Predicting Interconnect Uncertainty with A New Robust Model Order Reduction Method

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Abstract— As we scale toward nanometer technologies, the increase in interconnect parameter variations will bring significant performance variability. New design methodologies will emerge to facilitate construction of reliable systems from unreliable nanometer scale components. Such methodologies require new performance models which accurately capture the manufacturing realities. In this paper, we present a Linear Fractional Transform (LFT) based model for interconnect Parametric Uncertainty. This new model formulates the interconnect parameter uncertainty as a repeated scalar uncertainty structure. With the help of generalized Balanced Truncation Realization (BTR) based on Linear Matrix Inequalities (LMI's), the new model reduces the order of the original interconnect network while preserves the stability. This paper also shows that the LFT based model even guarantees passivity if the BTR reduction is based on solutions to a pair of Linear Matrix Inequalities (LMI's) which generalizes Lur'e equations .

1 Introduction

With decreasing MOS transistors for DSM (Deep Submicron) technologies, the influence of fluctuations in process parameters during manufacturing becomes increasingly important. Typically, the effect of process variations are captured by a set of worst-case device model parameters , and the circuit performance is evaluated at these worst-case corners. However, the context-dependent interconnect in DSM technologies complicates the feasibility of worst-case corner method by increasing the dimensionality of the problem. Moreover, the worst-case corner methods are known to create overly pessimistic results. Thus, the worst-case corner methods need to be replaced by more accurate methods that evaluate the uncertain nature of the system performance.

Because of the need to obtain accurate interconnect models with parametric uncertainty, interconnect variational analysis based on model order reduction has been an active research field in nanometer design automation over the past several years. Liu et al. in [1] studied the effect of interconnect parameter variations on three projection-based model order reduction techniques: Krylov subspace based, PACT [2] and PRIMA [3]. The paper basically combines the matrix perturbation theory [9] and the model order reduction methods. Heydari and Pedram in [8] proposed a BTR based interconnect variational analysis method. The BTR based method offers a weighted error bound.

Recently, it has become apparent that the methods in [1] and [8] directly approximate the projection matrices as perturbed matrices from the nominal ones. The reduced system is unable to preserve stability. As a result, subsequent analysis with nonlinear devices can cause instability [6].

In this paper, we discuss TBR-like LFT based interconnect uncertainty models that can preserve the stability and even the passivity. The new models have computable error bounds, and, unlike the existing variational analysis methods, pose no constraints on the internal structure of the state-space model. With LFT technique, We model the uncertain interconnect system as a repeated scalar uncertainty structure. We then reduce the original uncertain systems relying on the solution of two linear matrix inequalities (LMI's) with guaranteed error bounds.

The paper is organized as follows. In Section II, we briefly present the relevant Concepts, review balanced Realizations and recall available error bounds for the truncation of the models. In Section III, we discuss the uncertainty systems we will be treating. Section IV, we present a TBR-like methods that guarantee stable and even passive reduced models. In Section V, we show examples that illustrate the relevance of the algorithms presented in the paper. Finally, in Section VI, conclusions are presented.

2 Background

2.1 State-Space Models

Given a state-space model in descriptor form

$$E\frac{dx}{dt} = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) \tag{2}$$

where $E, A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times p}$, $C \in \mathcal{R}^{p\times p}$, $D \in \mathcal{R}^{p\times p}$, $y(t)$, $u(t) \in \mathcal{R}^{p}$, model reduction algorithms seek to produce a similar system

$$\tilde{E}\frac{d\tilde{x}}{dt} = \tilde{A}\tilde{x}(t) + \tilde{B}u(t) \tag{3}$$

$$\tilde{y}(t) = \tilde{C}x(t) + \tilde{D}u(t) \tag{4}$$

where $\tilde{E}$, $\tilde{A} \in \mathcal{R}^{q\times q}$, $\tilde{B} \in \mathcal{R}^{q\times p}$, $\tilde{C} \in \mathcal{R}^{p\times q}$, $\tilde{D} \in \mathcal{R}^{p\times p}$, of order q much smaller than the original order n, but for which the outputs $y(t)$ and $\tilde{y}(t)$ are approximately equal for inputs $u(t)$ of interest. To simplify the notation, in this paper, we set $E = I$. And we use M to denote the system matrices: $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$. The transfer functions

$$H(s) = D + C(sI - A)^{-1}B \tag{5}$$

$$\tilde{H}(s) = \tilde{D} + \tilde{C}(s\tilde{I} - \tilde{A})^{-1}\tilde{B} \quad (6)$$

are used as a metric for approximation: if $\|H(s) - \tilde{H}(s)\| < \varepsilon$, in some appropriate norm, for some given allowable error ε and allowed domain of the complex frequency variable s, the reduced model is accepted as accurate.

2.2 Passivity

When modeling passive systems, those that cannot produce energy internally, it is desired that the reduced models also be passive. Otherwise, the reduced models may cause non-physical behavior when used in later simulations, such as by generating energy at high frequencies that causes erratic or unstable time-domain behavior. Passivity is implied by positive-realness of the transfer function. The function $H(s)$ is positive-real(PR), if

$$\bar{H}(s) = H(\bar{s}) \quad (7)$$

$$H(s) \ \ is \ \ analytic \ \ in \ \ \{s : Re(s) > 0\} \quad (8)$$

$$H(s) + H(s)^H \geq in \ \ \{s : Re(s) > 0\} \quad (9)$$

2.3 BTR based Model Reduction Techniques

i. Balanced Truncation Realization (BTR):
The BTR procedure is centered around information obtained from the controllability Grammian W_c, which can be obtained from solving the Lyapunov equation

$$AW_cA^T - W_c + BB^T = 0 \quad (10)$$

for W_c, and the observability Grammian W_o, which can be obtained from the dual Lyapunov equation

$$AW_oA^T - W_o + CC^T = 0 \quad (11)$$

for W_o. Under a similarity transformation of the state-space model

$$A \to T^{-1}AT \ \ B \to T^{-1}B \ \ C \to CT \quad (12)$$

the input-output properties of state-space model, such as the transfer function, are invariant (only the internal variables are changed). The grammians, however, vary under the rules $W_c \to T^{-1}W_cT^{-T}$ and $W_o \to T^TW_oT$ and so are not invariant. The BTR procedure is based on two observations about W_o and W_c. First, the eigenvalues of the product W_cW_o are invariant. These eigenvalues, the Hankel singular values, contain useful information about the input and output behavior of the system. In particular, "small" eigenvalues of W_cW_o correspond to internal sub-systems that have a weak effect on the input-output behavior of the system and are, therefore, close to nonobservable or noncontrollable or both. Second, since the Grammians transform under congruence, and as any two symmetric matrices can be simultaneously diagonalized by an appropriate congruence transformation, it is possible to find a similarity transformation T that leaves the state-space system dynamics unchanged, but makes the transformed $\widehat{W_c}$ and $\widehat{W_o}$ equal and diagonal. In these coordinates, with $\widehat{W_c} = \widehat{W_o} = \Sigma$, we may partition Σ into

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \quad (13)$$

where Σ_1 describes the "strong" sub-systems to be retained and Σ_2 the "weak" sub-systems to be deleted. Conformally partitioning the transformed matrices as

$$\hat{A} = \begin{bmatrix} \hat{A_{11}} & \hat{A_{12}} \\ \hat{A_{21}} & \hat{A_{22}} \end{bmatrix} \ \ \hat{B} = \begin{bmatrix} \hat{B_1} \\ \hat{B_2} \end{bmatrix} \ \ \hat{C} = [\hat{C_1} \ \ \hat{C_2}] \quad (14)$$

and truncating the model, retaining $\hat{A} = \hat{A_{11}}$, $\hat{B} = \hat{B_1}$ and $\hat{C} = \hat{C_1}$ as the reduced system, therefore has the effect of deleting the "weak" internal subsystems.
One of the attractive aspect of BTR methods is that computable error bounds are available. If the i-th diagonal entry of the matrix Σ is given by σ_i, and the σ_i is ordered as $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_N$, then the error in the transfer function of the order-q reduced model is bounded by

$$\|H(s) - \hat{H}_q(s)\|_\infty \leq 2\Sigma_{k=q+1}^{N}\sigma_k \quad (15)$$

ii. Passivity-preserving BTR like procedure
In [7], a BTR based procedure that can assess the positive-realness of a state-space model in a global manner was presented. The authors proved that $H(s)$ is positive-real if and only if there exist matrices $X_c = X_c{}^T \geq 0$, J_c, K_c such that the Lur'e equations

$$AX_cA^T - X_c + K_cK_c{}^T = 0 \quad (16)$$

$$X_cC^T - B + K_cJ_c{}^T = 0 \quad (17)$$

$$J_cJ_c{}^T = D + D^T \quad (18)$$

and its dual

$$AX_oA^T - X_o + K_oK_o{}^T = 0 \quad (19)$$

$$X_oC^T - B + K_oJ_o{}^T = 0 \quad (20)$$

$$J_oJ_o{}^T = D + D^T \quad (21)$$

are satisfied. It is easy to verify that X_c and X_o transform under similarity transformation just as W_c and W_o. Their eigenvalues are invariant. And by Theorem 2 in [7], the reduced order system is positive-real if the original system is positive-real.

3 Interconnect Parametric Uncertainty

Characterization of the interconnect geometry variation is an important issue in deep-submicron VLSI technology. In order to accurately assess the performance of an interconnect system, it is essential to characterize the interconnect geometry, which in turn specifies the interconnect parasitics. From a designer point of view, one important source of the IC performance variability is the physical source of variability. For the purpose of design performance evaluation, we are concerned with two possible cases here. The first one includes the case where the interconnect parameters are constant within a die but vary within a wafer or a lot. In the second case, the interconnect parameters vary within a die. The inter-die variability can be minimized by using several techniques and corrections during the fabrication process. Due to its relatively low spatial frequency and smoothness, simple linear models can be used to describe the wafer level variations. As a result, for a given metal wire, if we know there is a width variation of w_1 and a height variation of w_2, then the resistance and capacitance of that particular metal wire are

$$r(w_1, w_2) = r_0 + r_1 w_1 + r_2 w_2 \tag{22}$$

$$c(w_1, w_2) = c_0 + c_1 w_1 + c_2 w_2 \tag{23}$$

or in general, for the conductance and capacitance/susceptance matrices of the interconnect system that are exposed to the process variations, we have:

$$G(w_1, w_2) = G_0 + G_1 w_1 + G_2 w_2 \tag{24}$$

$$C(w_1, w_2) = C_0 + C_1 w_1 + C_2 w_2 \tag{25}$$

In this paper, we derive the equations for RC circuit. RLC/RL/LC circuits can have the similar results. The MNA equations for RC circuit can be written as

$$\begin{aligned} (G(w_1, w_2) + sC(w_1, w_2))x &= Bu \\ y &= C_y x + D_y u \end{aligned} \tag{26}$$

We can rewrite the above equation as

$$sx = -C^{-1}(w_1, w_2)G(w_1, w_2)x + C^{-1}(w_1, w_2)Bu$$

thus, the system matrices are

$$\begin{aligned} \bar{A} &= -C^{-1}(w_1, w_2)G(w_1, w_2) \;\; \bar{B} = C^{-1}(w_1, w_2)B \\ \bar{C} &= C_y \quad \bar{D} = D_y \end{aligned} \tag{27}$$

where $\bar{A} \in \mathcal{R}^{n\times n}$, $\bar{B} \in \mathcal{R}^{n\times p}$, $\bar{C} \in \mathcal{R}^{p\times p}$, $\bar{D} \in \mathcal{R}^{p\times p}$, $\underline{x} \in \mathcal{R}^{n}$, y and $u \in \mathcal{R}^{p}$. $\bar{A}$ is a function of w_1 and w_2. So is $\bar{B}$. If we denote

$$dC = [C_1 \;\; C_2], \;\; dG = [G_1 \;\; G_2], \;\; w = [w_1 \;\; w_2]^T$$

we can write the Taylor series expansion at $w = 0$ (mean value) as

$$\begin{aligned} \bar{A}(w) &= -C_0^{-1}G_0 - (-C_0^{-1}dCC_0^{-1}G_0 \\ &+ C_0^{-1}dG)w + (C_0^{-1}dCC_0^{-1}dCC_0^{-1}G \\ &- C_0^{-1}dCC_0^{-1}dG)w + \cdots \end{aligned} \tag{28}$$

Similarly:

$$\begin{aligned} \bar{B}(w) &= C_0^{-1}B_0 + (-C_0^{-1}dCC_0^{-1}B_0 \\ &+ C_0^{-1}dG)w + (C_0^{-1}dCC_0^{-1}dCC_0^{-1}B_0 \\ &+ \cdots \end{aligned} \tag{29}$$

Both $\bar{A}$ and $\bar{B}$ have infinite number of terms. The transfer function is

$$H(s) = \bar{D} + \bar{C}(SI - \bar{A})^{-1}\bar{B} \tag{30}$$

For efficiency and accuracy reasons, existing variational model order reduction methods only approximate the the impact of parametric variations to the first or second order and ignore higher order terms. The resulting system matrices become

$$\begin{aligned} \bar{A}_t &= -C_0^{-1}G_0 - (-C_0^{-1}dCC_0^{-1}G_0 + C_0^{-1}dG)w \\ \bar{B}_t &= C_0^{-1}B_0 + (-C_0^{-1}dCC_0^{-1}B_0 + C_0^{-1}dG)w \\ \bar{C}_t &= \bar{C} \qquad\qquad \bar{D}_t = \bar{D} \end{aligned} \tag{31}$$

And the transfer function becomes

$$H_t(s) = \bar{D}_t + \bar{C}_t(SI - \bar{A}_t)^{-1}\bar{B}_t \tag{32}$$

The physical meaning of this procedure is that we first approximate the uncertainty system described from Eq(26) to Eq(30) as first order system in Eq(31) and Eq(32), them we apply model order reduction on the approximated uncertain system. The projection based transform matrix is estimated as:

$$T(w_1, w_2) = T_0 + T_1 w_1 + T_2 w_2 \tag{33}$$

Unfortunately, the first-order variational admittance macromodel is not a congruence transformation which is essential for macromodel passivity. This fact can be verified by following PRIMA algorithm and computing the first order variational admittance matrix for a single parameter as:

$$G_r(w_1) = T^T(w_1)G(w_1)T(w_1) \tag{34}$$

$$G_r(w_1) = (T_0 + dT_1 w_1)^T(G_0 + dG_1 w_1)(T_0 + dT_1 w_1) \tag{35}$$

$$\begin{aligned} G_r(w_1) = T_0^T G_0 X_0 + w_1 * (dT_1^T G_0 T_0 + \\ + T_0^T dG_1 T_0 + T_0^T G_0 dT_1 + h.o.t \end{aligned}$$

The main problem of the loss of passivity is because of the truncation of higher order terms. Therefore, unlike the nominal case, the previous variational reduced order models do not preserve the passivity and stability. Hence, their interfaces with general transistor-level analysis tools have potential divergent behavior.

4 LFT based uncertainty presentation

In this section, we will show, by means of RC circuits with 2 variation variable, that the Linear Fractional Transform (LFT) can represent the uncertain system in a "compact" way without ignoring the higher order terms.

Over the past decade, the LFT paradigm has been widely used as a mathematical representation for uncertainty in system models [4] [5]. This paradigm is represented pictorially in Figure 1. In LFT, M represents the nominal system model

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{36}$$

Δ represents the uncertainty. Because each perturbation source i.e. w_i as ith geometry perturbation, is likely to enter the real system at a different location, collecting these into one uncertain block results in Δ have a diagonal block structure. Furthermore, the perturbations are often assumed to be norm-bounded operators. The input/output (I/O) mapping from u to y is given by the LFT as

$$y = (\Delta * M)u \tag{37}$$

where

$$\Delta * M := D + C\Delta(I - A\Delta)^{-1}B \tag{38}$$

Because of the use of the feedback loop, we are able to represent the uncertain system with any number of higher order terms without storing all the high order coefficient matrices.

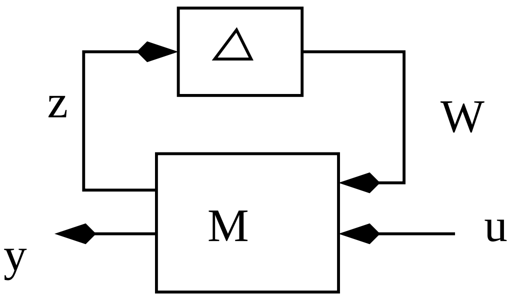

Figure 1: LFT presented uncertain system

For 2-D process variation cases, we collect the uncertain sources into one uncertain block

$$\bar{\Delta} = \begin{pmatrix} w_1 I_{n\times n} & 0 \\ 0 & w_2 I_{n\times n} \end{pmatrix}$$

where $\bar{\Delta} \in \mathcal{R}^{2n\times 2n}$. This means we consider the uncertain sources w_1 and w_2 will affect each internal state $x \in \mathcal{R}^n$. To form the feedback loop, we introduce two new variables z and $\bar{w}$, and

$$\bar{w} = \bar{\Delta} z = \begin{pmatrix} w_1 I & 0 \\ 0 & w_2 I \end{pmatrix} z$$

where z and $\bar{w} \in \mathcal{R}^{2n}$. Thus, Eq(26) and Eq(27) can be reorganized as

$$\begin{aligned} sx &= \bar{A}x + \bar{B_1}\bar{w} + \bar{B_2}u \\ z &= \bar{C_1}x + \bar{D_{11}}\bar{w} + \bar{D_{12}}u \\ y &= \bar{C_2}x + \bar{D_{21}}\bar{w} + \bar{D_{22}}u \end{aligned} \tag{39}$$

where

$$\bar{C_1} = \begin{pmatrix} C_0^{-1}C_1\bar{A_0} + C_0^{-1}G_1 \\ C_0^{-1}C_2\bar{A_0} + C_0^{-1}G_2 \end{pmatrix}$$

$$\bar{B_1} = \begin{pmatrix} I & I \end{pmatrix}$$

$$\bar{D_{11}} = \begin{pmatrix} C_0^{-1}C_1 & C_0^{-1}C_1 \\ C_0^{-1}C_2 & C_0^{-1}C_2 \end{pmatrix}$$

$$\bar{A} = -C_0^{-1}G_0 \quad \bar{B_2} = C_0^{-1}B_0$$

$$\bar{C_2} = C_y \quad \bar{D_{21}} = 0 \quad \bar{D_{22}} = D_y$$

The C_y and D_y are defined in Eq(26). We can use $\bar{M}$ to represent the system matrices,

$$\bar{M} = \begin{bmatrix} \bar{A} & \bar{B_1} & \bar{B_2} \\ \bar{C_1} & \bar{D_{11}} & \bar{D_{12}} \\ \bar{C_2} & \bar{D_{21}} & \bar{D_{22}} \end{bmatrix} \tag{40}$$

Figure 2 shows the LFT paradigm of the new presentation.

$$\Delta = \begin{bmatrix} I/s & 0 \\ 0 & \bar{\Delta} \end{bmatrix}$$

Theorem 1. The uncertain system described by Eq(39) is equivalent to the uncertain system described by Eq(26). By substituting the LFT definitions of system matrices into Eq39), we can easily prove Theorem 1. For example, substitute

$$\bar{\Delta}z = \bar{\Delta}\bar{C_1}x + \bar{\Delta}\bar{D_{11}}\bar{w} + \bar{\Delta}\bar{D_{12}}u$$

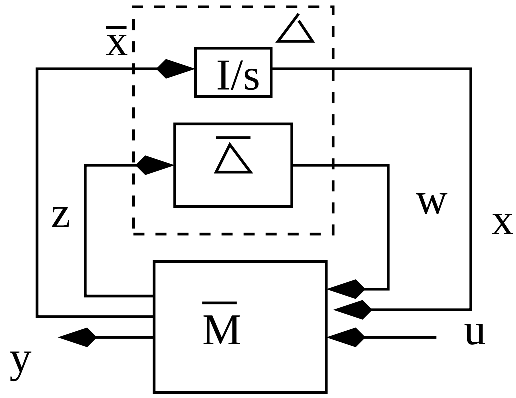

Figure 2: LFT presented uncertain system of 2-D process variations

into $\bar{w}$ of

$$sx = \bar{A}x + \bar{B_1}\bar{w} + \bar{B_2}u$$

we have

$$sx = \bar{A}x + \bar{B_1}(\bar{\Delta}\bar{C_1}x + \bar{\Delta}\bar{D_{11}}\bar{w} + \bar{\Delta}\bar{D_{12}}u) + \bar{B_2}u$$

We get the first order terms i.e. w_1x and w_2x. If we continue this substitution process, we are going to get $w_1{}^2x$ and $w_2{}^2x$ and all the other higher order terms.

Recently, major progress has been made in the model order reduction techniques for LFT represented uncertain systems. The following theorems and algorithm have been proved by two of the most important papers [4] [5] in this area.

Theorem 2. The system defined by the pair (Δ, M) is stable if $(I - A\Delta)$ is invertible.

It is easy to show that our LFT represented uncertain system satisfies "$(I - A\Delta)$ is invertible" condition. Therefore, the system (Δ, $\bar{M}$) is stable.

Theorem 3. (Δ, M) is stable if and only if there exist $Y \geq 0$ and $X \geq 0$ which satisfy the **Lyapunov inequalities**

$$\begin{aligned} AYA^T - Y + BB^T &\leq 0 \\ A^TXA - X + C^TC &\leq 0 \end{aligned} \tag{41}$$

For a stable uncertain systems, the existence of balanced realization is guaranteed by Theorem 3. However, the matrices Y and X are nonunique. We refer to these Y and X matrices as **Generalized Graminans**. We get the unique pair of X and Y by finding the minimality of the reduction through "minimize trace(XY)" as in Algorithm 1.

Theorem 4. Given a stable system representation (Δ, M), there exists a reduced, stable representation, (Δ_r, M_r) if and only if there exist $Y \geq 0$ and $X \geq 0$ which satisfy the **Lyapunov inequalities**

$$\begin{aligned} AYA^T - Y + BB^T &\leq 0 \\ A^TXA - X + C^TC &\leq 0 \end{aligned} \tag{42}$$

Truncating a balanced stable uncertain system realization results in a lower dimension realization that is balanced and stable which is easily seen by considering the system Lyapunov Inequalities. We now state the balanced truncation model reduction error bound theorem for uncertain and multidimensional systems. The difference between the original transfer function (realization) and the reduced order system's transfer function $(\Delta * M) - (\Delta_r * M_r)$ can be quantified by using the Structured Induced 2-norm (SI2-norm).

Theorem 5. Suppose (Δ_r, M_r) is the reduced model obtained from the balanced stable system (Δ, M), then

$$\|(\Delta, M) - (\Delta_r, M_r)\|_{SI2} \leq 2\Sigma i \in I\sigma_i \tag{43}$$

here I indicates the indices of neglected eigenvalues and σ_i indicates the square roots of the value XY defined in Algorithm 1.

Algorithm 1. Balanced Truncated Realization for Structure Uncertainty

1) Solve LMI equations

$$\begin{aligned} minimum\ \ trace(XY) \\ such\ \ that\ \ X \geq 0 \\ A^T XA - X + C^T C \leq 0 \\ Y \geq 0 \\ AYA^T - Y + BB^T \leq 0 \end{aligned} \tag{44}$$

2) Compute Cholesky factors $X = L_x L_x^T$, $Y = L_Y L_Y^T$,
3) Compute SVD of Cholesky product $U\Sigma V = L_x^T L_Y$ where Σ is diagonal positive and U,V have orthonormal columns,
4) Compute the balancing transformations $T = L_Y V \Sigma^{-1/2}$, $T^{-1} = \Sigma^{-1/2} U^{-T} L_x^T$,
5) Form the balanced realization as $\hat{A} = T^{-1}AT$,$\hat{B} = T^{-1}B$, $\hat{C} = CT$.

From the above procedure, it is obvious that we use LMI for generalized BTR to get the transform matrices for the uncertain system. Similarly, we can also change the Lur'e equations Eq(16)-Eq(21) to inequality and use LMI to get the transform matrices. The later one will lead to passive reductions as proved by [7] theorem 2 on page 6.

5 Experiment Results

In this section, we show examples that illustrate the applicability of our new algorithm presented in this paper. The first example is an RLC circuit. Figure 3 shows the diagram of the circuit. We set the variation percentage to 20%. We achieve 66.6% reduction. Figure 4 shows the far end waveform compared with SPICE result. From Figure 5, we observe that the frequency response of the reduced order system obtained by our new algorithm closely follows the frequency response of the original system.

Figure 7 shows a clock tree routed using the TSMC 0.25 μ technology. The clock tree is an H-tree clock distribution with tapered buffers [8]. Table 1 shows that we predict the 50% delays at the fanout end as accurate as SPICE results. We implemented algorithms in [1] and [8]. Our results are compared with those from [1] and [8]. In order to further demonstrate the accuracy of our model, we performed 50 tests. In each test, a set of normal distribution numbers is generated as the width variations of three metal layers. We compared the difference between the poles of our model and those in [1]. The distribution of error in the poles is shown in Figure 6. Even though the poles vary by as much as 300.7% to 0.85%.

6 Conclusion

In this paper, we present a Linear Fractional Transform (LFT) based model for interconnect Parametric Uncertainty. This new model formulates the interconnect parameter uncertainty as a repeated scalar uncertainty structure. With the help of generalized Balanced Truncation Realization (BTR) based on Linear Matrix Inequalities (LMI's), the new model reduces the order of the original interconnect network while preserves the stability. This paper also shows that the LFT based model even guarantees passivity if the BTR reduction is based on solutions to a pair of Linear Matrix Inequalities (LMI's) which generalizes Lur'e equations .

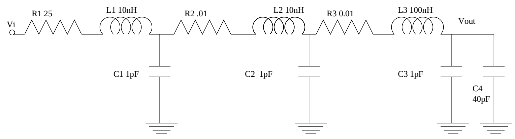

Figure 3: An RLC circuit with parametric variations

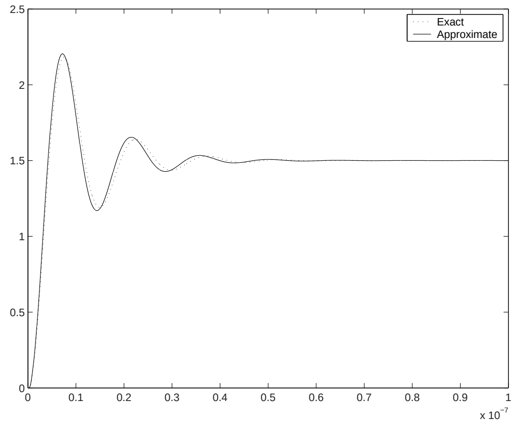

Figure 4: An RLC circuit with parametric variations

References

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Table 1: 50% delay of H shape clock tree at one sink

M4-w (%)	M4-h (%)	New (ns)	Spice (ns)	error (%)	[1] (ns)	[8] (ns)
-10.0	30.0	4.113	4.100	0.3	4.102	4.110
20.0	20.0	3.018	2.943	0.5	2.970	2.963
-5.0	20.0	3.489	3.505	0.6	3.505	3.521
-30.0	-30.0	4.983	4.990	0.1	4.980	4.990
30.0	-10.0	4.203	4.234	0.7	4.257	4.233
20.0	-5.0	3.442	3.425	0.6	3.454	3.440
4.0	4.0	3.447	3.448	0.08	3.448	3.448
10.0	-6.0	3.170	3.209	0.1	3.169	3.212
-20	10	2.896	2.830	0.6	2.890	2.830
25	-5	3.585	3.606	0.6	3.598	3.610
10	5	3.682	3.694	0.7	3.690	3.682

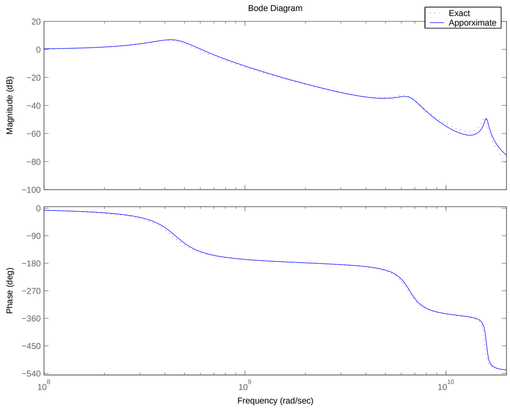

Figure 5: An RLC circuit with parametric variations

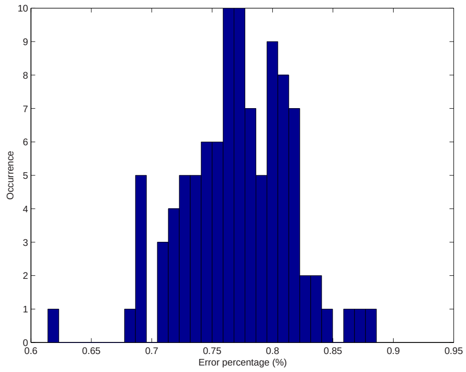

Figure 6: Error distribution of all the poles

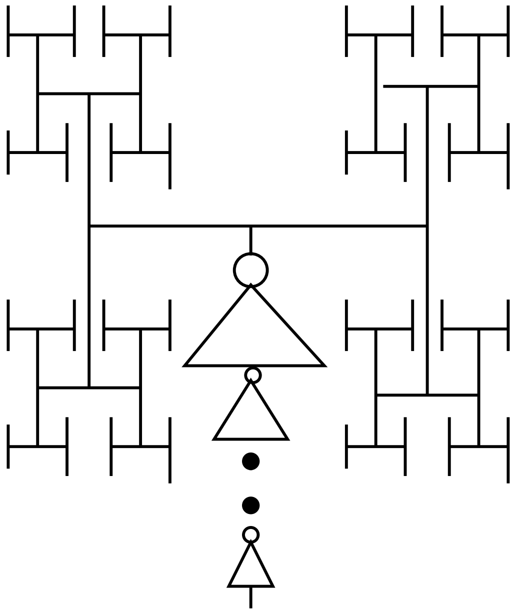

Figure 7: An H-tree clock distribution driven by a tapered buffer

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