatorname{Conv}(r_{t, \boldsymbol{s}}).$$
(16)

Considering all the demand types and from (16), we have

$$R_{\text{ave, o}}^* \ge \mathbb{E}_s\left[\min_{\mathcal{M}} \mathbb{E}_{\boldsymbol{d} \in \mathcal{D}_s}[R_{\text{o}}^*(\boldsymbol{d}, \mathcal{M})]\right] \ge \mathbb{E}_s[\text{Conv}(r_{t,s})].$$
(17)

Since $r_{t,s}$ is convex, we can change the order of the expectation and the Conv in (17), to obtain Theorem 1. Similarly, we can derive the converse bound on the worst-case load.

VI. NUMERICAL EVALUATIONS AND CONCLUSIONS

We compare the load achieved by the presented one-shot scheme with the achievable load in [5] and with the minimum achievable load for the shared-link model [2]. We also provide the converse bounds in [5], [15].

In this work, we characterized the load-memory trade-off for cache-aided D2D networks under the constraints of uncoded placement and one-shot delivery. We presented a caching scheme and proved its exact optimality in terms of both average and peak loads. The presented scheme is optimal within a factor of 2, when the constraint of one-shot delivery is removed [12].

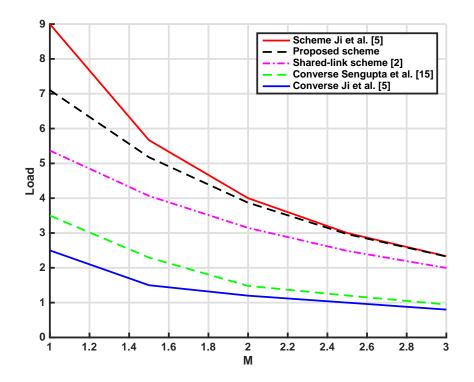


Fig. 1. N = 10, K = 20, worst-case demand

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