Non-Markovian Survivability Assessment Model for Infrastructure Wireless Networks

Lang Xie, Poul E Heegaard, Yuming Jiang
Department of Information Security and Communication Technology
NTNU, Norwegian University of Science and Technology, Trondheim, Norway

***Abstract*—Network design and operation of a mobile network infrastructure, especially its access points, need to consider survivability as a fundamental requirement. Quantifiable approaches to survivability analysis of such infrastructures are crucial. Most existing analytical models analyze the networks transient behaviors by applying homogeneous continuous-time Markov chain (CTMC). However, the distributions for transitions between states during a failure recovery are not exponential in many real cases. To address this problem, we first propose to use a non-Markovian model to characterize the transient behavior of the phased recovery of the network after a failure. Then, based on the proposed model, we conduct survivability analysis of the network. Moreover, numerical results are presented to validate the phase type (PH) approximation used in the proposed model. A case study illustrates the effects of different model parameters on the network's survivability. These results shed new insights not only on survivability analysis, e.g. the non-Markovian phased recovery model, but also on survivability provisioning, e.g. how the model parameters affect the network's survivability, of such a network against failure events.**

I. Introduction

With the explosive growth of Internet and mobile usage around the world, wireless communications plays a fundamental role in critical applications. However, wireless communication infrastructures, especially wireless access points (e.g., access point (AP), base transceiver station (BTS), NodeB and eNB for WLAN, GSM, UMTS and LTE respectively) are confronted with numerous threats, such as natural disasters, common mode software/hardware failures, and security attack. To evaluate the capability of a wireless network infrastructure in surviving failures tractably, it is essential to develop quantitative models for assessing the network performance during the (transient) period that starts from a failure occurs till the system fully recovers. In addition, the survivability evaluation models can be used to provide results and insights for network design and optimization to avert the impact of failures.

This paper focuses on exploiting state space models for assessing the network survivability. Most of the existing analytical models and the numerical solutions analyzed the infrastructure network survivability by applying homogeneous continuous-time Markov chain (CTMC) [1], [2], [3]. However, the actual recovery behavior of networks might be complex, i.e., dependent upon several factors such as resources, logistics and environments. In real cases, the exponential distribution assumption of the recovery time may not hold [4]. Such strong assumption may cause service providers to assess operational risk inaccurately, and consequently to inadequately plan under/overprovisioning of recovery resources. To overcome this problem, it is necessary to revisit the distribution of recovery time to better understand the recovery time characteristics.

The objective of this paper is to propose a model to better model the recovery time characteristics and based on the proposed model to conduct survivability analysis. The time-dependent performance metrics are computed given that the system initially is in a failed state. We apply phase type (PH) distributions technique to relax exponential assumptions related to the homogeneous CTMC. As such, non-exponential state sojourn time distributions can be taken into account.

The contributions of this paper are summarized as follows.

- We develop a non-Markovian survivability evaluation model that supports state transitions following non-exponential distributions, and conduct survivability analysis based on the proposed model.
- We apply PH distributions technique to approximate the non-exponential distributions, thereby inducing a Markovian structure, to simplify the analysis and obtain tractable analytical results.
- We provide numerical studies with real-life data on how the proposed approach analyzes the transient performance of an infrastructure wireless network.

Together, the results of this paper shed new insights not only on survivability analysis, e.g. the non-Markovian phased recovery model, but also on survivability provisioning, e.g. how the model parameters affect the network's survivability, of an infrastructure wireless network against failure events.

Sec. II presents some background knowledge and related work. In Sec. III, the survivability analysis of the system is performed. In Sec. IV, the PH approximation is validated. In addition, the effects of different model parameters on the network' survivability are investigated. Finally, conclusion is given in Sec. V.

II. Background and Related Work

A. Background

Regarding the term "survivability", different definitions have been proposed and applied under different scenarios [5], [6]. The many definitions of survivability can be summarized as "the system's ability to continuously deliver services in compliance with the given requirements in the presence of failures and other undesired events" [1].

Many network operators have increasingly invested in strengthening their network reliability and survivability [7].

However, wireless network systems are still vulnerable to undesired events, such as natural disasters, software/hardware failures, and security attacks. When an undesired event occurs in a network system, networking infrastructure may partially or even fully breakdown. For example, after Hurricane Maria hit Puerto Rico, the mobile network in this area almost entirely failed [8]. Our work focuses on infrastructure wireless network's massive failures caused by natural disasters. Following a disaster, critical infrastructure issues affecting a mobile network often include loss of cell sites due to damage to the site itself and lack of transmission or power. Additionally, in the aftermath of a disaster quite often logistics and transport are also challengeable [9].

One of the fastest methods of restoring networks is the rapid deployment of temporary and portable backup cell sites - cells on wheels (COW) or cells on light truck (COLT) [10], [11]. The deployment of backup cell sites can either reduce local network congestion or plug gaps in a damaged network. Furthermore, in the disaster areas where COWs and COLTs may struggle for access to, lightweight portable base stations (e.g., [9], [12]) give operators more deployment flexibility. Other emergency network solutions include unmanned aerial vehicles (UAVs) [13], and Device-to-Device (D2D) communication [14]. The introduction of various survivability methods to infrastructure wireless networks motivates the need for a quantitative assessment of network survivability against disasters.

B. Related Work

Quantitative analysis of survivability could help enhance the system's capability providing critical services when failure occurs to the system. To quantify the survivability of communication networks, the ANSI T1A1.2 [15] working group defined the survivability, which is concerned with the transient behavior of the system (performance) from the instant when a failure (attack or natural disaster) occurs until the system fully recovers. Liu et al. [6] proposed a general survivability quantification approach based on this definition. Starting from this framework, Xie et al. [3] extended it to the survivability analysis of a two-tier infrastructure wireless network. Both work assumed single phase recovery, i.e., multiple recovery actions are simply merged as one activity.

A refinement of the single phase recovery model is to sub-divide the recovery time into several phases. That is, multiple phases recovery [16], where each phase represents a new system/service restoration attempt. Multiple phases recovery has been considered for the survivability analysis of communication networks [1], [17]. In [2], the proposed model in [1] was further extended and adapted to capture the behavior of the situation with multiple failures due to disaster spreading. However, these studies were based on the strong assumption that state holding times are all exponentially distributed. In real cases, the distribution of the random time spent in each stage of the recovery may be either non-exponential or simply unknown. The assumption of exponentially distributed events might lead to inaccurate results and limits the model's real-world applicability. In line with that, our paper explores non-Markovian modeling approach to relax such assumption.

Although the non-Markovian models have been applied for survivability analysis of Intrusion Tolerant Database Systems [18] and distribution power grid networks [19], there is limited similar work on infrastructure wireless systems. We use PH distributions to approximate the non-exponential distributions, thereby inducing a Markovian structure, to simplify the analysis and obtain tractable analytical results. This complements survivability analysis by the methods described in [1], [2], [3].

III. Survivability Analysis

A. Non-Markovian Phased Recovery Model

Consider an infrastructure wireless network deployed in a small-sized geographic area to serve the total capacity required by its users. This network was stroked by a large-scale natural disaster. We assume that the disaster area is much larger than the area of interest, and the disaster event caused all the cell sites in this area entirely failed.

We consider a combination of the escalated levels of recovery process, and the deferred repair due to unavailability of the repair crews. An emergency backup network solution (deploying a portable cell site) is initiated by the mobile network operator after a preliminary assessment of the situation and will run until the manual repair is completed. Particularly, the backup network solution provides a minimum standard of communications, such as short message service (SMS) and emergency voice call to the disaster area.

Repair crews are dispatched to the field to deploy portable cell sites, and to do manual repair through a visit to non-functional cell sites. Promptness of failure recovery thus depends on environmental constraints, preparedness and resources. The survivability measure of interest is relative loss of capacity. Before the failure, the loss of capacity is 0. The backup solution will typically have a reduced capacity $0 < \alpha\% < 1$.

A state transition diagram of this 6-phase non-Markovian model is illustrated in Fig. 1, where each phase is clustered in three recovery stages. The system states (each state is assigned a number to simplify the notation) are defined as:

- Plan (1) - the recovery is planned and the repair crew has to be assigned and instructed.
- Init backup, Def rep (2) - initialization of backup while waiting for the repair crews.
- Init backup, Repair (3) - initialization of backup during system repair.
- Run backup, Def rep (4) - the backup is running and providing partial service with reduced capacity, while waiting for repairman.
- Run backup, Repair (5) - the backup is running and providing partial service with reduced capacity during system repair.
- OK (0) - system is running with 100% capacity.

For each phase, the system is assumed to be in a performance wise steady-state with unchanged operational conditions. The circular states represents the full capacity of the service, while the rectangular states represent the null capacity. During the service recovery, the system may visit octagonal states which represent reduced capacity. Let $c_i, i \in \{0, 1, \cdots, 5\}$

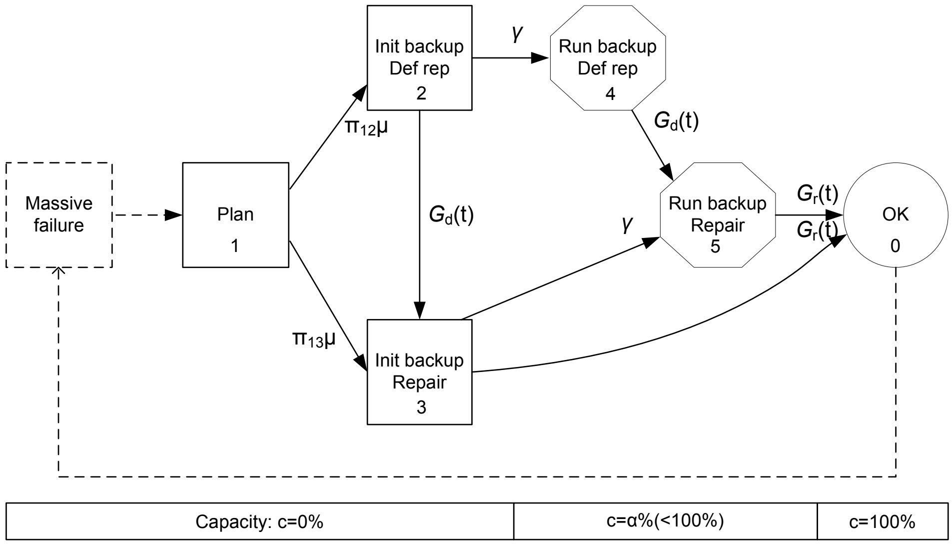


Fig. 1. Survivability model with 6 phases for the example.

TABLE I. MODEL PARAMETERS

Parameter	Description
π_{12}	probability that there is logistic delay after failure
π_{13}	probability that there is no logistic delay after failure
$1/\mu$	expected time to plan the disaster recovery
$1/\gamma$	expected time to initialize backup
$G_r(t)$	the distribution of repair time
$G_d(t)$	the distribution of time until repair crew is located

denote the capacity in the state i. We assume $c_0 = 1$ with full capacity, $c_i = 0$ when $i = 1, \cdots, 3$ and $c_i = \alpha$ with reduced capacity when $i = 4, 5$. The reward rates, $1 - c_i$, are assigned to each state according to the capacities. The transition from a state to another state is triggered by events such as failure detection, repair completion, etc.

This model is adapted from [16] but with non-exponential event time distributions. Since the time to initialize backup is predictable, we assume that these event times are exponentially distributed. But the time distributions of manual repair $G_r(t)$ and logistic delay $G_d(t)$ defined in Table I might be non-exponential.

Note that for survivability analysis, our focus is on the system's recovery behavior after the disaster. For this reason, this paper does not consider the effect of the failure types and rates. The time-dependent performance metrics are computed by assuming that the system initially is in a failed state. The model could be modified to refine the recovery phases to also model other failure modes including dynamic failures caused by disaster spreading as discussed in [2].

B. Analysis

Let $\mathbf{P}(t) = [p_i(t)]$ denote a row vector of transient state probabilities at time t. For a homogeneous CTMC, the transient probabilities $p_i(t)$ are easily determined by solving the linear system of ordinary differential equations $\mathbf{P}^{'}(t) = \mathbf{P}(t)\mathbf{Q}$, where $\mathbf{Q}$ is the transition rate matrix. Then the transient state probability vector can be obtained as follows:

$$\mathbf{P}(t) = \mathbf{P}(0)e^{\mathbf{Q}t}, \tag{1}$$

where $\mathbf{P}(0)$ is the initial state probability vector.

Combining the transient state probabilities $p_i(t)$ and the relative loss of capacity $1 - c_i$ associated with each state, the expected instantaneous reward $m(t)$ gives the relative loss of capacity of the system at time t, which is expressed as follows:

$$m(t) = \sum_{i=0}^{5}(1 - c_i) \cdot p_i(t). \tag{2}$$

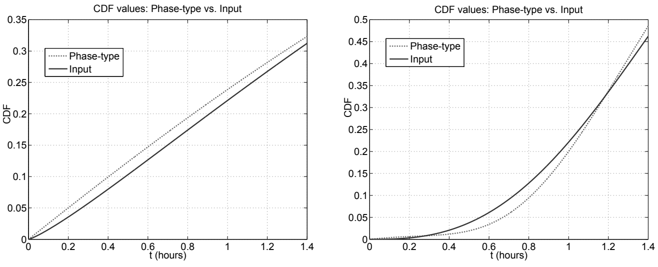
We highlight that to obtain Eq. (1) in above, an implicit exponential state holding time assumption has been made. To deal with situations where this assumption may not hold, the PH expansion approach proposed in [20] is used, through which the result of Eq.s (1) and (2) can still be applied.

Specifically, the non-exponential distributions such as $G_r(t), G_d(t)$ can be approximated by a PH distribution. That is, an absorbing Markov chain with $k+1$ states, where states $1, 2, \cdots, k$ are transient, and state $k+1$ is absorbing. Its infinitesimal generator matrix $\hat{\mathbf{T}}$ is in the form

$$\hat{\mathbf{T}} = \left[\begin{array}{c|c} \mathbf{T} & \mathbf{T}^o \\ \hline \mathbf{0} & 0 \end{array}\right],$$

where $\mathbf{T}$ is the $k \times k$ matrix of transition rates among the first k states, and $\mathbf{T}^o$ is the $k \times 1$ vector of transition rates out of the first k states into the absorbing state $k+1$ [20]. Given the $1 \times (k+1)$ initial probability vector $\boldsymbol{\beta}$, the cumulative distribution function (CDF) of a random variable following the PH distribution can be derived as $1 - \boldsymbol{\beta} \cdot e^{\mathbf{T}t} \cdot \mathbf{1}, t \geq 0$. The basis of choosing which PH distribution is the non-exponential distributions's squared coefficient of variation φ, which is its variance divided by the square of its expectation. If $\varphi > 1$, use three-moment, 2-phase Coxian distribution; if $0.5 \leq \varphi \leq 1$, use two-moment, 2-phase Coxian distribution; otherwise use two-moment, k_i-phase Erlang distribution.

With this approximation approach, the non-Markovian model is converted into a CTMC with a new (and expanded) infinitesimal generator matrix, which can be applied directly



(a) (b)

Fig. 2. PH approximation of the Weibull distribution. (a) $G_d(t)$ is Weibull (1, 0.01); (b) $G_r(t)$ is Weibull (1, 0.02).

to Eq. (1) to solve for $\mathbf{P}(t)$. Subsequently, the expected survivability performance is found from Eq. (2).

The procedure presented above for survivability analysis with a non-Markovian model is summarized in Algorithm 1.

Algorithm 1 Survivability analysis with a non-Markovian model

Step 1. Compute squared coefficient of variation (φ) of one non-exponential state holding time distribution.
Step 2. Approximate the time distribution with,
if $\varphi < 0.5$ **then**
  two-moment, k-phase Erlang distribution.
else if $0.5 \leq \varphi \leq 1$ **then**
  two-moment, 2-phase Coxian distribution.
else
  three-moment, 2-phase Coxian distribution.
end if
Step 3. Use PH representation to construct a new infinitesimal generator matrix.
Step 4. Calculate the expected survivability performance value by combining the new infinitesimal generator matrix and the performance associated with each state.

IV. Numerical Investigation

In this section, the validity of the PH approximation approach is first investigated, before results and insights from numerical experiments are presented.

A. Validity of PH Approximation

We assume that $G_d(t)$ and $G_r(t)$ both follow Weibull distribution in the example. Specifically, $G_d(t)$, the distribution of logistic delay is assumed to be Weibull(1,0.02), where 1 is the shape parameter and 0.02 is the scale parameter. The distribution of repair time $G_r(t)$ is Weibull(1,0.01).

As a sample illustration of the quality of the approximations, Fig. 2(a) depicts the CDF of $G_d(t)$ and its approximated CDF obtained using the procedure presented in Algorithm 1. In this scenario, coefficient of variation is 0.79. Then the type of approximation used is 2-moment, 2-phase Coxian distribution with an infinitesimal generator matrix

$$\hat{\mathbf{T}}_d = \begin{bmatrix} -0.038 & 0.025 & 0.013 \\ 0 & -0.025 & 0.025 \\ 0 & 0 & 0 \end{bmatrix},$$

and the initial probability vector $\boldsymbol{\beta} = [1, 0]$. Fig. 2(b) depicts the CDF of $G_r(t)$ and its approximated 2-moment, 2-phase Coxian distribution. Its infinitesimal generator matrix is

$$\hat{\mathbf{T}}_r = \begin{bmatrix} -0.021 & 0.011 & 0.01 \\ 0 & -0.011 & 0.011 \\ 0 & 0 & 0 \end{bmatrix}.$$

These results in Fig. 2 indicate that the PH distribution is very close to the original Weibull distribution. Then the original generator matrix $\mathbf{Q}$ is expanded to account for transitions among the phases of the approximated states. The average value of logistic delay and repair time during their event time are $1/0.038 + 1/0.025 = 1/0.015$ and $1/0.021 + 1/0.011 = 1/0.007$, used in the Markovian analysis in the next subsection.

B. Impact of Parameters on Survivability Performance

To gain more insights, we perform numerical experiments in this subsection to investigate the impact of different model parameters on the defined survivability performance.

In the numerical solution of the state space model of Fig. 1, we assume the probability that there is logistic delay π_{12} is set as 0.64, and the probability without logistic delay π_{13} is set as 0.36. The average disaster recovery planning time in the system is exponentially distributed with $\mu = 1/15$ (time unit is minutes). The parameterizations for the reduced capacity α and the time to initialize backup $1/\gamma$ refers to one commercial small cell based deployable LTE solutions [12], which needs about 10 to 15 minutes setup time. The small cell based deployable system can support up to 400 active users, while a macro cell based eNodeB supports up to 1000 active users. Based on the granularity of these real data, we set $\alpha = 0.4$ and $1/\gamma = 15$.

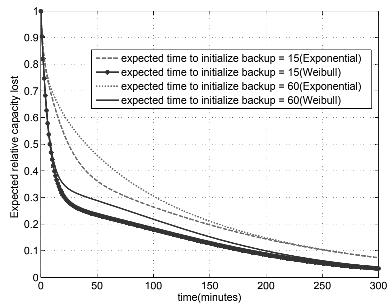


Fig. 3. Impact of mean time to initialize backup.

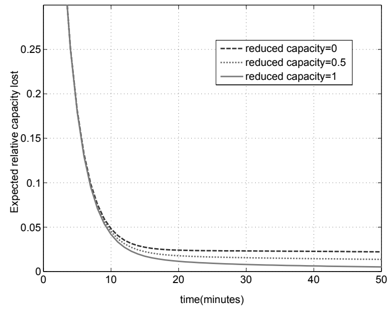


Fig. 4. Impact of the reduced capacity.

In Fig. 3, consider the scenario in which $1/\gamma$, the mean time to initialize backup is low ($1/\gamma = 15$). In this scenario, the expected relative capacity lost is lower. In contrast, if $1/\gamma$ is relatively higher ($1/\gamma = 60$), the expected capacity lost is higher. This indicates that the system can recover quickly under the lower mean time to initialize backup.

In Fig. 4 the different recovery phases from Fig. 1 can be observed. For $t < 15$ recovery planning dominates, and $t > 15$ (approx) the recovery stage with or without backup. The last stage shows the effect of investing in larger backup capacity, $\alpha = 1$, versus some, $\alpha = 0.5$, and no $\alpha = 0$.

The above results implies that the proposed model might be more applicable in real cases owing to relaxing the non-fitting exponential state holding time distribution assumption. In addition, with the proposed model and analysis approach, operators could use the quantification results to assess the fundamental trade-offs between survivable network planning and survivability performance. Based on such tradeoffs, operators might further implement optimization algorithms to achieve how to use the minimal cost (e.g. minimal number of wireless access points) while maximizing the network' survivability (e.g. maximizing the average capacity per area), or to strengthen some of the existing infrastructures to ensure that they will still function after a disruption of the system.

V. Conclusion

We have conducted analysis on the survivability of an infrastructure wireless network that is subject to catastrophic breakdown. The focus has been on the transient behavior of the network under large-scale failures, of which a non-Markovian model is established. To simplify the model analysis, the non-Markovian model has been converted to a CTMC using PH approximations so that the model solving methods applicable for CTMC can be used. Numerical investigation has been conducted to validate the PH approximation. In addition, a case study of large-scale disaster has shown how the proposed survivability analysis approach may be performed and how the model parameters may affect the network' survivability.

Although this paper focuses on the transient performance analysis of infrastructure wireless network, the proposed approach can be applied to the other systems, for example, power grids. Future work includes extensions of the analytical model to the situation with multiple-dependent failures.

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