Distortion-Based Outer-Bounds for Channels with Rate-Limited Feedback

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Abstract—We present a new technique to obtain outer-bounds on the capacity region of networks with ultra low-rate feedback. We establish a connection between the achievable rates in the forward channel and the minimum distortion that can be attained over the feedback channel.

Index Terms—Rate-limited feedback, erasure channels, channel capacity, Shannon feedback, sub-bit feedback.

I. INTRODUCTION

The introduction of massive Machine-Type Communications (mMTC) challenges many assumptions took for granted in network information theory. One of the main challenges is the increased cost of learning. For instance, in current systems, it is well justified to assume free access to small control packets (*e.g.*, ACK/NACK signals) when needed, as they are much smaller is size compared to payload packets. However, this is no longer the case in mMTC where payload and control packets will be of comparable sizes. To make matters more complicated, there is a growing concern about security attacks that aim to disrupt unprotected feedback channels.

We present new outer-bounds for networks with low-rate feedback. The outer-bound establishes a connection between rate-distortion theory and achievable rates in multi-terminal networks with feedback. Interestingly, we learn that the best use of the feedback channel may not be to minimize the error but rather the distortion in reconstructing channel information. More specifically, we first quantify how closely the channel state may be reconstructed using the rate-limited feedback link, and then, we define a space of indistinguishable channel realizations in which the outer-bound is optimized.

To illustrate the technique, we focus on the two-user broadcast erasure channel (BEC) and assume a sub-bit feedback link from *only* one receiver, $R \times_F$, while the other receiver, $R \times_N$, does not share any information with the other nodes. Receiver $R \times_f$ uses the rate-limited feedback link to causally provide its potentially encoded channel state information (CSI) to the other nodes. Although it remains open whether the bound is tight, it is the first of its kind and the surprising message is that to achieve the bound, the approach may be to purposefully induce distortion into the encoded CSI. We outline how existing results fall short of achieving these bounds and provide further insights and interpretations.

Related Literature: In [1], outer-bounds for wiretap channels with rate-limited output feedback were derived, which are tight for physically degraded channels. Shayevitz and Wigger observed in [2] that finding a general feedback capacity formula for memoryless broadcast channels (BCs) is very hard. For other channels such as Gaussian BCs, the capacity with single-user feedback is still unknown [3]. The block Markov coding of [4] provides interesting insights but involves characterizing complicated auxiliary random variables. To further understand how low-rate feedback affects the capacity region of multi-terminal channels, two-user broadcast erasure channels with intermittent [5], [6] and one-sided [7]–[9] delayed feedback have been studied. Interestingly, it was shown in [9], [10] that even when only one receiver provides its delayed CSI to the transmitter, the outer-bound with global delayed feedback

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can be achieved. This latter finding motivates us to further lower the feedback rate to sub-bit territory to understand the fundamental limits of communications with rate-limited feedback in its purest form. In the context of interference channels, in [11], [12], the capacity of two-user interference channels with rate-limited feedback was established; new coding schemes with noisy or intermittent output feedback were proposed in [13], [14]; and [15]–[17] generalize such ideas to two-way communications. Finally, locality of feedback was studied under different delay assumptions in [18]–[24].

II. PROBLEM FORMULATION

We consider the two-user broadcast erasure channel (BEC) of Fig. 1 in which a single-antenna transmitter, Tx, wishes to communicate two independent messages, W_F and W_N , to two single-antenna receiving terminals Rx_F and Rx_N (read feedback/silent receiver), respectively, over *n* channel uses. Each of the messages, W_F and W_N , is uniformly distributed over $\{1, 2, \ldots, 2^{nR_F}\}$ and $\{1, 2, \ldots, 2^{nR_N}\}$, respectively. At time instant *t*, the messages are mapped to channel input $X[t] \in \mathbb{F}_2$ (in the binary field), and the respective received signals at Rx_F and Rx_N are:

$$Y_F[t] = S_F[t]X[t] \quad \text{and} \quad Y_N[t] = S_N[t]X[t], \qquad (1)$$

where $\{S_F[t]\}\$ is the Bernoulli $(1 - \delta_F)$ process that governs the erasure at Rx_F , and $\{S_N[t]\}\$ is the Bernoulli $(1 - \delta_N)$ process that governs the erasure at Rx_N . In this manuscript, we assume the channels are distributed independently over time and across users, and we limit the scope to $\delta_F = \delta_N = \delta$ (*i.e.* homogeneous channels) where δ is known globally.

We assume that at time instant t, each receiver knows its channel value instantly, *e.g.*, at time instant t, R_{F} knows the realization of $S_F[t]$. When the channel realization is 1, the corresponding terminal receives X[t] noiselessly, and when it is 0, the terminal understands an erasure has occurred.

We further assume a one-sided feedback structure in which at time instant t, \mathbb{R}_N knows $S_N[t]$ but does *not* share this information with the other nodes. On the other hand, we

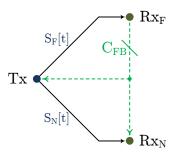


Fig. 1. Two-user BEC with rate-limited one-sided feedback.

assume R_{X_F} shares its channel state with receiver R_{X_N} and the transmitter through a rate-limited feedback channel of capacity C_{FB} . We assume the feedback encoder at R_{X_F} sends back feedback symbol $K[t] \in \mathcal{K}[t]$ at time t and this information becomes available to the other nodes at time t + 1. Here, $\mathcal{K}[t]$ is the feedback alphabet at time t. The feedback symbol depends causally on $S_F[t]$, and the cardinality of the feedback alphabets satisfies

$$\frac{1}{t} \sum_{\ell=1}^{t} \log_2\left(|\mathcal{K}[\ell]|\right) \le C_{\mathsf{FB}}, \qquad \forall \ 1 \le t \le n.$$
 (2)

The constraint imposed at the encoding function $f_t(.)$ at time index t is

$$X[t] = f_t \left(W_F, W_N, K^{t-1} \right).$$
(3)

Receivers R_{x_F} and R_{x_N} use decoding functions $\varphi_{F,n}(Y_F^n, S_F^n)$ and $\varphi_{N,n}(Y_2^n, K^n, S_N^n)$ to get estimates \widehat{W}_F of W_F and \widehat{W}_N of W_N , respectively. An error occurs whenever the estimate does not match the corresponding message. The average probabilities of error are given by

$$\lambda_{F,n} = \mathbb{E}[P(\widehat{W}_F \neq W_F)], \ \lambda_{N,n} = \mathbb{E}[P(\widehat{W}_N \neq W_N)], \ (4)$$

where the expectations are taken with respect to the random choice of the transmitted messages.

We say that a rate pair (R_F, R_N) is achievable if there exists a block encoder at the transmitter, and a block decoder at each receiver, such that the average probabilities of error go to zero as the block length n goes to infinity. The capacity region, C, is the closure of the set of achievable rate pairs.

III. MAIN RESULTS

In this section, we provide the new distortion-based outerbound for the two-user BEC with rate-limited feedback. Define $0 \le D^* \le \min \{\delta, 1 - \delta\}$ to be the unique value to satisfy

$$H(D) = \left(H(\delta) - C_{\mathsf{FB}}\right)^+,\tag{5}$$

and $\gamma_{\rm out}$ to be

$$\gamma_{\text{out}} \stackrel{\triangle}{=} \frac{D^*}{\min\left\{\delta, 1-\delta\right\}}.$$
(6)

Theorem 1. The capacity region, C, of the two-user BEC with one-sided rate-limited feedback is included in

$$\mathcal{C}_{\text{out}} \stackrel{\triangle}{=} \left\{ \left(R_F, R_N \right) \begin{vmatrix} R_F + \beta_{\text{out}} R_N \leq \beta_{\text{out}} \left(1 - \delta \right) \\ \beta_{\text{out}} R_F + R_N \leq \beta_{\text{out}} \left(1 - \delta \right) \end{vmatrix} \right\},\tag{7}$$

where

$$\beta_{\text{out}} = \gamma_{\text{out}} + (1 - \gamma_{\text{out}}) (1 + \delta).$$
(8)

Similar to the findings of [9], although only Rx_F provides feedback, the outer-bounds are symmetric. The results could also provide new capacity bounds for erasure interference channels based on their connection to BECs [25]–[28].

Figure 2 plots the outer-bound on the maximum sum-rate point for $\delta = 0.4$. When $C_{\mathsf{FB}} \geq H(\delta)$, this outer-bound matches the capacity region of the two-user BEC with global delayed feedback [29]. This recovers the results of [9], [10] where it is shown that perfect one-sided feedback is as good as global feedback. The interesting distinction between the results in [9], [10] and prior results is the fact that delayed CSI is harnessed at every step of the achievability. One can envision a Markov block structure to interleave different blocks and compress the feedback to $H(\delta) n$ bits and mimic the results of [9] to achieve of point A in Figure 2 where $H(\delta) = C_{FB}$. However, below this limit, the achievability can no longer be derived from such arguments. In fact, the argument presented above requires perfect delayed CSI, and for $C_{\text{FB}} < H(\delta)$, if we attempt to send perfectly a part of CSI back to the transmitter, we deviate from this outer-bound as in Figure 2. This suggests the possibility of new achievability ideas that purposefully induce distortion in the encoded CSI.

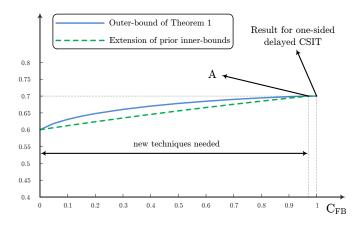


Fig. 2. Sum-rate outer-bound for $\delta = 0.4$. Point A is where $H(\delta) = C_{\text{FB}}$.

IV. PROOF OF THEOREM 1

Overview: The proof is based on the following three key ingredients: (1) for R_{x_F} , we find a set of channels realizations that result in the same feedback sequence as the original channel; (2) within this set, we find a candidate that has the same marginal distribution as S_F^n but is maximally correlated with S_N^n ; (3) as further discussed later, we use the fact that:

$$H(X^n|S_F^n, K^n) = H(X^n|K^n).$$
(9)

Derivation: To derive the outer-bounds, we enhance the knowledge at R_{X_N} by providing it with S_F^{t-1} rather than K^{t-1} . The transmitter is still informed causally of the CSI associated with R_{X_F} through a rate-limited feedback link of capacity C_{FB} . The proof is broken into several steps as discussed below.

Step 1: From Rate-Distortion Theory, we know that for a binary source distributed as i.i.d. $\mathcal{B}(1-\delta)$, given a channel capacity of C_{FB} , the minimum attainable Hamming distortion, $0 \leq D^* \leq \min{\{\delta, 1-\delta\}}$, between the source S_F^n and the estimate \hat{S}_F^n , is the solution to (5).

Fix the feedback strategy mapping S_F^n to K^n , and suppose this strategy results in distortion, $D_{\mathsf{FB}} \geq D^*$, between S_F^n and the estimate \hat{S}_F^n . We further denote the fraction of time instants in which $S_F[t] = 1$ and $\hat{S}_F[t] = 0$ by D_{10} . Similarly, D_{01} is the fraction of time instants in which $S_F[t] = 0$ and $\hat{S}_F[t] = 1$. Thus, we have

$$D_{10} + D_{01} = D_{\mathsf{FB}}.\tag{10}$$

Claim 1. There exists \tilde{S}_F^n such that: (1) we have

$$\tilde{S}_{F}[t] \stackrel{i.i.d.}{\sim} \mathcal{B}(1-\delta) \text{ and } \mathbb{E}\left(d\left(\hat{S}_{F}^{n}, \tilde{S}_{F}^{n}\right)\right) \leq D_{\mathsf{FB}}, \quad (11)$$

where $d(\cdot, \cdot)$ is the Hamming distance; (2) the feedback encoder maps \tilde{S}_{F}^{n} to the same feedback sequence K^{n} for S_{F}^{n} .

Proof. Suppose no such sequence exists. Then, the transmitter could have an estimate of S_F^n with a distortion smaller than D_{FB} , which would be in contradiction with Rate-Distortion Theory, thus, proving the result.

As noted, $\tilde{S}_F[t] \stackrel{i.i.d.}{\sim} \mathcal{B}(1-\delta)$. Denote the fraction of time instants in which $\tilde{S}_F[t] = 1$ and $\hat{S}_F[t] = 0$ by \tilde{D}_{10} , and the fraction of time instants in which $\tilde{S}_F[t] = 0$ and $\hat{S}_F[t] = 1$ by \tilde{D}_{01} . We have $\tilde{D}_{10} + \tilde{D}_{01} \leq D_{\text{FB}}$.

Step 2: In this step, we claim if we replace sequence S_F^n with \tilde{S}_F^n , the capacity region remains unchanged.

Claim 2. Any achievable (R_F, R_N) is included in the capacity region of a BEC with \tilde{S}_F^n instead of S_F^n while other parameters are kept the same and vice versa.

Proof.
$$I(W_F; Y_F^n | S_F^n) = I(W_F; Y_F^n | S_F^n, K^n)$$

= $I(W_F; \tilde{Y}_F^n | \tilde{S}_F^n, K^n) = I(W_F; \tilde{Y}_F^n | \tilde{S}_F^n),$ (12)

and $I(W_N; Y_N^n | S_F^n, S_N^n) = I(W_N; Y_N^n | S_F^n, S_N^n, K^n)$

$$= I(W_N; Y_N^n | S_N^n, K^n) = I\left(W_N; Y_N^n | \tilde{S}_F^n, S_N^n, K^n\right).$$
(13)

Step 3: In the rate-limited broadcast channel of Section II, $S_F[t]$ and $S_N[t]$ are distributed as independent Bernoulli random variables, and the feedback encoder at R_{X_F} is unaware of S_N^n . In this step, we create a "worst-case" scenario by creating maximum correlation between S_N^n and \tilde{S}_F^n .

Claim 3. Under the conditions expressed in Claim 1, we have

$$\max_{\tilde{D}_{10}+\tilde{D}_{01}\leq D_{\mathsf{FB}}} \Pr\left(\tilde{S}_F[t] = S_N[t] = 0\right)$$
$$= \begin{cases} \delta^2 + D_{\mathsf{FB}}\left(1-\delta\right), & \min\left\{\delta, 1-\delta\right\} = \delta, \\ \delta^2 + D_{\mathsf{FB}}\delta, & \min\left\{\delta, 1-\delta\right\} = 1-\delta. \end{cases}$$
(14)

Proof. We divide the proof into two parts based on δ .

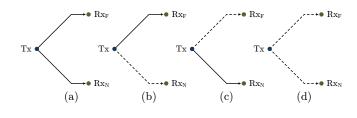


Fig. 3. Four possible channel realizations at each time.

• $\min \{\delta, 1 - \delta\} = \delta$: In this case, to maximize $\Pr \left(\tilde{S}_F[t] = S_N[t] = 0 \right)$, the transition from S_F^n to \hat{S}_F^n would be such that to minimize the times in which $\hat{S}_F[t] = 1$ and $S_N[t] = 0$ by setting $D_{10} (1 - \delta) n$ ones to zeros when $S_F[t] = 1$ and $S_N[t] = 0$. The transition from \hat{S}_F^n to \tilde{S}_F^n would be such that to maximize the times in which $\tilde{S}_F[t] = 1$ and $S_N[t] = 1$ by changing $D_{10} (1 - \delta) n$ zeros in \hat{S}_F^n to ones when $S_F[t] = 0$ and $S_N[t] = 1$. In effect, such transition from S_F^n to \tilde{S}_F^n , increases the probability of $\tilde{S}_F[t] = S_N[t] = 0$ by $D_{10} (1 - \delta)$, and since $D_{10} \leq D_{\text{FB}}$, the maximum occurs at $D_{10} = D_{\text{FB}}$. Based on Figure 3, the discussion above can be explained as follows. The transition from S_F^n to \hat{S}_F^n removes realizations from state (b) and adds them to state (d), while transition from \hat{S}_F^n to \tilde{S}_F^n moves realizations from state d to (c). Thus, end-to-end transitions are from state (b) to (c).

• $\min \{\delta, 1 - \delta\} = 1 - \delta$: In this case, to maximize $\Pr \left(\tilde{S}_F[t] = S_N[t] = 0 \right)$, the transition from S_F^n to \hat{S}_F^n would be such that to maximize the times in which $S_F[t] = 1$ and $S_N[t] = 1$ by changing $D_{01}\delta n$ zeros to ones when $S_F[t] = 0$ and $S_N[t] = 1$. The transition from \hat{S}_F^n to \tilde{S}_F^n would be such that to minimize the times in which $\tilde{S}_F[t] = 1$ and $S_N[t] = 0$ by changing $D_{01}\delta n$ ones in \hat{S}_F^n to zeros when $S_F[t] = 1$ and $S_N[t] = 0$. In effect, such transition from S_F^n to \tilde{S}_F^n , increase the probability of $\tilde{S}_F[t] = S_N[t] = 0$ by $D_{01}\delta$, and since $D_{01} \leq D_{\text{FB}}$, the maximum occurs at $D_{01} = D_{\text{FB}}$.

To prepare for the final step, we define

$$\tilde{\gamma}_{\text{out}} \stackrel{\triangle}{=} \frac{D_{\text{FB}}}{\min\left\{\delta, 1 - \delta\right\}}.$$
(15)

Based on this definition and (14), we have

$$\min_{\tilde{D}_{10}+\tilde{D}_{01}\leq D_{\mathsf{FB}}} \Pr\left(\{\tilde{S}_F[t] = S_N[t] = 0\}^c\right)$$

$$= \tilde{\gamma}_{\mathrm{out}} \left(1-\delta\right) + \left(1-\tilde{\gamma}_{\mathrm{out}}\right) \left(1-\delta^2\right).$$
(16)

Further, from rate-distortion theory, we have $D_{FB} \ge D^*$. Thus, for *any* feedback strategy, we have

$$A \stackrel{\Delta}{=} \max_{D_{\mathsf{FB}} \ge D^*} \min_{\tilde{D}_{10} + \tilde{D}_{01} \le D_{\mathsf{FB}}} \Pr\left(\left\{\tilde{S}_F[t] = S_N[t] = 0\right\}^c\right)$$
$$= \gamma_{\text{out}} \left(1 - \delta\right) + \left(1 - \gamma_{\text{out}}\right) \left(1 - \delta^2\right), \tag{17}$$

where γ_{out} is defined in (6), and this bound is attained for a code that achieves the rate-distortion bound. In other words, for *any* feedback strategy, we have

$$\Pr\left(\{\tilde{S}_F[t] = S_N[t] = 0\}^c\right)$$

$$\leq \gamma_{\text{out}} \left(1 - \delta\right) + \left(1 - \gamma_{\text{out}}\right) \left(1 - \delta^2\right)$$
(18)

where \tilde{S}_F^n satisfies the conditions in (11).

Step 4: For the final step, we first prove the following result.

Claim 4. For the two-user BEC with rate-limited feedback as described in Section II and for any input distribution, we have

$$H\left(Y_{F}^{n}|W_{N},S_{F}^{n},S_{N}^{n}\right) - \beta_{\text{out}}H\left(Y_{N}^{n}|W_{N},S_{F}^{n},S_{N}^{n}\right) \le 0,$$
(19)

where β_{out} is given in (8).

Proof.

$$\begin{split} H\left(Y_{N}^{n}|W_{N},S_{F}^{n},S_{N}^{n}\right) &= H\left(Y_{N}^{n}|W_{N},S_{F}^{n},K^{n},S_{N}^{n}\right) \stackrel{(a)}{=} H\left(Y_{N}^{n}|W_{N},K^{n},S_{N}^{n}\right) \\ &= H\left(Y_{N}^{n}|W_{N},\tilde{S}_{F}^{n},K^{n},S_{N}^{n}\right) + I\left(Y_{N}^{n};\tilde{S}_{F}^{n}|W_{N},K^{n},S_{N}^{n}\right) \\ \stackrel{(b)}{=} H\left(Y_{N}^{n}|W_{N},\tilde{S}_{F}^{n},K^{n},S_{N}^{n}\right) \stackrel{(c)}{=} H\left(Y_{N}^{n}|W_{N},\tilde{S}_{F}^{n},S_{N}^{n}\right) \\ \stackrel{(d)}{=} \sum_{t=1}^{n} H\left(Y_{N}[t]|Y_{N}^{t-1},W_{N},\tilde{S}_{F}^{t},S_{N}^{t}\right) \\ \stackrel{(e)}{=} \sum_{t=1}^{n} (1-\delta)H\left(X[t]|Y_{N}^{t-1},W_{N},\tilde{S}_{F}^{t},S_{N}^{t}\right) \\ \stackrel{(f)}{=} \sum_{t=1}^{n} (1-\delta)H\left(X[t]|Y_{N}^{t-1},W_{N},\tilde{S}_{F}^{t},S_{N}^{t}\right) \\ \stackrel{(g)}{\geq} \sum_{t=1}^{n} (1-\delta)H\left(X[t]|\tilde{Y}_{F}^{t-1},Y_{N}^{t-1},W_{N},\tilde{S}_{F}^{t},S_{N}^{t}\right) \\ \stackrel{(h)}{\geq} \sum_{t=1}^{n} \frac{(1-\delta)}{A}H\left(\tilde{Y}_{F}[t],Y_{N}[t]|\tilde{Y}_{F}^{t-1},Y_{N}^{t-1},W_{N},\tilde{S}_{F}^{t},S_{N}^{t}\right) \end{split}$$

$$\stackrel{(17)}{\geq} \sum_{t=1}^{n} \frac{(1-\delta)H\left(\tilde{Y}_{F}[t], Y_{N}[t]|\tilde{Y}_{F}^{t-1}, Y_{N}^{t-1}, W_{N}, \tilde{S}_{F}^{t}, S_{N}^{t}\right)}{\gamma_{\text{out}}(1-\delta) + (1-\gamma_{\text{out}})(1-\delta^{2})}$$

$$\stackrel{(8)}{=} \sum_{t=1}^{n} \frac{1}{\beta_{\text{out}}} H\left(\tilde{Y}_{F}[t], Y_{N}[t]|\tilde{Y}_{F}^{t-1}, Y_{N}^{t-1}, W_{N}, \tilde{S}_{F}^{t}, S_{N}^{t}\right)$$

$$\stackrel{(i)}{=} \sum_{t=1}^{n} \frac{1}{\beta_{\text{out}}} H\left(\tilde{Y}_{F}[t], Y_{N}[t]|\tilde{Y}_{F}^{t-1}, Y_{N}^{t-1}, W_{N}, \tilde{S}_{F}^{n}, S_{N}^{n}\right)$$

$$= \frac{H\left(\tilde{Y}_{F}^{n}, \tilde{Y}_{N}^{n} | W_{N}, \tilde{S}_{F}^{n}, S_{N}^{n}\right)}{\beta_{\text{out}}} \stackrel{(j)}{\geq} \frac{H\left(\tilde{Y}_{F}^{n} | W_{N}, \tilde{S}_{F}^{n}, S_{N}^{n}\right)}{\beta_{\text{out}}}$$

$$\stackrel{(20)}{=} \frac{1}{\beta_{\text{out}}} H\left(Y_{F}^{n} | W_{N}, S_{F}^{n}, S_{N}^{n}\right), \qquad (20)$$

where (a) & (b) hold since conditioned on K^{t-1} , X[t] is independent of all other channel parameters; (c) follows the fact that \tilde{S}_F^n results in K^n ; (d) follows from the chain rule and the causality of the channel; (e) holds since $S_N[t]$ is a Bernoulli $(1 - \delta)$ process; (f) is true since X[t] is independent of channel realizations at time instant t; (g) holds since conditioning reduces entropy; (h) follows by the definition of A in (17), normalizing by the probability that at least one signal is not erased, and ensuring the inequality holds for any feedback strategy; (i) follows the causality assumption; and (j) holds as the discrete entropy function is non-negative. \Box

Finally, we are ready to prove the outer-bounds.

$$n (R_{F} + \beta_{\text{out}} R_{N}) = H (W_{F}) + \beta_{\text{out}} H (W_{N})$$

$$= H (W_{F}|W_{N}, S_{F}^{n}, S_{N}^{n}) + \beta_{\text{out}} H (W_{N}|S_{F}^{n}, S_{N}^{n})$$

$$\stackrel{\text{Fano}}{\leq} I (W_{F}; Y_{F}^{n}|W_{N}, S_{F}^{n}, S_{N}^{n}) + n\epsilon_{n}$$

$$= H (Y_{F}^{n}|W_{N}, S_{F}^{n}, S_{N}^{n}) - \underbrace{H (Y_{F}^{n}|W_{F}, W_{N}, S_{F}^{n}, S_{N}^{n})}_{= 0}$$

$$+ \beta_{\text{out}} H (Y_{N}^{n}|S_{F}^{n}, S_{N}^{n}) - \beta_{\text{out}} H (Y_{N}^{n}|W_{N}, S_{F}^{n}, S_{N}^{n}) + n\epsilon_{n}$$

$$\stackrel{\text{Claim 4}}{\leq} \beta_{\text{out}} H (Y_{N}^{n}|S_{F}^{n}, S_{N}^{n}) + n\epsilon_{n} \leq n\beta_{\text{out}}(1 - \delta) + n\epsilon_{n},$$

$$(21)$$

where $\epsilon_n \to 0$ as $n \to \infty$. Dividing both sides by n and taking the limit for $n \to \infty$ completes the proof. The other outer-bound in Theorem 1 can be obtained similarly.

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