Secret Key-based Authentication With Passive Eavesdropper for Scalar Gaussian Sources

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Abstract—We analyze the fundamental trade-off of secret key-based authentication systems in the presence of an eavesdropper for correlated Gaussian sources. A complete characterization of the trade-off among secret-key, storage, and privacy-leakage rates of both generated and chosen secret models is provided. One of the main contributions is revealing that unlike the known results for discrete sources, there is no need for the second auxiliary random variable in characterizing the capacity regions for the Gaussian cases. In addition, it is shown that the strong secrecy for secrecy-leakage of the systems can be achieved by an information-spectrum approach, and the parametric expressions (computable forms) of the capacity regions are also derived.

Index Terms—Gaussian sources, strong secrecy, privacy-leakage, secret-key agreement, entropy power inequality.

I. INTRODUCTION

Secret key-based authentication (SKA) systems are generally designed to perform private authentication of users based on secret keys, usually generated from biometric identifiers [1] or physical unclonable functions [2]. In recent years, there has been a bunch of literature focusing on investigating the fundamental limits of SKA systems from information-theoretic perspectives. In the analysis of the SKA systems, a new condition called privacy constraint is added to the problem formulations of the well-known secret-key agreement (the source model with one-way communication only) discussed in, e.g., [3]–[6]. Therefore, many existing tools used for solving the key agreement problems are quite useful to characterize the capacity regions of the SKA systems as well.

The seminal works [7] and [8] independently investigated the trade-off relation between security and privacy-leakage in the SKA systems. Particularly, in [7], eight different systems were taken into consideration, but among them the generated secret (GS) and chosen secret (CS) models are two major models closely related to real-life applications and frequently analyzed in the researches taking place later on. Some extensions of the work [7] for GS and CS models considering a storage constraint and user identification can be found in, e.g., [9]–[11] and [12]–[14], respectively.

An SKA system in which an eavesdropper can observe both the helper data and correlated side information of the identified sequence was introduced in [15]. More specifically, the GS model was discussed in two scenarios; passive and active eavesdropper scenarios. In the passive case, the eavesdropper is interested in knowing the biometric identifier and the secret key based on the available information in his/her hand. On the other hand, the active eavesdropper tries to cheat the system or

decode the genuine secret key with a mock sequence generated by his/her own data. Basically, in this setup, the privacy requirement is more stringent than the one seen in [7], in which the eavesdropper only has the knowledge of the helper. The work [15] was further extended in [16], [17] to incorporate noisy enrollment and a cost-constrained action at the decoder. The capacity characterizations of each paper were derived via two auxiliary random variables (RVs) for discrete memoryless sources as seen in [5]. However, in practical applications, the biometric signal is usually represented in continuous forms.

Moreover, in information-theoretic security, the weak secrecy and the strong secrecy are commonly defined as metrics to assess the leakage of sensitive data, e.g, secret key used for private authentication. In [15], [16], the secrecy-leakage of the systems is evaluated under a weak secrecy criterion, where the secret-key information is allowed to leak to the eavesdropper in sub-linear order of the block length. From the security point of views, this is not preferable for the reason that the information leaked to the eavesdropper might grow unbounded with the block length [18]. A tighter security notion imposed on the secrecy-leakage is seen in [11], [17]. In the papers, an SKA system, in which the noisy identifiers and the identified sequences are observed through a broadcast channel [19, Chapter 8], is analyzed under a strong secrecy criterion for secrecy-leakage, where the amount of information leaked to the eavesdropper is demanded to be negligible regardless of the block length. However, the analyzing technique is different from the one adopted in this paper.

Motivated by these essential factors, for the same model in [15], we enhance the security criterion to the strong secrecy for secrecy-leakage, and characterize the capacity regions of secret-key, storage, and privacy-leakage rates for Gaussian sources from information-spectrum perspectives [20]. We solely focus on the passive eavesdropper case, but deal with both GS and CS models. The main contributions of this paper are summarized as follows:

• Show that unlike the results of discrete sources [15]–[17], a single auxiliary RV suffices to characterize the capacity regions of both models for Gaussian sources. Another interesting result is when the correlation coefficient of the channel to the decoder is smaller than that of the channel to the eavesdropper, the capacity regions of the two models coincide; the optimal rates of the secret key and storage become zero, but the minimum value of the privacy-leakage rate may be positive.

- Apply a privacy amplification technique developed in [6] to prove that the strong secrecy for secrecy-leakage of the SKA systems is achievable.
- Provide complete parametric expressions of the capacity regions.

As special cases, the capacity characterizations derived in this paper coincide with the results of [6] for a large enough privacy-leakage rate and [21] for no consideration of the storage rate.

II. SYSTEM MODELS AND PROBLEM FORMULATIONS

A. System Models

We basically use standard notation in [19]. The data flow of GS and CS models is depicted in Fig. 1. Arrows (g) and (c) indicate the directions of the secret key of the former and latter models. Assume that the biometric source $X \sim \mathcal{N}(0,1)$. The channel to the decoder $(X \to Y)$ and the channel to the eavesdropper $(X \to Z)$ are modeled as

$$Y = \rho_1 X + N_y, \qquad Z = \rho_2 X + N_2,$$
 (1)

where $|\rho_1|, |\rho_2| < 1$ are the correlation coefficients of each channel, $N_y \sim \mathcal{N}(0, 1 - \rho_1^2)$ and $N_2 \sim \mathcal{N}(0, 1 - \rho_2^2)$ are Gaussian RVs, and independent of each other and of other RVs. Let $S = [1:M_S]$ and $\mathcal{J} = [1:M_J]$ be the sets of secret keys and helper data, respectively. X^n and (Y^n, Z^n) denote the biometric identifier generated from source P_X , the outputs of X^n via the channel $P_{YZ|X}$, respectively. In GS model, observing X^n , the encoder e generates a helper data $J \in \mathcal{J}$ and a secret key $S \in \mathcal{S}$; $(J,S) = e(X^n)$. J is shared with the decoder via the noiseless public channel. Seeing Y^n , the decoder d estimates S from Y^n and the helper data J; $\widehat{S} = d(Z^n, J)$. In CS model, S is chosen uniformly from S and independent of other RVs. The encoder makes the helper data by $J = e(X^n, S)$. For the decoder, $\hat{S} = d(Y^n, J)$. The eavesdropper has (Z^n, J) and wants to learn about the biometric identifier X^n and the secret key S.

B. Problem Formulations

In this section, we provide the formal definitions of GS and CS models. First, we define the achievability definition of GS model.

Definition 1. A tuple of secret-key, storage, and privacy-leakage rates $(R_S, R_J, R_L) \in \mathbb{R}^3_+$ is said to be achievable for GS model if for any $\delta > 0$ and large enough n there exist pairs of encoders and decoders satisfying

$$\Pr{\hat{S} \neq S} \le \delta,$$
 (error probability) (2)

$$H(S) \ge n(R_S - \delta), \quad \text{(secret-key)}$$
 (3)

$$\log M_J \le n(R_J + \delta), \quad \text{(storage)}$$
 (4)

$$I(S; J, Z^n) \le \delta,$$
 (secrecy-leakage) (5)

$$I(X^n; J, Z^n) \le n(R_L + \delta).$$
 (privacy-leakage) (6)

Also, \mathcal{R}_G is defined as the closure of the set of all achievable rate tuples for GS model, called the capacity region.

The achievability definition of CS model is given below.

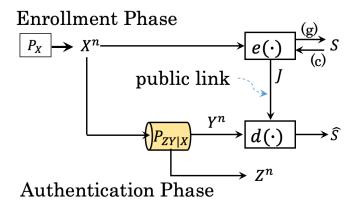


Fig. 1. System models

Definition 2. A tuple of $(R_S, R_J, R_L) \in \mathbb{R}^3_+$ is said to be achievable for CS model if for any $\delta > 0$ and large enough n there exist pairs of encoders and decoders satisfying all the requirements imposed in Definition 1. Let \mathcal{R}_C be the capacity region of CS model.

III. STATEMENT OF MAIN RESULTS

Before stating our main theorem, we would like to briefly mention the background why the general authentication (wiretap) channels can always be scaled down into the degraded versions for Gaussian sources.

Using [6, Lemma 6], in the case where

$$\rho_1^2 > \rho_2^2, \tag{7}$$

by setting $X'=X,\;Y'=Y,\;Z'=\frac{\rho_2}{\rho_1}Y'+N_z,$ where $N_z\sim\mathcal{N}(0,1-\rho_2^2/\rho_1^2)$ is Gaussian RV and independent of other RVs, the marginal densities of (X',Y') and (X',Z') coincides with (X,Y) and (X,Z), respectively. Contrary to the above condition, when

$$\rho_1^2 \le \rho_2^2,\tag{8}$$

for the RVs $X'=X,~Z'=Z,~Y'=\frac{\rho_1}{\rho_2}Z',$ it also follows that the marginal densities of (X',Y') and (X',Z') coincides with (X,Y) and (X,Z), respectively.

Since the constraints (2), (6), and (5) depend only on the marginal densities of (X,Y) and (X,Z), it suffices to derive our main theorem based on the joint sources (X',Y',Z') instead of (1). In the rest of discussions, we just use (X,Y,Z) to represent (X',Y',Z') for convenience.

Theorem 1. Under the condition of (7), i.e., the Markov chain X - Y - Z holds, we have that

$$\mathcal{R}_{G} = \cup_{P_{U|X}} \{ (R_{S}, R_{J}, R_{L}) \in \mathbb{R}^{3}_{+} : R_{S} \leq I(Y; U|Z), \\ R_{J} \geq I(X; U|Y), \\ R_{L} \geq I(X; U|Y) + I(X; Z) \}, \qquad (9)$$

$$\mathcal{R}_{C} = \cup_{P_{U|X}} \{ (R_{S}, R_{J}, R_{L}) \in \mathbb{R}^{3}_{+} : R_{S} \leq I(Y; U|Z), \\ R_{J} \geq I(X; U|Z), \\ R_{L} \geq I(X; U|Y) + I(X; Z) \}. \qquad (10)$$

For the case of (8), i.e., the Markov chain X - Z - Y holds, the capacity regions are characterized in the same form as

$$\mathcal{R}_G = \mathcal{R}_C = \{ (R_S, R_J, R_L) : R_S = 0, R_J \ge 0, R_L \ge I(X; Z) \}.$$
 (11)

A proof of Theorem 1 is given in Appendix A. Note that the above regions are uncomputable since the cardinality of auxiliary RV U is unbounded. In the following, we show that the parametric forms of Theorem 1 are determined by a single parameter.

Corollary 1. Under the case of (7), the capacity regions of GS and CS models can be computed as

$$\mathcal{R}_{G} = \cup_{\alpha \in (0,1]} \left\{ (R_{S}, R_{J}, R_{L}) \in \mathbb{R}_{+}^{3} : R_{S} \leq \frac{1}{2} \log \left(\frac{\alpha \rho_{2}^{2} + 1 - \rho_{2}^{2}}{\alpha \rho_{1}^{2} + 1 - \rho_{1}^{2}} \right), R_{J} \geq \frac{1}{2} \log \left(\frac{\alpha \rho_{1}^{2} + 1 - \rho_{1}^{2}}{\alpha} \right), R_{L} \geq \frac{1}{2} \log \left(\frac{\alpha \rho_{1}^{2} + 1 - \rho_{1}^{2}}{\alpha (1 - \rho_{2}^{2})} \right) \right\}, \quad (12)$$

$$\mathcal{R}_{C} = \cup_{\alpha \in (0,1]} \left\{ (R_{S}, R_{J}, R_{L}) \in \mathbb{R}_{+}^{3} : R_{S} \leq \frac{1}{2} \log \left(\frac{\alpha \rho_{2}^{2} + 1 - \rho_{2}^{2}}{\alpha \rho_{1}^{2} + 1 - \rho_{1}^{2}} \right), R_{J} \geq \frac{1}{2} \log \left(\frac{\alpha \rho_{2}^{2} + 1 - \rho_{2}^{2}}{\alpha (1 - \rho_{2}^{2})} \right), R_{L} \geq \frac{1}{2} \log \left(\frac{\alpha \rho_{1}^{2} + 1 - \rho_{1}^{2}}{\alpha (1 - \rho_{2}^{2})} \right) \right\}. \quad (13)$$

In light of (8), we have that

$$\mathcal{R}_{G} = \mathcal{R}_{C} = \{ (R_{S}, R_{J}, R_{L}) : R_{S} = 0, R_{J} \ge 0,$$

$$R_{L} \ge \frac{1}{2} \log \left(\frac{1}{1 - \rho_{2}^{2}} \right) \}.$$
(14)

The full proof of Corollary 1 is available in Appendix B. In Theorem 1, one can see that only auxiliary RV Usatisfying the Markov chain U - X - (Y, Z) is present in both regions. A similar conclusion was drawn in [6] for the secret-key agreement problem, but in the SKA systems, it is not trivial whether the constraint on privacy-leakage rate can be written by one auxiliary RV or not. In this paper, we have revealed that it is possible to do so. In addition, when the condition (8) is satisfied, the capacity regions of GS and CS models are given in the same form. The optimal values of the secret-key and storage rates are both zero, but that of the privacy-leakage rate can still be positive depending on the joint marginal densities of (X, Z). This is an interesting nature of the SKA systems, which was not seen in the secretkey agreement problems. Even when the encoding procedure is not needed (i.e., U is a constant), the uncontrollable information leaked to the eavesdropper via the channel $P_{Z|X}$ is at minimum rate I(Z;X), corresponding to the capacity of this channel.

As special cases, when Z is independent of other RVs $(\rho_2=0)$ and the storage rate is large enough $(R_J\to\infty)$, one can easily see that Corollary 1 is reduced to [21, Theorem 1] and [21, Theorem 2] for GS and CS models, respectively. Furthermore, for the case where $R_J\geq 0$ and $R_L\to\infty$, let us set $R_J=\frac{1}{2}\log\frac{\alpha\rho_1^2+1-\rho_1^2}{\alpha}$, implying $\alpha=(1-\rho_1^2)/(e^{2R_J}-\rho_1^2)$. Substituting the value of α into the right hand side of R_S , with some careful manipulation, it becomes

$$\frac{1}{2}\log\frac{\text{Var}[Y|X,Z]e^{-2R_J} + \text{Var}[Y|Z](1-e^{-2R_J})}{\text{Var}[Y|X,Z]},\quad(15)$$

where $\mathrm{Var}[\cdot]$ denotes the variance of an RV, and $\mathrm{Var}[Y|Z] = \frac{\rho_1^2 - \rho_2^2}{\rho_1^2}$, $\mathrm{Var}[Y|X,Z] = \frac{(1-\rho_1^2)(\rho_1^2 - \rho_2^2)}{\rho_1^2(1-\rho_2^2)}$. Equation (15) is exactly the upper bound on the secret-key rate for limited public communication rate $(R_J \geq 0)$ derived in [6, Theorem 4].

IV. CONCLUSION

In this paper, we characterized the capacity regions of GS and CS models for Gaussian sources, and showed that only one auxiliary RV was required for expressing the regions. Also, it was demonstrated that the strong secrecy for secrecy-leakage of SKA systems is achievable by information-spectrum methods [20]. For future work, we plan to clarify the optimal tradeoff for GS and CS models for vector Gaussian sources.

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APPENDIX A PROOF OF THEOREM 1

Due to the space limitations, we only prove GS model under the condition (7), which is the most difficult case for deriving Theorem 1. The proof for CS model is omitted since it can be done similarly to GS model with merely adding an extra procedure; one-time pad operation.

A. Converse Part

In [15], the capacity region of GS model for general discrete sources, denoted by \mathcal{R}'_G , is given by

Theorem 2. (Kittichokechai and Caire [15, Theorem 2])

$$\mathcal{R}'_{G} = \bigcup_{P_{V|U}, P_{U|X}} \{ (R_{S}, R_{J}, R_{L}) \in \mathbb{R}^{3}_{+} :$$

$$R_{S} \leq I(Y; U|V) - I(Z; U|V),$$

$$R_{J} \geq I(X; U|Y),$$

$$R_{L} \geq I(X; U, Y) - I(X; Y|V) + I(X; Z|V) \},$$
 (16)

where auxiliary RVs U, V satisfy the Markov chain V - U - X - (Y, Z), and $|\mathcal{V}| \leq |\mathcal{X}| + 3$ and $|\mathcal{U}| \leq (|\mathcal{X}| + 3)(|\mathcal{X}| + 2)$.

It can be easily verified that the above result also holds for Gaussian sources. One can see that the bounds of R_J

for both Equations (9) and (16) remain unchanged, so they can be shown in the same way. We need to check that other constraints, i.e., for R_S, R_L , hold. Consider the case of (7). For the degraded wiretap channels, e.g., V-U-X-Y-Z, it holds that $I(Y;V|Z)=I(Y;V)-I(Z;V)\geq 0$. Then, we can transform the bound on the secret-key rate as

$$R_{S} \leq I(Y; U|V) - I(Z; U|V)$$

$$\stackrel{\text{(a)}}{=} I(Y; U) - I(Y; V) - (I(Z; U) - I(Z; V))$$

$$= I(Y; U) - I(Z; U) - (I(Y; V) - I(Z; V))$$

$$\stackrel{\text{(b)}}{=} I(Y; U|Z) - I(Y; V|Z)$$

$$\leq I(Y; U|Z), \tag{17}$$

where (a) follows by the Markov chains V - U - Y and V - U - Z, and (b) is due to U - Y - Z and V - Y - Z. For the privacy-leakage rate,

$$R_{L} \geq I(X; U, Y) - I(X; Y|V) + I(X; Z|V)$$

$$\stackrel{\text{(c)}}{=} I(X; U|Y) + I(Y; V) + I(X; Z) - I(Z; V)$$

$$= I(X; U|Y) + I(X; Z) + I(Y; V|Z)$$

$$> I(X; U|Y) + I(X; Z), \tag{18}$$

where (c) is due to the Markov chains V - X - Y and V - X - Z. This wraps up the converse proof.

B. Achievability

Our proof technique is similar to the protocol used in [6]. Since the analyses on the error probability, secret-key and storage rates, and strong secrecy for secrecy-leakage are similar to the arguments discussed in [6], they will be mentioned briefly. However, we will describe the analysis on the bound of privacy-leakage rate, which was not taken into account in the literature, in detail.

Fix the test channel $P_{U|X}$ and let γ be small enough positive. Set $R_S = I(Y;U|Z) - 6\gamma$, $R_J = I(X;U|Y) + 4\gamma$, and $R_L = I(X;U|Y) + I(X;Z) + 3\gamma$, and the sizes of the set of helpers $|\mathcal{J}| = \exp\{nR_J\}$, and the set of secret keys $|\mathcal{S}| = \exp\{nR_S\}$. Define the sets

$$\mathcal{T}_{n} = \left\{ (u^{n}, x^{n}) : \frac{1}{n} \log \frac{P_{U^{n}|X^{n}}(u^{n}|x^{n})}{P_{U^{n}}(u^{n})} \leq I(X; U) + \gamma \right\},$$

$$\mathcal{A}_{n} = \left\{ (u^{n}, y^{n}) : \frac{1}{n} \log \frac{P_{Y^{n}|U^{n}}(y^{n}|u^{n})}{P_{Y^{n}}(y^{n})} \geq I(Y; U) - \gamma \right\},$$

$$\mathcal{B}_{n} = \left\{ (u^{n}, x^{n}, z^{n}) : \frac{1}{n} \log \frac{P_{X^{n}|U^{n}, Z^{n}}(x^{n}|u^{n}, z^{n})}{P_{Y^{n}|Z^{n}}(x^{n}|z^{n})} \geq I(X; U|Z) - \gamma \right\}.$$

Next we specify the codebook, and the enrollment and authentication procedures.

Generation of Codebook C_n : Generate $\exp\{n(I(X;U)+2\gamma)\}$ i.i.d. sequences of $\tilde{u}_i^n, i \in [1:\exp\{n(I(X;U)+2\gamma)\}]$, from P_U and denote the set of these sequences as Q_n . Let $g_n : \mathbb{R} \to Q_n \subset \mathbb{R}$ be the quantization function of the biometric source sequence x^n into \tilde{u}^n . The quantization rule of the function g_n is that it looks for a \tilde{u}_i^n such that $(\tilde{u}_i^n, x^n) \in \mathcal{T}_n$. In case there are multiple such \tilde{u}^n , the encoder picks one at random. More specially, $\mathcal{Q}_n = \{g_n(x^n)|x^n \in \mathbb{R}^n\}$ and the size $|\mathcal{Q}_n| = \exp\{n(I(X;U)+2\gamma)\}$. Prepare $M_J = e^{nR_J}$ bins. Randomly assign each $\tilde{u}^n \in \mathcal{Q}_n$ to one of the bins according to a function $\phi_n: \mathcal{Q}_n \to \mathcal{J}$. Let $j = \phi_n(\tilde{u}^n), j \in \mathcal{J}$, denote the index of the bin to which \tilde{u}^n belongs. A function $f_n: \mathcal{Q}_n \to \mathcal{S}$ is selected uniformly from \mathcal{F}_n so that it satisfies that $P_{F_n}(\{f_n \in \mathcal{F}_n: f_n(\tilde{u}^n) = f_n(\hat{u}^n)\}) \leq \frac{1}{|\mathcal{S}|}$, where P_{F_n} is a uniform distribution on \mathcal{F}_n , for any distinct sequences $\tilde{u}^n \in \mathcal{Q}_n$ and $\hat{u}^n \in \mathcal{Q}_n$.

In the encoding and decoding processes, we fix the set Q_n and the random functions ϕ_n and f_n .

Encoding: Observing x^n , the encoder utilizes the function g_n to quantize this sequence to \tilde{u}^n . Then, it computes the bin's index $j=\phi_n(\tilde{u}^n)$ and generates a secret key $s=f_n(\tilde{u}^n)$ by a function $f_n:\mathcal{Q}_n\to\mathcal{S}$, subsequently specified in Lemmas 3 and 4. The index j is shared with the decoder for authentication. If there is no such \tilde{u}^n , set (j,s)=(1,1).

Decoding: Seeing y^n and j, the decoder looks for a unique \hat{u}^n such as $j=\phi_n(\hat{u}^n)$ and $(y^n,\hat{u}^n)\in\mathcal{A}_n$. If such a \hat{u}^n is found, then the decoder sets $\psi_n(y^n,j)=\hat{u}^n$ by a function $\psi_n:\mathcal{J}\times\mathbb{R}^n\to\mathcal{Q}_n$, and distills the secret key $\hat{s}=f_n(\hat{u}^n)$. Otherwise, the decoder outputs $\hat{s}=1$ and error is declared.

The random codebook C_n consists of the set $Q_n = \{\tilde{U}_i^n : i \in [1:\exp\{n(I(X;U)+2\gamma)\}]\}$ and the functions (g_n,ϕ_n,ψ_n,f_n) , and it is revealed to all parties. In the achievability proof, we evaluate the averaged performance of the system, i.e., conditions (2)–(6), over all possible C_n .

Before proceeding to the detailed analysis, we introduce some important lemmas that will be used in the sequel.

Lemma 1. (Iwata and Muramatsu [22, Lemma 11]) It holds that

$$\mathbb{E}_{\mathcal{C}_n}[\Pr\{(g_n(X^n), Y^n) \notin \mathcal{A}_n \text{ or } (g_n(X^n), X^n, Z^n) \notin \mathcal{B}_n\}]$$

$$\leq 2\sqrt{\delta_n} + \Pr\{(U^n, X^n) \in \mathcal{T}_n\} + \exp\{-e^{n\gamma}\}, \tag{19}$$

where

$$\delta_n = \Pr\{(U^n, Y^n) \in \mathcal{A}_n \text{ or } (U^n, X^n, Z^n) \in \mathcal{B}_n\},$$
 (20)

and where $\mathbb{E}_{\mathcal{C}_n}[\cdot]$ denotes the expectation over the random codebook \mathcal{C}_n .

As mentioned in [6], it can be shown that $\Pr\{(U^n, X^n) \notin \mathcal{T}_n\}$ and δ_n go to zero exponentially by the Chernoff bound.

Lemma 2. (Iwata and Muramatsu [22, Proof of Theorem 1]) The error probability averaged over the random assemble is bounded by

$$\mathbb{E}_{\mathcal{C}_n}[\Pr\{g_n(X^n) \neq \psi_n(\phi_n(g_n(X^n)), Y^n)\}]$$

$$\leq e^{-\gamma n} + \mathbb{E}_{\mathcal{C}_n}[\Pr\{(g_n(X^n), Y^n) \notin \mathcal{A}_n\}]. \tag{21}$$

Here, we define a security measure

$$\mu_n = \int_{z^n} P_{Z^n}(z^n) \|P_{SJ|Z^n = z^n, \mathcal{C}_n} - P_{\tilde{S}} P_{J|Z^n = z^n, \mathcal{C}_n} \|dz^n,$$
(22)

where $||P_A - P_B||$ and $P_{\tilde{S}}$ denote the variational distance between probability distributions P_A and P_B , and the uniform distribution on the set S, respectively.

Lemma 3. (Watanabe and Oohama [6, Lemma 12]) An upper bound of the measure μ_n averaged over the random codebook is given by

$$\mathbb{E}_{\mathcal{C}_n}[\mu_n] \le e^{-\frac{n\gamma}{2}} + \mathbb{E}_{\mathcal{C}_n}[\Pr\{(g_n(X^n), X^n, Z^n) \in \mathcal{B}_n\}]. \tag{23}$$

Note that the right-hand side of Equation (23) decays exponentially since so does the second term (cf. (19)).

Lemma 4. (Naito et al. [23, Lemma 3]) We have that

$$H(S|J, Z^n, \mathcal{C}_n)$$

$$\geq (1 - \mathbb{E}_{\mathcal{C}_n}[\mu_n]) \log M_S + \mathbb{E}_{\mathcal{C}_n}[\mu_n] \log \mathbb{E}_{\mathcal{C}_n}[\mu_n]. \quad (24)$$

Using Lemma 2, the ensemble average of the error probability of encoding and decoding can be made exponentially vanishing for large enough n. For the analysis of the secret-key rate, this can be proved via Lemma 4. The bound on the storage rate is straightforward from the rate setting.

Analysis of Secrecy-Leakage: We can expand the left-hand side of (5) as

$$I(S; J, Z^{n} | \mathcal{C}_{n}) = H(S | \mathcal{C}_{n}) - H(S | J, Z^{n}, \mathcal{C}_{n})$$

$$\leq \log M_{S} - H(S | J, Z^{n}, \mathcal{C}_{n})$$

$$\leq \mathbb{E}_{\mathcal{C}_{n}}[\mu_{n}](\log M_{S} - \log \mathbb{E}_{\mathcal{C}_{n}}[\mu_{n}])$$

$$\leq \gamma$$
(25)

for sufficiently large n, where (a) follows from Lemma 4 and the last inequality is due to Lemma 3.

Analysis of Privacy-Leakage: For (6), we have that

$$I(X^{n}; J, Z^{n} | \mathcal{C}_{n})$$

$$= I(X^{n}; J | \mathcal{C}_{n}) + I(X^{n}; Z^{n} | J, \mathcal{C}_{n})$$

$$\stackrel{\text{(b)}}{=} H(J | \mathcal{C}_{n}) + h(Z^{n} | J, \mathcal{C}_{n}) - h(Z^{n} | J, X^{n}, \mathcal{C}_{n})$$

$$\stackrel{\text{(c)}}{=} H(J | \mathcal{C}_{n}) + h(Z^{n} | J, \mathcal{C}_{n}) - h(Z^{n} | X^{n})$$

$$\stackrel{\text{(d)}}{\leq} H(J | \mathcal{C}_{n}) + h(Z^{n}) - h(Z^{n} | X^{n})$$

$$\leq n(I(X; U | Y) + 4\gamma) + nI(X; Z)$$

$$= n(R_{L} + \gamma). \tag{26}$$

where (b) holds as J is a function of X^n , (c) holds because for a given codebook C_n , $J-X^n-Z^n$ forms a Markov chain and the codebook C_n is independent of (X^n, Z^n) , and (d) follows because conditioning does not increase entropy.

Finally, applying the selection lemma [18, Lemma 2.2], there exists at least one good codebook that satisfies all the conditions in Definition 1. \Box

APPENDIX B PROOF OF COROLLARY 1

In the same manner of Appendix A, we give only the proof of \mathcal{R}_G (cf. (12)) in the case of (7). For the achievability part, fix $0<\alpha\leq 1$. Let $U\sim\mathcal{N}(0,1-\alpha)$ and $\Theta\sim\mathcal{N}(0,\alpha)$. Assume that $X=U+\Theta$. Then, we have that $Y=\rho_1U+\rho_1\Theta+N_y$, $Z=\rho_2U+\rho_2\Theta+\frac{\rho_2}{\rho_1}N_y+N_z$. From these relations, it is not so difficult to see that

$$I(X;U) = \frac{1}{2}\log(\frac{1}{\alpha}), \quad I(Y;U) = \frac{1}{2}\log(\frac{1}{\alpha\rho_1^2 + 1 - \rho_1^2}),$$

$$I(Z;U) = \frac{1}{2}\log(\frac{1}{\alpha\rho_2^2 + 1 - \rho_2^2}). \tag{27}$$

Note that due to the Markov chains U - X - Y - Z, we can write that

$$I(Y;U|Z) = I(Y;U) - I(Z;U),$$

$$I(X;U|Y) = I(X;U) - I(Y;U).$$
(28)

Substituting all equations in (27) into the right-hand side of (9), one can see that any rate tuple contained in the right-hand side of (12) is achievable.

For the converse part, it is a bit more involved. Here, we prove this part by making use of conditional entropy power inequality (EPI) [25, Lemma II]. Note that each constraint in the right-hand side of (9) can be transformed as

$$R_{S} \leq I(Y; U|Z) = h(Y|Z) - h(Y|U, Z)$$

$$= \frac{1}{2} \log 2\pi e (1 - \rho_{2}^{2}/\rho_{1}^{2}) - h(Y|U, Z), \tag{29}$$

$$R_{J} \geq I(X; U|Y) \stackrel{\text{(a)}}{=} I(X; U|Z) - I(Y; U|Z)$$

$$= h(X|Z) - h(X|U, Z) - h(Y|Z) + h(Y|U, Z)$$

$$= \frac{1}{2} \log \frac{1 - \rho_{2}^{2}}{1 - \rho_{2}^{2}/\rho_{2}^{2}} - h(X|U, Z) + h(Y|U, Z), \tag{30}$$

$$R_{L} = h(X|Z) - h(X|U,Z) - h(Y|Z) + h(Y|U,Z) + I(X;Z)$$

$$\stackrel{\text{(b)}}{\geq} \frac{1}{2} \log \frac{1}{1 - \rho_{2}^{2}/\rho_{1}^{2}} - h(X|U,Z) + h(Y|U,Z), \quad (31)$$

where (a) is due to the Markov chain Z-Y-X-U, and (b) follows from the property that h(X)=h(Y) for Gaussian RVs with unit variances, and thus $h(X|Z)-h(Y|Z)+I(Z;X)=I(Z;Y)=\frac{1}{2}\log\frac{1}{1-\rho_2^2/\rho_1^2}.$ So as to bound the region \mathcal{R}_G , we need to find a lower

So as to bound the region \mathcal{R}_G , we need to find a lower bound on h(Y|U,Z) for fixed h(X|U,Z).

The following fine setting plays an important role to bound the conditional entropy h(Y|U,Z). Now let us set

$$h(X|U,Z) = \frac{1}{2}\log 2\pi e \left(\frac{\alpha(1-\rho_2^2)}{\alpha\rho_2^2 + 1 - \rho_2^2}\right)$$
(32)

for $0<\alpha\leq 1$. This setting comes from the fact that $h(X|U,Z)\leq h(X|Z)=\frac{1}{2}\log 2\pi e(1-\rho_2^2)$. Here, $\alpha=0$ is excluded since the right hand-side of (32) will go to infinity, but this value is impossible to achieve by Gaussian RVs with finite variances, which are always assumed in the analysis of Gaussian sources.

In the direction from X to Y, using the conditional EPI [25, Lemma II], we have that

$$e^{2h(Y|U,Z)} \ge e^{2h(\rho_1 X|U,Z)} + e^{2h(N_y|U,Z)}$$

$$= \rho_1^2 e^{2h(X|U,Z)} + e^{2h(N_y)}$$

$$\stackrel{\text{(c)}}{=} 2\pi e \left(\frac{\alpha \rho_1^2 (1 - \rho_2^2)}{\alpha \rho_2^2 + 1 - \rho_2^2} \right) + 2\pi e (1 - \rho_1^2)$$

$$= 2\pi e \left(\frac{\alpha \rho_1^2 (1 - \rho_2^2) + (1 - \rho_1^2)(\alpha \rho_2^2 + 1 - \rho_2^2)}{\alpha \rho_2^2 + 1 - \rho_2^2} \right), \quad (33)$$

where (c) follows from (32). Now let us focus only on the numerator of (33) (inside the biggest parenthesis). We continue scrutinizing it as

$$\begin{split} &\alpha\rho_{1}^{2}(1-\rho_{2}^{2})+(1-\rho_{1}^{2})(\alpha\rho_{2}^{2}+1-\rho_{2}^{2})\\ &=\alpha\rho_{1}^{2}-2\alpha\rho_{1}^{2}\rho_{2}^{2}+\alpha\rho_{2}^{2}+1-\rho_{2}^{2}-\rho_{1}^{2}+\rho_{1}^{2}\rho_{2}^{2}\\ &=(\alpha\rho_{1}^{2}+1-\rho_{1}^{2})(1-\frac{\rho_{2}^{2}}{\rho_{1}^{2}})+\frac{\rho_{2}^{2}}{\rho_{1}^{2}}(\alpha\rho_{1}^{2}+1-\rho_{1}^{2})\\ &+\rho_{2}^{2}(-2\alpha\rho_{1}^{2}-1+\alpha+\rho_{1}^{2})\\ &=(\alpha\rho_{1}^{2}+1-\rho_{1}^{2})(1-\frac{\rho_{2}^{2}}{\rho_{1}^{2}})+\frac{\rho_{2}^{2}}{\rho_{1}^{2}}(1-\rho_{1}^{2})(2\alpha\rho_{1}^{2}+1-\rho_{1}^{2})\\ &\stackrel{(d)}{\geq}(\alpha\rho_{1}^{2}+1-\rho_{1}^{2})(1-\frac{\rho_{2}^{2}}{\rho_{1}^{2}}), \end{split} \tag{34}$$

where (d) follows because $\alpha \in (0,1]$ and $\rho_1^2, \rho_1^2 < 1$. Plugging (34) into (33), we obtain that

$$e^{2h(Y|U,Z)} \ge 2\pi e \left(\frac{(\alpha\rho_1^2 + 1 - \rho_1^2)(1 - \rho_2^2/\rho_1^2)}{\alpha\rho_2^2 + 1 - \rho_2^2}\right).$$
 (35)

Therefore,

$$h(Y|U,Z) \ge \frac{1}{2} \log 2\pi e \left(\frac{(\alpha \rho_1^2 + 1 - \rho_1^2)(1 - \rho_2^2/\rho_1^2)}{\alpha \rho_2^2 + 1 - \rho_2^2} \right).$$
(36)

Finally, substituting (32) and (36) into (29)–(31), the converse proof of the region \mathcal{R}_G is completed.

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