Eco-Routing of Plug-In Hybrid Electric Vehicles in Transportation Networks

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Abstract— We study the problem of eco-routing Plug-In Hybrid Electric Vehicles (PHEVs) to minimize the overall energy consumption costs. Unlike the traditional Charge Depleting First (CDF) approaches in the literature where the power-train control strategy is fixed, we propose a Combined Routing and Power-train Control (CRPTC) algorithm which can simultaneously calculate the optimal energy route as well as the optimal power-train control strategy. To validate our method, we apply our eco-routing algorithm to a subnetwork of the Eastern Massachusetts (EMA) transportation network using actual traffic data provided by the Boston Region Metropolitan Planning Organization. As an alternative benchmark, we also simulate the traffic behavior of the network using the extracted flow data from the aforementioned traffic dataset. We show that the CRPTC approach outperforms the traditional CDF approach and we quantify the trade-off between saving energy and time in using eco-routing algorithms.

I. INTRODUCTION

Due to environmental concerns and high gas prices, there has been an increasing interest in vehicles using alternative energy sources such as electric vehicles (EV). However, EV adoption is limited by the all-electric range (AER) considering the battery capacity in current EVs. In this respect, Plug-In Hybrid Electric Vehicles (PHEVs) are viewed as a suitable alternative, as they can overcome range limitations by using both gas and electricity. Moreover, it is possible to decrease the energy consumption cost and the carbon footprint of PHEVs using smart eco-routing and power-train control strategies.

Unlike traditional vehicle routing algorithms which seek to find the minimum time or shortest path routes [1]–[3], eco-routing algorithms seek the paths that minimize the total energy consumption cost. Several routing algorithms have been proposed in the literature for conventional vehicles which are capable of finding the energy-optimal paths using historical and online traffic data [4]–[8]. Kubicka et al [9] performed a study to compare the objective values proposed in the eco-routing literature and showed that the performance of eco-routing algorithms is highly dependent on the method used to calculate the traveling cost of each link. Although eco-routing of conventional vehicles is well studied, there is little research that addresses the case of PHEVs [10]. Jurik et al [11] have addressed the eco-routing problem for HEVs based on the vehicle longitudinal dynamics. Sun et al [12] and Qiao et al [13] proposed the CDF approach to address the eco-routing for PHEVs. Furthermore, in [12], the authors have shown that energy-optimal paths typically take more time compared to the fastest route.

The contributions of this paper are summarized as follows. After reviewing the traditional CDF eco-routing approach, we propose a Hybrid-LP Relaxation algorithm to solve this problem by reducing it to a Linear Programming (LP) problem which guarantees convergence to a global optimum. Moreover, based on the energy model definition in Section II, we propose a Combined Routing and Power-train Control (CRPTC) eco-routing algorithm for PHEVs which can simultaneously find the optimal energy route as well as the optimal power-train control strategy for switching between charge depleting (CD) and charge sustaining (CS) modes. Unlike the previous methods where the power-train (PT) control strategy was considered a priori [12], [13], we do not make such an assumption and we let the optimizer choose the optimal control strategy. We formulate the problem as a mixed integer linear programing (MILP) problem and use actual traffic data from the Eastern Massachusetts transportation network (provided by the Boston Region Metropolitan Planning Organization) to validate the performance of our algorithm. As an alternative to such historical data, we also use the SUMO simulator to investigate traffic outcomes. We show that the CRPTC approach can lead to improved energy savings compared to the CDF approach while using the same energy models as in [13]. Finally to assess the performance of the CRPTC approach, we compare the energy cost and traveling time of the energy-optimal route with the fastest route and the routes obtained from actual traffic data. As in [12], we show the trade-off between saving energy and time in Section IV.

The remainder of this paper is organized as follows. The PHEV energy consumption model is presented in Section II. A MILP formulation is proposed in Section III to solve the eco-routing problem. In Section IV, we use actual historical data to validate the performance of the algorithm and also include simulation results to compare the energy cost and traveling times of different routing algorithms. Finally, conclusions and further research directions are outlined in Section V.

II. PHEV ENERGY CONSUMPTION MODELING

Unlike conventional vehicles where it is possible to use the empirical equations based on velocity and acceleration of the vehicle to estimate fuel consumption cost [14], estimating a PHEV's fuel consumption is a more involved process which

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over a finite time horizon can be expressed as follows:

$$\int_{t_0}^{t_f} (C_{gas}\dot{m}_{gas}(t) + C_{ele}P_{batt}(t))dt \qquad (1)$$

where $\dot{m}_{gas}$ is the fuel consumption rate, and P_{batt} is the total electrical power used/generated by the motor/generator units. Moreover, C_{gas} ($\$/gallon$) and C_{ele} ($\$/kWh$) are the cost of gas and electricity, respectively.

Due to the nature of our problem and to avoid unnecessary complexities, we use a simplified model proposed by Qiao et al [13] to calculate $\dot{m}_{gas}$ and P_{batt} in our eco-routing problem formulation. Instead of using real time driving data for a targeted vehicle, they calculate the average $\dot{m}_{gas}$ and P_{batt} per mile for different drive cycles using the software package *Autonomie/PSAT*. In this method, they consider two driving modes for a PHEV: charge-depleting (CD) and charge-sustaining (CS). The CD mode refers to the phase where the PHEV acts like an EV and consumes all of its propulsion energy from the battery pack. Once the state of the charge (SOC) of the battery reaches a target value, it switches to the CS mode in which the vehicle starts using the internal combustion engine as the main propulsion system and the battery and electric motors are only used to improve fuel economy as in HEVs [15].

Let us consider the traffic network as a directed graph (Fig. 1). Based on the traffic intensity on each link we can categorize the links into three modes: low, medium, and high traffic links. We can then assign different standard drive cycles to each link [13] (HWFET $\rightarrow$ low traffic links, UDDS $\rightarrow$ medium traffic links, and NYC $\rightarrow$ high traffic links). Qiao et al [13] modeled a PHEV20 (PHEV with 20 miles of AER) in *PSAT* and calculated the average electrical energy (μ_{CD}) and gas (μ_{CS}) used to drive one mile under CD and CS modes respectively under each of theses drive cycles (Table I):

$$\mu_{CD_{ij}} = \frac{d_{ij}}{P_{batt_{ij}}} \quad , \quad \mu_{CS_{ij}} = \frac{d_{ij}}{\dot{m}_{batt_{ij}}} \quad ,$$

where d_{ij} is the length of link (i,j). By knowing $\mu_{CD_{ij}}$ and $\mu_{CS_{ij}}$ on each link, as well as the network topology (length of each link), we can determine the average fuel consumption rate ($\dot{m}_{batt_{ij}}$) and electrical power demand from the battery ($P_{batt_{ij}}$) on each link (i,j). We can then use (1) to calculate the total energy cost for each trip.
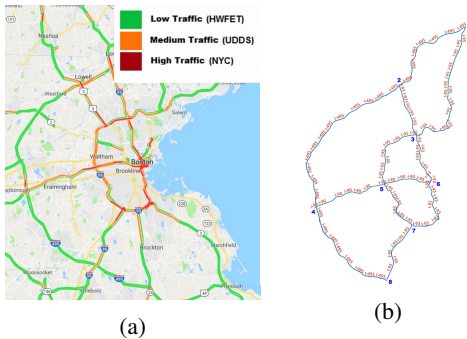

Fig. 1: (a) Typical Traffic in a network from Google Maps; (b) Interstate highway subnetwork of EMA [16]

TABLE I: Energy consumption of various drive cycles [13]

Vehicle Type	Symbol	Unit	HWFET	UDDS	NYC
PHEV20	μ_{CD}	mi/kWh	5.7	6.2	4.2
	μ_{CS}	mi/gal	58.6	69.4	45.7

III. SINGLE VEHICLE ECO-ROUTING

In this section, we first review the CDF eco-routing approach [12], [13]. Next, we propose the CRPTC algorithm to solve the eco-routing problem.

A. Problem Formulation

We model the traffic network as a directed graph $G = (\mathcal{N}, \mathcal{A})$ with $\mathcal{N} = 1,...,n$ and $|\mathcal{A}| = m$ with the arc (link) connecting node i to j denoted by $(i,j) \in \mathcal{A}$. The set of nodes that are incoming/outgoing to node i are defined as: $\mathcal{I}(i) = \{j \in \mathcal{N} | (j,i) \in \mathcal{A}\}$ and $\mathcal{O}(i) = \{j \in \mathcal{N} | (i,j) \in \mathcal{A}\}$, respectively. We consider the single-origin-single-destination eco-routing problem where origin and destination nodes are denoted by 1 and n respectively. The energy cost consumed by the vehicle on link (i,j) is denoted by c_{ij}. We use E_i to represent the vehicle's residual battery energy at node i. Moreover, we denote the selection of arc (i,j) by $x_{ij} \in \{0,1\}$. The problem objective is to determine a path from node 1 to n so as to minimize the total energy cost consumed by the vehicle to reach the destination. We consider two approaches to solve this problem as follows.

1) Charge Depleting First (CDF): A Hybrid-LP Relaxation Approach: In this approach, we assume that the PHEV always starts every trip in the CD mode and uses electricity to drive the vehicle until it drains all the energy out of the battery pack. Afterwards, it switches to the CS mode and starts using gas to drive the vehicle. Even though this is not the most accurate approach to solve the problem, it eliminates the need of using complicated control strategies for a PHEV power-train to switch between Internal Combustion Engine (ICE) and electric motors [13]. As a result, we can formulate the eco-routing problem using a mixed-integer nonlinear programming (MINLP) framework as follows:

$$\min_{x_{ij}, i,j \in \mathcal{N}} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}x_{ij} \qquad (2)$$

$$s.t. \quad c_{ij} = \begin{cases} C_{gas}\dfrac{d_{ij}}{\mu_{CS_{ij}}}; & E_i \leq 0 \\ C_{ele}\dfrac{d_{ij}}{\mu_{CD_{ij}}}; & E_i \geq \dfrac{d_{ij}}{\mu_{CD_{ij}}} \\ C_{ele}E_i + C_{gas}\dfrac{d_{ij} - \mu_{CD_{ij}}E_i}{\mu_{CS_{ij}}}; & \text{otherwise} \end{cases} \qquad (3)$$

$$E_j = \sum_{i \in \mathcal{I}(j)} \left(E_i - \frac{d_{ij}}{\mu_{CD_{ij}}}\right)x_{ij}, \quad \text{for } j = 2,...,n, \qquad (4)$$

$$\sum_{j \in \mathcal{O}(i)} x_{ij} - \sum_{j \in \mathcal{I}(i)} x_{ij} = b_i, \quad \text{for each } i \in \mathcal{N} \qquad (5)$$

$$b_1 = 1, b_n = -1, b_i = 0, \text{for } i \neq 1, n \qquad (6)$$

$$x_{ij} \in \{0,1\} \qquad (7)$$

where E_i is the remaining electrical energy at node i, and $\mu_{CD_{ij}}$ and $\mu_{CS_{ij}}$ are the conversion factors taken from Table I,

which are functions of the traffic intensity on each link (i,j). Note that (5)-(6) are the flow conservation constraints [17]. We assume that the vehicle has enough gas and electrical power to complete the trip and that $E_1 \geq 0$. Knowing the traffic density on each link, problem (2) was solved using Dijkstra's algorithm [18] in [13].

In what follows, we propose an alternative solution to this problem which we call Hybrid-LP Relaxation. In this approach we reduce the MINLP problem (2) to a simpler problem which can be solved using a combination of linear programming (LP) and a simple dynamic programming-like algorithm, in order to guarantee global convergence. The nonlinearities of the problem arise in (3) where c_{ij} is a function of x_{ij}. We show that we can reduce this piecewise constant function to a constant function, and the MINLP can be converted to a LP by using the properties of the minimum cost flow problem [19]. The proposed algorithm is as follows:

1) Find the shortest path (or shortest time path) and calculate the energy cost on this path and set it to ρ.
2) From the origin, construct all paths reaching node p such that $E_p \leq 0$ and stop constructing the path at this node. Disregard the paths with a total energy cost greater than ρ and save the remaining paths in a matrix.
3) From (3), the cost function for the paths outgoing from node p to n in the previous step is given by:

$$c_{ij} = C_{gas} \frac{d_{ij}}{\mu_{CS_{ij}}}$$

4) Assuming knowledge of traffic modes on each link, the least energy cost path from node p in step 2 to the destination node can be found from:

$$\min_{x_{ij}, i,j \in \mathcal{N}} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{8}$$

$$s.t. \quad c_{ij} = C_{gas} \frac{d_{ij}}{\mu_{CS_{ij}}} \tag{9}$$

$$\sum_{j \in \mathcal{O}(i)} x_{ij} - \sum_{j \in \mathcal{I}(i)} x_{ij} = b_i, \quad \text{for each } i \in \mathcal{N} \tag{10}$$

$$b_p = 1, b_n = -1, b_i = 0, \text{for } i \neq p, n \tag{11}$$

$$x_{ij} \in \{0,1\} \tag{12}$$

Note that constraint (11) ensures that by solving (8), we are finding the optimal path from p to n.

5) Using the property the minimum cost flow problem [19], problem (8) is equivalent to an LP problem with the integer restriction of x_{ij} relaxed:

$$0 \leq x_{ij} \leq 1 \tag{13}$$

6) Find the path from node 1 to p with the least energy cost. By the principle of optimality, the optimal path from 1 to n is the one determined in this manner followed by the path selected by steps 4 and 5 from node p to n.
7) Find the paths in step 6 for all nodes p such that $E_p \leq 0$, then choose the one with the minimum energy cost. The selected path would be the minimum energy cost path.

8) If there are paths without any node such that $E_p \leq 0$ (generated at step 2), compare their cost function values with the cost functions in step 6. The optimal route is the minimum among them.
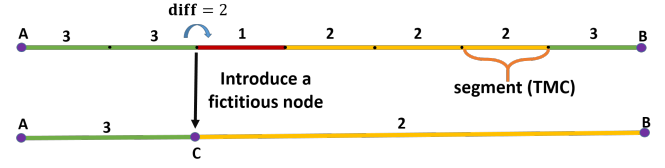

Fig. 2: Procedure for introducing new fictitious nodes into a link based on the traffic modes on the link's segments

2) Combined Routing and Power-Train Control (CRPTC): Based on Table I, the CD mode has the best efficiency on medium traffic links. As such, if we always consider using the CD mode at the beginning of each trip and then switch to the CS mode when we run out of battery, we miss the opportunity to harness the effectiveness of the CD mode on medium traffic links towards the end of the route. With this motivation, we propose a new algorithm which finds the routing decisions as well as the PT controller decision to switch between CD and CS modes. Let $y_{ij} \in [0,1]$ be an additional decision variable on link (i,j) which represents the fraction of the link's length over which we use the CD mode (thus, if we only use the CD mode over link (i,j), then $y_{ij} = 1$). Considering the new decision variable, we can formulate the CRPTC problem as follows:

$$\min_{x_{ij}, y_{ij}, i,j \in \mathcal{N}} \sum_{i=1}^{n} \sum_{j=1}^{n} [c_{gas} \frac{d_{ij}}{\mu_{CS_{ij}}} (1 - y_{ij}) + c_{ele} \frac{d_{ij}}{\mu_{CD_{ij}}} y_{ij}] x_{ij} \tag{14}$$

$$s.t. \sum_{j \in \mathcal{O}(i)} x_{ij} - \sum_{j \in \mathcal{I}(i)} x_{ij} = b_i, \quad \text{for each } i \in \mathcal{N} \tag{15}$$

$$b_1 = 1, b_n = -1, b_i = 0, \text{for } i \neq 1, n \tag{16}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{d_{ij}}{\mu_{CD_{ij}}} y_{ij} x_{ij} \leq E_1 \tag{17}$$

$$x_{ij} \in \{0,1\}, \quad y_{ij} \in [0,1] \tag{18}$$

Note that constraint (17) ensures that the total electrical energy used in the CD mode would be less than the initial available energy in the battery (E_1). Since we have the term $x_{ij} y_{ij}$ in the problem formulation, this is a MINLP problem and we may not be able to determine a global optimum. Hence, we transform (14) into a mixed integer linear programming problem (MILP) by introducing an intermediate decision variable $z_{ij} = x_{ij} y_{ij}$. We can then use the inequalities in (23) to transform the existing MINLP problem (14) into a MILP problem as follows:

$$\min_{x_{ij}, y_{ij}, z_{ij}, i,j \in \mathcal{N}} \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{gas} \frac{d_{ij}}{\mu_{CS_{ij}}} x_{ij} + (c_{ele} \frac{d_{ij}}{\mu_{CD_{ij}}} - c_{gas} \frac{d_{ij}}{\mu_{CS_{ij}}}) z_{ij}) \tag{19}$$

$$s.t. \sum_{j \in \mathcal{O}(i)} x_{ij} - \sum_{j \in \mathcal{I}(i)} x_{ij} = b_i, \quad \text{for each } i \in \mathcal{N} \tag{20}$$

$$b_1 = 1, b_n = -1, b_i = 0, \text{for } i \neq 1, n \tag{21}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{d_{ij}}{\mu_{CD_{ij}}}z_{ij} \leq E_1 \tag{22}$$

$$z_{ij} \geq 0, \qquad z_{ij} \leq y_{ij}$$
$$z_{ij} \leq x_{ij}, \qquad z_{ij} \geq y_{ij} - (1 - x_{ij}) \tag{23}$$

$$x_{ij} \in \{0,1\}, \quad y_{ij} \in [0,1] \tag{24}$$

This is a MILP problem which can be solved to determine a global optimum.

IV. NUMERICAL RESULTS

In order to evaluate the performance of the proposed algorithm, we conduct a data-driven case study using the actual traffic data from the EMA road network collected by *INRIX* [20], [21]. A sub-network including the interstate highways of EMA (Fig. 1b) is chosen for the case study. Details regarding this sub-network can be found in [20], [21].

A. Performance Measurement Baseline

For measuring the performance of each algorithm, we need to define a baseline against which to compare the energy cost obtained. We consider two different baselines to compare with our energy-optimal algorithm: time optimal paths and the actual paths from the INRIX dataset.

1) Energy-Optimal Path vs. Time-Optimal Path: In this approach, we first find the time-optimal paths using historical traffic data as follows:

$$\min_{x_{ij}, i, j \in \mathcal{N}} \sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij}x_{ij} \tag{25}$$

$$s.t. \quad t_{ij} = \frac{d_{ij}}{\bar{v}_{ij}}, \quad x_{ij} \in \{0,1\} \tag{26}$$

where $\bar{v}_{ij}$ and d_{ij} are the average speed and length of link (i,j) respectively, and t_{ij} is the travel time over link (i,j). We can then determine the energy costs for traveling through the shortest time path using (3) and compare the energy costs of traveling through the optimal energy paths (under CDF and CRPTC) with the costs of traveling through the minimum time path.

2) Energy-Optimal Path vs. Actual Routes: Using the INRIX dataset, Zhang et al [20], [21] transformed the average speeds to vehicle flows on each link. They then solved an inverse optimization problem [22] to find the origin-destination (O-D) matrices of the network including the probabilities of going through each route. They have reported these values based on four different time periods: AM (6 am to 9 am), MD (9 am to 3pm), PM (3 pm to 6pm), and NT (6 pm to 6 am). The values are calculated on an average sense over all days in April 2012. An example is shown in Table II for O-D pair (1,5).

TABLE II: Actual routes and their probabilities [20]

Origin	Dest.	Route	AM %	MD%	PM%	NT%
1	5	1→2→3→5	19.7	18.7	21	16.9
1	5	1→3→5	65.3	73	61	73
1	5	1→2→→3→6→5	15	8.3	18	10.1

Using the actual routes, we calculate the expected energy costs and travel times for each O-D pair based on the

traffic information at any given time. We use the following equations to calculate the total expected energy costs and traveling times for traveling through each O-D pair (i,j):

$$E(C_{ij}^{act}) = \sum_{k=1}^{m} c_{ij}^{k}p_{ij}^{k}, \quad E(\tau_{ij}^{act}) = \sum_{k=1}^{m} t_{ij}^{k}p_{ij}^{k} \tag{27}$$

where $E(\cdot)$ is the expected value, C_{ij}^{act} and τ_{ij}^{act} are the total energy cost and traveling time for traveling from node i to node j from (3) and (26), m is the number of possible routes, c_{ij}^{k} and t_{ij}^{k} are the energy cost and travel time for going through (i,j) using the kth possible route, and p_{ij}^{k} is the probability of the kth route for traveling through link (i,j).

B. Data Preprocessing

Each link consists of a number of road segments, and the average speed may differ over consecutive road segments. Meanwhile, to solve problem (19) we need to know the traffic mode on each link. Since different segments on a link have different traffic modes (low, medium, and heavy traffic), we need to come up with a strategy to assign a unified link mode to each link. Our approach is as follows:

1) Categorize each link segment into 3 modes based on the average speed (V_{ave}) of the segment: mode 1 ($V_{ave} < 20$), mode 2 ($20 \leq V_{ave} \leq 40$), mode 3 ($V_{ave} > 40$).
2) If the change in the traffic mode of two consecutive segments is 2, we introduce a fictitious node at that point into our network graph (Fig. 2).
3) Calculate the average mode of the segments of each link, and report the value as the traffic mode of that link.

Considering this approach, we end up with a new adjacency matrix including more nodes and edges than the original network. Using the new adjacency matrix, we can solve problems (2) and (19).

C. Comparison Results

Using the INRIX dataset, optimal energy paths (CDF and CRPTC), optimal time paths, and actual paths have been evaluated. We have also calculated the energy consumption costs and travel times for each of these paths. The cost of gas and electricity used in this case study are $C_{gas} = 2.75\$/gal$ and $C_{ele} = 0.114\$/kWh$ respectively [15]. Moreover, as in [13], we assumed the initial available battery energy to be $E_{ini} = 5.57kWh$. Energy and time comparison plots for traveling thorough node 1 to 5 can be found in Figs. 3 and 4.

As expected, CRPTC performs better than CDF in terms of energy saving. We repeated the same analysis for traveling from node 1 to 5 for a week in April 2012 using INRIX dataset and found that on average we can save 2.54% in terms of energy using CRPTC instead of CDF. We have also found that we can save on average 5.02% in terms of energy using CRPTC compared to the cost for traveling through the min time route, and also 6.89% compared to the cost for traveling through the actual routes. The trade-off between energy saving and travel time is such that on average it takes 4.79% more time to travel from node 1 to 5 while using the
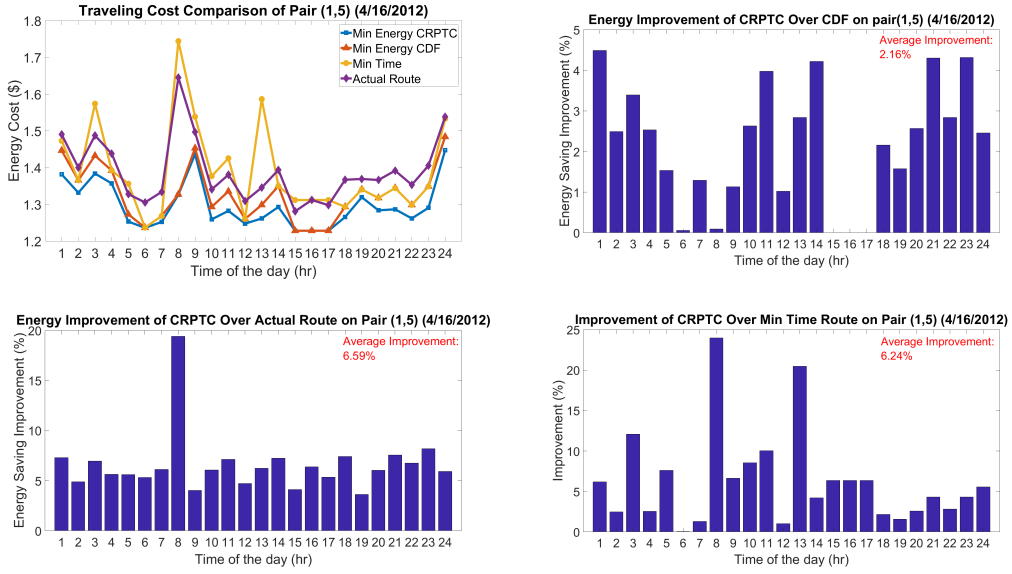
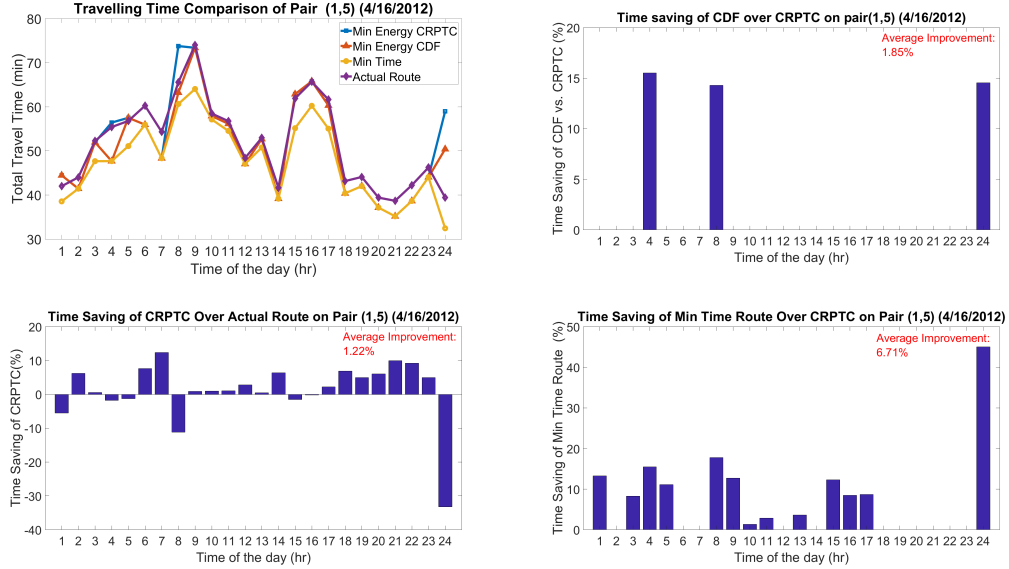Fig. 3: Energy comparison plots for traveling from node 1 to 5 (04/16/2012)

Fig. 4: Traveling Time plots for traveling from node 1 to 5 (04/16/2012)

CRPTC route compared to traveling through the minimum time route. It also takes 2.15% longer using the CRPTC route compared to the CDF route. We observe that users were not taking the time-optimal route in 2012; this may be because in 2012 navigation systems with real time traffic data were not as accessible to the public as they are today. In fact, we can save both in terms of energy and time if we take the CRPTC route instead of the actual routes users traveled at the time, and the average time saving is 2.15%.

D. Traffic Simulation

Since we did not want to rely solely upon the historical traffic data to validate our routing algorithm, we decided to simulate the traffic of the EMA sub-network (Fig. 1) using

SUMO (Simulation of Urban MObility) [23]. The flow data on each link are needed to start a simulation in SUMO. Hence, we used the INRIX data to extract the flow data for the subnetwork. The details of theses calculations may be found in [20], [24].

Using extracted flows, we simulated traffic in SUMO. We then aggregated every 5 segments in the map into a single link and recorded the average speed of that new link. In this respect, we ended up having a graph with 281 nodes and 300 edges. Using the new graph, we found the minimum time route as well as the CRPTC and CDF energy-optimal routes, and calculated the energy costs and traveling time for each of these routes for different O-D pairs. Considering

Table II, we have also calculated the expected energy costs and travel times over the actual paths taken by drivers. The comparison results for April 16, 2012 between energy costs and traveling time of 4 different O-D pairs are shown in Figs. 5a and 5b.
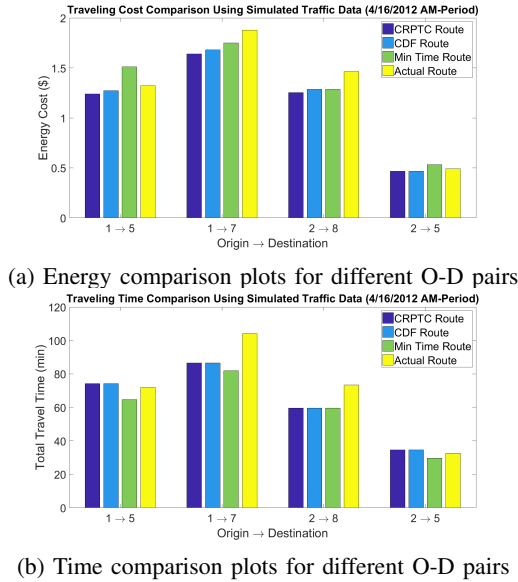

(a) Energy comparison plots for different O-D pairs

(b) Time comparison plots for different O-D pairs

Fig. 5: Comparison plots using SUMO simulated traffic data

V. CONCLUSIONS AND FUTURE WORK

In this paper, we reviewed the current approaches to solve the eco-routing problem for PHEVs, and proposed a method to solve the minimum energy cost problem for a single vehicle routing problem. The proposed CRPTC method is capable of finding both the optimal path and the optimal switching strategy between CD and CS modes on each link. Historical traffic data, as well as SUMO simulations, were used to validate the performance of our algorithm. Numerical results show improvements in terms of the total energy cost using the CRPTC approach compared to the previous CDF method. We have also shown that there is a trade-off between energy saving and time saving.

So far, we have not considered dynamically updating routing decisions at network nodes to account for sudden changes in traffic conditions (e.g., due to accidents). In on-going work, we are implementing and investigating dynamic eco-routing as well. Moreover, we have so far solved the problem for a single vehicle scenario with a known origin and destination. As a next step, we will consider connectivity among vehicles and determine the social optimum for the network considering a 100% penetration rate of connected automated vehicle. Moreover, we plan to include multiple vehicle architectures with different fuel consumption models and different initial energies to the problem, as well as add charging stations in the network to let vehicles recharge their batteries if necessary.

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