

Arterial Traffic Signal Coordination Utilizing Vehicular Traffic Origin-Destination Information

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Abstract— Traditional arterial traffic signal coordination methods are focused on providing uninterrupted flow along the arterial direction. This paper proposes a more versatile arterial traffic signal coordination method that can take into consideration of major turning traffic flows from and to cross streets. This new method is formulated as a mixed-integer linear program and solved by CPLEX. A case study based on AIMSUN simulation is conducted to demonstrate the proposed method’s superiority compared to Synchro and Transyt-7F.

I. INTRODUCTION

For smooth and efficient arterial traffic operations, closely spaced traffic signals along an arterial are often coordinated with a common cycle length and appropriate offsets such that a platoon of vehicles can travel across the entire arterial without stopping. The green interval that allows uninterrupted traffic flow along the entire arterial is called progression band. John Little [1] conducted one of the pioneer studies on arterial traffic signal coordination and proposed a progression bandwidth maximization model based on mixed-integer linear programming. This method was further enhanced by Little et al. [2] and is now widely known as MAXBAND, which generates uniform inbound and outbound bandwidths along an arterial and tries to maximize them by adjusting cycle time, offsets and phase sequence patterns.

A limitation of MAXBAND is that it does not take into consideration the actual traffic flow and capacity of each arterial segment. Intuitively, it would be better for the bandwidth to be related or proportional to the traffic flow and capacity of each directional segment instead of being the same for the entire arterial. A uniform bandwidth may not be able to produce the optimal control performance due to the mismatch between demand and supply. Some arterial segments may have excessive green times than needed, while others may have inadequate green times that result in the frequent interruptions of vehicle platoons.

To address this issue with MAXBAND, Gartner et al. [3] proposed a MULTIBAND model, which is able to generate progression bands with varying widths related to each arterial segment’s traffic characteristics. They demonstrated that

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MULTIBAND significantly outperforms MAXBAND. Built upon MULTIBAND, Stamatiadis and Gartner [4] developed a MULTIBAND-96 model, which is essentially a network version of MULTIBAND. Recently, Zhang et al. [5] proposed an AM-BAND model by relaxing the symmetrical progression band requirement in MULTIBAND. Such a relaxation allows the AM-BAND model to better utilize the available green times in each progression direction.

The above-referenced models were all designed to maximize the progression bandwidths along the arterial direction. For some arterials, the traffic flows turning into the arterial from one cross street then turning out to another cross street can be very significant. Intuitively, it would be helpful to create a progression band for these vehicles as well. With the technological advancements in traffic sensors, many cost-effective methods (e.g., matching license plates) can be used to obtain reliable vehicular traffic Origin-Destination (OD) information for an arterial. Given such information, the purpose of this paper is to develop an OD-based arterial progression BANDwidth (OD-BAND) maximization method that takes major turning traffic flows into consideration.

II. MODEL FORMULATION

TABLE I. OD MATRIX FOR THE ARTERIAL NETWORK

| Origin | Destination | Demand (vehs/h) |
|--------|-------------|-----------------|
| 1 | 8 | 400 |
| 2 | 3 | 240 |
| 3 | 2 | 240 |
| 4 | 5 | 240 |
| 5 | 4 | 240 |
| 6 | 7 | 240 |
| 7 | 6 | 240 |
| 2 | 7 | 740 |
| 8 | 1 | 400 |

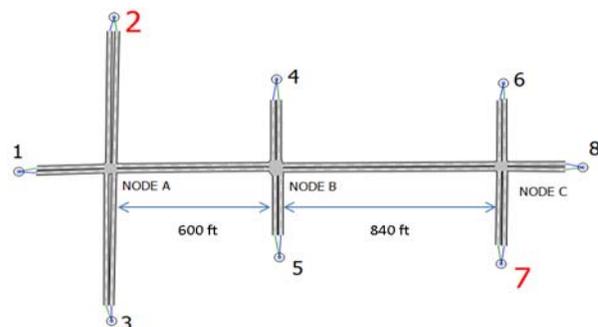


Figure 1. Arterial network for model formulation

The arterial network in Fig. 1 and the time-space diagram in Fig. 2 are used to illustrate how to formulate this

OD-BAND model. The numbers in Fig. 1 represent different origin/destination nodes. Table I shows the OD matrix considered in this study. It can be seen that there is a major traffic flow going from Origin 2 to Destination 7 via nodes/intersections A, B, and C. Therefore, it is necessary to create a progression band for OD pair 2→7 in addition to the arterial direction. The OD-BAND is formulated based on the MAXBAND model. All intersections in Fig. 1 are controlled by pre-timed strategy. Before introducing the formulation, the following symbols are first defined. Some of these symbols are also illustrated in Fig. 2.

m_i = loop integer;
 $b(\bar{b})$ = outbound (inbound) bandwidth (cycles);
 $a(\bar{a})$ = weight for outbound (inbound) bandwidth;
 C_1, C_2 = lower and upper limits on cycle length;
 $r_i(\bar{r}_i)$ = outbound (inbound) red time for arterial direction at intersection N_i (cycles);
 w_i = outbound interference variables (cycles), measured from the right end of outbound red to the left boundary of outbound arterial progression band;
 x_i = interference variables (cycles) measured from the left end of outbound red to the beginning of y_i band;

y_i = outbound progression bandwidth (cycles) for turning movements at intersection i ;
 $z = 1/C$ = signal frequency (cycles/second);
 k = ratios of inbound volume to outbound volume;
 $\phi_{i,i+1}[\bar{\phi}_{i,i+1}]$ = internode offsets (cycles);
 $t_{i,i+1}[\bar{t}_{i,i+1}]$ = travel time from S_i to S_{i+1} outbound [S_{i+1} to S_i inbound] (cycles);
 $d_{i,i+1}[\bar{d}_{i,i+1}]$ = distance between S_i and S_{i+1} outbound [inbound] (feet);
 $G_i(\bar{G}_i)$ = outbound (inbound) green time for through traffic at S_i (cycles);
 $e_i, f_i(\bar{e}_i, \bar{f}_i)$ = lower/upper limits on outbound (inbound) speed (feet/second);
 $g_i, h_i(\bar{g}_i, \bar{h}_i)$ = lower/upper limits on change in outbound (inbound) speed (feet/second);
 $V(\bar{V})$ = outbound (inbound) arterial through volume;
 V_{Ti} = outbound volume turned into main street from cross street at intersection N_i ;
 $S(\bar{S})$ = outbound (inbound) saturation flow rate; and
 S_{Ti} = outbound saturation flow for left-turn traffic (i.e., movement 2-A-B in Fig. 1) at intersection N_i

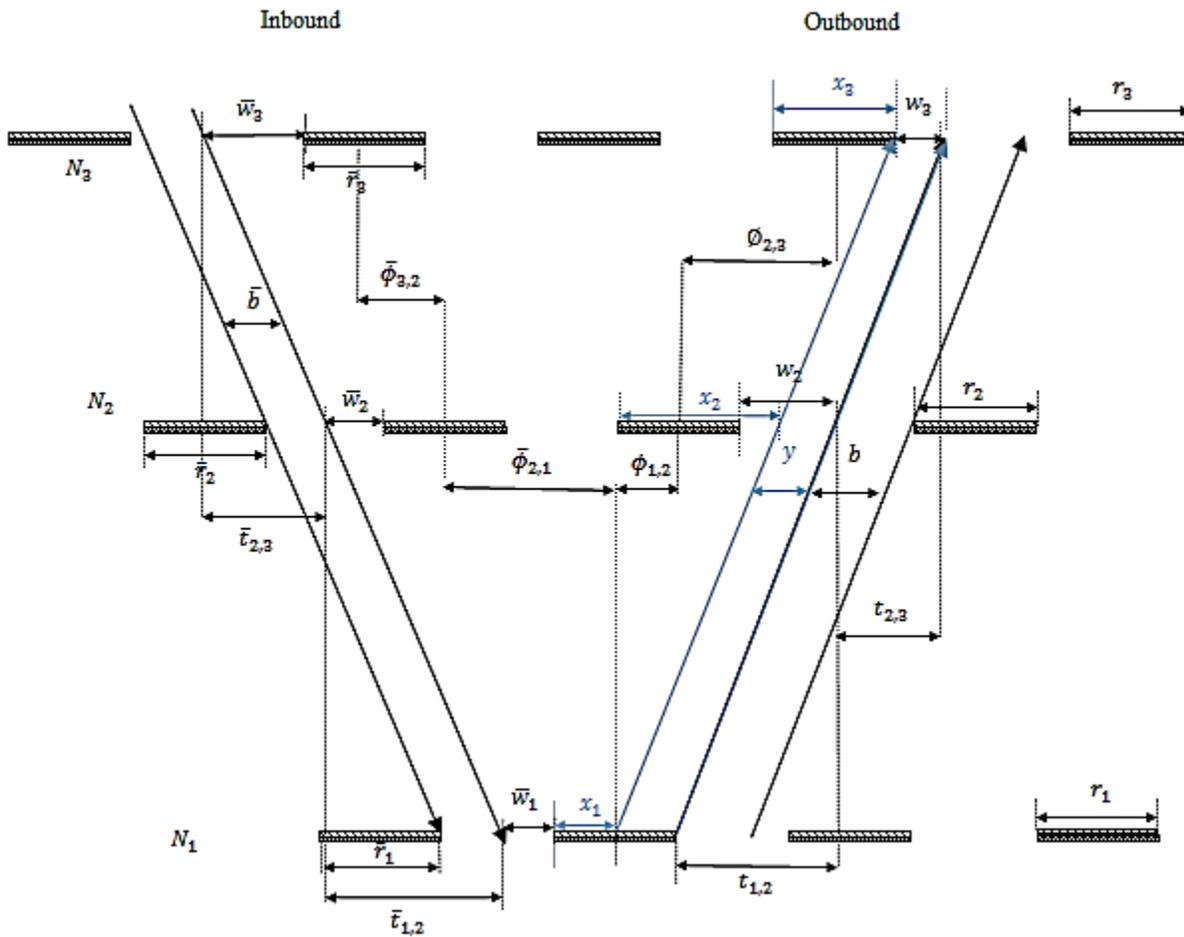


Figure 2. Time-space diagram for deriving the OD-BAND model formulation

Fig. 2 shows the time-space diagram along the arterial direction for the arterial network in Fig. 1. N_1, N_2 and N_3 represent intersections A, B, and C, respectively. To better

describe the OD-BAND model, only simultaneous left-turn phases are considered for the arterial direction. Since we only consider the cross-street left-turn movement from node 2, the

subscripts of y_i , V_{Ti} , and V_{Ti} are omitted. y is used to represent the width for the progression band going from Nodes 2 to 7. \hat{V} and \hat{S} are used for the corresponding flow and saturation flow rate. A lag left-turn is considered for movement 2-A-B. Therefore, progression band y starts in the middle of a red interval (not at the beginning of the red interval) for the arterial direction in Fig. 2. For the arterial network in Fig. 2, the OD-BAND is to maximize the objective in (1) by adjusting the values of $b, \bar{b}, z, w_i, \bar{w}_i, t_i, \bar{t}_i, x_i, y$ subject to the constraints in (3) through (13).

$$\text{Maximize } B = ab + \bar{a}\bar{b} + \hat{a}y \quad (1)$$

where,

$$a = \frac{v}{s} \quad \bar{a} = \frac{\bar{v}}{\bar{s}} \quad \hat{a} = \frac{\hat{v}}{\hat{s}} \quad (2)$$

subject to

$$1/C_2 \leq z \leq 1/C_1 \quad (3)$$

$$(1-k) * \bar{b} \geq (1-k) * k * b \quad (4)$$

$$\begin{cases} w_i + b_i \leq 1 - r_i \\ \bar{w}_i + \bar{b}_i \leq 1 - \bar{r}_i \end{cases} \quad i = 1, \dots, 3 \quad (5)$$

$$\begin{aligned} & (w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (t_i + \bar{t}_i) \\ & = \frac{1}{2}(r_{i+1} + \bar{r}_{i+1}) - \frac{1}{2}(r_i + \bar{r}_i) + m_i \quad i = 1, 2 \quad (6) \end{aligned}$$

$$\begin{cases} (d_i/f_i)z \leq t_i \leq (d_i/e_i)z \\ (\bar{d}_i/\bar{f}_i)z \leq \bar{t}_i \leq (\bar{d}_i/\bar{e}_i)z \end{cases} \quad i = 1, 2 \quad (7)$$

$$\begin{cases} (d_i/h_i)z \leq (d_i/d_{i+1})t_{i+1} - t_i \leq (d_i/g_i)z \\ (\bar{d}_i/\bar{h}_i)z \leq (\bar{d}_i/\bar{d}_{i+1})\bar{t}_{i+1} - \bar{t}_i \leq (\bar{d}_i/\bar{g}_i)z \end{cases} \quad i = 1 \quad (8)$$

$$y \leq \frac{(\hat{a} * b)}{(\hat{a} + a)} \quad (9)$$

$$x_i - x_{i+1} = w_i - w_{i+1} + r_i - r_{i+1} \quad i = 1, 2 \quad (10)$$

$$y + x_i \leq r_i \quad i = 1 \quad (11)$$

$$\begin{cases} y + b \leq 1 - r_i \\ x_i \geq r_i \end{cases} \quad i = 2, 3 \quad (12)$$

$$b, \bar{b}, z, w_i, \bar{w}_i, t_i, \bar{t}_i, x_i, y \geq 0 \quad i = 1, 2, 3 \quad (13)$$

The objective in (1) shows that OD-BAND tries to maximize a weighted sum of progression bands; the weights are calculated as each OD flow divided by its corresponding saturation flow rate as in (2); constraint (3) specifies the upper and lower limits of the cycle length; constraint (4) provides a wider band for the arterial direction with a higher traffic flow; constraints (5) are to ensure that the outbound and inbound bands do not infringe upon any portion of the red intervals for the arterial direction; constraints (6) are equivalent to the *loop integer constraint* in MAXBAND; constraints (7) specify the lower and upper limits on the speeds of each arterial segment; constraints (8) ensure that the speed change from one arterial segment to the next is not too drastic; constraint (9) sets a upper limit for the y bandwidth to prevent it from being extremely large; and constraints (10) are derived based on the following equations:

$$-r_i + x_i + t_i = -\frac{1}{2}r_i + \phi(i, i + 1) - \frac{1}{2}r_{i+1} + x_{i+1} \quad (14)$$

where,

$$\phi(i, i + 1) = \frac{1}{2}r_i + w_i + t_i - w_{i+1} - \frac{1}{2}r_{i+1} \quad (15)$$

By substituting Equation (15) into (14), constraints (10) can be obtained. Constraint (11) ensures that the y band starts when the arterial direction is having red signal; constraints (12) make sure that the y band does not infringe upon the red times of subsequent intersections along the arterial direction, so that the left-turn vehicles at intersection N_1 can pass intersections N_2 and N_3 without stopping; and constraints (13) restrict the decision variables to be nonnegative.

III. CASE STUDY

The formulated mixed-integer linear program (MILP) is coded in CPLEX [6]. It is used to optimize the signal control problem for the arterial in Fig. 1 and Table I. To evaluate the OD-BAND model's effectiveness, the same arterial network is coded in Transyt-7F [7] and Synchro [8]. The optimal control plans generated by OD-BAND, Transyt-7F, and Synchro are all programmed in AIMSUN [9] and each timing plan is simulated for 90 minutes with 20 replications. The average results are then used for comparing these models and are presented in the next section.

In addition to the data provided in Table I and Fig. 1, a loss time of 4 seconds per phase is considered for the network; the speed limits for all links are set to 34 mph; and the saturation flow rate is set to 1,900 vehicles per hour green time per lane. All segments along the arterial direction have two through lanes in each direction. For the right-turn vehicles at Intersection C, they share the right-most lane with through vehicles. All cross streets have two through lanes in each direction except for link 2→A, which has one exclusive through lane and one exclusive left-turn lane. As shown in Table I and Fig. 1, Intersection A has a significant amount of left-turn vehicles from Node 2 to Intersection B. Therefore, a three-phase control plan shown in Fig. 3 is adopted for this intersection. Since intersections B and C do not have any left-turn vehicles, they are controlled by a two-phase timing plan in Fig. 4.

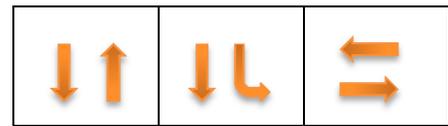


Figure 3. Control plan for intersection A



Figure 4. Control plan for intersections B and C

IV. RESULT ANALYSIS

To have a comprehensive comparison of the three sets of timing plans, different demand for Origin 2 and Destination 7 are considered while keeping the inbound and outbound through volumes constant. Specifically, the following demand values are evaluated: 740, 640, 540, 440, 240, and 140 vehicles per hour. In the rest of this section, these three types

of optimization strategies are compared in terms of overall network performance, path travel times for OD pairs, and time-space diagram.

A. Network Performance Comparison

Table II shows the overall network performance results of the timing plans generated by OD-BAND, Transyt-7F, and Synchro. The delay results in Table II suggest that when the demand for from Origin 2 to Destination 7 is relatively high (e.g., higher than the inbound and outbound through volumes), the performance of OD-BAND is substantially and consistently better than Transyt-7F and Synchro. When the demand from Origin 2 to Destination 7 is less than 300 vehicles per hour, the differences among the three methods become less significant and there is no consistent trend in terms of which method is the best.

The results in Table II are not difficult to explain. As the demand from Origin 2 to Destination 7 decreases, it becomes less important to provide a separate progression band for this OD pair. As illustrated in Fig. 2, providing a wider y bandwidth usually means less green time can be allocated for b and \bar{b} bands. Therefore, when the volume for band y is insignificant, it might be better to get rid of this band and reallocate the green time to other more important OD pairs. Intuitively, this conclusion applies to both cross-street OD pairs and OD pairs along the arterial direction. If the demand for OD pair $1 \rightarrow 8$ or $8 \rightarrow 1$ is insignificant, the corresponding progression band may be eliminated to save green time for other OD pairs.

TABLE II. OVERALL RESULTS COMPARISON

| Scenario | Left-Turn Volume (vehs/h) | Inbound and Outbound Through Volume (vehs/h) | Delay (sec/mi) | | |
|----------|---------------------------|--|----------------|------------|---------|
| | | | OD-BAND | Transyt-7F | Synchro |
| 1 | 740 | 400 | 106.98 | 138.75 | 139.76 |
| 2 | 640 | 400 | 104.21 | 126.07 | 146.98 |
| 3 | 540 | 400 | 103.57 | 126.43 | 110.45 |
| 4 | 440 | 400 | 100.07 | 128.66 | 108.65 |
| 5 | 240 | 400 | 97.72 | 96.96 | 105.51 |
| 6 | 140 | 400 | 95.27 | 101.02 | 86.83 |

B. Path Travel Time Comparison

Since the proposed OD-BAND method creates separate progression bands for cross-street OD pairs, it would be interesting to compare the path travel times generated by this new method with Transyt-7F and Synchro. For this purpose, the scenario with 740 vehicles per hour for OD pair $2 \rightarrow 7$ is selected and the path travel time results are presented in Table III. In AIMSUN, travel time is defined as the average time a vehicle needs to travel one mile inside the network or along a path and the unit of travel time is seconds per mile.

The results in Table III suggest that for OD pairs $1 \rightarrow 8$, $8 \rightarrow 1$, and $2 \rightarrow 7$, the proposed OD-BAND model generated significantly better path travel time results than Synchro and Transyt-7F. While for other cross-street OD pairs that pass only one intersection, the three signal timing methods produced approximately the same path travel times. It is not

surprising to see that the OD-BAND method generated the shortest path travel time for OD pair $2 \rightarrow 7$, since a progression band is dedicated to this particular OD pair. What is interesting is that it also generated the shortest path travel times for OD pairs $1 \rightarrow 8$ and $8 \rightarrow 1$. A possible reason for this result is that the OD-BAND model found a more appropriate cycle length. This is further explained in the next subsection in conjunction with the time-space diagram results.

TABLE III. PATH TRAVEL TIME COMPARISON

| Origin | Destination | Travel Time (sec/mi) | | |
|--------|-------------|----------------------|------------|---------|
| | | OD-BAND | Transyt-7F | Synchro |
| 1 | 8 | 66.77 | 72.78 | 86.15 |
| 2 | 3 | 25.87 | 28.03 | 25.56 |
| 3 | 2 | 49.91 | 56.70 | 58.22 |
| 4 | 5 | 34.29 | 35.58 | 34.52 |
| 5 | 4 | 35.19 | 36.92 | 33.83 |
| 6 | 7 | 36.68 | 48.09 | 36.43 |
| 7 | 6 | 38.72 | 49.04 | 36.48 |
| 2 | 7 | 69.14 | 84.98 | 102.80 |
| 8 | 1 | 65.42 | 65.86 | 108.51 |

C. Time-Space Diagram

Figs. 5~7 show the time-space diagrams for Scenario #1 in Table II. The band indicated by orange lines is for outbound arterial through movement. The black lines are for inbound arterial through movement. The progression band for OD pair $2 \rightarrow 7$ is indicated by the two blue lines in Fig. 5. All three signal timing methods can optimize the cycle length. The optimized cycle lengths and bandwidths are presented in Table IV.

TABLE IV. BANDWIDTH AND CYCLE LENGTH

| | Bandwidth (sec) | | | Cycle Length (sec) |
|------------|-----------------|-----------|----------|--------------------|
| | In Bound | Out Bound | OD Based | |
| OD-BAND | 12 | 12 | 9.54 | 60 |
| Transyt-7F | 22 | 18 | - | 90 |
| Synchro | 8.77 | 12.76 | - | 90 |

The results suggest that Transyt-7F generated the widest inbound and outbound bandwidths. However, its performance is still worse than OD-BAND. A possible reason is that it selected an unnecessarily larger cycle length than OD-BAND, which may have contributed to longer stopped delay to both arterial and cross-street vehicles. Synchro generated an inbound band of 8.77 seconds and an outbound band of 12.76 seconds. Both Transyt-7F and Synchro generated the same cycle length of 90 seconds. However, the bandwidths produced by Synchro are much smaller than those optimized by Transyt-7F. The absolute inbound and outbound bandwidths optimized by OD-BAND are about the same as those optimized by Synchro. However, if we take into account of the 90-second cycle length produced by Synchro and the 60-second cycle length for OD-BAND, the OD-BAND can provide larger total green bandwidth per cycle than Synchro. This probably explains why OD-BAND outperforms Synchro in terms of delay.

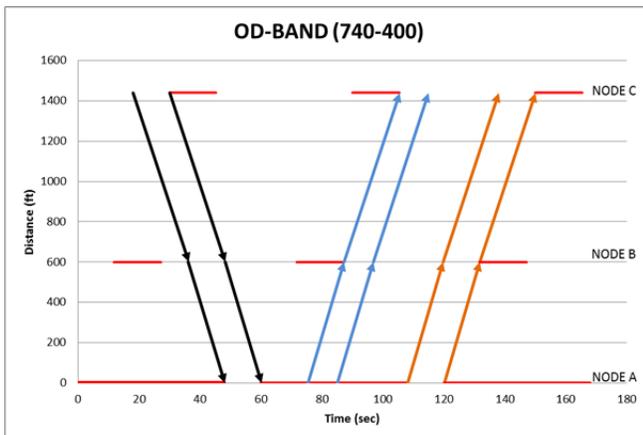


Figure 5. Time-space diagram for OD-BAND

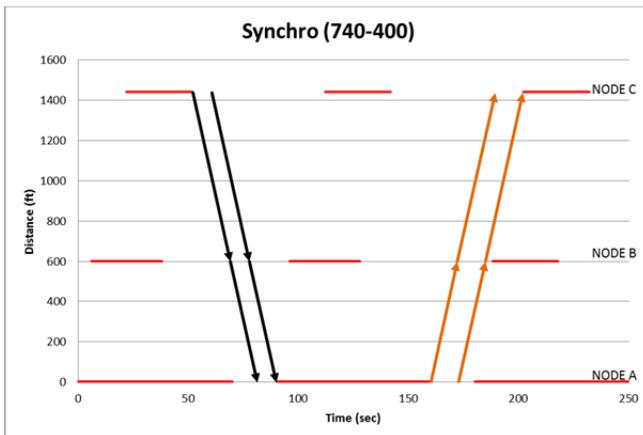


Figure 6. Time-space diagram for Synchro

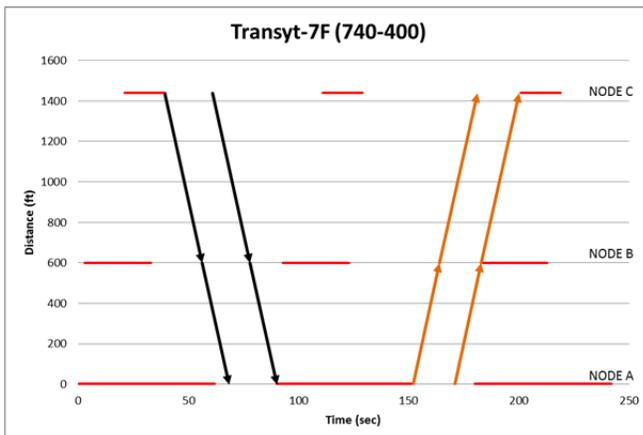


Figure 7. Time-space diagram for Transyt-7F

An interesting observation is that the inbound and outbound through movements have the same traffic flow (e.g., 400 vehicles per hour). However, Transyt-7F generated wider band for the inbound traffic than the outbound traffic, which does not appear to be very reasonable. However, since this is a proprietary software tool, it is hard to comment on what exactly happened during the optimization process.

On the other hand, Synchro generated a wider progression band for the outbound direction than the inbound direction.

This probably is because Synchro used the flow pattern for intersection B (see Fig. 8) to generate the progression bands for the arterial. When optimize the traffic signal timing plan in Synchro, we have to provide the turning movement counts of each intersection. At intersections B and C, the outbound (1→8) through traffic is combined with the traffic from 2 to 7 and the total is 1,140 vehicle per hour. While the inbound (8→1) through traffic is only 400 vehicles per hour. This may be the reason that a wider and *uniform* progression band was created for the entire outbound. Although at intersection A, the outbound through traffic is only 400 vehicles per hour and is equal to the inbound through traffic, the outbound band is still wider than the inbound band due to the uniform width nature of the progression bands generated by Synchro.

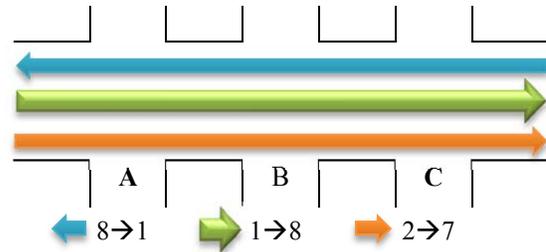


Figure 8. Flow pattern used in Synchro

As shown in Table IV, the inbound and outbound bands generated by the OD-BAND model have the same width of 12 seconds. This appears to be more reasonable, since the two arterial directions have the same amount of traffic flow. For the heavy outbound through movements at intersections B and C, a separate progression band is generated for them as shown in Fig. 9. This separate band starts at intersection A. However, it does not interfere with the green interval for the outbound through traffic at intersection A. As shown in Fig. 5, this progression band utilizes the red interval for the arterial direction at intersection A. This is why the inbound and outbound bands generated by the OD-BAND model for this case study is equal.

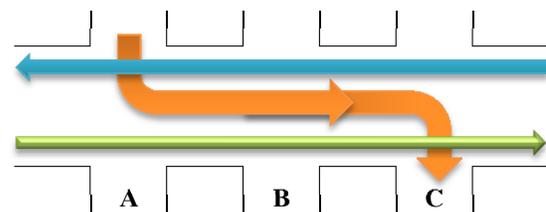


Figure 9. Flow pattern used in OD-BAND

Although the time-space diagrams in Figs. 5~7 look a little similar, the above analysis highlights the following fundamental differences between the OD-BAND, Transyt-7F, and Synchro models. Transyt-7F and Synchro use the turning movement counts at each intersection as the model input without questioning where these vehicles are from. They do check whether those flows are balanced. However, a balanced flow pattern cannot be used to uniquely determine the OD matrix for the arterial, which can be critical for developing efficient arterial traffic signal coordination plans. The OD-BAND model utilizes the arterial network vehicular OD matrix as the input directly. As the above analysis suggests, coordinating arterial traffic signals based on the OD matrix

and individual intersection's turning movement counts can give quite different results.

To further illustrate the necessity and advantage of introducing the OD-BAND model, the trajectories of the first and last possible vehicles for OD pair 2→7 based on the Synchro control plan are plotted in Fig. 10 below. In Fig. 10, the red intervals at intersection A are for left-turn vehicles from Node 2. The red intervals at intersections B and C are for the arterial through and right-turn vehicles. It can be clearly seen that some of the vehicles from OD pair 2→7 have to stop for a while at intersection B before they proceed to intersection C. While for the control plan generated by the OD-BAND model (see the left blue lines in Fig. 5), the vehicles from OD pair 2→7 can pass through all three intersections without stopping.

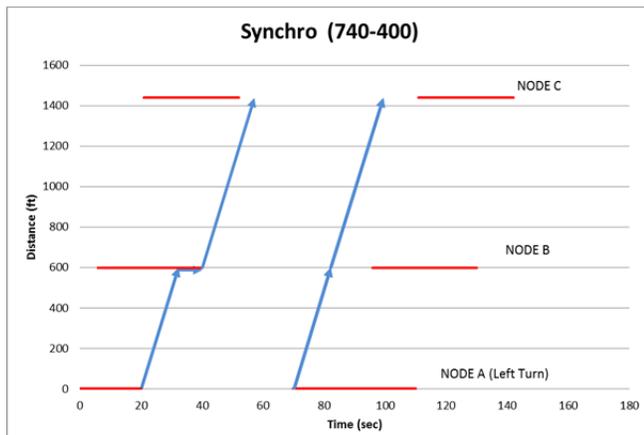


Figure 10. Trajectories of the first and last possible vehicles for OD pair 2→7 based on the Synchro control plan

V. CONCLUSION AND DISCUSSION

This paper proposed an arterial traffic signal coordination model based on the vehicular traffic origin-destination information. Previous arterial signal coordination methods try to create two either uniform or varying progression bands along the arterial direction only. This new model aims at creating a progression band for each OD pair with a significant amount of traffic. For arterials with significant cross-street turning traffic, this new modeling concept is both important and necessary.

The proposed new model was formulated as a mixed-integer linear program and solved using CPLEX. It was compared with Synchro and Transyt-7F based on AIMSUN simulation using a three-intersection arterial. When there exists heavy cross-street turning traffic, the OD-BAND model performs significantly better than Transyt-7F and Synchro for the case study arterial network. When the cross-street turning traffic is insignificant, the three methods generated results that are comparable to each other.

In Fig. 5, the right boundary of the generated y band can be further extended to reach the left boundary of the outbound arterial band b without causing problems to any movement. Also, if those left-turn vehicles from node 2 turn right at node B, the y band will stop at node B. In this case, if the extended y band and the b band are plotted together, one will see a

band with varying width along the arterial direction. This varying band is wider between intersections A and B and narrower between intersections B and C. This is reasonable because we assume that those left-turn vehicles from node 2 will turn right at intersection B. Therefore, there is no need to provide the same wide band between intersections B and C. This analysis shows the similarity between the results of OD-BAND and MULTIBAND. However, these two models are fundamentally different, as MULTIBAND only focuses on the arterial progression bands.

VI. FUTURE RESEARCH

For the case study network considered in this study, we only include one cross-street OD with turning movements (i.e., from origin 2 to destination 7) to better describe the proposed model. In future studies, additional cross-street OD pairs with heavy turning movements will be considered. Also, more complicated phasing schemes such as lead-lag left turns and overlapped left turns will be introduced for both the arterial direction and cross streets. Intuitively, this will make the model formulation much more complicated. Additionally, the phasing sequences of left turns can be incorporated into the model as decision variables similar to what was done in MULTIBAND [3, 4].

VII. ACKNOWLEDGMENT

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