

# Fast Identification of Critical Roads by Neural Networks Using System Optimum Assignment Information

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**Abstract**—Identification of critical segments in a road network is a crucial task for transportation system planners as it allows for in depth analysis of the robustness of the city’s infrastructure. The current techniques require a considerable amount of computation, which does not scale well with the size of the system. With recent advances in machine learning, especially classification techniques, there are methods, which can prove to be more efficient replacements of current approaches. In this paper we propose a neural network (NN) based approach for classification of critical roads under user equilibrium traffic (UE) assignment. We, furthermore, introduce a novel predictor attribute, which captures the contrast between UE and system optimum (SO) assignment on the network. Our results demonstrate that the neural network can achieve considerable identification precision of critical road segments and that the SO related attributes significantly increase the classification power. We, furthermore, demonstrate that the NN approach outperforms the commonly used approach of linear regression (LR) and another popular classification approach from the field of machine learning, namely support vector machines (SVM).

## I. INTRODUCTION

TRANSPORTATION systems are the medium, which allows the safe and reliable functioning of nation’s or city’s economy and further promote personal well-being. In case of a disaster the transportation system is the most critical module of a complex system as it supports all other lifelines by allowing them to transport people, supplies and tools to damaged sites. Therefore, it is vital that transportation network is 1) planned for robustness and 2) critical points are identified so that mitigation strategies can be tailored ahead of time and applied when necessary.

It is shown that daily traffic is highly predictable and that there exist regular patterns that can be exploited. This stability of choices made by traffic participants together with network topology also leads to traffic concentration on mainly a few links of the network as shown in [1].

This observation means that generic modelling techniques can be used to model the traffic assignment, as it typically exhibits regular patterns. It, however, also means that since there are only a few over utilized links, the system is expected

to be very vulnerable to damages to them. Furthermore, it is possible that some of the non-major roads might prove to be critical despite a lower experienced traffic volume due to the high degree of interconnection in the system. It is, therefore, not a trivial task to identify such a system and multiple parameters are needed in order to adequately estimate the criticality of a road.

When considering a UE traffic assignment a change in capacity of any road with positive flow can make a driver who has previously decided to take it, to choose an alternative option. Consequently, the flow on all new roads that the driver decides to utilize will be altered and therefore traffic condition on them might also change. Other drivers may then also decide to take alternative roads and so on until the system reaches an equilibrium point again. This type of phenomenon can lead to avalanche effects where small local changes create shifts of much larger traffic volumes. A road criticality classifier should therefore, somehow, grasp the dynamics of the system, which is a function of the demand and the topology of the network. This seems like an extremely complex task, which is why work on such techniques is scarce.

Typically, the impact of a capacity reduction of a road or group of roads is simulated and then explanatory parameters are sought in order find a correlation, which might be used later for classification of such roads. This approach resembles linear regression and therefore does not present any guarantees for generalization of the classification to unknown scenarios. In this work we demonstrate how the problem of critical road classification can be put into a machine learning framework, which allows not only for more accurate predictions of road classes but also for testing the reliability of those predictors when presented with a scenario, which has not been simulated yet. The main contribution of this work can be summarized as follows:

- Design of NN approach, which outperforms LR and SVM on the classification task of critical roads
- Addition of SO input attributes, which considerably improve the classification capabilities of all examined techniques
- Evaluation of classification accuracy magnitude gained from the different type of attributes used by the models

## II. LITERATURE REVIEW

The most commonly used indicator for the criticality of a road segment is the flow of vehicles on it under equilibrium

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traffic assignment conditions [2], [3] and the importance of a road is considered to be proportional to the traffic load on it [4]. A critical traffic volume is defined in [5], which is used to decide if the intersection connected to such a road should have signalling traffic control management. The work of [6] defines road criticality as a combination of three factors: 1) V/C ratio, describing the volume to capacity ratio (or the congestion factor) of the road, 2) information gain from sensing the road with respect to estimation of path travel times, and 3) centrality of the road in terms of percentage of Origin-Destination (OD) pairs, which utilize the road. A global network robustness index is presented in [7], which is a combination of link flows, capacity and network topology.

Graph theory further offers topological measures for criticality of edges, and their centrality, which is naturally correlated to the criticality. The most vital edges in a network are defined as the first  $n$  edges whose removal will lead to the biggest increase in average shortest-path distance within the set of all possible paths [8]. The work of [9] suggests three measures of centrality for a street: closeness, betweenness, and straightness. Those measures are found to be correlated to various economic activities in the respective areas. The work of [10] presents a substantial review of existing measure of heterogeneity, connectivity, accessibility, and interconnectivity in graphs. As the criticality of a road is a function of both the topology and the traffic demand the measure of entropy has proven useful in combining the information from those two factors. Family of graph measures based on entropy are summarized in [11].

Importance of road segments using entropy measures of nodes has been defined in [12] and used for optimal sensor positioning. The temporal flow variation has been examined [13] identifying nodes, which exhibit highly dynamic traffic conditions and are therefore critical to manage. Another factor, which should be taken into consideration for determining road criticality is the mismatch between the demand and capacity of the road [14].

A statistical analysis in [15], where data from various congested intersections in Shanghai during peak hour is processed and analysed shows that the characteristics of intersections varies evidently from site to site. Therefore, the approach of simply using a set of indicators and combining them in order to evaluate the criticality of a road would not be robust enough to cover all possible scenarios, which will make a road critical. A more elaborate approach would be to simulate the performance of the system reacting to capacity alterations of the road network. This methodology would replace the assessment of criticality based on chosen factors with actual measurements of the outcomes of capacity disturbances [16], [17], [18]. In [19] a capacity disruption approach is used in order to categorize and identify critical segments and the robustness of the network for different degrees of disruption.

The approaches based on direct measurement of the effect of capacity disruptions provide a clearer picture, however, they are computationally expensive. The user equilibrium traffic assignment, which is an computationally intensive has to be performed for every examined alteration. When a complete picture of the system robustness is needed the computation time for a realistic system can easily become not feasible.

Our work makes use of the established methodology for determining critical links and aims on developing a much faster way to evaluate the alterations.

### III. SIMULATION AND CASE STUDY

#### A. Macroscopic Simulation

The three main elements needed to enable our macroscopic simulation are the road network graph, the origin - destination pairs of the population and routes that commuters choose.

A road network of the simulated system is available to us, including speed limits, number of lanes on every road segment, and connectivity between the edges of the graph. A realistic number of drivers (375,000) are generated by sampling their origins and destinations from a survey data set of real start and end trip points. After this UE traffic assignment is performed to compute the routes of the drivers as described in [20].

After the routes are computed the number of drivers passing through every road segment during the simulation period  $T_S$  can be extracted. Knowing the flow  $F_i$ , length  $l_i$ , free flow speed  $v_i^f$ , number of lanes  $w_i$  and the coefficients  $\alpha_i$  and  $\beta_i$ , which are calibrated for road  $i$  (see a detailed description in [21]), we can estimate the average traverse time  $t_i$  along every road segment using the Bureau of Public Roads (BPR) function [22]:

$$t_i = \frac{l_i}{v_i^f} \left( 1 + \alpha_i \left( \frac{F_i}{2000w_iT_S} \right)^{\beta_i} \right) \quad (1)$$

Our case study examines the city of Singapore with population of 5.4 million people and around 1 million registered vehicles including taxis, delivery vans and public transportation vehicles [23]. It is an island city, thus making the examined system relatively closed. The road network graph comprises of 240,000 edges and 160,000 nodes.

Two datasets have been used in order to calibrate and validate the macroscopic simulation. The first one is the Household Interview Travel Survey (HITS) conducted in 2012 in the city of Singapore, which provides information about the traffic patterns of commuters. The second data set represents GPS traces of a 20,000 vehicles fleet for the duration of one month.

#### B. Experiment Description

The experiment performed in this paper consists of going through pre-selected links of interest and measuring the sensitivity of the system to their capacity. Let's assume that the computed total travel time of the system is  $T$ . Let link  $i$  have  $w_i$  lanes. The first step is to set the number of lanes to  $w_i - 1$ . New UE traffic assignment is performed and the total travel time  $T_i^{-1}$  is computed. The second step is to set the number of lanes to  $w_i + 1$  and assign the traffic once again to compute  $T_i^{+1}$ . By computing the difference between the computed travel times, the sensitivity of the system to the capacity of this road segment can be estimated. In a more abstract way, we are actually computing the partial derivative of the total travel time under UE with respect to the selected link. This procedure was done for roughly 2,500 road segments, which

**Data:**

$G$	Road network graph
$L$	Set of examined segments
$D$	OD demand
$UE$	Demand $\times$ Graph $\rightarrow$ Travel time
$Lane_{remove}$	Link $\times$ Graph $\rightarrow$ Graph
$Lane_{add}$	Link $\times$ Graph $\rightarrow$ Graph
$Lanes$	Link $\times$ Graph $\rightarrow \mathbb{N}$

**Result:** Set of average population travel times for respective lane additions and removals  $\Delta T_i$

```

// Compute UE traffic assignment
 $T \leftarrow UE(D, G)$ 
foreach  $l \in L$  do
   $G_l^{+1} \leftarrow Lane_{add}(l, G)$  // Add lane to  $l$ 
  // Re-calculcate UE traffic assignment with new
  graph
   $T_l^{+1} \leftarrow UE(D, G_l^{+1})$ 
  // Compute travel time difference
   $\Delta T_l^{+1} \leftarrow T_l^{+1} - T$ 
  if  $Lanes(l, G) > 1$  then
     $G_l^{-1} \leftarrow Lane_{remove}(l, G)$  // remove lane from
     $l$ 
    // Re-calculcate UE traffic assignment with
    new graph
     $T_l^{-1} \leftarrow UE(D, G_l^{-1})$ 
    // Compute travel time difference
     $\Delta T_l^{-1} \leftarrow T_l^{-1} - T$ 
  end
end

```

**Algorithm 1:** Quantifying population travel time change for reduction of capacity of roads

were chosen based on both their high throughput at UE or congestion factor. The sequence of actions is formalized in Algorithm 1

## IV. METHODOLOGY

There are three steps, which should be taken in order to exploit the suggested technique for road criticality prediction.

The first one would be the acquisition of data describing what would happen to the average travel time of the system if the capacity of a certain road is reduced. In this paper this data is gathered by means of simulation as described in Algorithm 1. It is also possible to acquire such data from real world measurements although this is considered a harder approach. In order to do that one would need to cross reference a data set describing average travel times in a system and a dataset describing road closures during this time period, car accidents, which block the road and other activities, which effectively reduce the capacity of a road. As this data is not easily accessible (requires a long computational time for the simulation approach and limited availability for real world data approach) it is important that the method needs as little training samples as possible in order to function adequately. In the next section a study on how the amount of gathered data influences the performance of the method will be presented.

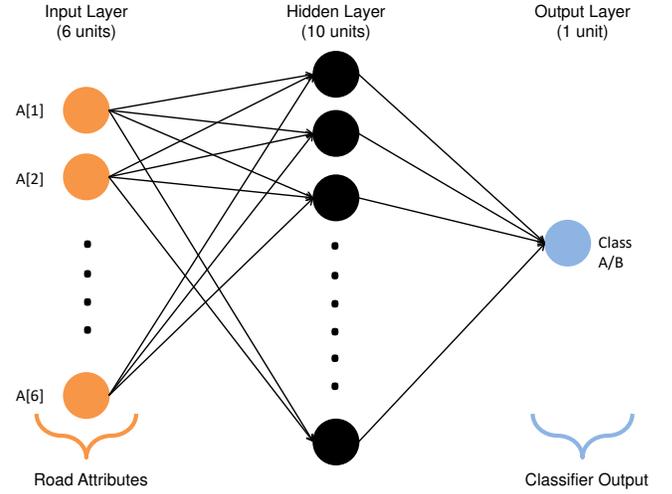


Fig. 1: Neural network architecture.

The second step of the proposed methodology is to train a set of neural networks to estimate whether a road is critical or not based on chosen attributes of the road using the collected data in the first step. The neural network, shown in Fig. 1 consists of input units, which are the values of the chosen road attributes, hidden layer of neurons, which is used for the combination of different inputs, and an output layer, where the outcome of the classification task can be read. The three layers of neurons are connected via weights, which are trained during the training phase using the data generated in the first step. After training, the values of the weights are fixed and the network is ready for exploitation.

The third step of the proposed methodology consists of running the neural network for any road the user would like to classify as critical or not. The user has to provide the attributes of the road, which should already be computed and then collect the classification class from the output layer of the network. In order to increase the performance of the method instead of training only one neural network on the data, the training procedure is repeated for several networks. During the last step (exploitation) the output of all networks is averaged in order to come up with the final classification of the road.

The input attributes of the neural network can be split into three main groups of information: 1) magnitude of the capacity reduction in terms of the length of the road, from which a lane is removed, 2) V/C ratio, total flow of vehicles under UE before the road was closed, and original capacity of the road, and 3) comparison to system optimum (SO) traffic assignment computed using the BISOS algorithm [24] in terms of flow difference on the road and ratio of expected travel time on the road between SO and UE traffic assignment.

The output, or the target, of the prediction module is a the class that is assigned to the road. There are two options for the class: Critical road or not critical road. For ease of representation we can consider the class as a binary unit, which is 0 if the road is not critical and 1 if it is. We define the road as critical if the increase of travel time when the road is closed  $Q_i = T_i^{+1} - T$  is bigger than the mean of all assessed roads plus two times the standard deviation. If we consider

the collection of all travel time increases to be the random variable  $Q$  then road class  $C_i$  can be represented as:

$$C_i = \begin{cases} 1 & \text{if } T_i^{+1} - T > \mu_Q + 2\sigma_Q \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

## V. CRITICALITY PREDICTION COMPARISON WITH OTHER METHODS

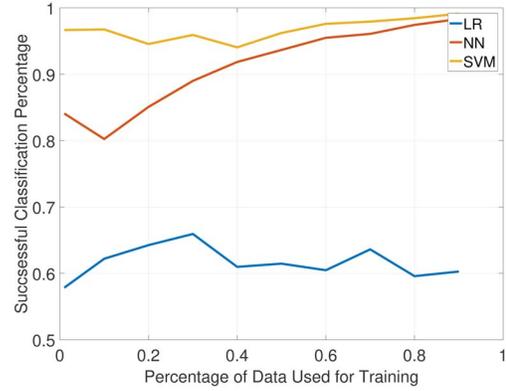
We compare the performance of 3 approaches in order to evaluate the classification power of the neural network approach:

- 1) Linear regression (LR), which represents the typical methodology used in the transportation field to access the criticality of a road. A portion of the data is used to estimate the regression coefficients (training set) and the rest of the data is used to estimate the classification power of the method.
- 2) Support vector machine (SVM), which is a standard classification method from the field of machine learning used to compare the method against a more flexible tool than LR. The SVM utilizes a radial basis kernel function.
- 3) A group of 100 neural networks (NNs), which are trained on the same task and their classification outputs are averaged in order to arrive at the final classification label. Each neural network has 10 neurons in the hidden layer and is trained using the Levenberg-Marquardt backpropagation algorithm [25], [26].

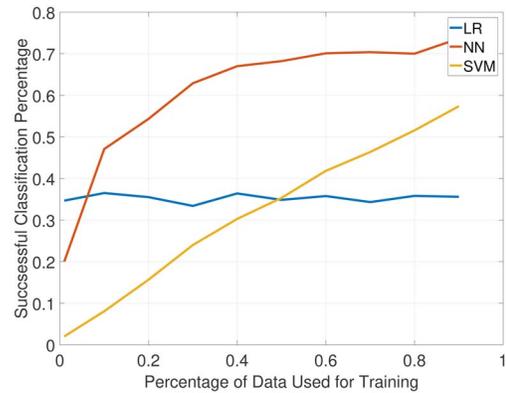
The main purpose of using machine learning techniques to evaluate the criticality of a road is to save computational resources due to the expensive UE computation. An actual computation of UE traffic assignment would require  $M \times N$  route computations, where  $M$  is the number of iterations needed for convergence and  $N$  is the number of agents. For the case of Singapore, this would mean (using  $M = 5$ ) 1,875,000 route computations. A route computation on the Singapore network takes about 40ms, which means around 20.8 hours on a single core machine. In comparison the classifier can output the predicted class of the road within milliseconds. The UE procedure, however, needs to be performed several times in order to generate training data. Therefore, it is crucial that the classifier can achieve satisfactory performance for as little training data as possible. Fig. 2 illustrates the performance of the classifier as a function of the amount of training data, which is provided to it. The figure shows averaged values over 100 training runs for every distinct percentage of randomly sampled training data. We are mainly interested in two classification performance indicators. First, the percentage of roads, which are classified as critical and are in fact critical:

$$\frac{[T = 1|C = 1]}{[T = 1|C = 1] + [T = 1|C = 0]} \quad (3)$$

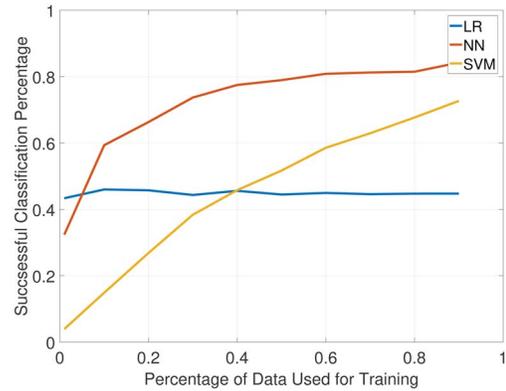
, where  $T = 1$  means that the target class of the road is “critical” and  $C = 1$  means that the road is classified as critical. This basically represents the certainty that a road is critical if the classifier has identified it as such. This is referred to as the precision of the classifier (Fig. 2a). Second,



(a) Classifier precision



(b) Classifier true positive rate



(c) Classifier F score

Fig. 2: Classifier performance as a function of amount of presented training data

the percentage of roads, which are critical and are classified as such:

$$\frac{[C = 1|T = 1]}{[C = 1|T = 1] + [C = 1|T = 0]} \quad (4)$$

This is referred to as the true positive rate (Fig. 2b). The F-score is a metric, which unifies the true positive rate and the precision into a scalar, which can be perceived as a weighted

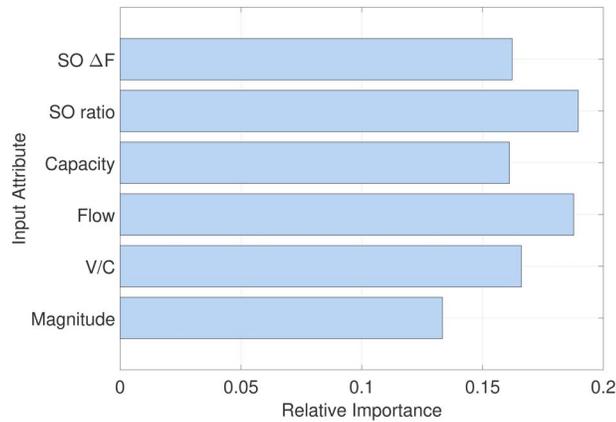


Fig. 3: Relative importance of input attributes according to the Garson test.

average of the two:

$$F = 2 \cdot \frac{\text{Precision} \cdot \text{TruePositive}}{\text{Precision} + \text{TruePositive}} \quad (5)$$

The F-score metric is used to evaluate the overall classification power of a classifier (Fig. 2c).

It can be observed that LR does not benefit significantly from new incoming data. It presents stable performance, which can be useful for extremely small amounts of data, however, as there are more learning samples the other tested methods outperform it considerably. From the results illustrated in Fig. 2 it can be observed that the high precision of the classifier is easier to achieve than a high true positive rate. The highest true positive rate is achieved by the NNs, which is 30% higher than the one achieved by the SVM.

Next, we examine the importance of the different types of input provided to the network and try to quantify the improvement in classification performance due to every type of input. First we perform the Garson relative importance test [27]. The results shown in Fig. 3 show that there is no significant difference in the level of utilization of the different inputs, which means that all types of inputs are useful for the network. In order to evaluate the contribution of the individual types of input, we evaluate the performance of the network without every single one of them.

Fig. 4 shows the contribution of the three types of parameters to the classification power of the NN. It can be observed that, the biggest amount of contribution is presented by the magnitude of the capacity reduction parameter. The contribution of this work can be quantified on an abstract level by the contribution of the SO related inputs to the classification power, as this is the novel type of parameter added to the set of inputs. It can be observed that it increases the F-score by about 20%, which is almost 50% increase.

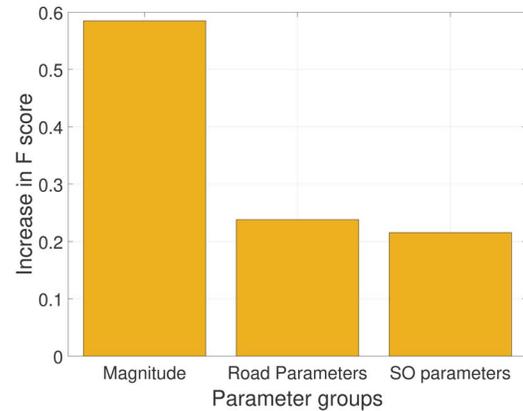


Fig. 4: Magnitude of F score improvement due to the different parameter groups

## VI. CONCLUSION

In this paper we have presented efforts to speed up the process of identification of critical links on urban road networks. We have presented approach based on neural networks, which is able to instantly classify whether a road is critical or not based on a set of computable attributes. We have tested and compared 3 methods for evaluation of criticality. The neural network approach was shown to offer the best classification capabilities over the various training data sizes in terms of both classification precision and true positive rate. In terms of classification F-score, it outperforms the linear regression current methodology by a factor of 2 and the support vector machine approach by a little over 35%. We further evaluated the importance and the magnitude of benefit coming from the different sets of attributes. Our results show that the information about the magnitude of the capacity disruption brings the biggest benefit to the classifier. The set of inputs connected to SO road properties were also shown to considerably improve the neural network approach classification power.

This set of promising results opens various possible future research opportunities. It must be tested whether the trained classifier can accurately predict transit systems with different topology and demand characteristics. A hybrid technique combining machine learning approaches might further improve the classification power. It is also desirable that the criticality of a group of roads can be assessed as well, as some extreme events tend to reduce the capacity of a certain region rather than just one road.

Last but not least, if the neural network can in fact grasp the properties of the UE, then it might be used to extract more specific information about the changes in traffic assignment as a result of capacity alterations. Currently we have demonstrated that the network can accurately classify the criticality of the alteration, however, this approach might also be used to evaluate quantitatively the change on a system level and also provide estimations of locally exhibited changes. This can allow the neural network approach to be used as a strong computationally efficient tool for UE estimation, thus saving considerable amount of time for system evaluation.

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