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# Adaptive Matching for Compact MIMO Systems

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**Abstract**—Compact MIMO systems using closely spaced antennas are faced with the well-known problem of antenna mutual coupling (MC) that can degrade the performance. Previous studies have shown that a proper choice of antenna load impedances can maximise the MIMO capacity or received power in the presence of MC. However, to calculate this optimum load, prior knowledge of the propagation channel matrix and the MC model, which is difficult to measure practically, are required. In this paper, we present an adaptive matching approach for the receiver that directly deals with the received signals rather than the channel and MC models, to find optimum load impedances which maximise the MIMO capacity or received power.

**Index Terms**—MIMO; performance; capacity; received power; mutual coupling; impedance matching; adaptive matching

## I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) technology, by using multiple antennas at both transmit and receive sides of the wireless link, offers a better link quality and higher data-rates [1]. However, applying MIMO at small wireless devices suffers from antenna mutual coupling (MC) which degrades the MIMO channel capacity [2]–[6]. Among previous studies, choosing the load impedances has been presented as a solution to control the radiation pattern of coupled antennas and thus the MIMO capacity and/or received power. There are two methods which either use a complex coupled matching network called a *multiport-conjugate match* [2], [3] or apply a simple uncoupled network called a *single/individual-port match* [4], [6]–[8]. It has been claimed that multiport matching network offers a significant capacity improvement but only for small bandwidths, while the individual-port matching network is simpler to implement and offers a broader bandwidth by finding an optimum load impedance for a given propagation environment [6], [8]. However, both methods require a prior knowledge of the propagation channel and an accurate MC model which is difficult to measure in practice. They use open-circuit voltages and scattering-parameters, respectively, to describe the MC among the transmit and receive antennas. In [9], [10] it has been claimed that those methods are not capable of modeling MC at the receiving array properly.

In this paper, we concentrate on the receive side of the MIMO system and propose an adaptive matching approach that directly deals with the received signals to find an uncoupled optimum load match which maximises the capacity or received power. Having used the received signals, the realistic effects of the propagation channel and the MC will be incorporated in the calculation process. We numerically

show how this method performs for different propagation environments.

The remainder of this paper is organised as follows. Section II gives a review of the Z-parameter representation of the MIMO system model. In section III, we derive capacity and received power expressions based on the received signals over a time interval. This is followed by a description of the adaptive matching method in section IV, which is treated as a random search for the uncoupled optimum load by using a smart step size. In section V, numerical results to optimise the mean capacity are performed to validate the proposed algorithm. We conclude the paper in Section VI.

## II. MIMO MODEL

We consider a MIMO system of  $M_T$  transmit and  $M_R$  receive antennas, communicating through a frequency-flat fading channel. The relationship between the transmit signal vector  $\mathbf{x}(t) \in \mathbb{C}^{M_T}$  and the receive signal vector  $\mathbf{y}(t) \in \mathbb{C}^{M_R}$  at time  $t$  is given by

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{n}(t) \quad (1)$$

where  $\mathbf{H} \in \mathbb{C}^{M_R \times M_T}$  is the channel gain matrix including MC effect, and  $\mathbf{n}(t) \in \mathbb{C}^{M_R}$  represents a vector of additive white Gaussian noise at the receiver, which is assumed to be a complex Gaussian noise with zero-mean and covariance matrix  $\sigma_n^2 \mathbf{I}_{M_R}$  where  $\mathbf{I}_{M_R}$  is a  $M_R \times M_R$  identity matrix. For the sake of simplicity, given a transmit power constraint  $P_T$ , we consider an equivalent model for (1) with a unit-variance noise and transmit power  $P_T/\sigma_n^2$ . Having  $\mathbf{R}_x$  as the covariance matrix of the input vector  $\mathbf{x}$ , the output covariance matrix  $\mathbf{R}_y$  associated with the received signal vector  $\mathbf{y}$  can be written as:

$$\begin{aligned} \mathbf{R}_y &= \mathcal{E}\{\mathbf{y}\mathbf{y}^H\} \\ &= \mathbf{H}\mathcal{E}\{\mathbf{x}\mathbf{x}^H\}\mathbf{H}^H + \mathcal{E}\{\mathbf{n}\mathbf{n}^H\} \\ &= \mathbf{H}\mathbf{R}_x\mathbf{H}^H + \mathbf{I}_{M_R} \end{aligned} \quad (2)$$

where  $(\cdot)^H$  is conjugate-transpose operator, and  $\mathbf{x}$  and  $\mathbf{n}$  are assumed to be uncorrelated. In other words,  $\mathcal{E}\{\mathbf{x}\mathbf{n}^H\} = \mathcal{E}\{\mathbf{n}\mathbf{x}^H\} = \mathbf{0}$ .

## III. MIMO CAPACITY AND RECEIVED POWER

The well-known MIMO channel capacity expression is given by

$$C = \log_2 \det(\mathbf{I}_{M_R} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H) \quad (3)$$

We note that accurately measuring the channel matrix for closely coupled antennas, taking MC modeling into account,

is difficult practically. So it is easier if we can work with the received signal  $\mathbf{y}$ . However, this is limited by the fact that in order to optimise the impedance we need to directly try out different impedance choices and see the effect on the capacity/received power.

Looking at (2) and (3), it is clear that we could use (an estimation of) the covariance matrix of the received signals to calculate the capacity, rather than estimating a channel model including MC effect. One way of implementing this idea is substituting a time averaging estimation of  $\mathbf{R}_y$  into the argument of  $\log_2 \det$  function at (3) as follows

$$C = \log_2 \det \left( \frac{1}{L} \sum_{i=t_0}^{t_0+L-1} \mathbf{y}[i] \mathbf{y}^H[i] \right) \quad (4)$$

where  $t_0$  is the starting sample time,  $L$  is the data-block length, and  $i$  is the time index for discrete-time samples. We further assume that the block length  $L$  is long enough for equation (2) to hold, and that  $\mathbf{H}$  does not change over each data block. Now, we have an expression for the capacity that includes propagation channel properties and MC effects by having a block of received data with no further parameters required.

Assuming  $\mathbf{y}(t)$  is the received voltage vector across the load terminals of receive antennas, the received power for  $i$ th antenna can be written as

$$P_{r,i} = \mathcal{E}\left\{\frac{y_i(t)y_i^*(t)}{R_{L,i}}\right\}/P_0, \quad i = 1, \dots, M_R \quad (5)$$

where  $(\cdot)^*$  denotes conjugate operator,  $R_{L,i}$  represents the real part of the load terminal  $Z_{L,i}$  for antenna  $i$ , and  $P_0$  is the power received by a conjugate matched isolated antenna which is used to normalise the MIMO received power. Here we assume all received antennas are terminated with identical loads. So, the total mean received power can be expressed as  $P_r = \mathcal{E}\{\mathbf{y}(t)\mathbf{y}^H(t)/R_L\}/P_0$ . Similar to the estimation procedure for the capacity, we can estimate the total received power from the following expression

$$P_r = \frac{1}{P_0} \left( \frac{1}{LR_L} \sum_{i=t_0}^{t_0+L-1} \mathbf{y}[i] \mathbf{y}^H[i] \right) \quad (6)$$

#### IV. ADAPTIVE MATCH ALGORITHM

In this section, we describe the proposed algorithm to find the optimum impedance match for an arbitrary propagation environment in the presence of the MC effects. As we mentioned in the previous section, we have to try out different load impedances and calculate the capacity and received power from (4) and (6) for each load. This allows us to find the optimum load which maximises the capacity or received power. One way is to try a possible range of load impedances and find the optimum load which corresponds to the maximum peak of the capacity or received power. This method has a high computational load and needs a large memory to keep all data. Furthermore, we have to repeat this process for different channel propagation conditions. Obviously, it is not practical specially for small portable wireless devices.

Instead, we propose an adaptive matching algorithm that uses fewer load impedances and has much lower computational burden. The algorithm starts a random search<sup>1</sup> for the optimum load impedance from an initial impedance  $Z_L^0$  (for instance  $50\Omega$ ), and for each step  $m$  selects a terminal impedance as follows

$$\begin{aligned} Z_L^m &= Z_L^{m-1} + (\Delta_R + j\Delta_X) \\ &= (R_L^{m-1} + \Delta_R) + j(X_L^{m-1} + \Delta_X) \end{aligned} \quad (7)$$

where  $R_L^{m-1}$  and  $X_L^{m-1}$  are the real and imaginary parts of the load impedance  $Z_L^{m-1}$  at step  $(m-1)$ , and  $\Delta_R, \Delta_X$  are randomly selected step sizes from the set  $\{-\Delta, 0, \Delta\}$  for a given  $\Delta$ , but are not equal to zero simultaneously. At each step, the mean capacity/received power, which is calculated by averaging the capacity/received power from equations (4) or (6) over  $K$  data-blocks, is compared with the previous value. The impedance which corresponds to the greater mean capacity/received power is hold as the optimum  $Z_L$  at each step.

In this work, we have considered a variable step size  $\Delta$  to have a faster convergence. We start from a large value such as  $\Delta = 16\Omega$  and then decrease it after having a specific number of unchanged choices for optimum  $Z_L$ , by dividing the present  $\Delta$  over 2 for  $\Delta \geq 2\Omega$ .

#### V. NUMERICAL RESULTS

To investigate the proposed adaptive matching algorithm, some simulations for a  $3 \times 3$  MIMO system of half-wavelength dipoles with identical loads at antenna spacing  $d = 0.05\lambda$  is carried out. We optimize the mean capacity under different propagation environments: 2D uniform, and 2D Laplacian defined by the mean  $\phi_0$  and the standard deviation  $\sigma$  of the distribution, for two signal-to-noise-ratios (SNR) 5 and 20dB at the receiver. We assume the transmit antennas to be separated far enough (negligible MC effect at the transmit side) and to be self-impedance conjugate matched. The mean capacity is calculated from (4) for both non-adaptive and adaptive matching methods. The received signal vector  $\mathbf{y}$  is calculated from (1) by generating a complex Gaussian transmit signal  $\mathbf{x}$  with zero-mean and  $\sigma_x^2 = SNR = \{5, 20 \text{ dB}\}$  variance, and the channel matrix given by [8]

$$\mathbf{H} = 2\sqrt{R_{11}R_L}(Z_L\mathbf{I} + \mathbf{Z}_R)^{-1}\Psi_R^{1/2}\mathbf{H}_w\Psi_T^{1/2} \quad (8)$$

where  $R_{11}$  and  $R_L$  denote the real parts of the self-impedance  $Z_{11}$  and terminal load  $Z_L$  of the antennas, and  $\mathbf{Z}_R$  represents the antennas mutual impedance matrix [13]. The matrix  $\mathbf{H}_w$  entries are complex Gaussian random variables of zero-mean and average power of unity,  $\Psi_T$  and  $\Psi_R$  are the spatial correlation matrices at the transmit and receive ends, respectively. Furthermore, we assume  $\Psi_T = \mathbf{I}$ , data block length  $L = 2000$ , and  $K = 2000$  data blocks.

Fig. 1 shows contour plots of the mean capacity versus real and imaginary parts of  $Z_L = R_L + jX_L$  where  $R_L \in (0, 100]\Omega$

<sup>1</sup>This idea is motivated by random phase selection [11] and random walk [12] algorithms.

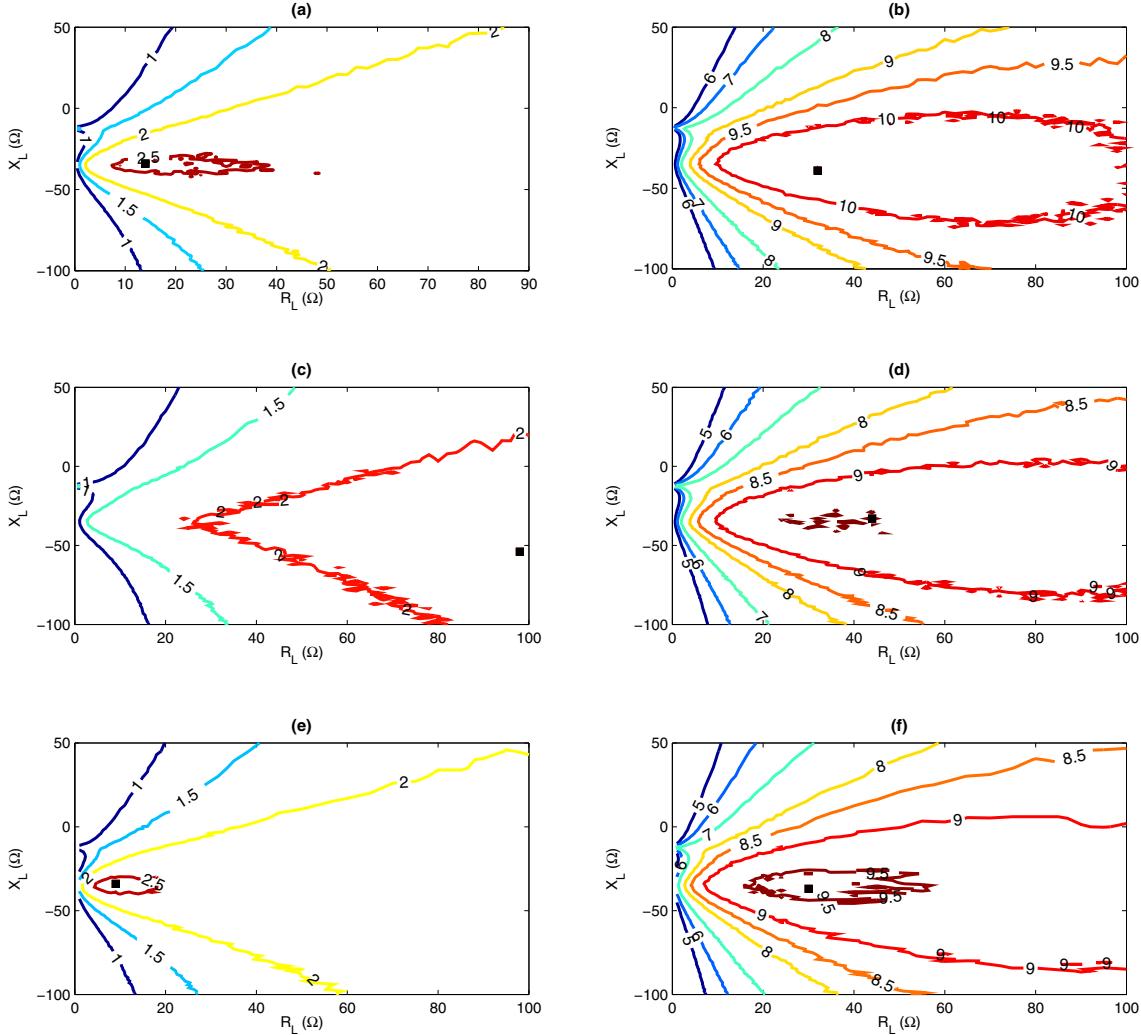


Fig. 1. Mean capacity versus real and imaginary parts of the antenna load impedance  $Z_L$  for uniform ((a) and (b)) and Laplacian distributions with  $(\phi_0, \sigma) = (0^\circ, 40^\circ)$  at (c)-(d), and  $(90^\circ, 67^\circ)$  at (e)-(f). Signal to noise ratio 5 dB for (a),(c),(e) and 20 dB for (b),(d) and (f) is considered. The optimum loads which maximise the mean capacity are marked by black squares for all cases.

and  $X_L \in [-100, 50]\Omega$ , for different propagation scenarios: uniform (a)-(b), and Laplacian with  $(\phi_0, \sigma) = (0^\circ, 40^\circ)$  for (c)-(d), and  $(90^\circ, 67^\circ)$  for (e)-(f). We note that the magnitudes of the correlation coefficient for these two set of Laplacian parameters are equal. The received  $SNR = 5\text{dB}$  for the left column (subfigures (a),(c),(e)) and  $20\text{ dB}$  for (b),(d) and (f) is considered. It can be seen that the mean capacity at any case can be maximized by selecting a proper load  $Z_L$  (black square marked points). Comparing the maximum values of the mean capacity and the corresponding loads in Table I reveals that the optimum load depends on different factors of the propagation environment. Therefore, existence of an adaptive matching approach would be necessary in practice.

TABLE I  
OPTIMIZED MEAN CAPACITY AND THE CORRESPONDING LOAD IMPEDANCES  $Z_L(\Omega)$  FOR THE UNIFORM AND LAPLACIAN  $(\phi_0, \sigma)$  SCATTERING DISTRIBUTIONS.

Capacity (bits/s/Hz)	Uniform	$(0^\circ, 40^\circ)$	$(90^\circ, 67^\circ)$
$SNR = 5\text{dB}$	2.5816	2.179	2.6114
$SNR = 20\text{dB}$	10.5135	9.5632	9.6118
Optimum $Z_L(\Omega)$	Uniform	$(0^\circ, 40^\circ)$	$(90^\circ, 67^\circ)$
$SNR = 5\text{dB}$	$14 - j34$	$98 - j54$	$9 - j34$
$SNR = 20\text{dB}$	$32 - j39$	$44 - j33$	$30 - j37$

The results of 100 Monte Carlo runs of the adaptive matching algorithm with initial load  $Z_0 = 50\Omega$  and 50 steps per execution, are shown with asterisk marked points in Fig.

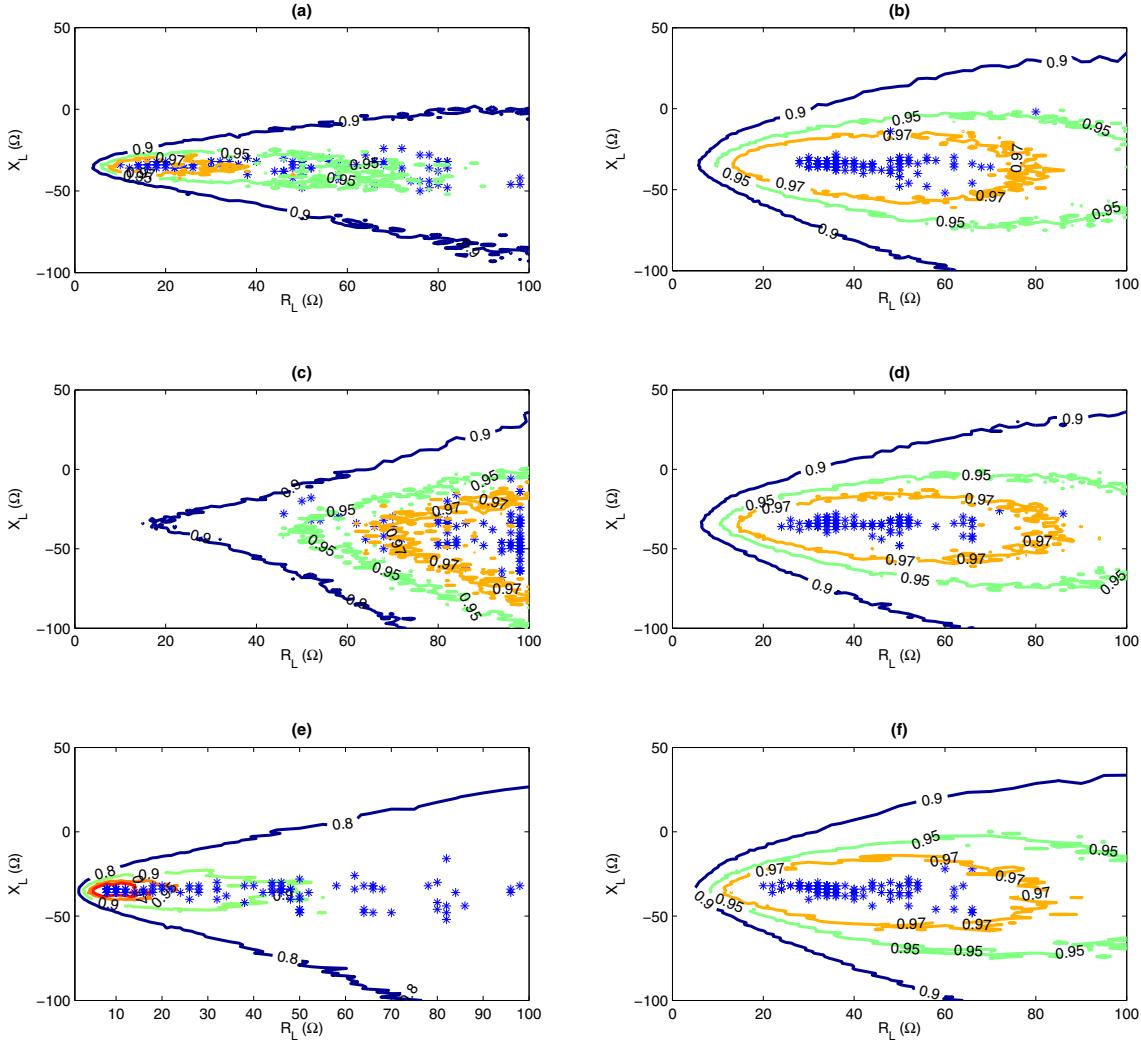


Fig. 2. Adaptive matching results for 100 runs (asterisk marked points) with initial load  $Z_0 = 50\Omega$  and normalized mean capacity contour for uniform ((a) and (b)) and Laplacian distributions with  $(\phi_0, \sigma) = (0^\circ, 40^\circ)$  at (c)-(d), and  $(90^\circ, 67^\circ)$  at (e)-(f).  $SNR = 5$  dB for (a),(c),(e) and  $20$  dB for (b),(d) and (f) is considered.

2. Additionally, the mean capacity contours normalized to their corresponding maximum values are plotted to evaluate the adaptive algorithm results. We observe that the adaptive algorithm has found an optimum load which gives a mean capacity higher than 97% of the maximum  $C_{mean}$  at Table I for  $SNR = 20$  dB. For the lower  $SNR$  case, algorithm still goes to the area of 97% of the maximum  $C_{mean}$  for some propagation scenario (c), but not for the others. This problem could be solved by trying different initial load impedances or longer block lengths  $L$ .

As it is shown in the simulation results, the proposed adaptive matching algorithm can be used to improve the compact MIMO performance by choosing a proper antenna

load impedance based on the received signals. This algorithm does not require any knowledge of the channel or MC model which are practical issues for previous studies. So, it could be a practical solution to deal with MC effects in compact MIMO systems.

## VI. CONCLUSION

In this paper, we investigated the effect of antenna load impedance on the MIMO performances based on the receiving signals. Then we proposed an adaptive matching algorithm that can find a proper load impedance to maximise the MIMO capacity and/or received power in the presence of MC. This optimisation is performed directly on the received signals and requires no prior knowledge of the channel matrix and MC

modeling which are the practical issues for the present studies. Simulation results are shown to illustrate the ability of the proposed algorithm to improve compact MIMO performance in the presence of MC.

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