Distributed Controller Design for Vehicle Platooning under Packet Drop Scenario

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***Abstract*—This paper proposes a distributed control strategy for homogeneous platoon systems with external disturbances under random packet drop scenario which can occur due to underlying network among the vehicles in a platoon. An linear matrix inequality (LMI) based approach is used to obtain the controller gains for ensuring the stability with bounded H_∞ norm for such systems. Effectiveness of the proposed method is demonstrated with numerical results considering different network topologies in a platoon under single packet drop. The variation of H_∞ norm bound for different number of platoon members under the different structure of network topologies and the packet drop has been studied in this paper.**

Keywords—vehicle platoon, distributed controller, LMI, packet drop

I. INTRODUCTION

Vehicle platoon is a group of two or more connected autonomous vehicles (CAVs) where all vehicles travel with the same velocity while maintaining a small inter-vehicular distance within themselves. In a highway, the platoon of CAVs offers number of benefits like road safety, highway utility, and fuel economy etc. [1]. In a platoon, vehicles exchange information with their neighbours through wireless vehicle-to-vehicle (V2V) [2] and vehicle-to-infrastructure (V2I) [3] communication systems. These communication systems utilize the concepts of network topologies e.g. (i) predecessor following (PF), (ii) predecessor leader following (PLF), (iii) bidirectional PF (BPF) [4], (iv) bidirectional predecessor leader following (BPLF), (v) two PF (TPF) [5], (vi) two BPF (TBPF) [4] and (vii) all-to-all [6] etc. relating the way of information exchange between vehicles in a platoon. However, the reliability of wireless communication network depends largely on bandwidth allocation, external disturbances, signal strength etc., which ultimately leads to packet drop and/or delay in data transmission. This packet drops and/or delay largely affects the stability and performance of the vehicle platoon system [7] due to presence of wireless communication network in it. Therefore, it is quite challenging to ensure the stability and maintain the desired performances (maintaining the same velocity and inter-vehicular distance) by designing the appropriate controller for the vehicle platoon system under different network topologies with packet drop.

The importance of network topology has been described for designing the controller to analyse the closed-loop stability and co-operative motion of the vehicles in a platoon in [5]. Recently, number of advanced control strategies have been implemented to achieve better performances of vehicle platoon using different network topologies [3, 8-11]. In [8], a distributed receding horizon controller has been designed for platoons under the PF topology while [9] extended the results of [8] with unidirectional topology. Using graph theory and Routh-Hurwitz criteria, the stabilizing threshold of controller gains has been derived in [2]. A distributed H_∞ controller has been designed for homogeneous platoon with undirected topology using LMI approach in [10]. An adaptive control strategy has been adopted for adjusting the gains of the distributed controller as a function of mismatched states of the platoon members in [6]. However, these above-mentioned methods do not ensure stability and performances under network imperfections (e.g. packet drop and/or delay), parametric uncertainty such as mass of passenger vehicles or engine time constant [12] and external disturbances such as lead vehicle's acceleration, wind gust or road slope [10, 12-13] acting on the vehicles in a platoon.

Although packet drop, communication delays, parametric uncertainty and external disturbances all have been investigated by researchers separately [7, 10] or in some combinations [13] for the vehicle platoon control problem but combination of all these issues remains a challenging research problem [9, 10]. For example, for short range wireless communication environment among the vehicles in a platoon, a decentralized model predictive controller has been designed under low and high communication latency for longitudinal platoon control problem in [14]. Tang *et al.* [7] have utilized the concept of network topology for communication systems [5] between vehicles in a platoon and have designed the LMI based distributed controller to achieve the co-ordinated motion of each vehicle under packet loss. Other approaches have used co-operative adaptive cruise control to analyse string stability and performance of platoons under packet drop [15, 16] and communication delay [17, 18] when vehicles are wirelessly connected. However, to the best our knowledge, very few researchers have proposed control strategies with networked packet drop or delay for vehicle platoon control under parametric uncertainty and/or external disturbances [13, 19]. An LMI based distributed H_∞ controller has been designed for heterogeneous vehicle platoon control under the parametric uncertainty, external disturbances and communication delays in [19]. However, the method proposed in [19] only considered PLF network topology for the communication among the vehicles in a platoon. As an extension of the work [19], our method proposes a distributed controller design methodology for vehicle platoon under random single packet drop and external disturbances considering the network topology for communication between vehicles are generic (i.e. valid for different network topologies (i)-(vii)) as reported in [4, 5].

This paper proposes a robust distributed control design methodology for homogeneous vehicle platoon under random packet drop. As in [10], the longitudinal dynamics of homogeneous platoon with disturbances has been considered in this work. However, to design the distributed controller for the homogeneous platoon with external disturbances, first, we recast the platoon control problem as a synchronization problem of a multi agent system [8, 11], i.e. leader-following consensus under single packet drop scenario using Bernoulli modelling of packet drop distribution is used. Next, we have used Lyapunov based LMI approach to obtain the controller gains with the satisfaction of certain bounded H_∞ norm which ensure the stability and maintain desired inter-vehicular distance for such systems. The effect of various network topologies with random packet drop rate on H_∞ norm bound for different numbers of platoon members has been studied in this paper. In addition, for different platoon members, the effect of leader vehicle's information to the follower vehicles in a platoon under network topologies and packet drop has also been studied by analysing the robust performance measures i.e. H_∞ norm bound γ.

The rest of the paper is organized as follows. Section II describes platoon modelling and control objectives of platoon for synchronization problem under single packet drop scenario. Then to achieve the control objectives for a platoon, the LMI based distributed controller design approach has been described in Section III. Section IV represents the detail derivation of proposed methodology. Numerical results have been shown in section V to analyse the effectiveness of proposed method. The final conclusions have been presented in Section VI followed by references.

II. PLATOON MODELING AND CONTROL OBJECTVES

In this paper, the platoon problem can be considered as a synchronization problem of a networked dynamical systems with a pinner node [5,11] where a set of agents (i.e. nodes) interact among themselves through network and these nodes are controlled in such a way that the dynamics of all nodes converge towards the pinner node i.e. leader of a platoon. The main components of a network of dynamical systems are (i) individual node dynamics which describes the evaluation of each node when not coupled with the network; (ii) different network topology which describe the interaction among nodes; (iii) control action to each node for steering the node dynamics to the pinner's trajectory. This section describes the modelling of the platoon network topology and longitudinal dynamics of the vehicles in a platoon which represent the node. The control objectives of platoon are also discussed in this section. Since this paper deals with a synchronization problem [6] under communication topology, the follower vehicles may be connected with a pinner node i.e. leader and it is further assumed that all the followers may receive information from the leader, but the followers will not send any information to the leader.

A. *Modelling of the Platoon Network Topology and Packet drop*

The communication topology of the N followers (nodes) can be represented by the graph $G_N = (V_N, E_N)$, where $V_N = \{1,2,...,N\}$ defines a set of vertices or nodes and $E \subseteq V_N \times V_N$ defines a set of arcs or edges. The pair $(j,i) \in E_N$ indicates the i^{th} vehicle receives the information from j^{th} vehicle. An adjacency matrix $A_N = [a_{ij}] \in \mathbb{R}^{N\times N}$ can be defined based on the edges E_N. The generic entry a_{ij} of the adjacency matrix defines $a_{ij} = 1$ when $(j,i) \in E_N$ and $a_{ij} = 0$ otherwise i.e. $a_{ij} = 1$ implies when the i^{th} vehicle receives the information from j^{th} vehicle while $a_{ii} = 0$ represents no self-loop in the network. The degree D_N of the graph is represented by the number of edges pointing to the follower. For such case the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ which can be represented by $L = D_N - A_N$ where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}\ \forall i \neq j$. If the network topology considers the leader (i.e. leader of the platoon or pinner of the network) then the graph G_N is augmented with node 0 and the modified graph is $G_{N+1} = (V_{N+1}, E_{N+1})$ where $V_{N+1} = \{0,1,...,N\}$ and $E_{N+1} \subseteq V_{N+1} \times V_{N+1}$, also representing the pinner of the network. The corresponding adjacency matrix is $A_{N+1} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ with $a_{0j} = 0,\ j = 0,1,...,N$ indicates that the followers are not sending information to the leader, $a_{i0} = 1,\ i = 1,2,...,N$ indicates that the followers are receiving information from the leader and finally $a_{i0} = 0$ indicates otherwise. N_i denotes the neighbour set in G_{N+1} for the node $i,\ \ i = 1,2,...,\mathrm{N}$ i.e. $N_i = \{j \in \{0,1,...,N\} : a_{ij} = 1\}$.

Let us consider the stochastic variable $\theta_{ji}(k) \in \{0,1\}$ is satisfying Bernoulli distribution of packet drop in the communication link among the vehicles. $\theta_{ji}(k) = 0$ represents the follower j will receive the information or packet from i^{th} follower and $\theta_{ji}(k) = 1$ represents packet or information is lost. Under such scenario the following holds:

$$Prob(\theta_{ij}(k) = 1) = E(\theta_{ij}(k)) = r \tag{1}$$

and

$$E\left(1-\theta_{ij}\left(k\right)\right)=1-r\ ,\tag{2}$$

where r is the packet drop rate in the communication link among the vehicles.

B. Modelling of the Longitudinal Vehicle Dynamics

In a vehicle platoon, the nonlinear longitudinal dynamics of the vehicles can be represented by a third order linearized differential equation due to its satisfactory trade-off between accuracy and simplicity [5, 10]. The third order model of i^{th} vehicle with disturbance input in homogeneous platoon considering the states $\mathbf{x}_i^T\left(t\right)=\left[s_i \quad v_i \quad a_i\right]$ is represented in state space form as:

$$\begin{aligned}\dot{\mathbf{x}}_i\left(t\right)&=\mathbf{A}\mathbf{x}_i\left(t\right)+\mathbf{B}u_i\left(t\right)+\mathbf{B}w_i\left(t\right)\\ y_i\left(t\right)&=\mathbf{C}\mathbf{x}_i\left(t\right)\end{aligned}\tag{3}$$

where $u_i\left(t\right)\in\mathbb{R}^m$, $w_i\left(t\right)\in\mathbb{R}^m\in L_2\left[0,\infty\right)$ and $y_i\left(t\right)\in\mathbb{R}^q$ represents system input, exogenous disturbance input and output matrices respectively with appropriate dimension and

$$\mathbf{A}=\begin{bmatrix}0&1&0\\0&0&1\\0&0&-1/\tau\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}0\\0\\1/\tau\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1&0&0\end{bmatrix}.\tag{4}$$

In (4), s_i, v_i, a_i, τ represents the position, velocity, acceleration and time lag respectively and $i=1,2,...,N$ represents the i^{th} follower vehicles.

The continuous time system (3) can be discretized using specified sampling time T_s and the corresponding equivalent zero order hold (ZOH) discretized system is

$$\begin{aligned}\mathbf{x}_i\left(k+1\right)&=\mathbf{A}_d\mathbf{x}_i\left(k\right)+\mathbf{B}_d u_i\left(k\right)+\mathbf{B}_d w_i\left(k\right)\\ y_i\left(k\right)&=\mathbf{C}\mathbf{x}_i\left(k\right)\end{aligned}\ ,\tag{5}$$

where $\mathbf{A}_d=e^{\mathbf{A}T_s}$ and $\mathbf{B}_d=\int_0^{T_s}e^{\mathbf{A}\tau}\mathbf{B}d\tau$.

In discrete time, the dynamics of leader vehicle (i = 0) with constant velocity can be represented as:

$$\begin{aligned}\mathbf{x}_0\left(k+1\right)&=\mathbf{A}_d\mathbf{x}_0\left(k\right)\\ y_0\left(k\right)&=\mathbf{C}\mathbf{x}_0\left(k\right)\end{aligned}\ .\tag{6}$$

C. Platoon Control Objectives

The control objective of the platoon is categorized in two ways [6]: (i) impose leader velocity to all followers; (ii) maintain specified inter-vehicular distance between two consecutive vehicles in a platoon. These control objectives can be described by the following definition:

Definition 1 [20]*:* platoon of vehicles represented by leader-following system is said to be consensus if it is satisfying the following under any initial conditions:

$$\lim_{k\to\infty}\left\|\mathbf{x}_i\left(k\right)-\mathbf{x}_0\left(k\right)\right\|=\lim_{k\to\infty}\left\|\mathbf{e}_i\left(k\right)\right\|=0\quad \forall i=1,2,...,N\ ,\tag{7}$$

where $\mathbf{e}_i\left(k\right)$ represents the tracking error.

Now, to achieve the above-mentioned control objective of a platoon, the controller is assumed to be static and identical for all the followers and then the corresponding distributed control law can be represented as:

$$u_i\left(k\right)=\mathbf{K}\sum_{\substack{j=0\\j\neq i}}^{N}a_{ij}\left[\mathbf{x}_i\left(k\right)-\mathbf{x}_j\left(k\right)\right],\quad i=1,2,...,N\tag{8}$$

where, $\mathbf{K}=\left[-K_s \quad -K_v \quad -K_a\right]$ represents the identical controller gain, $a_{ij}\in\left\{0,1\right\}$ is the $\left(i,j\right)$ entry of adjacency matrix $A_{N+1}=\left[a_{ij}\right]\in\mathbb{R}^{(N+1)\times(N+1)}$.

III. Stability Analysis and Controller Design under Packet Drop

This section describes the packet drop modelling in the communication link of a vehicle platoon and then LMI based approach is used to analyse the stability and design distributed identical controller with bounded H_∞ norm for such systems under the packet drop scenario.

A. Modelling of the Packet drop for Vehicle Platoon

Network packet drop modelled as a Bernoulli process can be used for the transmitted state vector $\mathbf{x}_j\left(k\right)$ as:

$$\bar{\mathbf{x}}_j\left(k\right)=\left(1-\theta_{ji}\left(k\right)\right)\mathbf{x}_j\left(k\right)+\theta_{ji}\left(k\right)\mathbf{x}_j\left(k-1\right)\tag{9}$$

Correspondingly,

$$\bar{\mathbf{x}}_i\left(k\right)=\left(1-\theta_{ij}\left(k\right)\right)\mathbf{x}_i\left(k\right)+\theta_{ij}\left(k\right)\mathbf{x}_i\left(k-1\right)\tag{10}$$

since follower i knows whether it has received the information in the form of packet from j^{th} follower or not at kT_s instant. Here, we assume $\theta_{ji}\left(k\right)=\theta_{ij}\left(k\right)$ which defines that a specific link failure affects the packet drop in the both direction and all the link failure is asynchronous with same failure rate.

B. Stability Analysis and Controller Design

This sub-section describes the stability analysis and identical controller design with bounded H_∞ norm for vehicles of platoon under the packet drop scenario. Considering the packet drop in the transmitted state vectors among the vehicles in a platoon and using (9), (10) and $\theta_{ji}\left(k\right)=\theta_{ij}\left(k\right)$ in (8), the distributed control law yields:

$$u_i\left(k\right)=\mathbf{K}\sum_{\substack{j=0\\j\neq i}}^{N}a_{ij}\begin{bmatrix}\left(1-\theta_{ij}\left(k\right)\right)\mathbf{x}_i\left(k\right)+\theta_{ij}\left(k\right)\mathbf{x}_i\left(k-1\right)\\-\left(1-\theta_{ij}\left(k\right)\right)\mathbf{x}_j\left(k\right)-\theta_{ij}\left(k\right)\mathbf{x}_j\left(k-1\right)\end{bmatrix}\tag{11}$$

$i=1,2,...,N.$

Aim of this paper is to design the distributed control law (8) for vehicle platoon systems under random packet drop by ensuring the mean square stability and bounded H_∞ norm γ to reach the consensus of the system (17).

Definition 2 [11]*:* The system (17) is said to be mean square stable (MSS) if $\lim_{k\to\infty}E\left(\left\|\mathbf{e}\left(k\right)\right\|^2\right)=0$ for any initial state $\mathbf{e}\left(0\right)\in\mathbb{R}^n$ [17].

Definition 3 [21]: The output error dynamics $\left(\mathbf{y}_e\right)$ are said to have a bounded H_∞ norm $\gamma>0$, when the closed-loop system is MSS and

$$\sum_{k=0}^{\infty}E\left(\left\|\mathbf{y}_e\left(k\right)\right\|^2\right)\leq\gamma^2\sum_{k=0}^{\infty}E\left(\left\|\mathbf{w}\left(k\right)\right\|^2\right)\ \forall\mathbf{w}\left(k\right)\neq 0,\text{and }\mathbf{e}\left(0\right)=0\tag{12}$$

Now, this paper addresses the distributed controller design methodology satisfying MSS with bounded H_∞ norm γ for the closed loop system (17) using LMI approach which is stated as Theorem 1, below.

Theorem 1 provides sufficient conditions for the control gain $\mathbf{K}$ in (8) to guarantee that the closed loop system is MSS with a given H_∞ norm bound γ.

Theorem 1: Given closed loop system (17) with packet loss modelled as in (1)-(2) with mean packet loss rate r and the control action , then closed-loop system is MSS with bounded H_∞ norm γ if there exists

$\mathbf{P}=\mathbf{P}^T=\mathbf{I}_N\otimes\mathbf{P}_0>\mathbf{0}, \mathbf{Q}=\mathbf{Q}^T=\mathbf{I}_N\otimes\mathbf{Q}_0>\mathbf{0}\in\mathbb{R}^{nN\times nN}$ such that

$$\begin{bmatrix} -\mathbf{P}^{-1} & * & * & * & * & * \\ \mathbf{0} & -\mathbf{P}^{-1}\mathbf{Q}\mathbf{P}^{-1} & * & * & * & * \\ \mathbf{0} & \mathbf{0} & -\gamma^2\otimes\mathbf{I}_N & * & * & * \\ \left((\mathbf{I}_N\otimes\mathbf{A}_d)\mathbf{P}^{-1}+\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{Y}\right) & \tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{Y} & \mathbf{I}_N\otimes\mathbf{B}_d & -\mathbf{P}^{-1} & * & * \\ \mathbf{P}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{Q}^{-1} & * \\ (\mathbf{I}_N\otimes\mathbf{C})\mathbf{P}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_N \end{bmatrix}<\mathbf{0} \tag{13}$$

and,

$$\begin{bmatrix} -\mathbf{M} & \mathbf{P}^{-1} \\ \mathbf{P}^{-1} & -\mathbf{Q}^{-1} \end{bmatrix}\le\mathbf{0} \tag{14}$$

where, $\mathbf{P}^{-1}=\overline{\mathbf{P}}, \mathbf{Q}^{-1}=\overline{\mathbf{Q}}$ and the control gain in (8) is selected as $\mathbf{K}=\mathbf{Y}\mathbf{P}_0=\mathbf{Y}\overline{\mathbf{P}}_0^{-1}\in\mathbb{R}^{1\times n}$.

IV. MAIN RESULTS AND PROOF

This section uses the error dynamic model of the vehicle platoon to establish the stability criteria and design the distributed identical controller based on Lyapunov theory. Hence, first we derive the model of closed loop error dynamics subsequently using Lyapunov-like function, *Theorem 1* presented in section III is proved for the platoon of vehicles.

A. Modelling of Error Dynamics

The followers (5) will be following the leader's dynamics (6) when the control law (11) is applied. Under this scenario, the error dynamics for the i^{th} follower can be represented as:

$$\begin{aligned}
&\mathbf{e}_i(k+1)=\mathbf{x}_i(k+1)-\mathbf{x}_0(k+1) \\
&=\mathbf{A}_d\mathbf{x}_i(k)+\mathbf{B}_d w_i(k)-\mathbf{A}_d\mathbf{x}_0(k) \\
&+\mathbf{B}_d\mathbf{K}\sum_{\substack{j=0\\j\neq i}}^{N}a_{ij}\begin{bmatrix}(1-\theta_{ij}(k))\mathbf{x}_i(k)+\theta_{ij}(k)\mathbf{x}_i(k-1) \\ -(1-\theta_{ij}(k))\mathbf{x}_j(k)-\theta_{ij}(k)\mathbf{x}_j(k-1)\end{bmatrix} \\
&=\mathbf{A}_d(\mathbf{x}_i(k)-\mathbf{x}_0(k))+\mathbf{B}_d w_i(k) \\
&+\mathbf{B}_d\mathbf{K}\sum_{\substack{j=0\\j\neq i}}^{N}a_{ij}\begin{bmatrix}(1-\theta_{ij}(k))(\mathbf{x}_i(k)-\mathbf{x}_0(k)-\mathbf{x}_j(k)+\mathbf{x}_0(k))- \\ \theta_{ij}(k)(\mathbf{x}_i(k-1)-\mathbf{x}_0(k-1)-\mathbf{x}_j(k-1)+\mathbf{x}_0(k-1))\end{bmatrix}
\end{aligned} \tag{15}$$

The error dynamics (15) for i^{th} the follower can be re-written using (7) as:

$$\begin{aligned}
\mathbf{e}_i(k+1)&=\mathbf{A}_d\mathbf{e}_i(k)+\mathbf{B}_d w_i(k) \\
&+\mathbf{B}_d\mathbf{K}\sum_{\substack{j=0\\j\neq i}}^{N}a_{ij}\begin{bmatrix}(1-\theta_{ij}(k))(\mathbf{e}_i(k)-\mathbf{e}_j(k)) \\ -\theta_{ij}(k)(\mathbf{e}_i(k-1)-\mathbf{e}_j(k-1))\end{bmatrix}
\end{aligned} \tag{16}$$

$$y_{e_i}(k)=y_i(k)-y_0(k)=\mathbf{C}(\mathbf{x}_i(k)-\mathbf{x}_0(k))=\mathbf{C}\mathbf{e}_i(k).$$

Therefore, the error dynamics of all the followers in closed loop can be represented with the augmented states $\mathbf{e}(k)=\left[\mathbf{e}_1^T(k),\mathbf{e}_2^T(k),...,\mathbf{e}_N^T(k)\right]^T$ and $\mathbf{w}(k)=\left[w_1^T(k),w_2^T(k),...,w_N^T(k)\right]^T$ as:

$$\begin{aligned}
\mathbf{e}(k+1)&=(\mathbf{I}_N\otimes\mathbf{A}_d)\mathbf{e}(k)+(\mathbf{L}_1\otimes\mathbf{B}_d\mathbf{K})\mathbf{e}(k) \\
&+(\mathbf{L}_2\otimes\mathbf{B}_d\mathbf{K})\mathbf{e}(k-1)+(\mathbf{I}_N\otimes\mathbf{B}_d)\mathbf{w}(k) \\
\mathbf{y}_e(k)&=(\mathbf{I}_N\otimes\mathbf{C})\mathbf{e}(k)
\end{aligned} \tag{17}$$

where,

$$\mathbf{L}_1=\begin{bmatrix}\sum_{j\in N_i}a_{1j}(1-\theta_{1j}(k)) & -a_{12}(1-\theta_{12}(k)) & ... & -a_{1N}(1-\theta_{1N}(k)) \\ -a_{21}(1-\theta_{21}(k)) & \sum_{j\in N_i}a_{2j}(1-\theta_{2j}(k)) & ... & -a_{2N}(1-\theta_{2N}(k)) \\ \vdots & \ddots & \ddots & \vdots \\ -a_{N1}(1-\theta_{N1}(k)) & -a_{N2}(1-\theta_{N2}(k)) & ... & \sum_{j\in N_i}a_{Nj}(1-\theta_{Nj}(k))\end{bmatrix}, \tag{18}$$

$$\mathbf{L}_2=\begin{bmatrix}\sum_{j\in N_i}a_{1j}\theta_{1j}(k) & -a_{12}\theta_{12}(k) & ... & -a_{1N}\theta_{1N}(k) \\ -a_{21}\theta_{21}(k) & \sum_{j\in N_i}a_{2j}\theta_{2j}(k) & ... & -a_{2N}\theta_{2N}(k) \\ \vdots & \ddots & \ddots & \vdots \\ -a_{N1}\theta_{N1}(k) & -a_{N2}\theta_{N2}(k) & ... & \sum_{j\in N_i}a_{Nj}\theta_{Nj}(k)\end{bmatrix}. \tag{19}$$

and

$$\tilde{\mathbf{L}}=\begin{bmatrix}\sum_{j\in N_i}a_{1j} & -a_{12} & ... & -a_{1N} \\ -a_{21} & \sum_{j\in N_i}a_{2j} & ... & -a_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ -a_{N1} & -a_{N2} & ... & \sum_{j\in N_i}a_{Nj}\end{bmatrix}. \tag{20}$$

Now, due to presence of random packet drop, expectation values of $(\mathbf{L}_1,\mathbf{L}_2)$ are, $E(\mathbf{L}_1)=\tilde{\mathbf{L}}(1-r)$ and $E(\mathbf{L}_2)=\tilde{\mathbf{L}}r$. And expectation value of (17) yields,

$$E\left(\mathbf{e}(k+1)\right)=\left(\mathbf{I}_N\otimes\mathbf{A}_d\right)E\left(\mathbf{e}(k)\right)+\left(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K}\right)E\left(\mathbf{e}(k)\right)$$
$$+\left(\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K}\right)E\left(\mathbf{e}(k-1)\right)+\mathbf{B}_dE\left(\mathbf{w}(k)\right). \tag{21}$$

Now, using the system (21) the proof of *Theorem 1* is described in the following.

B. *Proof of Theorem 1*

Defining the Lyapunov function candidate as:

$$V(k)=\mathbf{e}^T(k)\mathbf{P}\mathbf{e}(k)+\mathbf{e}^T(k-1)\mathbf{Q}\mathbf{e}(k-1) \tag{22}$$

for the inequality,

$$V(k+1)-V(k)+\mathbf{y}_e^T(k)\mathbf{y}_e(k)-\gamma^2\mathbf{w}^T(k)\mathbf{w}(k)<\mathbf{0}. \tag{23}$$

obtained from bounded real lemma (BRL) [21] which satisfies bounded H_∞ norm γ.

However, since, $E(\mathbf{L}_1)=\tilde{\mathbf{L}}(1-r)$ and $E(\mathbf{L}_2)=\tilde{\mathbf{L}}r$, therefore, taking expectation value of (23), the following holds [21]:

$$E\left(\Delta V(k)\right)+E\left(\mathbf{y}_e^T(k)\mathbf{y}_e(k)-\gamma^2\mathbf{w}^T(k)\mathbf{w}(k)\right)<\mathbf{0}. \tag{24}$$

From (24) as in [11],

$$\begin{aligned}E\left(\Delta V(k)\right)&=E\left(V(k+1)-V(k)\right)\\&=\mathbf{e}^T(k+1)\mathbf{P}\mathbf{e}(k+1)+\mathbf{e}^T(k)\mathbf{Q}\mathbf{e}(k)\\&\quad-\mathbf{e}^T(k)\mathbf{P}\mathbf{e}(k)+\mathbf{e}^T(k-1)\mathbf{Q}\mathbf{e}(k-1)\end{aligned}. \tag{25}$$

Using(17) in (25) yields,

$$\begin{bmatrix}\mathbf{e}^T(k) & \mathbf{e}^T(k-1) & \mathbf{w}^T(k)\end{bmatrix}\times$$
$$\begin{bmatrix}\begin{pmatrix}\left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)^T\mathbf{P}\\ \times\left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)\\ +\mathbf{Q}-\mathbf{P}\end{pmatrix} & \begin{pmatrix}(\mathbf{I}_N\otimes\mathbf{A}_d)+\\ (\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\end{pmatrix}^T \times\mathbf{P}\left(\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K}\right) & \left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)^T\mathbf{P}(\mathbf{I}_N\otimes\mathbf{B}_d)\\ * & \begin{pmatrix}(\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})^T\mathbf{P}\times\\ (\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})-\mathbf{Q}\end{pmatrix} & (\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})^T\mathbf{P}(\mathbf{I}_N\otimes\mathbf{B}_d)\\ * & * & (\mathbf{I}_N\otimes\mathbf{B}_d)^T\mathbf{P}(\mathbf{I}_N\otimes\mathbf{B}_d)\end{bmatrix}\begin{bmatrix}\mathbf{e}(k)\\ \mathbf{e}(k-1)\\ \mathbf{w}(k)\end{bmatrix}<\mathbf{0} \tag{26}$$

Again, as in [21],

$$\begin{aligned}&E\left(\mathbf{y}_e^T(k)\mathbf{y}_e(k)-\gamma^2\mathbf{w}^T(k)\mathbf{w}(k)\right)\\&=\begin{bmatrix}\mathbf{e}^T(k) & \mathbf{e}^T(k-1) & \mathbf{w}^T(k)\end{bmatrix}\times\\&\begin{bmatrix}(\mathbf{I}_N\otimes\mathbf{C})^T(\mathbf{I}_N\otimes\mathbf{C}) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & -\gamma^2\otimes\mathbf{I}_N\end{bmatrix}\begin{bmatrix}\mathbf{e}(k)\\ \mathbf{e}(k-1)\\ \mathbf{w}(k)\end{bmatrix}\end{aligned} \tag{27}$$

By adding (26) and (27) and for non-zero $\mathbf{z}^T=\begin{bmatrix}\mathbf{e}^T(k) & \mathbf{e}^T(k-1) & \mathbf{w}^T(k)\end{bmatrix}$ yields:

$$\begin{bmatrix}\begin{pmatrix}\left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)^T\mathbf{P}\\ \times\left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)\\ +\mathbf{Q}-\mathbf{P}+(\mathbf{I}_N\otimes\mathbf{C})^T(\mathbf{I}_N\otimes\mathbf{C})\end{pmatrix} & \begin{pmatrix}(\mathbf{I}_N\otimes\mathbf{A}_d)+\\ (\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\end{pmatrix}^T \times\mathbf{P}\left(\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K}\right) & \left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)^T\mathbf{P}(\mathbf{I}_N\otimes\mathbf{B}_d)\\ * & \begin{pmatrix}(\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})^T\mathbf{P}\times\\ (\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})-\mathbf{Q}\end{pmatrix} & (\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})^T\mathbf{P}(\mathbf{I}_N\otimes\mathbf{B}_d)\\ * & * & (\mathbf{I}_N\otimes\mathbf{B}_d)^T\mathbf{P}(\mathbf{I}_N\otimes\mathbf{B}_d)-\gamma^2\otimes\mathbf{I}_N\end{bmatrix}<\mathbf{0}. \tag{28}$$

Now (28) can be re-written as:

$$\begin{bmatrix}\mathbf{Q}-\mathbf{P} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{Q} & 0\\ \mathbf{0} & \mathbf{0} & -\gamma^2\otimes\mathbf{I}_N\end{bmatrix}+\begin{bmatrix}(\mathbf{I}_N\otimes\mathbf{C})^T\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\begin{bmatrix}(\mathbf{I}_N\otimes\mathbf{C}) & \mathbf{0} & \mathbf{0}\end{bmatrix}$$
$$+\begin{bmatrix}\left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)^T\\ (\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})^T\\ (\mathbf{I}_N\otimes\mathbf{B}_d)^T\end{bmatrix}\mathbf{P}\times\begin{bmatrix}\left((\mathbf{I}_N\otimes\mathbf{A}_d)+(\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K})\right)^T\\ (\tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K})^T\\ (\mathbf{I}_N\otimes\mathbf{B}_d)^T\end{bmatrix}^T<\mathbf{0}. \tag{29}$$

Using Schur complement [22] of (29) yields,

$$\begin{bmatrix}\mathbf{Q}-\mathbf{P} & * & * & * & *\\ \mathbf{0} & -\mathbf{Q} & * & * & *\\ \mathbf{0} & \mathbf{0} & -\gamma^2\otimes\mathbf{I}_N & * & *\\ \left((\mathbf{I}_N\otimes\mathbf{A}_d)+\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{K}\right) & \tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{K} & \mathbf{I}_N\otimes\mathbf{B}_d & -\mathbf{P}^{-1} & *\\ \mathbf{I}_N\otimes\mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_N\end{bmatrix}<\mathbf{0} \tag{30}$$

Now multiplying by $diag\left\{\mathbf{P}^{-1},\mathbf{P}^{-1},\mathbf{I}_N,\mathbf{I}_N,\mathbf{I}_N\right\}$ in both side of (30) and defining $\mathbf{P}^{-1}=\mathbf{I}_N\otimes\mathbf{P}_0^{-1}$ and $\mathbf{K}=\mathbf{Y}\mathbf{P}_0$ yields:

$$\begin{bmatrix}\mathbf{P}^{-1}\mathbf{Q}\mathbf{P}^{-1}-\mathbf{P}^{-1} & * & *\\ \mathbf{0} & -\mathbf{P}^{-1}\mathbf{Q}\mathbf{P}^{-1} & *\\ \mathbf{0} & \mathbf{0} & -\gamma^2\otimes\mathbf{I}_N\\ \left((\mathbf{I}_N\otimes\mathbf{A}_d)\mathbf{P}^{-1}+\tilde{\mathbf{L}}(1-r)\otimes\mathbf{B}_d\mathbf{Y}\right) & \tilde{\mathbf{L}}r\otimes\mathbf{B}_d\mathbf{Y} & \mathbf{I}_N\otimes\mathbf{B}_d\\ (\mathbf{I}_N\otimes\mathbf{C})\mathbf{P}^{-1} & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$
\left.\begin{matrix} * & * \\ * & * \\ * & * \\ -\mathbf{P}^{-1} & * \\ \mathbf{0} & -\mathbf{I}_N \end{matrix}\right] < \mathbf{0}. \tag{31}
$$

It is seen that (31) is a nonconvex LMI due to presence of constraints $\left(\mathbf{P}^{-1}, \mathbf{Q}^{-1}\right)$. This nonconvexity problem of (31) can be overcome by assuming

$$
\mathbf{P}^{-1}\mathbf{Q}\mathbf{P}^{-1} = \mathbf{M} \text{ and } \mathbf{P}^{-1} = \bar{\mathbf{P}}, \tag{32}
$$

where $\mathbf{Q} = \mathbf{I}_N \otimes \mathbf{Q}_0$.

Thus, using (32), (31) takes the LMI form as:

$$
\begin{bmatrix} \mathbf{M}-\bar{\mathbf{P}} & * & * & * & * \\ \mathbf{0} & -\mathbf{M} & * & * & * \\ \mathbf{0} & \mathbf{0} & -\gamma^2 \otimes \mathbf{I}_N & * & * \\ \left((\mathbf{I}_N \otimes \mathbf{A})\bar{\mathbf{P}} + \tilde{\mathbf{L}}(1-r)\otimes \mathbf{B}_d \mathbf{Y}\right) & \tilde{\mathbf{L}}r \otimes \mathbf{B}_d \mathbf{Y} & \mathbf{I}_N \otimes \mathbf{B}_d & -\bar{\mathbf{P}} & * \\ (\mathbf{I}_N \otimes \mathbf{C})\mathbf{P}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_N \end{bmatrix} < \mathbf{0} \tag{33}
$$

and (32) yields the LMI form

$$
\begin{bmatrix} -\mathbf{M} & \bar{\mathbf{P}} \\ \bar{\mathbf{P}} & -\bar{\mathbf{Q}} \end{bmatrix} \le \mathbf{0} \tag{34}
$$

where $\bar{\mathbf{Q}} = \mathbf{Q}^{-1}$.

Now, (33) can be re-written as:

$$
\begin{bmatrix} -\bar{\mathbf{P}} & * & * & * \\ \mathbf{0} & -\mathbf{M} & * & * \\ \mathbf{0} & \mathbf{0} & -\gamma^2 \otimes \mathbf{I}_N & * \\ \left((\mathbf{I}_N \otimes \mathbf{A})\bar{\mathbf{P}} + \tilde{\mathbf{L}}(1-r)\otimes \mathbf{B}_d \mathbf{Y}\right) & \tilde{\mathbf{L}}r \otimes \mathbf{B}_d \mathbf{Y} & \mathbf{I}_N \otimes \mathbf{B}_d & -\bar{\mathbf{P}} \end{bmatrix}
+ \begin{bmatrix} \bar{\mathbf{P}}^T \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \mathbf{Q} \begin{bmatrix} \bar{\mathbf{P}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} +
\begin{bmatrix} \bar{\mathbf{P}}^T (\mathbf{I}_N \otimes \mathbf{C})^T \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} (\mathbf{I}_N \otimes \mathbf{C})\bar{\mathbf{P}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} < \mathbf{0}. \tag{35}
$$

Using Schur complement of (35), (32) and $\bar{\mathbf{P}} = \mathbf{P}^{-1}, \bar{\mathbf{Q}} = \mathbf{Q}^{-1}$, the LMI can be obtained the form (13). □

V. NUMERICAL RESULTS

This section represents the results to demonstrate the effectiveness of the proposed methodology for achieving cooperative motion of platoon considering seven different network topologies i.e. PF, PLF, BPF [4], BPLF, TPF [5], TBPF [4] and all-to-all [6] with different number of homogeneous follower vehicles in discrete time domain with specified sampling time $T_s = 0.1s$. A constant spacing policy of 25m [4] is used among the vehicles in a platoon. Again, for all the homogeneous followers it is assumed that there is no collision at initial time t = 0s and the initial states are random in nature. The time lag is chosen as $\tau = 0.4s$ [4], [6]. The Leader's speed is considered constant at 72 km/hr [23]. In this paper, the external disturbance input to the acceleration of all followers has been considered as a square wave of amplitude 1, starting at $t = 120s$ and thereafter continued for the period of $20s$ as shown in Figure 1, with limited energy which means $\left\|w_i(k)\right\|_{L_2}$ is bounded. Under such scenario, the γ measures the robustness of a platoon which is defined by $\gamma \ge \sup\left(\left\|\mathbf{y}_e(k)\right\|_{L_2} / \left\|\mathbf{w}(k)\right\|_{L_2}\right)$. Therefore, γ describes the sensitivity or attenuation effect of the energy of external disturbances.

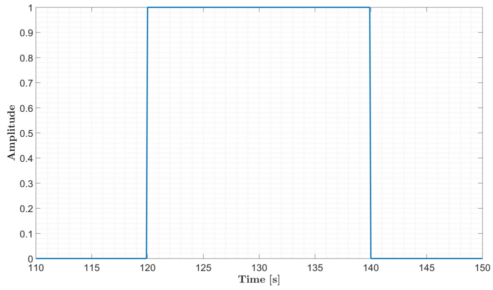
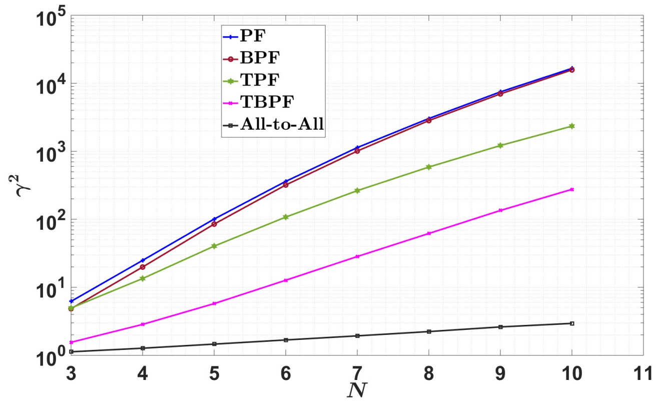
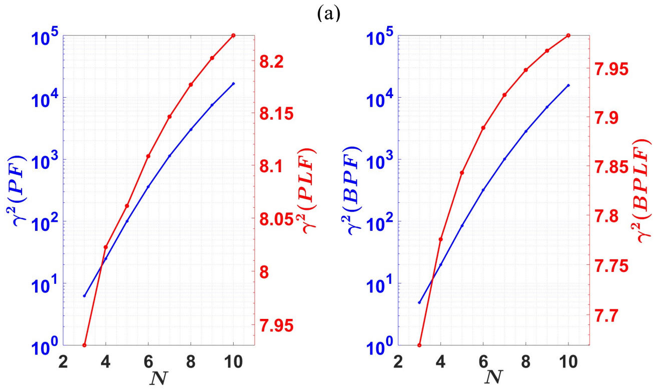


Figure 1: Disturbance input

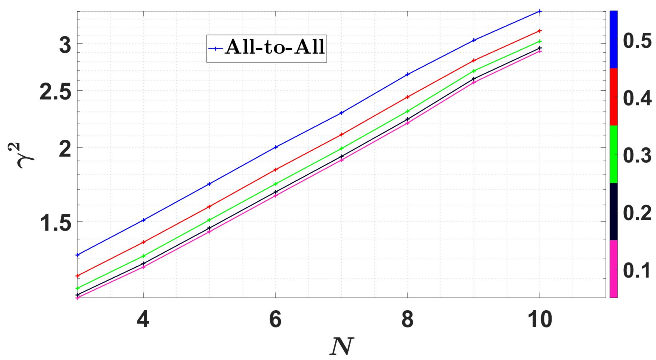
Now, the variation of γ^2 has been depicted in Figure 2 to demonstrate the effect of network topologies, platoon members and packet drop in platoon by solving the designed LMI (13). The LMI has been solved using YALMIP toolbox [24] and SeDuMi solver [25] considering seven different network topologies [5, 9], different number of platoon members (N = 3 to 10) and under varying packet drop rate (r = 0.1 to 0.5 representing 10% to 50 %) (colour bar). It can be observed form the Figure 2 (a) and (b) that under 20% packet drop, the value of γ^2 is increasing when the number of platoon members (N) are higher and this is same for all the seven different network topologies i.e. PF, PLF,BPF, BPLF, TPF, TBPF, all-to all. In Figure 2 (a) and (b), the value of γ^2 is higher when the amount of information exchange between platoon members is less. Also, it is evident that the magnitude of γ^2 is more for the topologies where the leader is connected only with the first follower in a platoon compared to all-to-all network topology under 20% packet drop. Next, Figure 2 (b) compares the variation of γ^2 for N = 3 to 10 under 20% packet drop scenario for four different network topologies i.e. PF with PLF (left) and BPF with BPLF (right). It is seen form the Figure 2 (b) the value of γ^2 is increasing with the platoon number (N) and this value is higher when leader is connected only with first follower i.e. PF and BPF compared to all leader following i.e. PLF and BPLF under 20% packet drop as expected. In Figure 2 (c), all-to-all network topology has been considered to depict the variation of γ^2 for different platoon (N) i.e. 3 to 10 and different packet drop rate (r) i.e. 10% to 50% (colour bar). It is seen from the Figure 2 (c) that the value of γ^2 is becoming larger when number of vehicles in a platoon and rate of packet drop is increasing. It can be noted that with the variation of the rate of packet drop, the value of γ^2 is not varying a large amount because the proposed algorithm is designed and validated under the consideration of single packet drop.



(a)



(b)



(c)

Figure 2: Variation of γ^2 for different network topologies with different number of follower vehicles under different packet drop rate (a) effect of network topology under 20% packet drop (b) effect of leader information under 20% packet drop (c) effect of packet drop (i.e. 10% to 50%).

Next, Figure 3-5 shows the effectiveness of the designed controller to achieve control objectives (7) and that has been demonstrated in terms of time responses considering 10 followers in a platoon for two different network topologies i.e. BPF and BPLF under 20% packet drop [7]. By solving the LMI (13), the controller gains $\mathbf{K}$ = [-0.5528, -6.5034, -2.5130] and $\mathbf{K}$ = [-3.0506, -3.9947, -1.5223] are obtained for BPF and BPLF topology, respectively. It is assumed that the follower states are mismatched at time instant $t = 0s$ e.g., up-to 2m inter vehicular distance error $\left(e_{i-1,i}\right)$ among the consecutive platoon members and hence the variation in velocities (v_i) of followers. It is seen from Figure 3-4 that all vehicle dynamics are converging to desired behaviours i.e. zero inter-vehicular distance error, matching to leader's velocity, zero accelerations (a_i) when the controllers are activated. It is also seen that the converging time i.e. 40s of the follower states in the BPF topology is higher than the converging time (3.5s) of states in the BPLF topology under 20% packet drop since leader vehicle is connected to all the followers in BPLF topology as in [5]. Figure 4 depicts the behaviour of follower's states when the external disturbances shown in Figure 1 is applied to each follower under 20% packet drop. Under such scenario, the inter-vehicular distance error (maximum 15.3 m) and settling time is higher when BPF topology is used compared to BPLF topology where maximum inter-vehicular distance error is 0.35m. For BPLF topology the inter-vehicular distance error exist only for the first follower and other followers are maintaining almost zero error. This is because all the followers have zero initial error (at steady state) and they are also pinned to the leader, thereby maintaining same dynamic evaluation [5]. Again, the amplification of velocity is also higher for BPF topology compared to BPLF topology. It is also noted that each vehicle has maintained almost same velocity and acceleration when BPLF topology is considered.

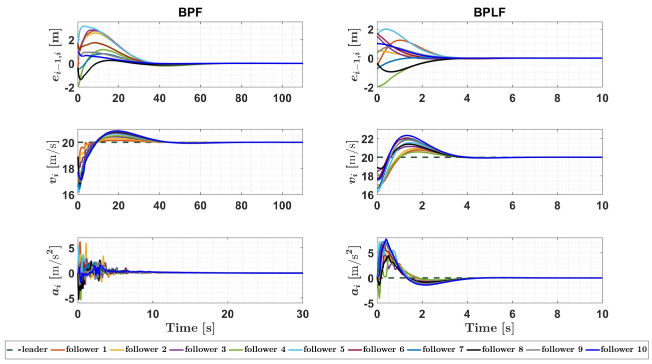


Figure 3: Steady state response for 10 follower vehicles in a platoon under 20% packet drop rate with BPF (left panel) and BPLF topology (right panel).

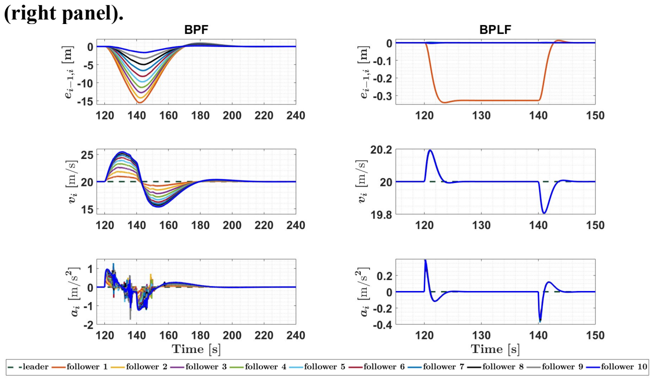


Figure 4: Time response of the follower's states under external disturbance and 20% packet drop for BPF (left panel) and BPLF topology (right panel).

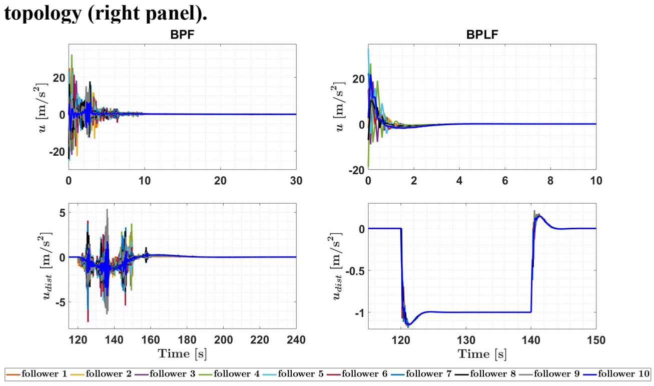


Figure 5: Control inputs to the followers at steady state (upper panel) and external disturbances (lower panel) under 20% packet drop.

From Figure 5, it is seen that all the control inputs are converging to zero for both topologies before and after acting of external disturbances to each follower. Also, highly oscillatory nature of control inputs for the BPF topology describes the requirement of high control effort compared to the BPLF topology.

VI. CONCLUSION

In this paper, an LMI based distributed state feedback controller satisfying bounded H_∞ norm has been designed for vehicle platoon under single packet drop. The variation of bounded H_∞ norm for different network topologies and number of platoon members under packet drop, has also been presented. The designed controller has been tested for 10 follower vehicles in a platoon considering two different network topologies under 20% packet drop, as well. The simulation results demonstrated the effectiveness of the proposed controller for maintaining desired performances for vehicle platooning under random packet drop and external disturbances. As a scope of future work, the proposed method can be extended under the scenario of multiple packet drop.

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