AoI in Source-Aware Preemptive M/G/1/1 Queueing Systems: Moment Generating Function

Mohammad Moltafet, Markus Leinonen, and Marian Codreanu

***Abstract*—We consider a multi-source status update system consisting of multiple independent sources, one server, and one sink. The packets of the sources are generated according to Poisson processes and served according to a generally distributed service time. We consider a system with no waiting buffer and model it as a multi-source M/G/1/1 queueing model. We introduce a source-aware preemptive packet management policy and subsequently derive the moment generating functions (MGFs) of the age of information (AoI) and peak AoI of each source. According to the policy, when a packet arrives, the possible packet of the same source in the system is replaced by the fresh packet. Simulation results show the performance of the packet management policy.**

I. Introduction

Timely delivery of the status updates of various real-world physical processes plays a critical role in enabling the time-critical Internet of Things (IoT) applications. The age of information (AoI) was introduced in the seminal work [1] as a destination-centric metric to measure the information freshness in status update systems. A status update packet contains the measured value of a monitored process and a time stamp representing the time at which the sample was generated. Due to wireless channel access, channel errors, fading, etc. communicating a status update packet through the network experiences a random delay. If at a time instant t, the most recently received status update packet contains the time stamp $U(t)$, AoI is defined as the random process $\Delta(t) = t - U(t)$. Thus, the AoI measures for each source node the time elapsed since the last received status update packet was generated at the source node.

The first queueing theoretic work on AoI is [2] where the authors derived the average AoI for M/M/1, D/M/1, and M/D/1 first-come first-served (FCFS) queueing models. In [3], the authors proposed peak AoI as an alternative metric to evaluate the information freshness. The work in [4] was the first to investigate the AoI in a multi-source setup in which the authors derived an approximate expression for the average AoI in a multi-source M/M/1 FCFS queueing model.

It has been shown that an appropriate packet management policy – in the waiting queue or/and server – has a great potential to improve the information freshness in status update systems [5], [6]. AoI under various packet management policies in queueing systems with exponentially distributed service time and Poisson process arrivals has been extensively studied [7]–[16].

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Besides exponentially distributed service time and Poisson arrivals, AoI has been studied under various arrival processes and service time distributions in both single- and multi-source systems. In [17], the authors derived various approximations for the average AoI in a multi-source M/G/1 FCFS queueing model. The work in [18] derived the distribution of the AoI and peak AoI for the single-source PH/PH/1/1 and M/PH/1/2 queueing models. The authors of [19] analyzed the AoI in a single-source D/G/1 FCFS queueing model. The authors of [20] derived a closed-form expression for the average AoI of a single-source M/G/1/1 preemptive queueing model with hybrid automatic repeat request. The stationary distributions of the AoI and peak AoI of single-source M/G/1/1 and G/M/1/1 queueing models were derived in [21]. In [22], the authors derived a general formula for the stationary distribution of the AoI in single-source single-server queueing systems. The work in [23] considered a single-source last-come first-served (LCFS) queueing model where the packets arrive according to a Poisson process and the service time follows a gamma distribution. They derived the average AoI and average peak AoI for the preemptive LCFS policy and the non-preemptive LCFS policy, wherein, when the server is busy, any arriving packet is blocked and cleared. The work in [24], [25] derived the average AoI expression for a single-source G/G/1/1 queueing model under two packet management policies. The authors of [26] considered a multi-source M/G/1 queueing system and optimized the arrival rates of each source to minimize the peak AoI. The average AoI and average peak AoI of a multi-source M/G/1/1 queueing model under the source-agnostic preemption policy were derived in [27]. In [28], the authors derived the average AoI for a queueing system with two classes of Poisson arrivals with different priorities under a general service time distribution. They assumed that the system contains at most one packet and a newly arriving packet replaces the possible currently-in-service packet with the same or lower priority. The average AoI and average peak AoI of a multi-source M/G/1/1 queueing model under the source-agnostic non-preemptive policy were derived in [29].

We consider a multi-source status update system of capacity one packet (i.e., no waiting buffer), where the source packets are generated according to Poisson processes and served according to a generally distributed service time. For this multi-source M/G/1/1 queueing system, we derive the MGFs of the AoI and peak AoI under a source-aware preemptive policy. According to the policy, when a packet arrives, the possible packet of the same source in the system is replaced by the fresh packet. By using the MGFs of the AoI and peak AoI, the

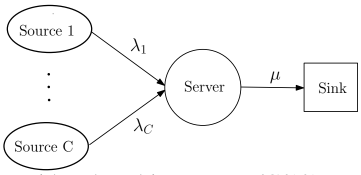

Fig. 1: The considered multi-source M/G/1/1 queueing system.

average AoI and average peak AoI of a two-source M/G/1/1 queueing system are derived. The numerical results show that, depending on the system parameters, the proposed source-aware preemptive packet management policy can outperform the source-agnostic preemptive and non-preemptive policy proposed in [27] and [29], respectively, from the perspective of average AoI.

II. System Model and Main Results

We consider a status update system consisting of a set $\mathcal{C} = \{1, \ldots, C\}$ of independent sources, one server, and one sink, as depicted in Fig. 1. Each source sends status information about a random process to the sink as status update packets containing the measured value of the monitored process and a time stamp representing the time when the sample was generated. We assume that the packets of source $c \in \mathcal{C}$ are generated according to the Poisson process with rate λ_c. Since the source packets are generated according to independent Poisson processes, the packet generation in the system follows the Poisson process with rate $\lambda = \sum_{c' \in \mathcal{C}} \lambda_{c'}$. The server serves the packets according to a generally distributed service time with rate μ. We assume that the service times of packets are independent and identically distributed (i.i.d.) random variables following a general distribution. Finally, we consider that the capacity of the system is one (i.e., there is no waiting buffer) and thus, the considered setup is referred to as a multi-source M/G/1/1 queueing system.

Source-Aware Preemptive Policy: According to the policy, a new arriving packet preempts the possible packet of the same source in the system. Whenever the new arriving packet finds a packet of another source under service, the arriving packet is blocked and cleared.

A. AoI Definition

For each source, the AoI at the sink is defined as the time elapsed since the last successfully received packet was generated. Formally, let $t_{c,i}$ denote the time instant at which the ith delivered status update packet of source c was generated, and let $t'_{c,i}$ denote the time instant at which this packet arrives at the sink. Let $\bar{t}_{c,i}$ denote the generation time of the ith packet of source c that does not complete service because of the packet management policy (i.e., the packet is either preempted by another packet or it is blocked and cleared).

At a time instant τ, the index of the most recently received packet of source c is given by

$$N_c(\tau) = \max\{i' \mid t'_{c,i'} \leq \tau\}, \tag{1}$$

and the time stamp of the most recently received packet of source c is $U_c(\tau) = t_{c,N_c(\tau)}$. The AoI of source c at the sink is defined as the random process $\delta_c(t) = t - U_c(t)$.

Let the random variable

$$Y_{c,i} = t'_{c,i+1} - t'_{c,i} \tag{2}$$

represent the ith interdeparture time of source c, i.e., the time elapsed between the departures of ith and $i+1$th (delivered) packets from source c. From here onwards, we refer to the ith delivered packet from source c simply as "packet c, i". Moreover, let the random variable

$$T_{c,i} = t'_{c,i} - t_{c,i} \tag{3}$$

represent the system time of packet c, i, i.e., the duration this (delivered) packet spends in the system.

One of the most commonly used metrics for evaluating the AoI of a source at the sink is the peak AoI [3]. The peak AoI of source c at the sink is defined as the value of the AoI immediately before receiving an update packet. Accordingly, the peak AoI concerning the ith successfully received packet of source c, denoted by $A_{c,i}$, is given by

$$A_{c,i} = Y_{c,i-1} + T_{c,i-1}. \tag{4}$$

We assume that the considered status update system is stationary so that $T_{c,i} =^{\text{st}} T_c$, $Y_{c,i} =^{\text{st}} Y_c$, and $A_{c,i} =^{\text{st}} A_c, \forall i$, where $=^{\text{st}}$ means stochastically identical (i.e., they have an identical marginal distribution). We further assume that the AoI process for each source is ergodic.

The main results of our paper are presented in Theorem 1.

Theorem 1. *Let S be the random variable representing the service time of any packet in the system. The MGFs of the AoI and peak AoI of source c under the source-aware preemptive packet management policy, denoted by $M_{\delta_c}(s)$ and $M_{A_c}(s)$, respectively, are given as*

$$M_{\delta_c}(s) = \frac{M_S(s-\lambda_c)(M_{Y_c}(s)-1)}{sL_{\lambda_c}M'_{Y_c}(0)}, \tag{5}$$

$$M_{A_c}(s) = \frac{M_S(s-\lambda_c)M_{Y_c}(s)}{L_{\lambda_c}}, \tag{6}$$

where $L_{\lambda_c} = \mathbb{E}[e^{-\lambda_c S}]$, $M_S(s-\lambda_c) = \mathbb{E}[e^{(s-\lambda_c)S}]$ is the MGF of the service time S at $s-\lambda_c$, $M_{Y_c}(s)$ is the MGF of the interdeparture time Y_c under the policy, which is given as

$$M_{Y_c}(s) = \frac{a_c M_S(s-\lambda_c)}{(1-a'_c)\left(1 - \sum_{c' \in \mathcal{C}\setminus\{c\}} \dfrac{a_{c'}M_S(s-\lambda_{c'})}{1-a'_{c'}}\right)}, \tag{7}$$

where $a_c = \dfrac{\lambda_c}{\lambda - s}$ and $a'_c = \dfrac{\lambda_c(1-M_S(s-\lambda_c))}{\lambda_c - s}$, and $M'_{Y_c}(0)$ is the first derivative of the MGF of Y_c, evaluated at $s=0$, i.e.,

$$M'_{Y_c}(0) = \frac{\mathrm{d}(M_{Y_c}(s))}{\mathrm{d}s}\Big|_{s=0}.$$

Remark 1. The mth moment of the AoI (peak AoI) is derived by calculating the mth derivative of the MGF of the AoI (peak AoI) when $s \to 0$.

Corollary 1. *The average AoI and average peak AoI of source 1 in a two-source M/G/1/1 queueing model under the source-aware preemptive packet management policy are given as*

$$\bar{\Delta}_1=\frac{L^2_{\lambda_1}(\lambda(1-L_{\lambda_2})-\lambda_1\lambda_2L'_{\lambda_2})+L^2_{\lambda_2}(\lambda_2(1-L_{\lambda_1}))-\Psi}{\lambda_1\lambda_2L_{\lambda_1}L_{\lambda_2}(L_{\lambda_1}+L_{\lambda_2}-L_{\lambda_1}L_{\lambda_2})},$$

$$\bar{A}_1=\frac{L_{\lambda_1}+L_{\lambda_2}-L_{\lambda_1}L_{\lambda_2}+\lambda_1L_{\lambda_2}L'_{\lambda_1}}{\lambda_1L_{\lambda_1}L_{\lambda_2}},$$

where $\Psi = \lambda_1\lambda_2L_{\lambda_1}L'_{\lambda_1} + L_{\lambda_1}L_{\lambda_2}\lambda_2(1+\lambda_1L'_{\lambda_1})$ *and* $L'_{\lambda_c}=\mathbb{E}[Se^{-\lambda_cS}]$.

Remark 2. *The average AoI under the source-aware preemptive policy, presented in Corollary 1, generalizes the existing results in [20] and [12]. Specifically, when confining to a single-source case by letting* $\lambda_2\to 0$*, the average AoI becomes equal to that of the single-source M/G/1/1 queueing model with preemption derived in [20]. Moreover, when we consider an exponentially distributed service time, the average AoI expression coincides with that of the multi-source M/M/1/1 queueing model with preemption derived in [12].*

III. Derivation of the MGFs of the AoI and Peak AoI

In this section, we prove Theorem 1. To begin, we first provide Lemma 1 which presents the MGF of the AoI of source c in the considered multi-source M/G/1/1 queueing model as a function of the MGFs of the system time of source c, T_c, and interdeparture time of source c, Y_c.

Lemma 1. *The MGFs of the AoI and peak AoI of source c in a multi-source M/G/1/1 queueing model under the source-aware preemptive policy, denoted by* $M_{\delta_c}(s)$ *and* $M_{A_c}(s)$*, respectively, can be expressed as*

$$M_{\delta_c}(s)=\frac{M_{T_c}(s)(M_{Y_c}(s)-1)}{s\mathbb{E}[Y_c]}, \tag{8}$$

$$M_{A_c}(s)=M_{T_c}(s)M_{Y_c}(s), \tag{9}$$

where $M_{T_c}(s)$ *is the MGF of the system time of a delivered packet of source c and* $M_{Y_c}(s)$ *is the MGF of the interdeparture time of source c.*

Proof. See [30, Lemma 1]. □

According to Lemma 1, the main challenge in calculating the MGFs of the AoI (see (8)) and peak AoI (see (9)) reduces to deriving the MGF of the system time of source c, $M_{T_c}(s)$, and the MGF of the interdeparture time of source c, $M_{Y_c}(s)$. Note that when we have $M_{Y_c}(s)$, we can easily derive $\mathbb{E}[Y_c]$ (as showed in Remark 1).

To derive the MGF of the system time of source c, we first derive the probability density function (PDF) of the system time, $f_{T_c}(t)$, which is given by the following lemma.

Lemma 2. *The PDF of the system time of source c,* $f_{T_c}(t)$*, is given by*

$$f_{T_c}(t)=\frac{f_S(t)e^{-\lambda_ct}}{L_{\lambda_c}}. \tag{10}$$

Proof. The system time of a delivered packet from source c is equal to the service time of the packet. Let X_c be a random variable representing the interarrival time between two consecutive packets of source c. Thus, the distribution of T_c is given by $\Pr(T_c>t)=\Pr(S>t\mid S<X_c)$. Hence, $f_{T_c}(t)$ is calculated as

$$\begin{aligned}f_{T_c}(t)&=\lim_{\epsilon\to0}\frac{\Pr(t<T_c<t+\epsilon)}{\epsilon}\\&=\lim_{\epsilon\to0}\frac{\Pr(t<S<t+\epsilon\mid S<X_c)}{\epsilon}\\&=\lim_{\epsilon\to0}\frac{\Pr(t<S<t+\epsilon)\Pr(S<X_c\mid t<S<t+\epsilon)}{\epsilon\Pr(S<X_c)}\\&=\frac{f_S(t)\Pr(X_c>t)}{\Pr(S<X_c)}\overset{(a)}{=}\frac{f_S(t)e^{-\lambda_ct}}{L_{\lambda_c}},\end{aligned} \tag{11}$$

where (a) follows because i) the interarrival times of the source c packets follow the exponential distribution with parameter λ_c and thus, $\Pr(X_c>t)=1-F_{X_c}(t)=e^{-\lambda_ct}$, where $F_{X_c}(t)$ is the cumulative distribution function (CDF) of the interarrival time X_c and ii) $\Pr(S<X_c)$ is given as

$$\begin{aligned}\Pr(S<X_c)&=\int_0^\infty\Pr(S<X_c\mid X_c=t)f_{X_c}(t)\mathrm{d}t\\&=\int_0^\infty F_s(t)\lambda_ce^{-\lambda_ct}\mathrm{d}t\overset{(b)}{=}L_{\lambda_c},\end{aligned} \tag{12}$$

where $F_s(t)$ is the CDF of the service time S, and (b) follows from a feature of the Laplace transform that for any function $f(y), y\ge0$, we have [31, Sect. 13.5]

$$L_{\int_0^y f(b)\mathrm{d}b}(s)=\frac{L_{f(y)}(s)}{s}, \tag{13}$$

where $L_{f(y)}(s)$ is the Laplace transform of $f(y)$. □

Using Lemma 2, the MGF of the system time of source c, $M_{T_c}(s)=\int_0^\infty e^{st}f_{T_c}(t)\mathrm{d}t$, is given as

$$M_{T_c}(s)=\frac{1}{L_{\lambda_c}}\int_0^\infty e^{(s-\lambda_c)t}f_S(t)\mathrm{d}t=\frac{M_S(s-\lambda_c)}{L_{\lambda_c}}. \tag{14}$$

The next step is to derive the MGF of the interdeparture time Y_c, $M_{Y_c}(s)$, which is given by the following proposition.

Proposition 1. *The MGF of the interdeparture time of source c,* $M_{Y_c}(s)$*, is given by*

$$M_{Y_c}(s)=\frac{a_cM_S(s-\lambda_c)}{(1-a'_c)\left(1-\sum_{c'\in\mathcal{C}\setminus\{c\}}\dfrac{a_{c'}M_S(s-\lambda_{c'})}{1-a'_{c'}}\right)}, \tag{15}$$

where a'_c *and* a_c *were defined below* (7).

Proof. See [30, Proposition 1]. □

Finally, substituting the MGF of the system time of source c derived in (14) and the MGF of the interdeparture time of source c derived in (15) into (8) results in the MGF of the AoI under the source-aware preemptive policy, $M_{\delta_c}(s)$, given in Theorem 1. Similarly, substituting (14) and (15) into (9) results in the MGF of the peak AoI under the source-aware preemptive policy, $M_{A_c}(s)$, given in Theorem 1.

IV. Numerical Results

We use Corollary 1 to validate the derived results for the average AoI under the source-aware preemptive packet management policy in a two-source system and subsequently compare the average AoI performance against those of the source-agnostic preemptive policy [27] and the non-preemptive policy [29]. We investigate two service time distributions: i) gamma distribution and ii) Pareto distribution.

- The PDF of a random variable S following a gamma distribution is $f_S(t) = \dfrac{\beta^\kappa t^{\kappa-1}\exp(-\beta t)}{\Gamma(\kappa)}$, $t > 0$, for parameters $\kappa > 0$ and $\beta > 0$, where $\Gamma(\kappa)$ is the gamma function at κ. The service rate is $\mu = 1/\mathbb{E}[S] = \beta/\kappa$.
- The PDF of a random variable S following a Pareto distribution is $f_S(t) = \dfrac{\alpha\omega^\alpha}{t^{\alpha+1}}$, for $t \in [\omega, \infty]$ and parameters $\omega > 0$ and $\alpha > 1$. The service rate is $\mu = \dfrac{\alpha-1}{\alpha\omega}$.

In all the figures, we have $\lambda = \lambda_1 + \lambda_2 = 1$. Next, we investigate the contours of achievable average AoI pairs.

Fig. 2 illustrates the contours of achievable average AoI pairs $(\Delta_1,\ \Delta_2)$ for the proposed source-aware preemptive packet management policy, the source-agnostic preemptive policy, and the non-preemptive policy under the gamma distribution with service rate $\mu = 1$ for the parameters $\kappa = \beta = 0.5$, $\kappa = \beta = 1.7$, and $\kappa = \beta = 3$. Note that for a fixed service rate, increasing β makes the gamma distribution to have a lighter tail. For the parameters $\kappa = \beta = 0.5$, the source-agnostic preemptive policy outperforms the others and the non-preemptive is the worst policy (Fig. 2(a)); for the parameters $\kappa = \beta = 1.7$, the source-aware preemptive policy outperforms the others and the non-preemptive is the worst policy (Fig. 2(b)); and for the parameters $\kappa = \beta = 3$, the non-preemptive policy outperforms the others and the source-agnostic preemptive policy is the worst one (Fig. 2(c)). In addition, Fig. 2(a) shows that the simulated curve for the source-aware preemptive packet management policy matches with the derived expression in Corollary 1.

Fig. 3 illustrates the contours of achievable average AoI pairs $(\Delta_1,\ \Delta_2)$ for the packet management policies under the Pareto distribution with $\mu = 10$ for the sets of parameters $(\alpha = 2.4,\ \omega = 0.0583)$, $(\alpha = 2.7,\ \omega = 0.630)$, and $(\alpha = 4,\ \omega = 0.750)$. Note that for a fixed service rate, increasing α makes the Pareto distribution to have a lighter tail. Similar to the observations made for the gamma distribution, for the parameters $(\alpha = 2.4,\ \omega = 0.0583)$, the source-agnostic preemptive policy outperforms the others and the non-preemptive policy is the worst one (Fig. 3(a)); for the parameters $(\alpha = 2.7,\ \omega = 0.630)$, the source-aware preemptive policy outperforms the others and the non-preemptive policy is the worst one (Fig. 3(b)); and for the parameters $(\alpha = 4,\ \omega = 0.750)$, the non-preemptive policy outperforms the others and the source-agnostic preemptive policy is the worst one (Fig. 3(c)).

Figs. 2 and 3 show that for a fixed mean service time and the set of parameters that make the tail of the distribution heavy enough, the source-agnostic preemptive policy is the best one; and for the parameters that the tail of the distribution is light enough, the non-preemptive policy is the best one. This is due to the fact that for a fixed mean service time, the heavier the tail, the higher the chance of serving a packet with service time that is substantially longer than the mean service time. In this case, the preemption enables discarding the packets that would otherwise keep the server inefficiently busy for a long time period and, in turn, enables switching to serve a more fresh packet which has a high chance of experiencing shorter service time. On the other hand, when the tail of the distribution is light enough, it is better to block new arrivals. This is because preemption would cause infrequent updating due to excessively switching the packet under service so that any packet rarely completes service.

V. Conclusions

We derived the MGFs of the AoI and peak AoI in a multi-source M/G/1/1 queueing model under the proposed source-aware preemptive packet management policy. Using the derived MGFs, we derived the average AoI and average peak AoI of a two-source M/G/1/1 queueing system. We numerically compared the average AoI of the source-aware preemptive policy with those of the source-agnostic preemptive and non-preemptive policies. The results showed that, depending on the system parameters, i.e., the packet arrival rates and the distribution of the service time, each policy can outperform the others. In particular, for a given service rate, when the tail of the service time distribution is sufficiently heavy, the source-agnostic preemptive policy is the best policy, whereas for a sufficiently light tailed distribution, the non-preemptive policy is the best one.

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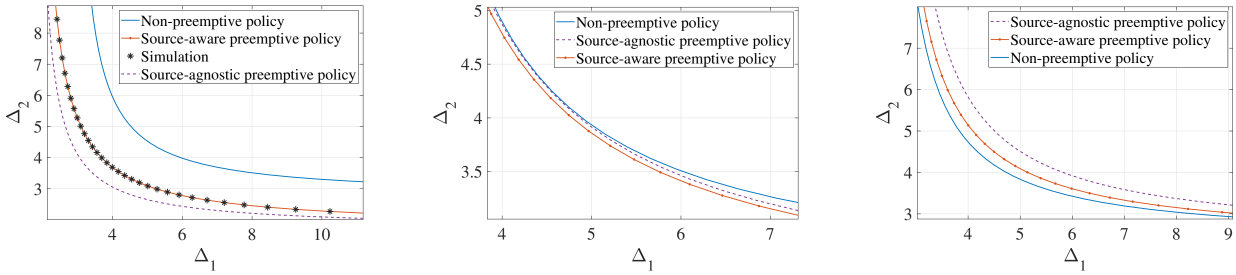
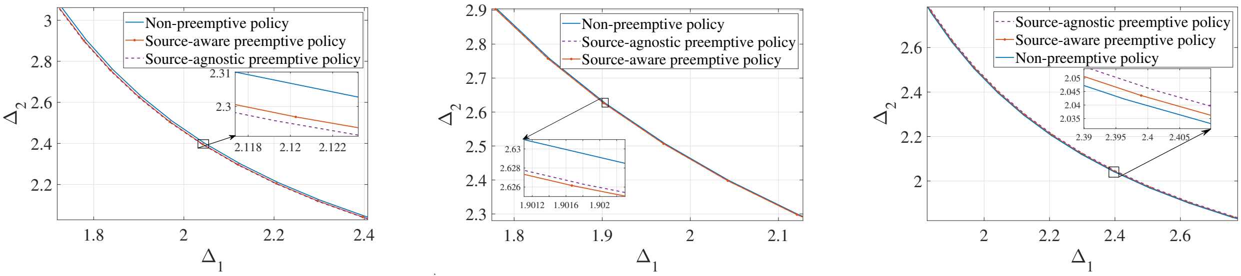


(a) $\kappa = \beta = 0.5$ (b) $\kappa = \beta = 1.7$ (c) $\kappa = \beta = 3$

Fig. 2: The contours of achievable average AoI pairs under the gamma distribution for the different sets of parameters with $\mu = 1$.



(a) $\alpha = 2.4, \omega = 0.0583$ (b) $\alpha = 2.7, \omega = 0.630$ (c) $\alpha = 4, \omega = 0.750$

Fig. 3: The contours of achievable average AoI pairs under the Pareto distribution for the different sets of parameters with $\mu = 1$.

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