# Preliminary Estimate of Highway Capacity Benefit Attainable with IntelliDrive Technologies 

Daiheng Ni, University of Massachusetts - Amherst
Jia Li
Steven Andrews
Haizhong Wang

# Preliminary Estimate of Highway Capacity Benefit Attainable with IntelliDrive Technologies 

Daiheng Ni, Jia Li, Steven Andrews, and Haizhong Wang


#### Abstract

Recent development in IntelliDrive and associated Vehicular Ad Hoc Networks (VANET) has stimulated tremendous interests among decision-makers, practitioners, and researchers due to the potential safety and mobility benefits provided by these technologies. A primary concern regarding the deployment of IntelliDrive is degree of market penetration required for effectiveness. This paper proposes an approach to analyze the benefit of highway capacity gained from IntelliDrive. To fulfill this purpose, a model incorporating the effects of IntelliDrive on car following is formulated, based on which a rough estimate of the resulting capacity gain is derived. A simulation study is conducted to verify the model and an illustrative example is provided to show the order of magnitude of the capacity gain. This work provides decisionmakers and practitioners a basic understanding of the mobility benefit obtained from IntelliDrive and how such benefit varies as market penetration changes.


## I. INTRODUCTION

Recent development in IntelliDrive, formally known as Vehicle Infrastructure Integration (VII), has stimulated tremendous interests among decision-makers, practitioners, and researchers due to the potential safety and mobility benefits provided by these technologies. Supported by the Dedicated Short Range Communication (DSRC) standard and Vehicular Ad Hoc Networks (VANET), IntelliDrive will enable road vehicles to communicate with each other as well as to roadside infrastructure in the future. Thus, highways and streets will become an environment that encompasses ubiquitous computing and communication. Consequently, a new class of applications can be developed to dramatically increase safety, throughput, and energy efficiency. All of these possibilities depend on large-scale deployment of IntelliDrive. However, a deployment decision has to take many factors into consideration. Among others, a primary factor is the infrastructure needed for success or, alternatively, degree of market penetration (i.e. percent of vehicles equipped with IntelliDrive technologies) required for effectiveness.

The above question is very difficult to answer because of the following: field experiments require a large-scale IntelliDrive testbed which has yet to be deployed; simulation is unavailable since existing traffic simulation packages are not designed to model traffic enabled by IntelliDrive; analytical modeling is prohibitive because of the complexity and interdependency involved in IntelliDrive. To bypass these

[^0]difficulties, this paper carries a humble goal by following a simplified modeling approach which is complemented by Monte Carlo simulation. In addition, the focus is to explore a feasible approach to conduct preliminary estimation of the mobility benefit of IntelliDrive, i.e. the increase of highway capacity brought about by IntelliDrive and how the result changes as market penetration varies. There are two building blocks in this approach. The first is to incorporate the effects of IntelliDrive into driving behavior modeling. For such a purpose, a simplified Gipps model [1] was used to attribute the effects of IntelliDrive to the change in the distribution of drivers' perception-reaction time. Recognizing that IntelliDrive may bring other profound changes in traffic operations than merely perception-reaction time, the model has to be kept as tractable as possible to make the analysis feasible yet capturing the major effect of IntelliDrive. Based on the model, the second building block is a probabilistic analysis to provide an estimate of highway capacity. In this part the major tools utilized are Wald's formula in probability theory and a theorem regarding the product moment of stopping time. An analytical approximate formula for capacity is obtained therein. A Monte Carlo simulation study is conducted to provide an alternative to verify this estimate since field tests are not possible at this time. The result obtained in this article may offer decision-makers and practitioners some insights into the question at the beginning.

## II. EXISTING STUDIES

The idea of studying traffic flow benefits due to advanced technologies such as Adaptive Cruise Control (ACC) Systems and Automated Highway Systems (AHS) has been addressed in the past. A great deal of studies have been identified which provided insights into highway capacity and traffic stability. A good survey of these studies can be found in [2]. A few additional references that present the necessary context for this study are added here. In their early studies on flow benefits of AHS, [3], [4] investigated how ACC affected traffic flow and found that the improvement in capacity is small. Also focused on ACC, [5] studied the impact of ACC on traffic flow stability. [6] studied a more challenging case which involves mixed traffic consisting of ACC automated vehicles and manually operated vehicles. They found that the capacity benefit became significant when ACC equipped cars exceeded $50 \%$ market penetration. When all cars were equipped with the technology, they found a $33 \%$ increase in capacity. [7] further considered Cooperative ACC (CACC) involving inter-vehicular communication and concluded that AACC could only have a small impact on highway capacity
(at most a $7 \%$ increase), while significant capacity gain could be expected with increased CACC market penetration (potentially more than doubling the capacity). [8] studied similar subject matter with a focus on the impacts of CACC on a highway-merging scenario. Based on the traffic flow simulation model MIXIC, they found improved traffic stability and a slightly increased capacity compared to the nonAAC equipped scenario.

Inspired by these original studies, our work considers a more general scenario which incorporates three types of driving modes enabled by IntelliDrive: namely non-IntelliDrive, IntelliDrive-assisted, and IntelliDrive-automated. In the nonIntelliDrive mode, drivers operate their vehicles without any assistance from IntelliDrive, just as what a regular driver does. In the IntelliDrive-assisted mode, drivers receive IntelliDrive assistances such as driver advisories (e.g. downstream congestion) and safety warnings (e.g. emergency brake), but these drivers still assume full control of their vehicles. The IntelliDrive-automated mode denotes that a vehicle is operated by IntelliDrive-enabled automatic driving features; however, the driver may break the loop and take over at any time as the need arises. In relation to these modes, existing studies emphasized the IntelliDrive-automated mode since ACC, AACC, and CACC can be considered as special cases of this mode. This paper broadens the perspective by also consider the effect due to IntelliDrive-enabled assistance to drivers (such as driver advisories and warnings). Moreover, this research takes a probabilistic approach and analytically relate the capacity benefit to the attributes of these driving modes and their market penetration.

## III. Incorporating Intellidrive Effects

## A. Assumptions and simplifications

IntelliDrive can bring about many fundamental changes to transportation systems such as ubiquitous situational awareness, more efficient system control, more advanced safety features, etc. These changes will affect drivers who actually control their vehicles. Among others, the primary effect of IntelliDrive on drivers is the distribution of their perceptionreaction time. For example, in the non-IntelliDrive mode, a driver typically needs to go through the full perceptionreaction process and thus may necessitate a relatively long perception-reaction time (perhaps, a few seconds) on average. In addition, drivers without any assistance have less situational awareness which results in more uncertainty in their responses. This may give rise to a larger variance in their perception-reaction time. In the IntelliDrive-automated mode, the perception process is taken care of by IntelliDrive and the reaction process is handled by the automatic driving system. Thus, the resulting perception-reaction time can be minimal. Also, human drivers are not involved in the driving loop; therefore, the variance of perception-reaction time may be close to zero. In the IntelliDrive-assisted mode, a wide range of possibilities may occur to the distribution. On one hand, it seems intuitive that IntelliDrive-assistances such as advisories and warnings can greatly reduce drivers' perception time, which is supported by evidences in psychology
literature such as [9]. On the other hand, such a new service may demand more attention to understand and familiarize and thus require a longer perception time, which is particularly true during confidence-building process. Before experimental data become available, the above discussion on perception-reaction time and their distributions remain open to debate. Nevertheless, it is reasonable to assume that the perception-reaction time of non-IntelliDrive, IntelliDriveassisted, and IntelliDrive-automated drivers follow different distributions, as illustrated in Figure 1. Note that no assumption is made regarding their actual distributions and relation. This keeps the subsequent formulation generic and flexible for analysts to customize their models. For example, analysts can plug in suitable perception-reaction time distributions based their own understanding or experiments in the field or on driving simulators.


Fig. 1. Perception-reaction time under different driving modes
In addition, it is assumed that IntelliDrive-automated and IntelliDrive-assisted modes are always able to reap the benefits of VANET, i.e. such vehicles are always assumed to be in a vehicular ad-hoc network. It is recognized that such an assumption is not very true, especially under low IntelliDrive market penetration. Fortunately, this assumption is acceptable considering the following. First, it tends to overestimate highway capacity when there are not many IntelliDrive-equipped vehicles. Though not desirable, such an estimate does provide an upper bound of the capacity gained by IntelliDrive. Second and perhaps more importantly, the validity of such an assumption increases when the deployment of IntelliDrive is relatively significant, a scenario at which IntelliDrive aims and under which IntelliDrive makes the most sense. In order to fully account for this limitation, one must consider the dynamics of and interdependence between vehicular ad-hoc networks and vehicular traffic. If this complication were to be taken into account, an analytical approach would no longer be adequate. Therefore, the goal of this research is to conduct a preliminary estimation of capacity benefit. Considering that field data are rare and the actual effects of IntelliDrive are still subject to discussion, an easily understood and tractable approach seems more desirable to fulfill the purpose.
It is further assumed that IntelliDrive market penetration
(i.e. the percent of total vehicles operating in each of the three modes) is known. With the above setup, it is straightforward to derive a car-following model with perception-reaction time as a parameter. Compared with the original [1] model, the new model rectifies the perception-reaction time which considers IntelliDrive-enabled driving modes and incorporates their market penetration rates. This model is then used to derive an equilibrium flow-density relationship, from which maximum flow rate (i.e. the capacity) can be derived. Considering the random nature of the perception-reaction time, a probabilistic analysis is performed to investigate the properties of the capacity and a Monte Carlo simulation is used to verify the results obtained above.

## B. Model formulation

Figure 2 shows two vehicles in car following. According to Gipps model ([1]), a vehicle should leave enough room in front of it in order to be able to stop safely behind its leading vehicle in the event that the leading vehicle applies emergency brake. A slightly modified Gipps model in a simpler form can be the following:

$$
S_{i}(t)=x_{i-1}(t)-x_{i}(t) \geq l_{i-1}+\dot{x}_{i} \tau_{i}-\frac{\dot{x}_{i}^{2}(t)}{2 b_{i}}+\frac{\dot{x}_{i-1}^{2}(t)}{2 B_{i-1}}
$$

where $S_{i}(t)$ is the spacing between the follower $i$ and its leader $i-1$ at time $t . x_{i}(t)$ and $\dot{x}_{i}(t)$ are location and speed of $i$ at time $t$ and the same notations apply to $i-1$. $\tau_{i}$ and $b_{i}$ are $i$ 's perception-reaction time and comfortable deceleration. $l_{i-1}$ and $B_{i-1}$ are $i-1$ 's length and maximum deceleration.


Fig. 2. Vehicles in car following
Under equilibrium conditions, vehicles tend to behave uniformly and thus lose their identities. After suppressing time $t$ and adding an additional delay $\theta$ for extra protection, the spacing becomes
$S=l+\dot{x}(\tau+\theta)-\frac{\dot{x}^{2}}{2 b}+\frac{\dot{x}^{2}}{2 B}=\left(\frac{1}{2 B}-\frac{1}{2 b}\right) \dot{x}^{2}+(\tau+\theta) \dot{x}+l$ Thus, the safe spacing is explicitly expressed as a function of speed $v \equiv \dot{x}$ (under equilibrium conditions, it is also the traffic speed) with parameters $\tau, \theta, B, b$ and $l$. Among all the parameters, $\tau$ and $\theta$ characterize the behavior of drivers and are independent of speed $v$ and spacing $S . B, b$, and $l$ are vehicle properties and can be assumed as constants. Since density $k$ is related to spacing $S$ as $k=\frac{1}{S}$, flow $q$ is obtained by substituting $k$ and $v$ into the fundamental relation,

$$
q=k v=\frac{v}{G v^{2}+\tau^{\prime} v+l}
$$

where $\tau^{\prime}=\tau+\theta$ and $G=\frac{1}{2 B}-\frac{1}{2 b}$. In this relation, $v$ can be viewed as the primary input. $v$ and $\tau^{\prime}$ are independent variables. The maximum attainable $q$ is of interest. To find the maximum $q$ (denoted $q_{m}$ ), we solve the equation

$$
\left.\frac{d q}{d v}\right|_{v_{m}}=-\left.\frac{G-\frac{l}{v^{2}}}{\left(G v+\tau^{\prime}+\frac{l}{v}\right)^{2}}\right|_{v_{m}}=0
$$

we get the root, $v_{m}=\sqrt{l / G}$, and correspondingly, $q_{m}=$ $1 /\left(2 \sqrt{G l}+\tau^{\prime}\right)$. To verify that $q_{m}$ is indeed a maximum as $v$ varies, one may simply check the second derivative of $q$ at $v_{m}$. It turns out that this is true.

## IV. Probabilistic analysis

## A. The stopping time formulation with random $\tau$

Note that the above discussion does not incorporate the random nature of perception-reaction time $\tau$ nor its distribution in different driving modes. Denote $f_{n o}(t)$ the probability density of $\tau$ of drivers under the non-IntelliDrive with mean $\tau_{n o}$ and variance $\operatorname{Var}\left(\tau_{n o}\right)$. Similarly, the probability density of $\tau$ of drivers under the IntelliDrive-assisted mode is $f_{a s}(t)$ with mean $\tau_{a s}$ and variance $\operatorname{Var}\left(\tau_{a s}\right)$; the probability density of $\tau$ of drivers under the IntelliDrive-automated mode is $f_{a u}(t)$ with mean $\tau_{a u}$ and variance $\operatorname{Var}\left(\tau_{a u}\right)$. In addition, market penetration rates of road vehicles operating in nonIntelliDrive, IntelliDrive-assisted, and IntelliDrive-automated modes are denoted as $p_{n o}, p_{a s}$, and $p_{a u}$ respectively. They satisfy the following relationships: $0 \leq p_{n o}, p_{a s}, p_{a u} \leq 1$ and $p_{n o}+p_{a s}+p_{a u}=1$. Therefore, the perception-reaction time of an individual driver $i, \tau_{i}$, is a random variable which can be modeled by drawing first from the percent/probability of market penetration to determine which driving mode this driver uses and then from the distribution of perceptionreaction time of that particular mode.

Henceforth, we will investigate the properties of $q$ and $q_{m}$ as $\tau$ takes on random values. Usually, the first order second moment analysis (FOSM) is sufficient to fulfill this purpose. However, since FOSM is based on the Taylor expansion of functions, the accuracy of approximation relies heavily on the convergence rate of the Taylor series in the neighborhood of the expansion. For the higher order moment, this is especially true. In this situation, it is unfortunate that the expression of $q_{m}$ is ill-posed to adopt the FOSM. This is because $q_{m}$, written in the form of $f(x)=1 /(a+b x)$, corresponds to a slowly converging expansion series when $|a+b x| \sim 0$, a result of comparable values of $a$ and $b$.

Thus, we tackle the problem in a different way. In particular, we introduce the stopping time concept such that the expansion-based analysis like FOSM is avoided. The procedure is as follows. First, we redefine the flow as,

$$
q=k v=\frac{N}{L} v
$$

where $v$ is the traffic speed, $L$ is the length of a segment of highway in consideration, and $N$ is the number of vehicles within the length. Flow $q$, can be written as $\frac{N}{L / v}$ and be interpreted as the number of vehicles occupying a certain
length of road divided by the time they take to traverse the road. Under equilibrium conditions, this definition is equivalent to the original definition. Then we can adopt the concept of random walk. It is easy to see that $N$ is actually the stopping time (stopping time, a standard concept in probability theory, can be roughly regarded as a 'random time' whose value depends on current and historical values of a stochastic process). A rigorous definition is found in [10]) where a random walk $\sum_{i=1}^{n} S_{i}$ with positive drift $E\left(S_{i}\right)$ has:

$$
N=\inf _{n}\left\{n: \sum_{i=1}^{n} S_{i}>L\right\}
$$

where inf indicates the infinimum of a set, and

$$
S_{i}=G v^{2}+\tau^{\prime} v+l
$$

Then

$$
\mu_{q} \equiv E(q)=\frac{v}{L} E(N)
$$

Moreover,

$$
E(N)=\frac{L}{E\left(S_{i}\right)}=\frac{L}{G v^{2}+\mu_{\tau^{\prime}} v+l}
$$

where the first equality is due to Wald's equation, with its form and derivation given in [10]. Application of this equation requires $E\left(S_{i}\right)<\infty$, which is obviously true from a realistic point of view. Thus, we obtain the approximation of expected capacity when speed is $v$,

$$
\mu_{q} \equiv E(q) \sim \frac{v}{G v^{2}+\mu_{\tau^{\prime}} v+l}
$$

where $\mu_{\tau^{\prime}}=E\left(\tau^{\prime}\right)$. Plugging in the optimal speed $v=v_{m}=$ $\sqrt{l / G}$, we obtain the maximum value of approximation of expected capacity,

$$
\mu_{q, m} \equiv E\left(q_{m}\right) \sim \frac{1}{2 \sqrt{G l}+\mu_{\tau^{\prime}}}
$$

To obtain the variance of $q$ and $q_{m}$, we need to utilize a formula regarding the variance of stopping time given in [11]. In the current scenario, it is,

$$
\operatorname{Var}(N)=\mu^{-3} \sigma^{2} L+\mu^{-2} K+o(1)
$$

where $K$ is a constant independent of $L$, and

$$
\begin{gathered}
\mu=E\left(S_{i}\right)=G v^{2}+\mu_{\tau^{\prime}} v+l \\
\sigma^{2}=\operatorname{Var}\left(S_{i}\right)=\sigma_{\tau^{\prime}}^{2} v^{2}
\end{gathered}
$$

By substituting them into the definition of $q$, by letting $L$ be large enough, and by only keeping the dominating term, we get the variance of flow in the general case,

$$
\sigma_{q}^{2} \equiv \operatorname{Var}(q) \sim \frac{\sigma_{\tau^{\prime}}^{2} v^{4}}{\left(G v^{2}+\mu_{\tau^{\prime}} v+l\right)^{3} L}
$$

Plugging in the optimal speed $v=v_{m}=\sqrt{l / G}$, we obtain the approximate variance of the maximum flow, i.e. capacity,

$$
\begin{aligned}
\sigma_{q, m}^{2} \equiv \operatorname{Var}\left(q_{m}\right) & \sim \frac{\sigma_{\tau^{\prime}}^{2} l^{2}}{\left(2 l+\mu_{\tau^{\prime}} \sqrt{l / G}\right)^{3} G^{2} L} \\
& =\frac{\sigma_{\tau^{\prime}}^{2} G l}{\left(2 G l+\mu_{\tau^{\prime}} \sqrt{G l}\right)^{3}} \frac{l}{L}
\end{aligned}
$$

We see two quantities, $\sqrt{G l}$ and $l / L$, together with characteristics of perception-reaction time $\tau^{\prime}$ determine the variance. It is notable that the involved quantities are all easily measured, indicating the advantage of our estimate formula in terms of calibration.

## B. Simulation verification

In this section, a simulation study is conducted to verify the above approximation formulas from the numerical perspective. This is necessary since approximate formulas themselves do not guarantee satisfactory numerical performance, as only the orders of approximation accuracy are known. It is notable that this simulation itself is not a realization of the Gipps car-following model influenced by IntelliDrive which is beyond the scope of current study. We designed a Monte Carlo simulation, with one trial as follows:

1) Select road length in consideration, denoted $L$, initialize $L_{0}=0$;
2) Randomly sample the perception-reaction time $\tau$, calculate the cumulative length, which is defined as, $L_{j+1}=L_{j}+S$
3) If $L_{j}>L$, denote $N=\max j$, calculate $q=\frac{v N}{L}$; else return to (2).
Each trial in the simulation mimics the instantaneous vehicle spatial distribution on the road, such that the count of vehicles $N$ at each moment is obtained. In the simulation, we assume the random perception-reaction time has a density of the following form,

$$
f_{\tau}(t)=p_{a u} f_{a u}(t)+p_{a s} f_{a s}(t)+p_{n o} f_{n o}(t)
$$

where $\sum_{i} p_{i}=1$ and $i \in\{a u, a s, n o\}$. Here $f_{i}(\cdot)$ 's represents the density of the perception-reaction time distribution of the $i$-th group, and $p_{i}$ 's are interpreted as the market penetration of the corresponding groups. For the purpose of illustration, we consider an ideal and simplified case. We assume the $f_{i}$ 's are the density of uniform random variables. The function $f_{i}$ is of the form,

$$
f_{i}(t)=\frac{1}{u_{i}-l_{i}} I\left(t \in\left[l_{i}, u_{i}\right]\right)
$$

It is easy to see, a random variable with the above density $f_{i}$ has an expectation $\mu_{i}=\left(u_{i}+l_{i}\right) / 2$ and a variance $\sigma_{i}=$ $\left(u_{i}-l_{i}\right)^{2} / 12$. We then have,

$$
\begin{gathered}
\mu_{\tau}=\sum_{i=1}^{3} p_{i} \mu_{i} \\
\sigma_{\tau}=\sqrt{\sum_{i=1}^{3} p_{i} \mu_{i, 2}-\mu_{\tau}^{2}}=\sqrt{\sum_{i=1}^{3} p_{i}\left(\sigma_{i}^{2}+\mu_{i}^{2}\right)-\mu_{\tau}^{2}}
\end{gathered}
$$

Let the first, second, and third term in expression of $f_{\tau}$ represent the IntelliDrive-automated, IntelliDrive-assisted, and non-IntelliDrive group, respectively. Then we assume,

$$
\begin{gathered}
\mu_{a u}=0.5, \mu_{a s}=1.0, \mu_{n o}=1.5 \\
\sigma_{a u}=0, \sigma_{a s}=0.2, \sigma_{n o}=0.5 \\
p_{a u}=0.2, p_{a s}=0.5, p_{n o}=0.3
\end{gathered}
$$

Moreover, we fix $B=-4 \mathrm{~m} / \mathrm{sec}^{2}, b=-2 \mathrm{~m} / \mathrm{sec}^{2}, l=$ $4.5 \mathrm{~m}\left(15\right.$ feet) and $\tau^{\prime}=1.5 \tau$ (then there is $\mu_{\tau^{\prime}}=1.5 \mu_{\tau}$ and $\sigma_{\tau^{\prime}}=1.5 \sigma_{\tau}$ ). The number of iterations in each loop is 1000. The lengths of the segments on the one-lane highway vary from 5 km to 125 km , with a step of 5 km . The traffic speed varies from $v_{o p t}-10.5 \mathrm{~km} / \mathrm{hr}$ to $v_{o p t}+10.5 \mathrm{~km} / \mathrm{hr}$, with a step of $3.5 \mathrm{~km} / \mathrm{hr}$. The $v_{m}$ is the optimal speed defined above.


Fig. 3. The comparison of simulated and approximate expectation of flow $\mu_{q}$ (in $p c p h p l$ ) (top plot: simulated value; bottom plot: approximate value)

Observation of the simulation results and the conclusions drawn are as follows. First, in the case under consideration, the approximate expectation $\mu_{q}$ and standard deviation $\sigma_{q}$ of the flow are close to the simulation results, as shown in Figure 3 and 4. Second, in particular, the comparison of $\mu_{q, m}$ and $\sigma_{q, m}$ with the simulation is shown in Figure 5. The relative error of $\mu_{q, m}$ is very small, around $1 \%$. For the standard deviation $\sigma_{q, m}$, we see the fit of approximation to simulation is also near perfect, especially as the distance $L$ gets larger. To summarize, the simulation of the case


Fig. 4. The comparison of simulated and approximate standard deviation of flow $\sigma_{q}$ (in $p c p h p l$ ) (top plot: simulated value; bottom plot: approximate value)
when $\tau$ takes a specific distribution, numerically justifies the approximations obtained by probabilistic analysis, and it intuitively illustrates the quality of these approximations.

## V. An ILLUSTRATIVE EXAMPLE

The simulation study in the above section is able to provide an estimate of the capacity gain in absolute terms. Such a result, however, is lower than the typical capacity under ideal conditions, i.e. 2400 pcphpl for a basic freeway section. This is due to the conservative nature of the Gipps model, which is less capable of capturing the close-following behavior in reality. Developing a more realistic model may resolve the problem, but the mathematical tractability may be lost as well. Therefore, it is reasonable to describe the capacity benefit in relative terms, as discussed below.

To answer the question at the beginning of this paper (i.e. degree of market penetration required for effectiveness), we provide the following illustrative example. This example consists of four cases and in each case the ratio $p_{a u} / p_{a s}$ is assumed to be constant. In addition, we define the relative change in capacity as

$$
\begin{aligned}
& r\left(p_{a u} / p_{a s}, p_{n o}\right)=\frac{q_{m}\left(p_{a u}, p_{a s}, p_{n o}\right)}{q_{m}(0,0,1)} \\
& =\frac{q_{m}\left(\left(1-p_{n o}\right) \frac{p_{a u} / p_{a s}}{p_{a u} / p_{a s}+1},\left(1-p_{n o}\right) \frac{1}{p_{a u} / p_{a s}+1}, p_{n o}\right)}{q_{m}(0,0,1)}
\end{aligned}
$$



Fig. 5. The comparison of simulated and approximate performances (top plot: expectation of maximum flow $\mu_{q, m}$; bottom plot: standard deviation of maximum flow $\sigma_{q, m}$ ) (distance $L, m$; flow $q, p c p h p l$; dots: simulated values; solid line: approximate values)
where $q_{m}(\cdot, \cdot, \cdot)$ is the capacity corresponding to market penetration $\left(p_{a u}, p_{a s}, p_{n o}\right)$, and the second equality is $p_{a u}+p_{a s}+p_{n o}=1$. This formula could be interpreted as the ratio of increased capacity over the original capacity (i.e. $p_{n o}=100 \%$ ). By employing this definition, we will hopefully overcome the lower estimate by the Gipps model. We obtain the values of $r$ in four cases, i.e., when $p_{a u} / p_{a s}=$ $0.1,1,10,100$. The results are as shown in Figure 6. It is found that the relative increase of capacity ranges between $20 \%$ to $50 \%$ when IntelliDrive is fully deployed (i.e. $p_{n o}=$ 0 ), with the former case corresponding to $p_{a u} / p_{a s}=0.1$, the latter case $p_{a u} / p_{a s}=100$. Note that the above example is a only a rough estimate for illustration purpose. Nevertheless, the example does indicate that the benefit from employing IntelliDrive could be quite significant even when the market penetration of IntelliDrive-automated vehicles is small. As more accurate information regarding the involved parameters becomes available, the estimate can be fine-tuned and more accurate results are expected. The outcomes can be used to help make the decision on IntelliDrive deployment in future.

## VI. Concluding remarks

The purpose of this study is to provide a preliminary estimate of the capacity benefit obtained from IntelliDrive deployment. To fulfill this purpose, we present a modification to the classical Gipps model and a probabilistic approach to analyze highway capacity by incorporating the effects


Fig. 6. The relative benefits of IntelliDrive as function of market penetration of IntelliDrive in different cases ( $x$-axis: market penetration of non-assisted vehicles; $y$-axis: ratio of increased capacity to the original capacity)
of IntelliDrive. In particular, we obtain the approximate formulas of expectation and variance of the capacity in a random setting. We also find that the derived approximate expectation and variance formulas are numerically credible through a simulation study. In the future, the study may be extended to more complicated and realistic scenarios (e.g. non-equilibrium flow and non-homogeneous types of vehicles) where more involved simulation is expected before field experiments in a large-scale testbed become feasible.

## REFERENCES

[1] P. Gipps, "A behavioral car following model for computer simulation," Transportation Research Part B, no. 15, pp. 105-111, 1981.
[2] J. VanderWerf, S. Shladover, N. Kourjanskaia, M. Miller, and H. Krishnan, "Modeling effects of driver control assistance systems on traffic," Transportation Research Record, vol. 1748, pp. 167-174, 2001.
[3] B. Rao and P. Varaiya, "Flow benefits of autonomous intelligent cruise control in mixed manual and automated traffic," Transportation Research Record, vol. 1408, pp. 36-43, 1993.
[4] B. Rao, P. Varaiya, and F. Eskafi, "Investigations into achievable capacities and stream stability with coordinated intelligent vehicles," Transportation Research Record, vol. 1408, pp. 27-35, 1993.
[5] S. Darbha and K. Rajagopal, "Intelligent cruise control systems and traffic flow stability," Transportation Research Part C, vol. 7, no. 6, pp. 329-352, 1999.
[6] T. Chang and I. Lai, "Analysis of characteristics of mixed traffic flow of autopilot vehicles and manual vehicles," Transportation Research Part C, vol. 5, no. 6, pp. 333-348, 1997.
[7] J. VanderWerf, S. Shladover, M. Miller, and N. Kourjanskaia, "Evaluation of the effects of adaptive cruise control systems on highway traffic flow capacity and implications for deployment of future automated systems," Pre-Print CD-ROM of 81st TRB Annual Meeting, 2001.
[8] B. van Arem, C. van Driel, and R. Visser, "The impact of cooperative adaptive cruise control on traffic flow characteristics," IEEE Transactions on Intelligent Transportation Systems, vol. 7, no. 4, pp. 429-436, 2006.
[9] R. D. Luce, Response Times: Their Role in Inferring Elementary Mental Organization. Oxford University Press, New York., 1986.
[10] R. Durrett, Probability: Theory and Examples. Duxbury Press, 1996.
[11] T. Lai and D. Siegmund, "A nonlinear renewal theory with applications to sequential analysis. II," The Annuals of Statistics, vol. 7, no. 1, pp. 60-76, 1979.


[^0]:    Daiheng Ni is an Assistant Professor with the Department of Civil and Environmental Engineering, University of Massachusetts Amherst, MA 01003, U.S.A. ni@ecs.umass.edu

    Jia Li, Steven Andrews, and Haizhong Wang were graduate students with the Department of Civil and Environmental Engineering, University of Massachusetts Amherst

