The Generalized Degrees-of-Freedom of the Asymmetric Interference Channel with Delayed CSIT

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Abstract—In this paper, we investigate the generalized degreesof-freedom (GDoF) of the asymmetric interference channel with delayed channel state information at the transmitter (CSIT), where each transmitter has two antennas, each receiver has one antenna, and the strength for each interfering link can vary. The optimal sum-GDoF is characterized by matched converse and achievability proof. Through our results, we also reveal that in our antenna setting, the symmetric GDoF lower bound in [Mohanty et. al, TIT 2019] can be elevated, and the symmetric GDoF upper bound in [Mohanty et. al, TIT 2019] is tight in fact.

Index Terms—Delayed CSIT, sum-GDoF, interference channel.

I. INTRODUCTION

The degrees-of-freedom (DoF) characterization with delayed channel state information at the transmitter (CSIT) has attracted a plenty of research interests in the past decade. For example, the DoF region of multiple-input multiple-output (MIMO) interference channel with delayed CSIT was derived in [1]. The study of DoF of MIMO broadcast channel with delayed CSIT can be found in [2]–[5], whose exact value was still not completely obtained. The linear DoF region of MIMO X channel with delayed CSIT was characterized in [6].

One limitation of DoF is that the strength of each link is assumed to be equal, whereas the link strength can vary tremendously in wireless. Nevertheless, the generalized degrees-offreedom (GDoF) overcomes this drawback by considering the different strength of each link, which was first proposed in [7]. The GDoF characterization with delayed CSIT can be found in [8]–[11]. In [8], the GDoF was studied in the two-user multiple-input single-output (MISO) broadcast channel under alternating delayed, perfect, and no CSIT, where sum-GDoF upper and lower bounds were shown to be partially coincided. In [9], the secure GDoF was investigated in the two-user MISO broadcast channel with an external eaversdropper and alternating delayed, perfect, and no CSIT. The GDoF region of the MIMO Z channel with delayed CSIT was characterized in [10]. For MIMO interference channel with delayed CSIT and symmetric interfering link strengths, symmetric GDoF upper and lower bounds were derived in [11], where the symmetric GDoF was characterized partially, and the upper and lower bounds do not change with antenna ratio, i.e., number of



Fig. 1. The considered scenario of asymmetric interference channel, where

each transmitter has 2 antennas and each receiver has 1 antenna.

antenna at each transmitter over number of antennas at each receiver, if the antenna ratio is equal to or larger than two¹.

In this paper, we study the GDoF of the asymmetric interference channel with delayed CSIT, where the transmitter has two antennas, the receiver has one antenna, and the strength for each interfering link can vary. We characterize the sum-GDoF by presenting matched converse and achievability proof. Then, via this result, we reveal that in our antenna setting, the symmetric GDoF lower bound in [11] can be elevated, and the symmetric GDoF upper bound in [11] is tight in fact. The idea of elevating the GDoF lower bound comes from [9], where the authors in [9] fully exploit the receiver signal space by sending new fresh symbols in each time slot. Our transmission scheme generalizes the scheme in [9, Appendix C], which was designed for achieving the corner points of GDoF region of MISO broadcast channel with delayed CSIT.

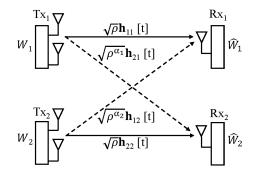
II. SYSTEM MODEL

The considered asymmetric interference channel has two transmitters, denoted by Tx_1 and Tx_2 , each with two antennas, and two receivers, denoted by Rx_1 and Rx_2 , each with one antenna, as depicted in Fig. 1. The transmitter Tx_i , i = 1, 2, sends private message W_i to the receiver Rx_i . The received signals at two receivers and time slot t can be written as

$$y_1[t] = \sqrt{\rho} \mathbf{h}_{11}[t] \mathbf{x}_1[t] + \sqrt{\rho^{\alpha_2}} \mathbf{h}_{12}[t] \mathbf{x}_2[t] + n_1[t], \qquad (1a)$$

$$y_2[t] = \sqrt{\rho^{\alpha_1}} \mathbf{h}_{21}[t] \mathbf{x}_1[t] + \sqrt{\rho} \mathbf{h}_{22}[t] \mathbf{x}_2[t] + n_2[t], \quad (1b)$$

¹A representative antenna setting for this antenna configuration case is that the transmitter has 2 antennas and the receiver has 1 antenna.



where $\mathbf{x}_i[t] \in \mathbb{C}^{2 \times 1}$ denotes the transmitted signal from transmitter Tx_i at time slot $t, y_i[t] \in \mathbb{C}^1, j = 1, 2$, denotes the received signal at receiver Rx_j and time slot t, $\mathbf{h}_{ji} \in \mathbb{C}^{1 \times 2}$ denotes the channel matrix from transmitter Tx_i to receiver $\operatorname{Rx}_i, n_i \sim \mathcal{CN}(0, \sigma^2)$ denotes the additive White Gaussian noise (AWGN) at receiver Rx_j , the channel gains of desired links and interfering links are denoted by $\sqrt{\rho}$ and $\sqrt{\rho^{\alpha_i}}$ with $0 < \rho$ and $0 \le \alpha_i$, respectively. Without loss of generality, we assume that $\alpha_2 \leq \alpha_1$. The transmit signals follow an average power constraint, given by $\frac{1}{n} \sum_{t=1}^{n} \operatorname{tr} \left(\mathbb{E} \{ \mathbf{x}_{i}[t] \mathbf{x}_{i}[t]^{H} \} \right) \leq 1$. All entries of channel matrices are independent and identical distributed (i.i.d.) across space and time slot. The signal-tonoise ratio (SNR) at each receiver is ρ , and interference-tonoise ratio (INR) at receivers Rx_1 and Rx_2 are ρ^{α_2} and ρ^{α_1} , respectively. We define the following assembly of channel matrices: $\mathcal{H}[t] \triangleq \{\mathbf{h}_{ji}[t]\}_{i,j=1,2}$, and $\mathcal{H}^{\tau} \triangleq \{\mathcal{H}[t]\}_{t=1}^{\tau}$.

Due to feedback delay, the transmitter has delayed channel state information. Specifically, at time slot t, the transmitters know \mathcal{H}^{τ} , namely all the channel matrices up to time slot t-1. Two receivers have instantaneous knowledge of channel matrices. The encoding function at transmitter Tx_i and time slot t is denoted by $e_{i,t}(W_i, \mathcal{H}^{t-1})$. The decoding function at receiver Rx_j after n time slots is denoted by $c_{j,n}(W_i, \mathcal{H}^n)$.

The rate tuple is written as $(R_1(\rho, \alpha_1, \alpha_2), R_2(\rho, \alpha_1, \alpha_2))$, where rate $R_i = \frac{\log |\mathcal{W}_i|}{n}$ is the cardinality of message set \mathcal{W}_i . The rate is achievable, if there are a sequence of codebook pairs $\{\mathcal{C}_{1,t}, \mathcal{C}_{1,t}\}_{t=1}^n$ and decoding functions $\{c_{1,n}, c_{2,n}\}$ such that the error probability $P_e^{(n)}$ goes to zero when n goes to infinity. The sum-capacity is defined as the supremum of sum of achievable rates, i.e., $C_{\text{sum}} = \sup \sum_{i=1}^2 R_i(\rho, \alpha_1, \alpha_2)$. Then, the sum-GDoF is defined as the pre-log factor of sumcapacity, i.e., $\sum_{i=1}^2 d_i = \lim_{\rho \to \infty} \frac{C_{\text{sum}}}{\log \rho}$.

III. MAIN RESULTS AND DISCUSSION

Theorem 1: For the considered asymmetric interference channel with delayed CSIT, defined in Section-II, the sum-GDoF is given as follows:

$$\sum_{i=1}^{2} d_{i} = \begin{cases} 2 - \frac{\alpha_{1} + \alpha_{2}}{3}, & \alpha_{1}, \alpha_{2} \leq 1, \\ \min\left\{\frac{4 + \alpha_{1} - \alpha_{2}}{3}, 2\right\}, & 1 < \alpha_{1} \& \alpha_{2} \leq 1 \\ \& 2 \leq \alpha_{1} + 2\alpha_{2}, \\ \min\left\{\frac{2 + \alpha_{1} + \alpha_{2}}{3}, 2\right\}, & 1 < \alpha_{1}, \alpha_{2}. \end{cases}$$
(2)

Proof: Please refer to Section-IV for the converse proof and Section-V for the achievability proof.

Remark 1: This sum-GDoF in (2) degenerates to optimal sum-DoF in [1], by setting $\alpha_1 = \alpha_2 = 1$, whose value is the celebrated 4/3. Furthermore, in our antenna setting, it can be verified that the symmetric GDoF upper bound in [11] is tight.

IV. CONVERSE PROOF OF THEOREM 1

The key steps of this proof follows the that in [11]. To begin with, we define the following virtual received signals,

which are obtained from removing the impact of $\mathbf{x}_1[t]$ in the received signals at each receiver:

$$\overline{y}_1[t] = \sqrt{\rho^{\alpha_2}} \mathbf{h}_{12}[t] \mathbf{x}_2[t] + n_1[t], \tag{3a}$$

$$\overline{y}_2[t] = \sqrt{\rho} \mathbf{h}_{22}[t] \mathbf{x}_2[t] + n_2[t].$$
(3b)

Before the next step, we define the following assembly of channel matrices: $\overline{\mathcal{Y}}_{i}^{\tau} \triangleq \{\overline{y}_{i}[t]\}_{t=1}^{\tau}, \mathcal{Y}_{i}^{\tau} \triangleq \{y_{i}[t]\}_{t=1}^{\tau}, \text{ and } \mathcal{X}_{i}^{\tau} \triangleq \{\mathbf{x}_{i}[t]\}_{t=1}^{\tau}, i = 1, 2$. Since the error probability $P_{e}^{(n)}$ goes to zero as n goes to infinity, we denote $n\epsilon_{n} \triangleq 1 + nR_{i}P_{e}^{(n)}$ so that $\lim_{n\to\infty} \epsilon_{n} = 0$. According to [11], the rate of receiver 1 can be bounded as

$$n(R_1 - \epsilon_n) \le \sum_{t=1}^n h(y_1[t]|\mathcal{H}[t]) - \sum_{t=1}^n h(\overline{y}_1[t]|\mathcal{U}[t], \mathcal{H}[t]), \quad (4)$$

where $\mathcal{U}[t] \triangleq \{\overline{\mathcal{V}}_1^{t-1}, \overline{\mathcal{V}}_2^{t-1}, \mathcal{H}^{t-1}\}$. Next, according to [11], the rate of receiver 2 can be bounded as

$$n(R_2 - \epsilon_n) \le \sum_{t=1}^n h(\overline{y}_1[t], \overline{y}_2[t] | \mathcal{U}[t], \mathcal{H}[t]).$$
(5)

Henceforth, we define $\mathbf{S}[t] \triangleq [\sqrt{\rho^{\alpha_2}} \mathbf{h}_{12}[t], \sqrt{\rho} \mathbf{h}_{22}[t]]^T$, $\mathbf{K}[t] \triangleq \mathbb{E}\{\mathbf{x}_2[t]\mathbf{x}_2[t]^H | \mathcal{U}[t]\}, \mathbf{L}[t] \triangleq \mathbb{E}\{\mathbf{x}_1[t]\mathbf{x}_1[t]^H | \mathcal{H}^{t-1}\},$ $\mathcal{V}[t] \triangleq \{\mathcal{U}[t], \mathcal{H}[t]\}$, where the transmit covariance matrix $\mathbf{K}[t]$ is independent of $\mathbf{h}_{12}[t], \mathbf{h}_{22}[t]$, and $\mathbf{S}[t]$. Applying the extremal inequality in [12] for the physically degraded channel $\mathcal{X}_2^{\tau} \to (\overline{\mathcal{Y}}_1^n, \overline{\mathcal{Y}}_2^n) \to \overline{\mathcal{Y}}_1^n$, we have the following inequality:

$$\frac{h(\overline{y}_{1}[t], \overline{y}_{2}[t] | \mathcal{V}[t])}{2} - h(\overline{y}_{1}[t] | \mathcal{V}[t]) \\
\leq \max_{\mathbf{K}[t] \geq 0} \mathbb{E} \left\{ \log \left| \mathbf{I}_{2} + \mathbf{S}[t] \mathbf{K}[t] \mathbf{S}[t]^{H} \right| / 2 \\
\operatorname{tr} \{\mathbf{K}[t]\} \leq 1 \\
- \log \left| 1 + \rho^{\alpha_{2}} \mathbf{h}_{12}[t] \mathbf{K}[t] \mathbf{h}_{12}[t]^{H} \right| \right\}.$$
(6)

To proceed, we can approximate the first term of (6) as

$$\begin{split} &\log \left| \mathbf{I}_{2} + \mathbf{S}[t] \mathbf{K}[t] \mathbf{S}[t]^{H} \right| \\ &\stackrel{(a)}{=} \log \left| \mathbf{I}_{2} + \left[\frac{\sqrt{\rho^{\alpha_{2}}} \tilde{\mathbf{h}}_{12}[t]}{\sqrt{\rho} \tilde{\mathbf{h}}_{22}[t]} \right] \left[\frac{\sqrt{\rho^{\alpha_{2}}} \tilde{\mathbf{h}}_{12}[t]}{\sqrt{\rho} \tilde{\mathbf{h}}_{22}[t]} \right]^{H} \right| \\ &\stackrel{(b)}{=} \log \left| \mathbf{I}_{2-k_{t}} + \rho^{\alpha_{2}} \tilde{\mathbf{h}}_{12}[t]^{H} \tilde{\mathbf{h}}_{12}[t] + \rho \tilde{\mathbf{h}}_{22}[t]^{H} \tilde{\mathbf{h}}_{22}[t] \right| \\ &\stackrel{(c)}{=} f(2 - k_{t}, (\alpha_{2}, 1), (1, 1)) \log \rho + \mathcal{O}(1) \\ &= \begin{cases} (\min\{2 - k_{t}, 1\} + \\ \min\{[1 - k_{t}]^{+}, 1\} \alpha_{2}) \log \rho + \mathcal{O}(1), & \alpha_{2} \leq 1, \\ (\min\{2 - k_{t}, 1\} \alpha_{2} + \\ \min\{[1 - k_{t}]^{+}, 1\}) \log \rho + \mathcal{O}(1), & 1 < \alpha_{2}, \end{cases}$$
(7)

where (a) is from SVD of $\mathbf{K}[t]$, i.e., $\mathbf{K}[t] = \mathbf{U}[t]\boldsymbol{\Sigma}[t]\mathbf{U}[t]^H$ with unitary matrix $\mathbf{U}[t] \in \mathbb{C}^{2\times(2-k_t)}$ and diagonal matrix $\boldsymbol{\Sigma}[t] \in \mathbb{C}^{(2-k_t)\times(2-k_t)}$, and $\tilde{\mathbf{h}}_{j2}[t] \triangleq \mathbf{h}_{j2}[t]\mathbf{U}[t]\boldsymbol{\Sigma}[t]^{1/2}$, where $k_t \in \{0, 1, 2\}$ denotes the number of zero singular values; (b) is from $|\mathbf{I} + \mathbf{AB}| = |\mathbf{I} + \mathbf{BA}|$; (c) is from [11, Lemma 1]. The second term of (6) can be approximated as

$$\log \left| 1 + \rho^{\alpha_2} \mathbf{h}_{12}[t] \mathbf{K}[t] \mathbf{h}_{12}[t]^H \right|$$

$$\stackrel{(a)}{=} \min\{2 - k_t, 1\} \alpha_2 \log \rho + \mathcal{O}(1), \quad (8)$$

where (a) is from SVD of $\mathbf{K}[t]$ and [11, Lemma 1]. Next, we can approximate the first term of (4) as

$$\begin{aligned} h(y_{1}[t]|\mathcal{H}[t]) & \stackrel{(a)}{\leq} \log \left| 1 + \rho \mathbf{h}_{11}[t] \mathbf{L}[t] \mathbf{h}_{11}[t]^{H} + \rho^{\alpha_{2}} \mathbf{h}_{12}[t] \mathbf{K}[t] \mathbf{h}_{12}[t]^{H} \right| \\ \stackrel{(b)}{\leq} \log \left| 1 + \rho \mathbf{h}_{11}[t] \mathbf{h}_{11}[t]^{H} + \rho^{\alpha_{2}} \widetilde{\mathbf{h}}_{12}[t] \widetilde{\mathbf{h}}_{12}[t]^{H} \right| \\ \stackrel{(c)}{=} f(1, (1, 2), (\alpha_{2}, 2 - k_{t})) \log \rho + \mathcal{O}(1), \\ = \begin{cases} \log \rho + \mathcal{O}(1), & \alpha_{2} \leq 1, \\ (\min\{2 - k_{t}, 1\}\alpha_{2} + & (9) \\ \min\{[1 - (2 - k_{t})]^{+}, 2\}) \log \rho + \mathcal{O}(1), & 1 < \alpha_{2}, \end{cases} \end{aligned}$$

where (a) is from Gaussian input maximizing the entropy with covariance constraints; (b) is from $\mathbf{L}[t] \leq \mathbf{I}_2$ for tr{ $\mathbf{L}[t]$ } ≤ 1 and the SVD of $\mathbf{K}[t]$; and (c) is from [11, Lemma 1].

As such, the upper bound of weighted sum of achievable rates from (4) and (5) is given in (10), shown on the top of next page, where (a) is from (4), (5) and (9); (b) is from (7) and (8); (c) is from maximizer is $k_t = 0$ by exhausting $k_t \in \{0, 1, 2\}$. Then, we rewrite (10) into GDoF expression,

$$d_1(\alpha_1, \alpha_2) + \frac{d_2(\alpha_1, \alpha_2)}{2} \le \begin{cases} \frac{3 - \alpha_2}{2}, & \alpha_2 \le 1, \\ \frac{1 + \alpha_2}{2}, & 1 < \alpha_2, \end{cases}$$
(11)

Due to the symmetry, we have another GDoF inequality, i.e.,

$$d_2(\alpha_1, \alpha_2) + \frac{d_1(\alpha_1, \alpha_2)}{2} \le \begin{cases} \frac{3 - \alpha_1}{2}, & \alpha_1 \le 1, \\ \frac{1 + \alpha_1}{2}, & 1 < \alpha_1, \end{cases}$$
(12)

Moreover, considering single-user GDoF bound for MIMO point-to-point channel, we have

$$d_i(\alpha_1, \alpha_2) \le 1, \quad i = 1, 2.$$
 (13)

Combing (11) with (12) and (13), we derive the sum-GDoF upper bound in Theorem 1 (see (2)). This ends the proof.

V. ACHIEVABILITY PROOF OF THEOREM 1

A. Proposed Transmission Scheme for $1 < \alpha_1, \alpha_2$ and $1 < \alpha_1 \& \alpha_2 \le 1 \& 2 \le \alpha_1 + 2\alpha_2$ Cases

The proposed transmission scheme is with block-Markov structure, and has B blocks with s time slot each block. Without loss of generality, we assume s = 1.

In the $b^{th}(1 \le b < B)$ block, the transmitter Tx_i encodes the message $w_i[b]$ desired by receiver Rx_i using the vector $\mathbf{u}_i(w_i[b]) \in \mathbb{C}^{2\times 1}$, such that $\mathbf{u}_i \sim \mathcal{CN}(0, \rho^{-A_i}\mathbf{I}_2)$ with $0 \le A_i \le \alpha_1$. The common message $l_i[b-1]$ is encoded using the vector $\mathbf{x}_{ic}(l_i[b-1]) \in \mathbb{C}^{2\times 1}$, which is transmitted at transmitter Tx_i with power $\mathcal{O}(\rho^0)$. The transmit signal at block b and transmitter Tx_i can be written as follows:

$$\mathbf{x}_{i}[b] = \underbrace{\mathbf{u}_{i}(w_{i}[b])}_{\mathcal{O}(\rho^{-A_{i}})} + \underbrace{\mathbf{x}_{ic}(l_{i}[b-1])}_{\mathcal{O}(\rho^{0})}, \tag{14}$$

where i = 1, 2. The received signal at block b and receiver Rx_i is given as follows:

$$y_{i}[b] = \underbrace{\sqrt{\rho}\mathbf{h}_{ii}[b]\mathbf{x}_{ic}(l_{i}[b-1])}_{\mathcal{O}(\rho^{1})} + \underbrace{\sqrt{\rho^{\alpha_{j}}}\mathbf{h}_{ij}[b]\mathbf{x}_{jc}(l_{j}[b-1])}_{\mathcal{O}(\rho^{\alpha_{j}})} + \underbrace{\sqrt{\rho}\mathbf{h}_{ii}[b]\mathbf{u}_{i}(w_{i}[b])}_{\mathcal{O}(\rho^{1-A_{i}})} + \underbrace{\sqrt{\rho^{\alpha_{j}}}\mathbf{h}_{ij}[b]\mathbf{u}_{j}(w_{j}[b])}_{\eta_{i}[b]\sim\mathcal{O}(\rho^{\alpha_{j}-A_{j}})} + n_{i}[b], (15)$$

where $i \neq j$, and $\eta_i[b]$ denotes the interference at receiver Rx_i , which can be reconstructed at block b + 1. From the rate distortion theorem [13], this interference $\eta_i[b]$ can be quantized using a source codebook with size $\mathcal{O}(\rho^{\alpha_j - A_j})$, such that the average distortion does not exceed the noise power level and can be ignored in GDoF analysis. The quantization index of interference $\eta_i[b]$ is denoted by $l_j[b]$, which is transmitted as $\mathbf{x}_{jc}(l_j[b])$ from transmitter Tx_j in block b + 1.

In the B^{th} block, the transmitter Tx_i sends common messages only, namely

$$\mathbf{x}_i[B] = \underbrace{\mathbf{x}_{ic}(l_i[B-1])}_{\mathcal{O}(a^0)},\tag{16}$$

where i = 1, 2. The received signal at block B and receiver Rx_i is given as follows:

$$y_{i}[B] = \underbrace{\sqrt{\rho} \mathbf{h}_{ii}[B] \mathbf{x}_{ic}(l_{i}[B-1])}_{\mathcal{O}(\rho^{1})} + \underbrace{\sqrt{\rho^{\alpha_{j}}} \mathbf{h}_{ij}[B] \mathbf{x}_{jc}(l_{j}[B-1])}_{\mathcal{O}(\rho^{\alpha_{j}})} + n_{i}[B], \quad (17)$$

where i = 1, 2. The decoding procedure is backward and begins with block B, where the common messages are firstly decoded. After that, at the $(B-1)^{th}$ block, the interference from transmitter Tx_j can be canceled and extra information about $\mathbf{u}_i(w_i[B-1])$ can be provided. Generally, the equivalent channel for decoding can be written as follows:

$$\underbrace{\begin{bmatrix} y_i[b] - \eta_i[b] \\ \eta_j[b] \end{bmatrix}}_{Y'_i} = \underbrace{\begin{bmatrix} \sqrt{\rho} \mathbf{h}_{ii}[b] \\ \mathbf{0} \end{bmatrix}}_{\mathbf{S}_{ic}} \underbrace{\mathbf{x}_{ic}(l_i[b-1])}_{d_{\eta_j}} \\ + \underbrace{\begin{bmatrix} \sqrt{\rho^{\alpha_j}} \mathbf{h}_{ij}[b] \end{bmatrix}}_{\mathbf{S}_{jc}} \underbrace{\mathbf{x}_{jc}(l_j[b-1])}_{d_{\eta_i}} \\ + \underbrace{\begin{bmatrix} \sqrt{\rho} \mathbf{h}_{ii}[b] \\ \sqrt{\rho^{\alpha_i}} \mathbf{h}_{ji}[b] \end{bmatrix}}_{\mathbf{S}_i} \underbrace{\mathbf{u}_i(w_i[b])}_{d_i[b]} + \begin{bmatrix} n_i[b] \\ 0 \end{bmatrix}, \quad (18)$$

where i, j = 1, 2 and $i \neq j$. Note that (18) is equivalent to the three-user multiple-access channel (MAC). Applying capacity region of three-user MAC, we have the following general condition for achievable GDoF tuple to our problem:

Proposition 1: For the GDoF tuple $(d_{\eta_1}, d_{\eta_2}, d_1[b], d_2[b])$, which denote the GDoF carried in $\mathbf{x}_{2c}(l_2[b-1])$, $\mathbf{x}_{1c}(l_1[b-1])$

$$n\left(R_{1} + \frac{R_{2}}{2} - \epsilon_{n}\right) \stackrel{(a)}{\leq} \sum_{t=1}^{n} f(N, (1, M), (\alpha_{2}, M - k_{t})) \log \rho + \frac{1}{2} \sum_{t=1}^{n} h(\overline{y}_{1}[t], \overline{y}_{2}[t]|\mathcal{V}[t]) - \sum_{t=1}^{n} h(\overline{y}_{1}[t]|\mathcal{V}[t]) + n\mathcal{O}(1)$$

$$\stackrel{(b)}{\leq} \begin{cases} \sum_{t=1}^{n} ((1 + \min\{2 - k_{t}, 1\}/2 + \min\{[1 - k_{t}]^{+}, 1\}\alpha_{2}/2 - \min\{2 - k_{t}, 1\}\alpha_{2}) \log \rho + \mathcal{O}(1)), & \alpha \leq 1, \\ \sum_{t=1}^{n} ((\min\{[1 - (2 - k_{t})]^{+}, 2\} + \min\{2 - k_{t}, 1\}\alpha_{2}/2 + \min\{[1 - k_{t}]^{+}, 1\}/2) \log \rho + \mathcal{O}(1)), & 1 < \alpha_{2}. \end{cases}$$

$$\stackrel{(c)}{\leq} \begin{cases} \frac{3 - \alpha_{2}}{2} n \log \rho + \mathcal{O}(1), & \alpha_{2} \leq 1, \\ \frac{1 + \alpha_{2}}{2} n \log \rho + \mathcal{O}(1), & 1 < \alpha_{2}, \end{cases}$$

$$(10)$$

1]), $\mathbf{u}_1(w_1[b])$, and $\mathbf{u}_2(w_2[b])$, respectively, we have (19)-(29), where $f(\cdot)$ and $g(\cdot)$ are defined in [11, Lemmas 1 & 2].

$$d_{\eta_1} \le \min\{\alpha_2, 1\},\tag{19}$$

$$d_{\eta_2} \le \min\{\alpha_1, 1\},\tag{20}$$

$$d_1[b] \le f(2, (1 - A_1, 1), (\alpha_1 - A_1, 1)), \tag{21}$$

$$d_2[b] \le f(2, (1 - A_2, 1), (\alpha_2 - A_2, 1)), \tag{22}$$

$$d_{\eta_1} + d_{\eta_2} \le f(1, (1, 2), (\alpha_2, 2)), \tag{23}$$

$$d_{\eta_1} + d_1[b] \le \alpha_1 - A_1 + g(1, (\alpha_2, 2), (1 - \alpha_1, 1), (1 - A_1, 1)), \quad (24)$$

$$d_{\eta_2} + d_2[b] \le \alpha_2 - A_2$$

$$+g(1, (\alpha_1, 2), (1 - \alpha_2, 1), (1 - A_2, 1)),$$
 (25)

$$d_{\eta_2} + d_1[b] \le \alpha_1 - A_1 + 1, \tag{26}$$

$$d_{\eta_1} + d_2[b] \le \alpha_2 - A_2 + 1, \tag{27}$$

$$d_{\eta_1} + d_{\eta_2} + d_1[b]$$

.

$$\leq \alpha_1 - A_1 + f(1, (1, 2), (\alpha_2, 2)), \tag{28}$$

$$d_{\eta_1} + d_{\eta_2} + d_2[b]$$

$$\leq \alpha_2 - A_2 + f(1, (1, 2), (\alpha_1, 2)),$$
 (29)

Proof: The proof is similar to that in [11]. Thus, it is omitted for simplicity.

For the block-Markov transmission, the achievable GDoF of receiver *i* is calculated as $d_i = \lim_{b\to\infty} \frac{1}{B} \sum_{b=1}^{B} d_i[b] = d_i[B]$. Moreover, d_{η_i} is allocated as $\alpha_j - A_j$, since the common message from transmitter Tx_i need to be decoded at receiver Rx_j . In the following, we analyze the achievable sum-GDoF case by case, by means of Proposition 1.

l) 1 < α_1, α_2 *Case:* According to Proposition 1, we present the achievable GDoF condition in this case as follows:

$$\alpha_1 - A_1 \le 1, \quad \alpha_2 - A_2 \le 1,$$
 (30)

$$d_1 \le \alpha_1 + 1 - 2A_1, \tag{31}$$

$$d_2 \le \alpha_2 + 1 - 2A_2, \tag{32}$$

$$\alpha_1 + \alpha_2 \le A_1 + A_2 + 1, \tag{33}$$

$$d_1 \le \alpha_1 - A_1 + A_2, \tag{34}$$

$$d_2 \le \alpha_2 - A_2 + A_1, \tag{35}$$

$$d_1 \le 1,\tag{36}$$

$$d_2 \le 1,\tag{37}$$

$$d_1 \le A_2,\tag{38}$$

$$d_2 \le A_1. \tag{39}$$

Therefore, we are able to formulate the following sum-GDoF lower bound maximization problem:

$$\max_{A_1,A_2} \min\{A_1 + A_2, \alpha_1 + \alpha_2 + 2 - 2(A_1 + A_2), 2\}, \quad (40)$$

where the maximizer is $A_1^* + A_2^* = \min\{(2 + \alpha_1 + \alpha_2)/3, 2\}$. This leads to the sum-GDoF lower bound $\min\{(2 + \alpha_1 + \alpha_2)/3, 2\}$ achievable.

2) $1 < \alpha_1 \& \alpha_2 < 1 \& 2 \le \alpha_1 + 2\alpha_2$ *Case:* According to Proposition 1, we present the achievable GDoF condition in this case as follows:

$$0 \le A_2, \quad \alpha_1 - A_1 \le 1,$$
 (41)

$$d_1 \le \alpha_1 + 1 - 2A_1, \tag{42}$$

$$d_2 \le \alpha_2 + 1 - 2A_2, \tag{43}$$

$$\alpha_1 + \alpha_2 \le 1 + A_1 + A_2, \tag{44}$$

$$d_1 \leq \begin{cases} \alpha_1 - A_1 + A_2, & 1 - A_1 \leq \alpha_2, \\ \alpha_1 - \alpha_2 + A_2 + 1 - 2A_1, & 1 - A_1 > \alpha_2, \end{cases} (45)$$

$$d_2 \le \alpha_2 - A_2 + A_1, \tag{46}$$

$$d_1 \le 1,\tag{47}$$

$$d_2 \le 1,\tag{48}$$

$$d_1 \le A_2 - \alpha_2 + 1, \tag{49}$$

$$d_2 \le A_1. \tag{50}$$

Therefore, due to $2 \le \alpha_1 + 2\alpha_2$, we are able to formulate the following sum-GDoF lower bound maximization problem:

$$\max_{A_1,A_2} \min\{A_1 + A_2 - \alpha_2 + 1, \alpha_1 + \alpha_2 + 2 - 2(A_1 + A_2)\},$$
(51)

where the maximizer is $A_1^* + A_2^* = (\alpha_1 + 2\alpha_2 + 1)/3$. This leads to the sum-GDoF lower bound $(4 + \alpha_1 - \alpha_2)/3$ achievable.

B. Proposed Transmission Scheme for $\alpha_1, \alpha_2 \leq 1$ Case

In the 1st time slot, the transmitter Tx_1 sends three symbols for receiver Rx_1 and transmitter Tx_2 sends one symbol for receiver Rx_2 . Let us denote the symbols desired by receiver Rx_1 transmitted in time slot 1 by a_1, a_2, a_3 , and denote the symbol desired by receiver Rx_2 transmitted in time slot 1 by b_1 . The transmit signal at transmitter Tx_1 is designed as

$$\mathbf{x}_{1}[1] = \begin{bmatrix} a_{1} \\ a_{2} \end{bmatrix} + \begin{bmatrix} a_{3}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}.$$
 (52)

The transmit signal at transmitter Tx₂ is designed as

$$\mathbf{x}_{2}[1] = \begin{bmatrix} b_{1}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}.$$
 (53)

As such, the received signals at each receiver are expressed as

$$y_{1}[1] = \underbrace{\sqrt{\rho}\mathbf{h}_{11}[1] \begin{bmatrix} a_{1} \\ a_{2} \end{bmatrix}}_{\mathcal{O}(\rho^{1-(1-\alpha_{1})}) = \mathcal{O}(\rho^{\alpha_{1}})} + \underbrace{\sqrt{\rho}\mathbf{h}_{11}[1] \begin{bmatrix} a_{3}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-\alpha_{1}})} + \underbrace{\sqrt{\rho^{\alpha_{2}}}\mathbf{h}_{12}[1] \begin{bmatrix} b_{1}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{0})},$$
(54a)

$$y_{2}[1] = \underbrace{\sqrt{\rho^{\alpha_{1}}} \mathbf{h}_{21}[1] \begin{bmatrix} a_{1} \\ a_{2} \end{bmatrix}}_{\mathcal{O}(\rho^{\alpha_{1}})} + \underbrace{\sqrt{\rho^{\alpha_{1}}} \mathbf{h}_{21}[1] \begin{bmatrix} a_{3}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{0})} + \underbrace{\sqrt{\rho} \mathbf{h}_{22}[1] \begin{bmatrix} b_{1}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-\alpha_{1}})}.$$
 (54b)

It can be seen that a part of interference fall into noise level. Moreover, each receiver can retrieve the profile of $\mathcal{O}(\rho^{\alpha_1})$ part and decode the information of $\mathcal{O}(\rho^{1-\alpha_1})$ part immediately.

In the 2nd time slot, the transmitter Tx_2 sends three symbols for receiver Rx_2 and transmitter Tx_1 sends three symbols for receiver Rx_1 . Let us denote the symbols desired by receiver Rx_2 transmitted in time slot 2 by b_2, b_3, b_4 , and denote the symbol desired by receiver Rx_1 transmitted in time slot 2 by a_4 . The transmit signal at transmitter Tx_1 is designed as

$$\mathbf{x}_1[2] = \begin{bmatrix} a_4 \rho^{-\alpha_2/2} \\ \phi \end{bmatrix}.$$
 (55)

The transmit signal at transmitter Tx₂ is designed as

$$\mathbf{x}_{2}[2] = \begin{bmatrix} b_{2} \\ b_{3} \end{bmatrix} + \begin{bmatrix} b_{4}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix}.$$
 (56)

As such, the received signals at each receiver are expressed as

$$y_{1}[2] = \underbrace{\sqrt{\rho}\mathbf{h}_{11}[2] \begin{bmatrix} a_{4}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-\alpha_{2}})} + \underbrace{\sqrt{\rho^{\alpha_{2}}}\mathbf{h}_{12}[2] \begin{bmatrix} b_{2} \\ b_{3} \end{bmatrix}}_{\mathcal{O}(\rho^{\alpha_{2}})} + \underbrace{\sqrt{\rho^{\alpha_{2}}}\mathbf{h}_{12}[2] \begin{bmatrix} b_{4}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho)},$$
(57a)

$$y_{2}[2] = \underbrace{\sqrt{\rho^{\alpha_{2}}} \mathbf{h}_{21}[2] \begin{bmatrix} a_{4}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{0})} + \underbrace{\sqrt{\rho} \mathbf{h}_{22}[2] \begin{bmatrix} b_{2} \\ b_{3} \end{bmatrix}}_{\mathcal{O}(\rho^{1-(1-\alpha_{2})}) = \mathcal{O}(\rho^{\alpha_{2}})} + \underbrace{\sqrt{\rho} \mathbf{h}_{22}[2] \begin{bmatrix} b_{4}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-\alpha_{2}})}.$$
(57b)

It can be seen that a part of interference fall into noise level. Moreover, each receiver can retrieve the profile of $\mathcal{O}(\rho^{\alpha_2})$ part and decode the information of $\mathcal{O}(\rho^{1-\alpha_2})$ part immediately.

In the 3rd time slot, each transmitter has delayed CSI of past two time slots. Thus, the transmitter Tx_1 re-constructs the interfering signals and treat it as a common symbol, i.e., $c_1 \triangleq \sqrt{\rho^{\alpha_1}}\mathbf{h}_{21}[1][a_1;a_2]$, where c_1 is a valid codeword if a_1, a_2 are selected from lattice. The transmitter Tx_2 re-constructs the interfering signals and treat it as a common symbol, i.e., $c_2 \triangleq \sqrt{\rho^{\alpha_2}}\mathbf{h}_{12}[2][b_2;b_3]$, where c_2 is a valid codeword if b_2, b_3 are selected from lattice. The transmit signals at each transmitter are designed as

$$\mathbf{x}_{1}[3] = \begin{bmatrix} c_{1} \\ \phi \end{bmatrix} + \begin{bmatrix} a_{5}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix},$$
(58a)

$$\mathbf{x}_{2}[3] = \begin{bmatrix} c_{2} \\ \phi \end{bmatrix} + \begin{bmatrix} b_{5}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix},$$
(58b)

where a_5, b_5 are symbols desired by receivers Rx_1 and Rx_2 , respectively. As such, the received signals at each receiver are expressed as

$$y_{1}[3] = \underbrace{\sqrt{\rho}\mathbf{h}_{11}[3] \begin{bmatrix} c_{1} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-(1-\alpha_{1})}) = \mathcal{O}(\rho^{\alpha_{1}})} + \underbrace{\sqrt{\rho}\mathbf{h}_{11}[3] \begin{bmatrix} a_{5}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-\alpha_{1}})} + \underbrace{\sqrt{\rho^{\alpha_{2}}\mathbf{h}_{12}[3] \begin{bmatrix} c_{2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{0})} + \underbrace{\sqrt{\rho^{\alpha_{2}}\mathbf{h}_{12}[3] \begin{bmatrix} b_{5}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{0})}, \quad (59a)$$

$$y_{2}[3] = \underbrace{\sqrt{\rho}\mathbf{h}_{22}[3] \begin{bmatrix} c_{2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-(1-\alpha_{2})}) = \mathcal{O}(\rho^{\alpha_{2}})} + \underbrace{\sqrt{\rho}\mathbf{h}_{22}[3] \begin{bmatrix} b_{5}\rho^{-\alpha_{2}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{1-\alpha_{2}})} + \underbrace{\sqrt{\rho^{\alpha_{1}}\mathbf{h}_{21}[3] \begin{bmatrix} a_{5}\rho^{-\alpha_{1}/2} \\ \phi \end{bmatrix}}_{\mathcal{O}(\rho^{0})}, \quad (59b)$$

where the impact of $\sqrt{\rho^{\alpha_2}} \mathbf{h}_{12}[3][c_2; \phi]$ can be subtracted from $y_1[3]$ and the $\sqrt{\rho^{\alpha_1}} \mathbf{h}_{21}[3][c_1; \phi]$ can be subtracted from $y_2[3]$. It can be seen that a part of interference fall into noise level. Moreover, receiver \mathbf{Rx}_j can retrieve the profile of $\mathcal{O}(\rho^{\alpha_j})$ part and decode the information of $\mathcal{O}(\rho^{1-\alpha_j})$ part immediately.

The achievable sum-GDoF is calculated as follows: Receiver Rx_1 acquires $1 - \alpha_1$, $1 - \alpha_2$, and $(1 - \alpha_1) \log \rho + O(1)$ immediately in time slot 1, 2 and 3, respectively. Additionally, receiver Rx_1 has $2\alpha_1 \log \rho + O(1)$ via delayed CSIT. Thus, $d_1 \ge 1 - \alpha_2/3$ is achievable. Likewise, $d_2 \ge 1 - \alpha_1/3$ is achievable. To sum up, $d_1+d_2 \ge 2-(\alpha_1+\alpha_2)/3$ is achievable.

VI. CONCLUSION

The sum-GDoF was characterized in the asymmetry interference channel with delayed CSIT, where each transmitter has 2 antennas and each receiver has 1 antenna. In the future, it is interesting to design a better transmission scheme in $1 < \alpha_1 \& \alpha_2 < 1 \& \alpha_1 + 2\alpha_2 < 2$ Case.

REFERENCES

- C. S. Vaze and M. K. Varanasi, "The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSIT," *IEEE Trans. Inf Theory*, vol. 58, no. 7, pp. 4396–4417, 2012.
- [2] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, "On the degrees of freedom of three-user MIMO broadcast channel with delayed CSIT," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, St. Petersburg, Russia, 2011, pp. 209–213.
- [3] T. Zhang, X. W. Wu, Y. F. Xu, Y. Ge, and P. C. Ching, "Threeuser MIMO broadcast channel with delayed CSIT: A higher achievable DoF," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.* (ICASSP), 2018, pp. 3709–3713.
- [4] M. J. Abdoli, "Feedback and cooperation in wireless networks," in *PhD Dissertation, University of Waterloo*, 2012.
- [5] T. Zhang and R. Wang, "Achievable DoF regions of three-user MIMO broadcast channel with delayed CSIT," *IEEE Trans. Commun.*, vol. 69, no. 4, pp. 2240–2253, 2021.
- [6] D. T. H. Kao and A. S. Avestimehr, "Linear degrees of freedom of the MIMO X-channel with delayed CSIT," *IEEE Trans. Inf Theory*, vol. 63, no. 1, pp. 297–319, 2017.
- [7] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, 2008.
- [8] J. Chen, P. Elia, and S. A. Jafar, "On the two-user MISO broadcast channel with alternating CSIT: A topological perspective," *IEEE Trans. Inf Theory*, vol. 61, no. 8, pp. 4345–4366, 2015.
- [9] Z. H. Awan and A. Sezgin, "Secure MISO broadcast channel: An interplay between CSIT and network topology," *IEEE J. Sel. Areas Inf. Theory*, vol. 2, no. 1, pp. 121–138, 2021.
- [10] K. Mohanty and M. K. Varanasi, "The generalized degrees of freedom region of the MIMO Z-interference channel with delayed CSIT," *IEEE Trans. Inf. Theory*, vol. 64, no. 1, pp. 531–546, 2018.
- [11] —, "On the generalized degrees of freedom of the MIMO interference channel with delayed CSIT," *IEEE Trans. Inf. Theory*, vol. 65, no. 5, pp. 3261–3277, 2019.
- [12] T. Liu and P. Viswanath, "An extremal inequality motivated by multiterminal information-theoretic problems," *IEEE Trans. Inf Theory*, vol. 53, no. 5, pp. 1839–1851, 2007.
- [13] T. M. Cover, *Elements of information theory*. John Wiley & Sons, 1999.