

RLS-Based Detection for Massive Spatial Modulation MIMO

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Abstract — Most detection algorithms in spatial modulation (SM) are formulated as linear regression via the regularized least-squares (RLS) method. In this method, the transmit signal is estimated by minimizing the residual sum of squares penalized with some regularization. This paper studies the asymptotic performance of a generic RLS-based detection algorithm employed for recovery of SM signals. We derive analytically the asymptotic average mean squared error and the error rate for the class of *bi-unitarily invariant* channel matrices.

The analytic results are employed to study the performance of SM detection via the box-LASSO. The analysis demonstrates that the performance characterization for i.i.d. Gaussian channel matrices is valid for matrices with non-Gaussian entries, as well. This justifies the partially approved conjecture given in [1]. The derivations further extend the former studies to scenarios with non-i.i.d. channel matrices. Numerical investigations validate the analysis, even for practical system dimensions.

I. INTRODUCTION

Recovery algorithms based on the regularized least-squares (RLS) regression method are doubtless among the most studied schemes for ill-posed signal recovery in linear models. Examples of such algorithms are the least absolute shrinkage and selection operator (LASSO) [2] and Tikhonov regularization method [3] which are used in various applications, e.g. sparse recovery [4]. The main task in these applications is to recover a signal \mathbf{x} from an underdetermined set of linear and possibly noisy observations \mathbf{y} . The RLS-based algorithms solve this problem by minimizing the residual sum of squares (RSS) penalized via a regularization term, i.e. $\|\mathbf{H}\mathbf{x} - \mathbf{y}\|^2 + f(\mathbf{x})$ for some penalty $f(\cdot)$ and the known projection \mathbf{H} .

This study investigates the characteristics of a generic class of RLS-based algorithms by considering their applications to the spatial modulation (SM) technique recently proposed for multiple-input multiple-output (MIMO) transmission with restricted hardware complexity [5], [6]. For scenarios in which the number of active transmit antennas is smaller than the total number of available antenna elements, SM utilizes the sparsity of the transmit signal to convey information via both the non-zero symbols and the support of the transmit signal. To this end, the information bits are divided into two subsets. One subset is used to select a unique subset of transmit antennas which are set active during transmission, and the other subset is transmitted over these active antennas via a conventional

modulation technique, e.g. phase shift keying (PSK). At the receive side, the receiver needs to jointly recover the index of the active antennas, and the transmitted symbols.

Following analogy between the detection task in SM and compressive sensing [4], [7], several detection schemes were proposed in the literature based on sparse recovery techniques, e.g. [1], [8], [9]. These schemes often lie in the class of RLS-based recovery algorithms; hence, their performance is characterized by studying these recovery algorithms.

Contributions and Related Work

Recent developments in multi-antenna technologies suggest the utilization of massive MIMO settings in the next generation of mobile networks [10]. In this respect, we characterize the RLS-based recovery algorithms in the large-system limit. The analysis differs from earlier studies on mathematically similar models, e.g. [11], in the fact that the *SM signals are not independent and identically distributed (i.i.d.)* in general. We address this via a conditional form of the *asymmetric decoupling property* derived for RLS recovery in [12]. Closed-form expressions for the average mean squared error (MSE) and error rate in a massive multiuser multiple-active spatial modulation (MA-SM) MIMO system are derived.

Our derivations extend available results in the literature in various respects. We demonstrate this by studying the example of box-LASSO recovery. For this example, our results extend the earlier derivations for i.i.d. Gaussian channel matrices in [1] to the class of *bi-unitarily invariant* random matrices. Our investigations further show that the asymptotic distortion is of the same form for all i.i.d. channel matrices. This justifies the universality conjecture which was partially approved in [1].

Notations

We represent scalars, vectors and matrices with non-bold, bold lower case and bold upper case letters, respectively. \mathbf{I}_K denotes the $K \times K$ identity matrix. \mathbf{H}^T and \mathbf{H}^H are the transposed and transposed conjugate of \mathbf{H} . The real axes and its non-negative subset are denoted by \mathbb{R} and \mathbb{R}_0^+ , respectively. The complex plane is shown by \mathbb{C} . We use $\|\cdot\|$ and $\|\cdot\|_1$ to show the ℓ_2 - and ℓ_1 -norm, respectively. $\|\mathbf{x}\|_0$ denotes the “ ℓ_0 -norm” of \mathbf{x} defined as the number of non-zero entries. For random variable x , the probability distribution is shown by $p(x)$. $\mathbb{E}\{\cdot\}$ is the expectation operator. We use the shortened notation $[N]$ to represent $\{1, \dots, N\}$.

II. PROBLEM FORMULATION

Consider uplink transmission in a Gaussian multiple access MIMO channel with K transmitters and a single receiver. Each transmitter is equipped with M_u antennas and L_u radio frequency (RF) chains. Hence, in each channel use, only L_u transmit antennas are active at transmit terminals. We denote the fraction of active antennas by $\eta = L_u/M_u$.

At the receive side, an antenna array of size N is employed. Hence, the receive vector $\mathbf{y} \in \mathbb{C}^N$ reads

$$\mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{n}. \quad (1)$$

Here, \mathbf{x} represents the vector of transmit signals, i.e.

$$\mathbf{x} = [\mathbf{x}_1^\top, \dots, \mathbf{x}_K^\top]^\top \quad (2)$$

where $\mathbf{x}_k \in \mathbb{C}^{M_u}$ is the signal transmitted by terminal k . As a result, \mathbf{x} is of size $M = KM_u$. $\mathbf{H} \in \mathbb{C}^{M \times N}$ denotes the channel matrix. It is assumed that the channel is estimated prior to data transmission, and hence \mathbf{H} is known at the receiver. $\mathbf{n} \in \mathbb{C}^N$ represents noise and reads $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$.

A. Channel Model

The channel is assumed to experience frequency-flat fading with slow time variations. We consider a generic fading model in which \mathbf{H} is a *bi-unitarily invariant* random matrix. This means that for any pair of unitary matrices $\mathbf{U} \in \mathbb{C}^{N \times N}$ and $\mathbf{V} \in \mathbb{C}^{M \times M}$, which are independent of \mathbf{H} , the joint distribution of the entries of \mathbf{H} is identical to that of $\mathbf{U}\mathbf{H}\mathbf{V}$ [13].

This ensemble comprises a variety of fading models including the well-known i.i.d. Rayleigh fading model.

B. Spatial Modulation

Let \mathbf{d}_k denote a sequence of information bits at terminal k . Without loss of generality, let \mathbf{d}_k be an i.i.d. binary sequence with uniform distribution. Assume that the active antennas at each terminal transmit symbols from alphabet \mathbb{S} which contains 2^S symbols. \mathbf{x}_k is then constructed as follows:

- 1) *Assigning modulation indices*: Consider all possible subsets of L_u transmit antennas selected out of M_u available antennas at terminal k . The transmitter selects 2^I distinct subsets randomly and uniformly, where

$$I = \left\lceil \log_2 \binom{M_u}{L_u} \right\rceil. \quad (3)$$

To each of these subsets a *modulation index* is assigned.

- 2) *Index modulation*: For given sequence \mathbf{d}_k , the transmitter chooses index $i_k \in [2^I]$, such that the first I bits of \mathbf{d}_k be the binary representation of i_k . We denote the subset of L_u antenna indices corresponding to i_k with $\mathbb{L}(i_k)$.
- 3) *Modulating multiple streams*: Terminal k considers L_u independent blocks of \mathbf{d}_k , each of length S , and maps them into the symbols $s_k(m) \in \mathbb{S}$ for $m \in \mathbb{L}(i_k)$, using a standard modulation scheme, e.g. PSK. These symbols are then transmitted on the antennas corresponding to i_k .

Considering the above SM scheme, the m -th transmit signal entry of terminal k , i.e. $x_{k,m}$ for $m \in [M_u]$, reads $x_{k,m} = s_k(m) \mathbf{1}\{m \in \mathbb{L}(i_k)\}$, where $\mathbf{1}\{m \in \mathbb{L}(i_k)\}$ is the indicator

function, returning one, if $m \in \mathbb{L}(i_k)$ and zero, if $m \notin \mathbb{L}(i_k)$. Hence, \mathbf{x}_k is an L_u -sparse vector, i.e. only L_u entries are non-zero. As a result, \mathbf{x} is L -sparse, where $L = KL_u$.

We define the *activity factor* as $\text{AF} := \|\mathbf{x}\|_0/M$. This factor describes the total fraction of active transmit antennas in the network. Noting that $M = KM_u$, we have $\text{AF} = \eta$.

C. RLS-Based Detection Algorithms

For data recovery, the receiver requires to detect both the support of \mathbf{x} and the transmitted symbols from \mathbf{y} . To this end, an RLS-based algorithm is employed. This algorithm determines a *soft estimation* of \mathbf{x} , for given \mathbf{H} , as

$$\mathbf{x}^* := \underset{\mathbf{v} \in \mathbb{X}_0^M}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{v}\|^2 + f_{\text{reg}}(\mathbf{v}), \quad (4)$$

where $f_{\text{reg}}(\cdot)$ is a regularization function, and \mathbb{X} is a subset of \mathbb{C} including the alphabet \mathbb{S} , i.e. $\mathbb{S} \subseteq \mathbb{X} \subseteq \mathbb{C}$. The notation $\mathbb{X}_0 := \{0\} \cup \mathbb{X}$ is further defined for brevity.

Given \mathbf{x}^* , the detected vector is then given by mapping the soft estimation into a vector in \mathbb{S}_0^M . This means $\hat{\mathbf{x}} = f_{\text{dec}}(\mathbf{x}^*)$ where $f_{\text{dec}}(\cdot) : \mathbb{X}_0^M \mapsto \mathbb{S}_0^M$ is a decisioning function.

Special Cases: The RLS-based recovery schemes recover various MA-SM detection algorithms. As an example, let $\mathbb{X} = \mathbb{S}$ and $f_{\text{reg}}(\mathbf{x}) = -\sigma^2 \ln p(\mathbf{v})$ with $p(\cdot)$ denoting the prior distribution of \mathbf{x} . Moreover, set $f_{\text{dec}}(\mathbf{x}) = \mathbf{x}$. The algorithm in this case reduces to the optimal Bayesian detection scheme, i.e. *maximum-a-posterior (MAP)*.

Noting that dimensions are rather large in massive MIMO systems, the non-convex choices of \mathbb{X}_0 and $f_{\text{reg}}(\mathbf{x})$ result in computationally intractable detection algorithms. As a result, convex forms are considered in practice. A well-known example is the so-called *box-LASSO*. In box-LASSO, \mathbb{X}_0 is set to a convex subset of the complex plane which contains the symbols in \mathbb{S}_0 . $f_{\text{reg}}(\mathbf{x})$ is further chosen proportional to the ℓ_1 -norm following the regularization approach proposed by Tibshirani in [2]. In this case, the soft estimation comprises symbols from \mathbb{X}_0 . $f_{\text{dec}}(\cdot)$ is hence set to be an entry-wise thresholding operator which maps each estimated entry into either zero or a transmit symbol in \mathbb{S} .

D. Performance Metrics

Using the MA-SM scheme, each terminal transmits $I + L_u S$ information bits per channel use. Comparing to conventional modulation schemes, the spectral efficiency in this case is increased by I bits per channel use. This is acquired at the expense of diversity. In fact, the index modulation in MA-SM performs as a random selection algorithm which reduces the diversity gain, compared to other selection techniques.

In order to characterize the performance of MA-SM transmission, a distortion metric is further required. The common metric is the *average error rate* defined as the probability of bit flip averaged over the transmit block size, i.e. M .

Definition 1 (Average error rate): Let $\hat{\mathbf{x}}$ denote the vector of detected signals. The average error rate is defined as

$$\bar{P}_E(M) := \frac{1}{M} \sum_{m=1}^M \Pr\{\hat{x}_m \neq x_m\}. \quad (5)$$

For RLS-based detectors, the distortion can also be defined with respect to the soft estimation. In this respect, we further consider the average MSE as another distortion metric.

Definition 2 (Average MSE): Consider the soft estimation \mathbf{x}^* . The average MSE is defined as

$$\text{MSE}(M) := \frac{1}{M} \mathbb{E} \|\mathbf{x}^* - \mathbf{x}_m\|^2. \quad (6)$$

E. The Large-System Limit

We aim to determine the asymptotic limit of the distortion metrics. In this respect, for bounded and fixed L_u and M_u , we consider a sequence of settings with N receive antennas and K transmit terminals. We assume that K is a deterministic sequence of N , such that

$$\alpha = \lim_{N \uparrow \infty} \frac{K}{N} \quad (7)$$

is bounded. As a result, the sequence of *channel loads*, defined as $\xi := N/M$, converges to $\xi = 1/\alpha M_u$, as $N \rightarrow \infty$. The activity factor of this sequence of settings is constant in N and reads $\text{AF} = \eta$ for all choices of N .

For a given setting with N receive antennas, we define $\mathbf{F}_{\mathbf{J}}^N$ to be the empirical cumulative distribution of the eigenvalues of $\mathbf{J} = \mathbf{H}^H \mathbf{H}$. It is assumed that the sequence of $\mathbf{F}_{\mathbf{J}}^N$ has a deterministic limit $\mathbf{F}_{\mathbf{J}}$, when N grows large. The Stieltjes transform of $\mathbf{F}_{\mathbf{J}}$ is given by

$$G(z) = \int \frac{d\mathbf{F}_{\mathbf{J}}(\lambda)}{\lambda - z} \quad (8)$$

for some z in the upper half complex plane. The R-transform is further defined as $R(\omega) = G^{-1}(-\omega) - 1/\omega$ for some $\omega \in \mathbb{C}$, such that $R(0) = \int \lambda d\mathbf{F}_{\mathbf{J}}(\lambda)$. Here, $G^{-1}(\cdot)$ denotes the inverse with respect to composition.

III. MAIN RESULTS

The analytic derivations in this section follows the results given in [12] via the replica method. The consistency of the results relies on the validity of some conjectures, such as replica continuity and replica symmetry. Although these conjectures lack mathematical proofs, several studies have confirmed the consistency for some particular examples, e.g. [14].

We state the main results using the *decoupled setting*. This is a tunable scalar setting whose distortion metrics are analytically calculated for all tuning parameters. It is shown that for a specific tuple of the tuning parameters, the distortion metrics of this setting give the large-system limits of the average error rate and MSE.

Definition 3 (Decoupled setting): For given c and q , define

$$\tau(c) := \frac{1}{R(-c)} \quad (9a)$$

$$\theta(c, q) := \frac{1}{R(-c)} \sqrt{\frac{\partial}{\partial c} [(\sigma^2 c - q) R(-c)]}. \quad (9b)$$

Let $\mathbf{x} = \psi \mathbf{s}$, where \mathbf{s} is uniformly distributed on \mathbb{S} , and ψ is a Bernoulli random variable for which $\Pr\{\psi = 1\} = 1 - \Pr\{\psi = 0\} = \eta$. Define the decoupled output $\mathbf{y}(c, q)$ as

$$\mathbf{y}(c, q) = \mathbf{x} + \theta(c, q) \mathbf{z}$$

with $\mathbf{z} \sim \mathcal{CN}(0, 1)$. Then, the decoupled soft estimation is

$$\mathbf{x}^*(c, q) := \underset{\mathbf{v} \in \mathbb{X}_0}{\operatorname{argmin}} \frac{1}{\tau(c)} |\mathbf{y}(c, q) - \mathbf{v}|^2 + f_{\text{reg}}(\mathbf{v}). \quad (10)$$

The decoupled detected symbol is moreover defined in terms of \mathbf{x}^* as $\hat{\mathbf{x}}(c, q) := f_{\text{dec}}(\mathbf{x}^*(c, q))$. For this setting, the error probability reads $Q_E(c, q) = \Pr\{\hat{\mathbf{x}}(c, q) \neq \mathbf{x}\}$ and the MSE is given by $\Gamma(c, q) = \mathbb{E}\{|\mathbf{x}^*(c, q) - \mathbf{x}|^2\}$.

Unlike $\bar{P}_E(M)$ and $\text{MSE}(M)$, the derivation of $Q_E(c, q)$ and $\Gamma(c, q)$ deals with a scalar optimization problem which is tractable for any \mathbb{X} and $f_{\text{reg}}(\cdot)$. Proposition 1 indicates that for specific choices of c and q , the average error rate and the average MSE are given by the error probability and the MSE of the decoupled setting, respectively. The values of c and q , for which this equivalency happens, is given in the proposition via a system of fixed-point equations.

Proposition 1: Consider the sequence of settings illustrated in Section II-E. As M grows large, we have

$$\lim_{M \uparrow \infty} \bar{P}_E(M) = Q_E(c^*, q^*) \quad (11)$$

and

$$\lim_{M \uparrow \infty} \text{MSE}(M) = \Gamma(c^*, q^*) \quad (12)$$

where c^* and q^* are solutions to the fixed-point equations

$$c \theta(c, q) = \tau(c) \mathbb{E}\{\mathbb{R}_E\{(\mathbf{x}^*(c, q) - \mathbf{x}) \mathbf{z}^*\}\} \quad (13a)$$

$$q = \mathbb{E}\{|\mathbf{x}^*(c, q) - \mathbf{x}|^2\}. \quad (13b)$$

Sketch of the Proof. The proof follows the *asymmetric* form of the *decoupling property* derived for RLS recovery in [12]. To start with the proof, let $\mathbf{i} = [i_1, \dots, i_K]^T$ be the vector of modulation indices for a setting with transmit dimension M . For a given distortion function $F_D(\cdot; \cdot) : \mathbb{X}_0 \times \mathbb{S}_0 \mapsto \mathbb{R}_0^+$, define the conditional average distortion as

$$D_M(\mathbf{i}) := \frac{1}{M} \sum_{m=1}^M \mathbb{E}\{F_D(\mathbf{x}_m^*; x_m) | \mathbf{i}\}. \quad (14)$$

For a given \mathbf{i} , let $\text{Supp}(\mathbf{i}) = \{m \in [M] : x_m \neq 0\}$ contain the indices of the non-zero entries in \mathbf{x} . It is then straightforward to conclude that conditioned to \mathbf{i} , the entries x_m for $m \in \text{Supp}(\mathbf{i})$ are independent and uniformly distributed on \mathbb{S} . As a result, the conditional distribution $p(\mathbf{x} | \mathbf{i})$ reads

$$p(\mathbf{x} | \mathbf{i}) = \prod_{m \in \text{Supp}(\mathbf{i})} 2^{-S} \prod_{m \notin \text{Supp}(\mathbf{i})} \mathbf{1}\{x_m = 0\}. \quad (15)$$

This distribution follows the asymmetric signal model that considered in [12] with two blocks of i.i.d. sequences.

By standard derivations, it is concluded from the asymptotic decoupling property in [12] that for $m \in \text{Supp}(\mathbf{i})$ and $m \notin$

Supp (i), $p(x_m^* | x_m, \mathbf{i})$ converges to $p(x^*(c^*, q^*) | x = s)$ and $p(x^*(c^*, q^*) | x = 0)$, respectively, as N tends to ∞ . Here, $p(x^*(c^*, q^*) | x = u)$ denotes the conditional distribution of the decoupled soft estimation defined in (10), for $c = c^*$ and $q = q^*$, given that the decoupled input is set to $x = u$. s is moreover distributed uniformly over \mathbb{S} . Substituting into (14), after some lines of calculations, we have

$$\lim_{M \uparrow \infty} D_M(\mathbf{i}) = \eta \mathbb{E}_s \{ \mathbb{E} \{ F_D(x^*(c^*, q^*); s) | x = s \} \} + (1 - \eta) \mathbb{E} \{ F_D(x^*(c^*, q^*); 0) | x = 0 \}. \quad (16)$$

Noting that the limit in (16) is constant in \mathbf{i} , we finally have

$$\lim_{M \uparrow \infty} D_M = \lim_{M \uparrow \infty} D_M(\mathbf{i}), \quad (17)$$

where $D_M := \mathbb{E}_{\mathbf{i}} \{ D_M(\mathbf{i}) \}$.

By setting the distortion function to $F_D(x^*; x) = |x^* - x|^2$ and $F_D(x^*; x) = \mathbf{1} \{ f_{\text{dec}}(x^*) \neq x \}$, the asymptotic average MSE and error rate are derived from (16), respectively. More details are given in the extended version of the paper. \square

IV. A CLASSIC APPLICATION OF THE RESULTS

The asymptotic characterization of the RLS-based detection schemes enables us investigating MA-SM in various aspects. In this section, we discuss a particular application; namely, we study a *box-LASSO* detector. This special form was investigated earlier in [1], where its asymptotic performance was characterized analytically for i.i.d. Gaussian channel matrices. The analysis in this section hence extends the derivations in [1] in several aspects, e.g. channel model.

Applications of the results are not restricted to this particular example. For instance, the asymptotic characterizations can be utilized to derive theoretical bounds on the performance of MA-SM transmission. Due to page limitation, we skip further applications and present them later in an extended version.

A. Box-LASSO Detection for SSK Transmission

We consider the following special case of the setting:

- 1) $\mathbb{S} = \{\sqrt{P}\}$ for some power P . This equivalently means $S = 0$. This is an extreme case of SM, known as space shift keying (SSK), in which the information symbols are completely conveyed via index modulation.
- 2) The RLS-based detector has the following specifications:
 - The regularization function is set to $f_{\text{reg}}(\mathbf{v}) = \lambda \|\mathbf{v}\|_1$ for some *regularization parameter* λ which is tunable.
 - $\mathbb{X} = [-\ell, u]$ for some $\ell \geq 0$ and $u \geq \sqrt{P}$.
 - The decisioning function is $f_{\text{dec}}(x) = \sqrt{P} \mathbf{1} \{ x \geq \epsilon \}$ for some *threshold* ϵ .

This setting describes box-LASSO detection for SSK signaling. The detector, in this case, relaxes the optimal Bayesian detector by approximating the exponent of the transmit signal's prior distribution with the ℓ_1 -norm and convexifying the set $\mathbb{S}_0 = \{0, \sqrt{P}\}$ with the interval $\mathbb{X}_0 = [-\ell, u]$.

The asymptotic performance of this particular setting was investigated in [1] for i.i.d. Gaussian channel matrices. Based

on simulations and universality results¹, the authors conjectured that the analysis is further valid for *non-Gaussian* i.i.d. channel matrices. This conjecture was partially approved in [1] using the Lindeberg principle.

Invoking our asymptotic derivations, the conjecture in [1] is straightforwardly justified. In fact, from Proposition 1, it is observed that the channel matrix plays rule in the asymptotic characterization via the limiting distribution F_J . From random matrix theory, we know that this distribution is the same for all i.i.d. channel matrices and follows the Marcenko-Pastur law [13]. It is hence concluded that the asymptotic characterizations of the performance for i.i.d. Gaussian matrices extends to i.i.d. matrices with other entry distributions, as well.

Remark 1: The asymptotic results in this paper are given for bi-unitarily invariant random matrices. Hence, the analysis not only justifies the conjecture in [1], but also extends the results beyond the i.i.d. matrices. For non-i.i.d. matrices, it is obvious from Proposition 1, that the derivations in [1] are not valid anymore. The performance in such cases is straightforwardly characterized via Proposition 1.

B. Decoupled Setting of the Box-LASSO Detector

For the box-LASSO detector, the decoupled input reads $x = \sqrt{P}\psi$ with ψ being a Bernoulli random variable described in Definition 3. The decoupled soft estimation is further given by

$$x^*(c, q) = \begin{cases} u & y(c, q) \geq \theta + u \\ y(c, q) - \theta & \theta \leq y(c, q) \leq \theta + u \\ 0 & -\theta \leq y(c, q) \leq \theta \\ y(c, q) + \theta & -\theta - \ell \leq y(c, q) \leq -\theta \\ -\ell & y(c, q) \leq -\theta - \ell \end{cases} \quad (18)$$

where $\theta := \tau(c) \lambda / 2$ and $y(c, q) = \sqrt{P}\psi + \theta(c, q)z$.

The asymptotic values for the MSE and error rate are derived by substituting (18) into the fixed point equations, given in Proposition 1, and calculating c^* and q^* .

Noting that z is a zero-mean unit-variance complex Gaussian random variable, the expectations on the right hand side of (13a) and (13b) are analytically derived for a given pair of c and q as sums of Gaussian integrals. Hence, they are straightforwardly calculated and replaced into the fixed-point equations. The resulting equations are then solved numerically.²

C. Numerical Results

The analytic derivations are validated via numerical investigations considering the example of box-LASSO detection. To this end, we consider a scenario with $K = 20$ transmit terminals, each equipped with $M_u = 8$ antennas and a single RF chain, i.e. $L_u = 1$. Consequently, $\eta = L_u / M_u = 0.125$. Using the single RF-chain, each terminal transmits three information bits in each channel use by SSK signaling. We also set $P = 1$.

¹See [15] for some results on universality.

²An alternative approach is to iteratively find the stability point of the corresponding *replica simulator*; see [11] for detailed discussions.

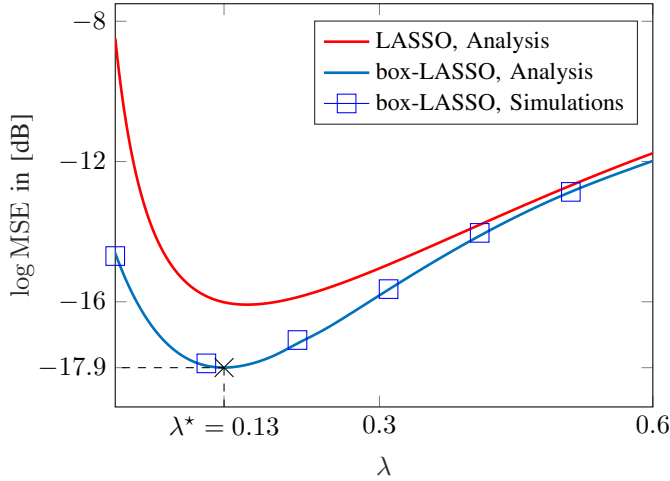


Fig. 1: MSE vs. λ . The simulations closely track analytical results. For box-LASSO detection, the MSE is minimized at $\lambda^* = 0.13$.

At the receiver, an antenna array of size $N = 80$ is employed. Thus, $\alpha = K/N = 0.25$, and the channel load is $\xi = 1/M_u \alpha = 0.5$. For signal recovery, a box-LASSO detector is used in which $\ell = 0$ and $u = \sqrt{P} = 1$. The threshold in the decisioning function is set to $\epsilon = 0.5$.

The analytical results are given for an i.i.d. channel matrix whose entries are zero-mean with variance $1/M$.³ In this case, we have $R(c) = \xi/(1-c)$ [13]. The simulations are given for Rayleigh fading model in which the channel entries are Gaussian. We set the noise variance with respect to the signal-to-noise ratio (SNR) which is defined as $\text{SNR} = P/\sigma^2$.

Fig. 1 and 2 depict the average MSE and the error rate, achieved by the box-LASSO detector, against regularization parameter λ , when $\log \text{SNR} = 14$ dB. For sake of comparison, the plots for *standard LASSO* are further given. By standard LASSO, we mean the extreme case of box-LASSO detection in which $u, \ell \uparrow \infty$. The solid lines in these figures indicate the analytic results given by Proposition 1. The squares are given by numerical simulations with 1000 realizations. As the figures depict the simulated points closely track the large-system characterization. This observation validates the consistency of the analytic results in practical dimensions.

As Fig. 1 and 2 demonstrate, for a given SNR, the performance is optimized at some λ^* . This value is analytically found via Proposition 1, as Fig. 3 illustrates. In this figure, λ^* is plotted against the SNR. At each SNR, λ^* is found such that the asymptotic MSE, derived via Proposition 1, is minimized. The regularization parameter in the box-LASSO detector is then set to λ^* and the achieved MSE is plotted against the SNR. As the figure depicts, numerical simulations are tightly consistent with the analytic derivations. This implies that the analytic derivations provide an easy and fast tool for efficient tuning of RLS-based detectors, even in practical dimensions.

³The entries can have an arbitrary distribution.

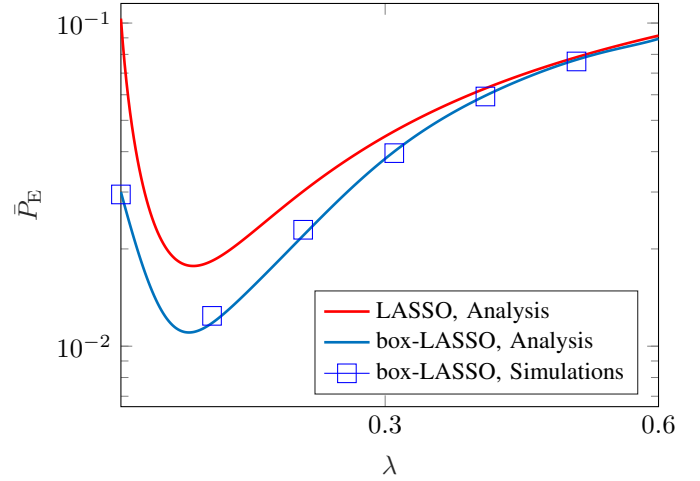


Fig. 2: Average error rate vs. λ considering both analytical results, given by Proposition 1, and numerical simulations.

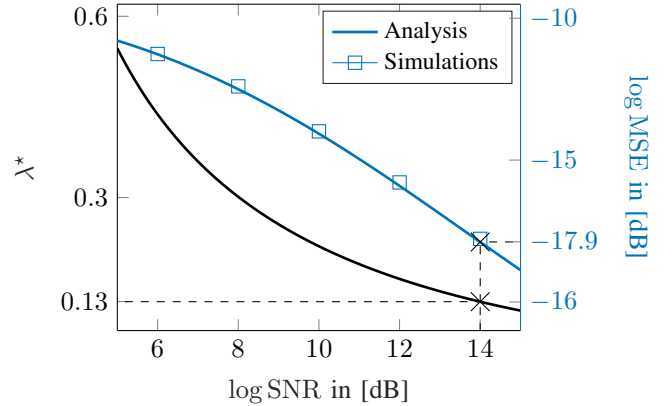


Fig. 3: Optimal regularization parameter and minimum achievable MSE via box-LASSO detection vs. SNR. The curves match the result seen in Fig. 1.

V. CONCLUSION

The average MSE and error rate were analytically derived for massive SM MIMO settings when a generic RLS-based algorithm is employed for detection. The analysis was given for the large class of bi-unitarily invariant random matrices which includes various fading models. The asymptotic results were employed to study box-LASSO detectors for SSK signaling. This particular application of the results extended the analysis in [1] to a larger set of channel matrices. Numerical simulations showed close consistency with the analytic results, even in practical dimensions. Other applications of the results are skipped due to the page limitation and will be presented in an extended version which is currently under preparation.

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