Game Theory to Study Interactions between Mobility Stakeholders

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Abstract—Increasing urbanization and exacerbation of sustainability goals threaten the operational efficiency of current transportation systems and confront cities with complex choices with huge impact on future generations. At the same time, the rise of private, profit-maximizing Mobility Service Providers leveraging public resources, such as ride-hailing companies, entangles current regulation schemes. This calls for tools to study such complex socio-technical problems. In this paper, we provide a game-theoretic framework to study interactions between stakeholders of the mobility ecosystem, modeling regulatory aspects such as taxes and public transport prices, as well as operational matters for Mobility Service Providers such as pricing strategy, fleet sizing, and vehicle design. Our framework is modular and can readily accommodate different types of Mobility Service Providers, actions of municipalities, and low-level models of customers' choices in the mobility system. Through both an analytical and a numerical case study for the city of Berlin, Germany, we showcase the ability of our framework to compute equilibria of the problem, to study fundamental tradeoffs, and to inform stakeholders and policy makers on the effects of interventions. Among others, we show tradeoffs between customers' satisfaction, environmental impact, and public revenue, as well as the impact of strategic decisions on these metrics.

I. INTRODUCTION

In past decades, cities worldwide have observed a dramatic urbanization. Today, 55 % of the world's population resides in urban areas, and by 2050 the proportion is expected to reach 68 % [1]. A direct consequence of the population density growth is the increase of urban travel, and of the externalities it produces [2]. In this rapidly expanding setting, cities have to take important decisions to adapt their transportation system to welcome larger travel demands. This is a very complex task for at least three reasons. First, cities need to accommodate the changing travel needs of the population, by predicting them [3], and by ensuring fairness and equity [4]. Second, designed policies not only have to account for the citizens' satisfaction, but also for their impact on private Mobility Service Providers (MSPs) such as ride-hailing companies, micromobility (μ M), and, in a near future, Autonomous Mobility-on-Demand (AMoD) systems [5]. Indeed, such services gained a considerable share of the transportation market in recent years; e.g., in NYC, ride-hailing companies have increased their daily trips by 1,000% from 2012 to 2019 [6]. While offering more choices to travellers, these systems operate benefiting from public resources (such as roads and public spaces), are profit-oriented, and often lead to potentially disruptive consequences for the efficiency of the transportation system and for society at large [7]–[9]. In this avenue, cities gain an important, onerous regulatory role. Third, policies have to be

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designed while meeting global sustainability goals. It is not surprising that cities are estimated to be responsible for 78 \% of the world's energy consumption and for over $60\,\%$ of the global greenhouse emissions (30 % of which is produced by transportation, in US) [10]. Indeed, sustainability is central in policy-making worldwide: NYC plans to increase sustainable trips from 68% to 80% [6], and EU plans a 90% reduction of emissions by 2050 [11].

Taken together, the aforementioned perspectives highlight the complexity of this socio-technical problem, and imperatively call for methods to inform and drive policy makers.

The goal of this paper is to lay the foundations for a framework through which one can model sequential, competitive interactions between stakeholders of the mobility ecosystem, and characterize their equilibria. Specifically, we leverage game theory to frame the problem in a modular fashion, and provide both an analytical and a numerical case study to showcase our methodology.

A. Related Literature

Our work lies at the interface of applications of gametheory in transportation science, and policy making related to future mobility systems. Game theory has been leveraged to solve various mobility-related problems. Main application areas include optimization of pricing strategies for MSPs [12]-[17], analysis of interactions between MSP and users [18]– [20], interactions between authorities and MSPs [21]–[28], and tolls and incentives to regulate congested networks [29]– [36]. While [12], [13] use game theory to determine prices for public transport, [14], [15] focus on subsidies and management of shared fleets of electric vehicles, and [16], [17] focus on pricing strategies at the network level. The competition among MSPs is studied in [18] through a realtime gaming framework, in [19] focusing on AMoD systems, and via evolutionary game theory with a focus on ride-sourcing in [20]. When studying interactions between authorities and MSPs, shared mobility systems received much attention. In particular, [21] proposes a unified gametheoretic framework for policy making related to shared mobility systems, [22] focuses on carpooling systems and evolutionary stable policies, [23] analyzes the dynamic interactions between shared autonomous vehicles (AVs) and public transit, and [24] proposes a modeling framework for competing carsharing operators in Zurich, Switzerland. Furthermore, game-theoretic frameworks to study interactions of MSPs and public transportation systems are proposed in [25], [26] (for AVs), in [27] (for ride-hailing companies), and in [28] (for bike-sharing systems). General game-theoretic approaches for the regulation of congestion via policies and incentives have been studied in [29]-[31], and via clever routing and price-making in [32], [33]. Moreover, [34] focuses on game-theoretic urban planning to reduce externalities of transportation systems, [35] leverages mobility patterns to propose travel incentives, and [36] studies tolling policies for the transportation of hazardous materials.

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The first two authors contributed equally to this work. This work was supported by the Swiss National Science Foundation under NCCR Automation, grant agreement 51NF40_180545.

There has been research on policy making for future mobility systems not involving game theory [37]–[44]. Strategies to reduce externalities (including tolls, subsidies, electrification) are proposed in [37]–[40], and socially efficient arguments are made in [41], [42]. Finally the economics of ride-hailing, AVs, and carpooling companies is studied in [43], [44].

Overall, all these works either focus on specific problems, neglect some mobility stakeholders, or ignore game-theoretic dynamics. So, to the best of our knowledge, there does not exist a comprehensive framework which allows one to formulate and solve mobility problems involving interactions between different stakeholders, at different time-scales, all the way from municipalities to customers, through MSPs.

B. Statement of Contribution

To fill this gap, we study the interactions between a central municipality, MSPs, and customers from a game-theoretic perspective. Our contribution is threefold. First, we provide a general game-theoretic framework to model the sequential and simultaneous interactions between a municipality and MSPs. Second, we instantiate our framework with two low-level models of the mobility system: a parallel-arc congestion game and a game-theoretic model including ride-hailing companies. Third, we present numerical results for the city of Berlin, and derive insights to inform stakeholders of the mobility ecosystem.

C. Organization

The remainder of this paper is as follows. We specify our problem setting and model in Section II. In Section III, we detail our case study and present numerical results. We draw conclusions and present an outlook on future research in Section IV. Proofs are relegated to the appendix.

II. MOBILITY INTERACTIONS AS SEQUENTIAL GAMES

As outlined in Section I, urban mobility systems feature a broad variety of complex interactions. We classify such interactions based on the time scale at which they occur. We identify four time scales: a day, a month, a year, and five years. While day-to-day interactions include specific operational conditions such as dynamic pricing and rebalancing policies for MSPs, monthly interactions cover changes in the regions served by a particular MSP. When looking at a horizon of one year, one can consider general price plans for public transport, taxation systems on ride-hailing companies, as well as logistic decisions for MSPs, such as fleet sizing and fleet diversification. Finally, a horizon of five years could include infrastructural changes, land use planning, and public contracts for transportation systems. In the following, we detail a model for the yearly horizon. This way we consider long-term horizon interventions (e.g., transportation network topology) as fixed parameters and include short-term horizon dynamics (e.g., mode selection, rebalancing, etc.) in what we call low-level model of the mobility system. Nonetheless, our methodology readily applies to the other settings as well.

A. Game-theoretic Model

We consider a one-stage sequential game between a municipality and $N \in \mathbb{N}$ MSPs, also called single-leader multifollower Stackelberg game. Herein, the municipality first decides on policies such as taxes, public transport prices, and number of released vehicles licenses, to maximize social

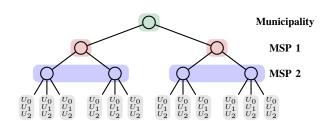


Fig. 1: Sequential interactions of the game in the case of a municipality with two available actions and two MSPs having two and three available actions, respectively. First, the municipality chooses its action. Then, MSPs simultaneously decide on their action. The payoff of all stakeholders follows accordingly. The boxes depict the so-called information sets: MSP 1 knows the action of the municipality, but does not know the action of MSP 2.

welfare. The profit-oriented MSPs then selfishly co-design their fleet and their pricing strategies (see Fig. 1). Finally, the outcome of the game results from a low-level model of the mobility system, which includes the dynamics happening on a short-term horizon.

Formally, the game is specified as follows. The municipality chooses its action u_0 from the (possibly uncountable) set of actions U_0 . Since the municipality plays first, actions and strategies coincide: $\gamma_0 = u_0$ means that the municipality plays action $u_0 \in \mathcal{U}_0$; so, the set of strategies Γ_0 and the set of actions \mathcal{U}_0 coincide. For instance, if the municipality only designs the (flat) price of public transport, then $\mathcal{U}_0 = \Gamma_0 \coloneqq \mathbb{R}_{\geq 0}$. MSP $j \in \{1,\dots,N\}$ selects their action u_j from the (possibly uncountable) set of actions $\mathcal{U}_j(\gamma_0)$, possibly dependent on the strategy of the municipality. Since MSPs play after the municipality, the strategy γ_j is a map $\gamma_j : \Gamma_0 \to \bigcup_{\gamma_0 \in \Gamma_0} \mathcal{U}_j(\gamma_0)$, where $\gamma_j(\gamma_0) \in \mathcal{U}_j(\gamma_0)$. We denote by Γ_j the set of all such maps. For instance, $\gamma_j(\gamma_0) \in \mathcal{U}_j$ is the action that MSP j plays if the municipality played $\gamma_0 \in \Gamma_0$. For simplicity, we neglect the influence of MSP j to the set of actions of MSP $k \neq j$. Yet, our framework can be readily extended to accommodate such interactions. Finally, the payoffs of all agents result from a low-level model of the mobility system comprising, among others, day-to-day behavior of the customers, and dynamic pricing of MSPs. While numerical results very much depend on this model, the latter can be easily replaced, making our framework modular. Formally, we associate to the municipality (j = 0) and to each MSP $j \in \{1, ..., N\}$ a payoff function

$$U_j: \Gamma_0 \times \Gamma_1 \times \ldots \times \Gamma_N \to \mathbb{R}$$
$$\langle \gamma_0, \gamma_1, \ldots, \gamma_N \rangle \mapsto U_j(\gamma_0, \gamma_1, \ldots, \gamma_N).$$

In our case studies, we will focus on U_j being the profit for MSPs and U_0 being the social welfare for the municipality. Nonetheless, our framework accommodates players with different interests (e.g., return on investment).

B. Equilibria

To characterize equilibria of our game, we use the classical notion of pure equilibrium in sequential games. Intuitively, a tuple of strategies is an equilibrium of the game if no agent is willing to unilaterally deviate from its strategy. Formally:

Definition 1 (Equilibrium). The tuple $\langle \gamma_0^*, \gamma_1^*, \ldots, \gamma_N^* \rangle \in \prod_{i \in \{0, \ldots, N\}} \Gamma_i$ is an *equilibrium* of the game if for all players $j \in \{0, \ldots, N\}$: $U_j(\gamma_j^*, \gamma_{-j}^*) \geq U_j(\gamma_j, \gamma_{-j}^*), \forall \gamma_j \in \Gamma_j$, where the subscript -j represents all players but j.

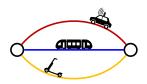


Fig. 2: Example of a network with a municipality offering public transport an AMoD system, and a μ M operator offering mobility service.

Definition 1 emphasizes why we distinguish between actions and strategies. In particular, Definition 1 would fail if expressed in terms of actions, as it would ignore the sequential nature of the game. Conversely, strategies, defined as maps from the "current information" to the set of actions, capture the sequential nature of the game. As well-known in game theory, equilibria need not exist: one may easily come up with examples of sequential games with no equilibrium. Nonetheless, we will see that, when one fixes a low-level model of the mobility system, it may be possible to study sufficient conditions for the existence of equilibria.

C. Discussion

First, we do not a priori fix the low-level model of the mobility system, allowing one to choose the instance which best suits a desired analytical setting. Examples of low-level models include congestion games [30], mobility simulators [45], and approaches based on reaction curves, ubiquitous in economics [46]. Second, we tacitly assumed a market with a fixed number of MSPs. This assumption is realistic for the yearly time scale of our game and can be relaxed for other time scales. Third, we assumed a sequential game with sequence as in Fig. 1. Arguably, one could think about MSPs acting first, making the municipality a reactive player. We believe that the proposed sequence well aligns with the yearly time horizon, forcing MSPs willing to enter the market to follow rules established by a municipality. Nevertheless, our formulation can accommodate permutations in the action sequences.

D. Specifying Low-level Models

To showcase our framework, we instantiate it with two different low-level models of the mobility system. First, we consider a simplistic mobility system resulting from decoupled congestion games on parallel-arc networks. This allows for a clear exposition of the analyzed dependencies, and a thorough analytical study of equilibria. Second, we consider a mobility system whereby a MSP applies dynamic pricing, and can therefore select its pricing strategy in real-time, and a MSP which strategically decides operational matter, such as fleet sizing and composition. Customers strategically react to minimize their overall travel cost, resulting from fares paid throughout the trip and monetary value of time.

1) Analytical Parallel-arc Congestion Game: We consider one demand (per unit time) between two non-identical nodes, connected by multiple parallel arcs (see Fig. 2). Each arc denotes an homogeneous mode of transportation, which (possibly combined with walking) leads customers from the origin to the destination node. The municipality chooses a non-negative price from the compact set $\mathcal{U}_0 \coloneqq [0, p_0^{\max}]$, for some $p_0^{\max} \in \mathbb{R}_{>0}$, while MSPs co-design the price of the ride p_j and their fleet size f_j , i.e., $\mathcal{U}_j \coloneqq \mathbb{R}^2_{>0}$. For simplicity, we assume that MSPs can choose arbitrarily large fleet sizes

and prices. To each MSP $j \in \{1, \dots, N\}$ (and therefore to each arc) we assign a non-decreasing smooth delay function

$$\ell_i(\gamma_i): \mathbb{R}_{>0} \to \mathbb{R}_{>0} \cup \{+\infty\},$$

which captures the total cost of using that arc to reach the destination. We use the notation $\ell_j(\gamma_j)$ to emphasize the dependency of the delay function of the strategy of provider j^1 . Since customers minimize their individual cost, given as the sum of fares and monetary value of time, we consider delays function of the form

$$\ell_0(\gamma_0)(z) = \ell_j(p_0)(z) = p_0 + V_{\mathrm{T}} \cdot \bar{\ell}_0,$$

$$\ell_j(\gamma_j)(z) = \ell_j((p_j, f_j))(z) = \begin{cases} p_j + V_{\mathrm{T}} \cdot \bar{\ell}_j(z) & \text{if } z \leq f_j \\ +\infty & \text{else}, \end{cases}$$

where $V_T \in \mathbb{R}_{>0}$ is the customers' (average) value of time, $\bar{\ell}_0 \in \mathbb{R}_{\geq 0}$ is the total time required to reach the destination with public transport, and $\bar{\ell}_j \colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0} \cup \{+\infty\}$ is a non-negative non-decreasing smooth function accounting for congestion. For instance, in the case of an MSP operating on the road network, one may construct $\bar{\ell}_j$ based on the BPR function [47]. In this setting, we model the outcome of the interactions among customers via the well-known notion of Wardrop equilibrium:

Definition 2 (Wardrop equilibrium). $x^* \in [0,1]^{N+1}$ with $\sum_{i=0}^N x_i = 1$ is a *Wardrop equilibrium* if for all $i,j \in \{0,\ldots,N\}$ one has $\ell_i(x_i) > \ell_j(x_j) \Rightarrow x_i = 0$.

In other words, this condition ensures that no agent has an incentive to travel through another arc: at equilibrium an arc can admit a larger delay than other arcs only if no one travels through it. Also, without loss of generality, we assume a total flow of 1. It is well-known that congestion games have a Wardrop equilibrium, which can be recovered from the solution of an optimization problem:

Proposition 1 (Equilibria of congestion games). The congestion game possesses a Wardrop equilibrium. Moreover, all equilibria coincide with the solutions of

$$\min_{x \in [0,1]^{N+1}} \sum_{j=0}^{N} \int_{0}^{x_{j}} \ell_{j}(z) dz$$

$$\sum_{j=0}^{N} x_{j} = 1, \quad x_{j} \leq f_{j} \quad \forall j \in \{1, \dots, N\}.$$
(1)

Finally, if ℓ_j for all $j \in \{1, ..., N\}$ is strictly increasing, then the Wardrop equilibrium is unique.

To formally talk about *the* equilibrium we make the following assumption:

Assumption 1. The functions $\bar{\ell}_j$ are convex and strictly increasing for all $j \in \{1, \dots, N\}$.

Assumption 1 encompasses relevant delay functions, such as the BPR function [47]. Strict monotonicity, via Proposition 1, ensures uniqueness of the equilibrium of the congestion game. Hence, we unambiguously denote the Wardrop equilibrium by $x^*(\gamma_0, \gamma_1, \ldots, \gamma_N)$. Armed with this result, we can establish existence of an equilibrium for our game. To

 $^{^{1}\}mbox{Formally},\,\ell_{j}$ is a function from the set of strategies to the set of nonnegative smooth functions.

do so, we first introduce utilities for all players. The sociallyaware municipality maximizes social welfare, defined in terms of cost for the customers (including fares and monetary value of time), cost of emissions, and public revenue:

$$U_0(\gamma_0, \gamma_{-0}) := -k_1 \cdot \sum_{j=0}^{N} \ell_j(x_j^*(\gamma_0, \gamma_{-0})) \cdot x_j^*(\gamma_0, \gamma_{-0})$$
$$-k_2 \cdot \sum_{j=1}^{N} \varepsilon_j \cdot x_j^*(\gamma_0, \gamma_{-0}) + k_3 \cdot \gamma_0 \cdot x_0^*(\gamma_0, \gamma_{-0}),$$

where k_1 , k_2 , and k_3 are strictly positive weights, ε_j is the marginal cost of emissions of MSP j, and γ_{-j} collects the strategies of all players but j. For ease of exposition, we consider profit-maximizing MSPs:

$$U_j(\gamma_j, \gamma_{-j}) := (p_j - c_j)x_j^*(\gamma_j, \gamma_{-j}) - \bar{c}_j f_j,$$

where c_i denotes the cost of serving a trip and \bar{c}_i is the cost of buying a vehicle. We write $\tilde{c}_j \coloneqq c_j + \bar{c}_j$. To show existence of an equilibrium of the sequential game (c.f. Definition 1), we rely on the following assumption:

Assumption 2. All $\bar{\ell}_i$ satisfy

$$\sum_{j=1}^{N} \bar{\ell}_{j}^{-1} \left(\frac{p_{0}^{\max}}{V_{T}} + \bar{\ell}_{0} \right) \leq 1.$$

Assumption 2 ensures that for all realizations of $\gamma_0, \gamma_1, \dots, \gamma_N$ at least a portion of the customers travels with public transport, and leads to the following theorem.

Theorem 2 (Equilibria of the Sequential Game). Let Assumptions 1 and 2 hold. Then, the sequential game possesses an equilibrium.

The proof of Theorem 2 provides some insights on the equilibrium itself, which we summarize as follows.

Corollary 3. Let Assumptions 1 and 2 hold. For $j \in$ $\{1,\ldots,N\}$, define γ_i^* as:

- 1) If $\tilde{c}_j + V_T \cdot \bar{\ell}_j(0) \ge \gamma_0 + V_T \cdot \bar{\ell}_0$, then: $\gamma_j^*(\gamma_0) = \langle 0, 0 \rangle$.
- 2) Else, let $p_j^* \in [\tilde{c}_j, \gamma_0 + V_{\rm T} \cdot (\bar{\ell}_0 \bar{\ell}_j(0))]$ be the unique solution of

$$-\frac{p_j^* - \tilde{c}_j}{V_{\rm T}} \frac{\mathrm{d}\bar{\ell}_j^{-1}}{\mathrm{d}z} \bigg|_{\frac{\gamma_0 - p_j^*}{V_{\rm T}} + \bar{\ell}_0} + \bar{\ell}_j^{-1} \left(\frac{\gamma_0 - p_j^*}{V_{\rm T}} + \bar{\ell}_0 \right) = 0.$$

Then,
$$\gamma_j^*(\gamma_0) = \left\langle p_j^*(\gamma_0), \bar{\ell}_j^{-1} \left(\frac{\gamma_0 - p_j^*(\gamma_0)}{V_{\mathrm{T}}} + \bar{\ell}_0 \right) \right\rangle$$
.

Further, let γ_0^* be a minimizer of $U_0(\gamma_0, \gamma_{-0}^*(\gamma_0))$. Then, $\langle \gamma_0^*, \gamma_1^*, \dots, \gamma_N^* \rangle$ is an equilibrium.

If we further assume that delay functions are affine, then we can compute equilibria in closed form.

Corollary 4. Let Assumptions 1 and 2 hold and assume that $\bar{\ell}_j$ is affine, i.e., $\bar{\ell}_j(z) = \alpha_j + \beta_j z$, with $\alpha_j, \beta_j \geq 0$, and $\tilde{c}_j + V_{\mathrm{T}} \cdot \alpha_j < V_{\mathrm{T}} \cdot \bar{\ell}_0$. Then,

$$\gamma_j^*(\gamma_0) = \left\langle \frac{\gamma_0 + V_{\mathrm{T}}(\bar{\ell}_0 - \alpha_j) - \tilde{c}_j}{2}, \frac{\gamma_0 + V_{\mathrm{T}}(\bar{\ell}_0 - \alpha_j) - \tilde{c}_j}{2V_{\mathrm{T}}\beta_j} \right\rangle$$

$$\gamma_0^* = \Pi_{[0, p_0^{\text{max}}]} \left[\frac{1 - \frac{k_1}{k_3} - \sum_{j=1}^{N} \frac{\frac{k_2}{k_3} \varepsilon_j + (\tilde{c}_j + V_{\text{T}}(\alpha_j - \bar{\ell}_0))}{2V_{\text{T}} \beta_j}}{\sum_{j=1}^{N} \frac{1}{V_{\text{T}} \beta_j}} \right], \tag{4}$$

where $\Pi_A[x]$ denotes the projection of x in the set A.

Corollary 4 shows intuitive features in extreme cases:

- A municipality prioritizing the cost for customers (i.e., $k_1 = 1, k_2, k_3 \rightarrow 0$) will choose $\gamma_0^* \rightarrow 0$.
- A municipality prioritizing the environmental impact (i.e., $k_2 = 1, k_1, k_3 \to 0$) will choose $\gamma_0^* \to 0$.
- A municipality prioritizing its revenue (i.e., k_3 $1, k_1, k_2 \rightarrow 0$) will choose

$$\gamma_0^* \to \Pi_{[0,p_0^{\text{max}}]} \left[\frac{1 - \sum_{j=1}^N \tilde{c}_j + V_{\text{T}}(\alpha_j - \bar{\ell}_0))}{\sum_{j=1}^N \frac{1}{V_{\text{T}}\beta_i}} \right].$$

The generalization of this approach to general congestion games is an active field of research [48], and we leave its treatment to future work.

2) Game-theoretic Model of the Mobility System: We adapt the game-theoretic model of the mobility system from [26]. In particular, we consider a mobility system with an AMoD operator, an μ M operator, a taxi company, and public transport. We assume that the AMoD operator changes her prices dynamically, while all other MSPs and the municipality strategically choose their prices on a longer time horizon. In addition to travelling with MSPs or public transport, customers can also walk from their origin to their destination. Formally, we model the mobility system as a multigraph $\mathcal{G} = \langle \mathcal{V}, \mathcal{A}, s, t \rangle$, where \mathcal{V} is the set of vertices, \mathcal{A} is the set of arcs, and $s \colon \mathcal{A} \to \mathcal{V}$ and $t \colon \mathcal{A} \to \mathcal{V}$ map each arc to its origin and destination vertices, respectively. We define arc subsets A_{AMoD} , $A_{\mu M}$, A_{PT} , A_{Taxi} , and A_{W} , each inducing a subgraph. We model customers demand by a triple $\langle o, d, \alpha \rangle$, where o is the origin of the trip, d is its destination, and α is the rate of customers per unit time, and consider M demands. We assume that customers of demand $i \in \{1, ..., M\}$ select their trip via a mobile app, offering them two options. First, the AMoD ride, which takes time $t_{AMoD,i}$ and results from the shortest path from the customer's origin to the customer's destination on \mathcal{G}_{AMoD} . Second, the most convenient option between μ M, public transport, taxi, and walking. The most convenient option is defined as the one minimizing the sum of ticket prices and monetary value of time, and takes time $t_{\text{alt},i}$ (whereby the time is again computed via shortest path). For each demand, we assume a linear reaction curve: the rate of customers choosing the AMoD ride decreases linearly with the price of the AMoD ride. The parameters of the linear curve depend on the customers' distribution of value of time; see [26], [56]. We consider a transportation system in steady state and suppose that the AMoD operator offers mobility service on her subgraph \mathcal{G}_{AMoD} . To prevent imbalances in her fleet, the AMoD operator rebalances her fleet via empty vehicles. Formally, we dictate that the sum of vehicle flows entering a node equals the sum of the vehicle flows exiting it. When taking operational decisions, the AMoD operator has a limited number of AVs and it is subject to two types of taxes imposed by the municipality: a distance-based tax on all AVs and an additional distance-based tax on AVs driving empty, without customers. Then, the AMoD operator selects

Parameter	Variable			Va	lue			Units	Source
Vehicle operational cost Vehicle fixed cost Vehicle emissions Vehicle life	$c_{ m v,o} \ c_{ m v,f} \ e_{ m v} \ l_{ m v}$	5.78 19,000 0.16	5.86 31,000 0.18	5.89 29,000 0.13	AV ICEV 0.38 89,000 0.16 ,000	AV Hybrid 0.45 101,000 0.18	0.48 99,000 0.13	USD/mile USD/car kg/mile miles	[49], [50] [49], [51] [49], [51], [52] [49]
μM operational cost μM speed μM emissions	$egin{array}{c} c_{ ext{m,o}} \ v_{ ext{m}} \ e_{ ext{m}} \end{array}$			ES 0.79 5.0 0.101	SB 1.58 7.5 0.033	-		USD/mile mph kg/mile	[51], [53] [54] [51], [53]
PT waiting time	$t_{ m w,p}$		U-Bahn 5.0	S-Bahn 5.0	Trams 7.0	Buses 10.0	-	min	[55]

TABLE I: Parameters, variables, numbers, and units for the case studies.

prices to maximize profit. Formally, prices result from the quadratic convex optimization problem

$$\max_{\substack{x_i \in [0,\alpha_i], \\ f_0 \in \mathbb{R}_{>0}^{|\mathcal{A}_{\text{AMoD}}|}}} \sum_{i=1}^{M} \left[x_i(m_i x_i + q_i) - \sum_{j \in \mathcal{A}_{\text{AMoD}}} c_j f_{i,j}^* x_i + c_{0,j} f_{0,j} \right]$$

s.t.
$$\mathbf{B}^{\top} (f_i^* x_i + f_0) = 0$$
 (5a)

$$\sum_{i} t_{\text{AMoD},i} x_i + \sum_{j} t_j f_{0,j} \le N_{\text{fleet}}, \tag{5b}$$

where x_i denotes the flow of customers served by the AMoD operator and f_0 are rebalancing vehicle flows. Here, $m_i < 0$ and q_i are parameters defining the reaction curve, depending on the value of time, α_i , $t_{AMoD,i}$, and $t_{alt,i}$. The parameters c_j and $c_{0,j}$ model operational costs, including taxes, corresponding to arc $j \in \mathcal{A}_j$. The vector $f_j \in \{0,1\}^{|\mathcal{A}_{AMoD}|}$ denotes the shortest path on \mathcal{G}_{AMoD} from o_i to d_i . Constraint (5a), with B being the incidence matrix of \mathcal{G}_{AMoD} , ensures vehicle conservation at each node. Constraint (5b) upperbounds the fleet size. Finally, prices result from combining x_i with the customers' reaction curve. The profit of other MSPs, defined as excess of revenue over costs, results from the solution of this optimization problem, via the demand function. Finally, social welfare results from the weighted combination of the cost for the customers, CO₂ emissions, and public revenue. Specifically, the cost for the customers results from the fares and the monetary value of time. CO₂ emissions result from the distance driven by AVs, taxi, and μM vehicles; here, we neglect emissions of the public transportation systems, as we consider them as fixed and independent from the system's load, therefore not influencing strategic decisions. Finally, public revenue results from taxes and public transport tickets.

Consistently with the described low-level model of the mobility system, we consider a game among a municipality deciding on public transport prices and taxes (specific values, but also generic taxation strategies) for AMoD vehicles. In line with current public transport prices in many cities, we parametrize fares via a short-distance price (SDP) in $P_{\rm p,s}\subseteq\mathbb{R}_{\geq 0}$, a long-distance price (LDP) in $P_{\rm p,l}\subseteq\mathbb{R}_{\geq 0}$, and a cutoff distance in $D\subseteq\mathbb{R}_{\geq 0}$. We consider two types of taxes: a distance-based tax on AVs in $T_{\rm m}\subseteq\mathbb{R}_{\geq 0}$ and an additional distance-based tax on empty AVs in $T_{\rm e}\subseteq\mathbb{R}_{\geq 0}$, resulting from the scaling of the first tax. So, overall, the strategy space of the municipality is $\Gamma_0=P_{\rm p,s}\times P_{\rm p,l}\times D\times T_{\rm m}\times T_{\rm e}$. The AMoD operator chooses the propulsion type of AVs from a set of options E (e.g., Internal Combustion Engine

Vehicle (ICEV), Hybrid Electric Vehicle (HEV), or Battery Electric Vehicle (BEV)), the automation level from A (e.g., standard vehicle (SV), AV), and the fleet size from $F \subseteq \mathbb{N}_{>0}$. Our framework can be easily extended to capture different degrees of vehicle automation [42], [57], [58]. As AMoD applies dynamic pricing, decisions on prices do not happen at the level of our game, but are rather embedded in the lowlevel model of the mobility system. The action space of the AMoD operator is $\mathcal{U}_1(\gamma_0) = \mathcal{U}_1 = E \times A \times F$. Hence, Γ_1 consists of all maps from Γ_0 to \mathcal{U}_1 . For instance, the action of the AMoD operator if the municipality played γ_0 is $\gamma_1(\gamma_0) = \langle e, a, n \rangle \in E \times A \times F$. The μM operator decides on prices by choosing base and variable, mileagedepedent, prices from $P_{\mathrm{m,b}} \times P_{\mathrm{m,v}} \subseteq \mathbb{R}^2_{\geq 0}$. She also chooses the type of vehicles from M (e.g., e-scooter (ES) or shared bike (SB)), giving $\mathcal{U}_2(\gamma_0) = \mathcal{U}_2 = P_{\mathrm{m,b}} \times P_{\mathrm{m,v}} \times M$. Finally, the taxi company decides on base and variable prices from $P_{t,b} \times P_{t,v} \subseteq \mathbb{R}^2_{>0}$, giving $\mathcal{U}_3(\gamma_0) = \mathcal{U}_3 = P_{t,b} \times P_{t,v}$. In the following, we instantiate the model in a numerical case study.

III. RESULTS

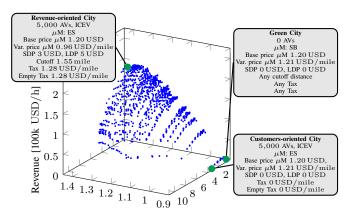
Our case study bases on a real-world setting of the city center of Berlin, Germany. We derive the road network and its features from OpenStreetMap [59], and import the public transport network, including U-Bahn, S-Bahn, tram, and bus lines, together with its schedules from GTFS [60]. We consider a set of 129,560 real travel requests [45]. To compute t_{AMoD} we use an average waiting time of $3 \min [61]$. To compute $t_{\rm alt}$ we use public transport schedules, velocity of μ M vehicles, and an average walking velocity of 3.13 mph. We account for congestion effects by increasing the nominal travel time of each interested arc by 56 %, corresponding to congestion levels in the evening peaks in Berlin [62]. In line with [56], we assume the customers' value of time to be uniformly distributed between 10 USD/h and 17 USD/h. We report the parameters, such as operational costs and emissions in Table I, and the action spaces of the players in Table II.

We compute equilibria of the sequential game of Section II via backward induction. First, we look for Nash Equilibria (NE) of the simultaneous game between MSPs². We report all of them in Fig. 3, locating them with respect to the three metrics defining social welfare: cost for the customers, emissions, and public revenue. Second, we compute the equilibrium of the sequential game by selecting the NE maximizing social welfare. As this depends on the weight of

²We report a more exhaustive list of figures at gioele.science/itsc21.

Pl	Parameter	Actions	Units	Source
MSP1	Fleet size Engine Automation	$\begin{array}{l} F \! = \! \{0, 1,\!000, \ldots, 16,\!000\} \\ E \! = \! \{\text{ICEV}, \text{HEV}, \text{BEV}\} \\ A \! = \! \{\text{SV}, \text{AV}\} \end{array}$	cars - -	[63] - -
MSP2	Vehicle type Base price Variable price	$\begin{array}{l} M\!=\!\{\rm ES,SB\} \\ P_{\rm m,b}\!=\!\{1.20\} \\ P_{\rm m,v}\!=\!\{0.72,0.96,1.21,1.45,1.69\} \end{array}$	USD USD/mile	- [64] [63], [64]
MSP3	Base price Variable price	$P_{t,b}$ ={4.72} $P_{t,v}$ ={1.17, 1.95, 3.89, 5.84, 7.79}	USD USD/mile	[65] [65]
City	Miles tax	$\begin{array}{l} P_{\mathrm{p,s}}\!=\!\{0,1.0,2.0,3.0,4.0\} \\ P_{\mathrm{p,l}}\!=\!\{0,1.0,2.0,3.0,4.0,5.0\} \\ D\!=\!\{0,1.55,3.10\} \\ T_{\mathrm{m}}\!=\!\{0,0.16,0.32,\ldots,1.60\} \\ T_{\mathrm{e}}\!=\!T_{\mathrm{m}}\times\{0,1,10\} \end{array}$	USD USD miles USD/mile USD/mile	

TABLE II: Parameters, actions, units, and sources for the case study.



Cost for customers [100k USD/h] Cost emissions [100k USD/h]

Fig. 3: Equilibria of the game with respect to cost for customers, cost of emissions, and public revenue. Each point is a NE of the simultaneous game between MSP. The equilibrium of the sequential game directly results from the weights of the three metrics in municipality's social welfare.

each metric, which is a purely political question, we think of these results as drivers to inform policy making.

Ideally, from the perspective of a socially-aware municipality, it is desirable to obtain high revenue, low emissions, and low cost for customers. However, Figures 3 and 4 show that the trade-offs characterizing the game do not allow for such scenarios. For instance, lowest emissions are reachable with no public revenue and a cost for customers 0.9% larger than the potentially obtainable one. Similarly, cost for customers and public revenue are monotonically related. Therefore, one needs to come to terms with the fundamental tradeoffs characterizing this system. Nevertheless, our framework provides a way to rigorously reason about possible solutions.

First, fixed the weights of each metric, we provide the equilibrium strategy and the corresponding social welfare. For instance, as can be seen in Fig. 3, a municipality minimizing CO₂ emissions incurs in emissions cost of 4,711 USD/h, and should make public transport free and ban AMoD vehicles. A customer-centric city also opts for free public transport and does not introduce taxes, at the price of no public revenue. Conversely, a city maximizing public revenue should make public transport expensive, but not as expensive as possible, and impose a tax of 1.28 USD/mile of AMoD vehicles, with an additional tax of 1.28 USD/mile on empty AMoD vehicles. These values represent the sweet spot

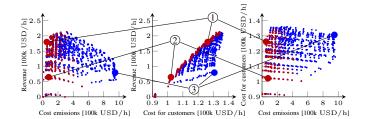


Fig. 4: Two-dimensional projections of the equilibria reported in Fig. 3. In red, the *rational* equilibria, dominating the blue ones in the projections.

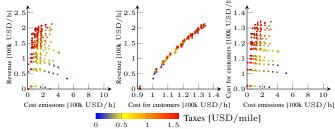


Fig. 5: Rational equilibria, classified by the entity of the taxes on AMoD.

between free public transport and no taxes and too expensive public transport and unsustainable taxes (which also lead to low public revenue). These are three extreme scenarios; for each choice of weights, we can compute the corresponding equilibrium.

Second, while we cannot a priori give an equilibrium, we can identify solutions which are always inconvenient. In general, NE are incomparable: is NE ① better than NE ②? It depends on the weights of the metrics in social welfare: NE ① yields larger public revenue, but also larger cost for customers. Hence, we call NE ① and NE ② *incomparable*. However, some equilibria are objectively better than others. For instance, NE ① dominates NE ③, as it outperforms it in all three metrics. We call non-dominated equilibria *rational*, and depict them in red in Fig. 4. Interestingly, all NE yielding high emissions are dominated, i.e., never rational.

Third, we can study fundamental tradeoffs of the system. For instance, lowering public revenue from $200,000\,\mathrm{USD/h}$ to $150,000\,\mathrm{USD/h}$ (i.e., $25\,\%$ reduction) leads to $50\,\%$ lower emissions and $10\,\%$ lower cost for customers.

Fourth, we can "zoom-in" and analyze the actions corresponding to each solution, as shown in Fig. 5 for distance-based taxes. As expected, high taxes correlate with high public revenue. However, they also correlate with larger costs for customers, confirming the well-known principle that the tax load partially falls on customers, and not only on sellers.

Fifth, we can evaluate the impact of interventions on other metrics. For instance, Fig. 6 shows the AMoD modal share. As expected, higher emissions correlate with larger AMoD modal share. Interestingly, though, we do not observe a correlation with the cost for customers: there are NE yielding to low costs and low AMoD modal share as well as NE yielding to low costs and large AMoD modal share.

Sixth, we analyzed results from the perspective of a socially-aware municipality. Yet, our framework can be directly used by (profit-oriented) MSPs to reason on strategic decisions.

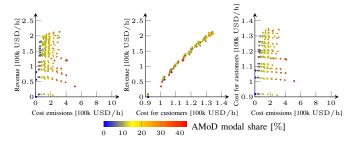


Fig. 6: Rational equilibria, classified by the modal share of AMoD vehicles.

IV. CONCLUSION

In this paper, we proposed a game-theoretic framework to study interactions between stakeholders of the mobility ecosystem. Our framework relies on the theory of sequential games, and can modularly accommodate different lowlevel models of the mobility system. We instantiated our framework in two case studies, a parallel arc congestion game and a game-theoretic model of the transportation system, and study them both analytically and numerically. With our framework, we arm stakeholders of the mobility ecosystem with analytical tools to reason about interventions and tradeoffs in mobility systems. Our work opens the field for various future research streams. First, we would like to instantiate our framework for various classes of low-level models of the mobility system, by explicitly characterizing equilibria and studying algorithms to efficiently compute them. Second, we want to exploit our framework to model and study interactions happening at different time scales. Third, we aim at applying our methodology to study other settings with similar structures, such as energy and global maritime shipping markets.

REFERENCES

- [1] D. o. E. a. S. A. United Nations, "68% of the world population projected to live in urban areas by 2050, says un," UN, Tech. Rep., 2021. [Online]. Available: https://www.un.org/development
- [2] M. Czepkiewicz, J. Heinonen, and J. Ottelin, "Why do urbanites travel more than do others? A review of associations between urban form and long-distance leisure travel," Environmental Research Letters, vol. 13, no. 7, p. 073001, 2018.
- [3] C. Calastri, S. Borghesi, and G. Fagiolo, "How do people choose their commuting mode? An evolutionary approach to travel choices, Economia politica, vol. 36, no. 3, pp. 887-912, 2019.
- [4] S. Ranchordas, "Smart mobility, transport poverty, and the right to inclusive mobility," Smart Urban Mobility-Law, Regulation and Policy (Springer, 2020), University of Groningen Faculty of Law Research Paper, no. 13, 2020.

 [5] G. Zardini, N. Lanzetti, M. Pavone, and E. Frazzoli, "Analysis and
- control of autonomous mobility-on-demand systems: A review, nual Review of Control, Robotics, and Autonomous Systems, 2021, submitted.
- T. city of New York, "Onenyc 2050 report," NYC, Tech. Rep., 2021. [Online]. Available: http://onenyc.cityofnewyork.us/strategies
- T. Berger, C. Chen, and C. B. Frey, "Drivers of disruption? Estimating the uber effect," European Economic Review, vol. 110, pp. 197-210,
- [8] B. Rogers, "The social costs of Uber," U. Chi. L. Rev. Dialogue, vol. 82, p. 85, 2015.
- [9] T. Yigitcanlar, M. Wilson, and M. Kamruzzaman, "Disruptive impacts of automated driving systems on the built environment and land use: An urban planner's perspective," *Journal of Open Innovation: Technology, Market, and Complexity*, vol. 5, no. 2, p. 24, 2019.

 [10] U. Nations, "Cities and pollution," UN, Tech. Rep., 2021. [Online].
- Available: https://www.un.org/en/climatechange
- Trasport, St. Mobility and strategy," EU. "Sustaiable C. and for EU, smart mobility 2021. [On-Available: https://ec.europa.eu/transport/sites/transport/files/ 2021-mobility-strategy-and-action-plan.pdf

- [12] Z. Mingbao, C. Ying, Z. Ning, and Z. Xiaojun, "Pricing of urban rail transit for different operation stages based on game theory," in 2010 2nd IEEE International Conference on Information and Financial Engineering. IEEE, 2010, pp. 139–142.
 [13] H. Gong and W. Jin, "Analysis of urban public transit pricing adjust-
- ment program evaluation based on trilateral game," *Procedia-Social and Behavioral Sciences*, vol. 138, pp. 332–339, 2014.

 [14] J. Yang, Y. Lin, F. Wu, and L. Chen, "Subsidy and pricing model of
- electric vehicle sharing based on two-stage Stackelberg game—a case study in China," *Applied Sciences*, vol. 9, no. 8, p. 1631, 2019.
- [15] T. D. Chen and K. M. Kockelman, "Management of a shared autonomous electric vehicle fleet: Implications of pricing schemes,' Transportation Research Record, vol. 2572, no. 1, pp. 37-46, 2016.
- A. K. Kuiteing, P. Marcotte, and G. Savard, "Network pricing of congestion-free networks: The elastic and linear demand case," Transportation Science, vol. 51, no. 3, pp. 791-806, 2017.
- K. Bimpikis, O. Candogan, and D. Saban, "Spatial pricing in ride-sharing networks," *Operations Research*, vol. 67, no. 3, pp. 744–769,
- [18] F. Dandl, K. Bogenberger, and H. S. Mahmassani, "Autonomous mobility-on-demand real-time gaming framework," in 2019 6th International Conference on Models and Technologies for Intelligent Transportation Systems (MT-ITS). IEEE, 2019, pp. 1–10.
- [19] B. Turan and M. Alizadeh, "Competition in electric autonomous mobility on demand systems," arXiv preprint arXiv:2007.06051, 2020.
- L. Lei and S. Gao, "Evolutionary game analysis of ridesourcing industry between transportation network companies and passengers under new policies of ridesourcing," *IEEE Access*, vol. 6, pp. 71918– 71 931, 2018.
- [21] X. Di and X. J. Ban, "A unified equilibrium framework of new shared mobility systems," *Transportation Research Part B: Methodological*, vol. 129, pp. 50–78, 2019.
- [22] R. Hernández, C. Cárdenas, and D. Muñoz, "Game theory applied to transportation systems in smart cities: analysis of evolutionary stable strategies in a generic car pooling system," *International Journal on* Interactive Design and Manufacturing (IJIDeM), vol. 12, no. 1, pp. 179-185, 2018.
- [23] B. Mo, Z. Cao, H. Zhang, Y. Shen, and J. Zhao, "Competition between shared autonomous vehicles and public transit: A case study in Singapore," Transportation Research Part C: Emerging Technologies, vol. 127, p. 103058, 2021.
- [24] M. Balac, H. Becker, F. Ciari, and K. W. Axhausen, "Modeling competing free-floating carsharing operators—a case study for Zurich, Switzerland," Transportation Research Part C: Emerging Technologies, vol. 98, pp. 101-117, 2019.
- [25] N. Lanzetti, G. Zardini, M. Schiffer, M. Ostrovsky, and M. Pavone, "Do self-driving cars swallow public transport? a game-theoretical perspective on transportation systems," in *Program Book: INFORMS* Annual Meeting 2019. Informs, 2019, pp. 483–483.

 [26] N. Lanzetti, M. Schiffer, M. Ostrovsky, and M. Pavone, "On the
- interplay between self-driving cars and public transportation," in ACM Conference on Economics and Computation, 2021, submitted.
- Q. Sun, Y. He, Y. Wang, and F. Ma, "Evolutionary game between government and ride-hailing platform: Evidence from China," *Discrete* Dynamics in Nature and Society, vol. 2019, 2019.
- [28] Z. Wang, L. Zheng, T. Zhao, and J. Tian, "Mitigation strategies for overuse of Chinese bikesharing systems based on game theory analyses of three generations worldwide," *Journal of Cleaner Production*, vol. 227, pp. 447-456, 2019. [29] C. Swamy, "The effectiveness of stackelberg strategies and tolls for
- network congestion games," *ACM Transactions on Algorithms (TALG)*, vol. 8, no. 4, pp. 1–19, 2012.
- D. Paccagnan, R. Chandan, B. L. Ferguson, and J. R. Marden,
- "Optimal taxes in atomic congestion games," 2021.
 [31] D. A. Lazar and R. Pedarsani, "Optimal tolling for multitype mixed autonomous traffic networks," *IEEE Control Systems Letters*, 2020.
- W. Krichene, J. D. Reilly, S. Amin, A. M. Bayen, T. Basar, and G. Zaccour, "Stackelberg routing on parallel transportation networks,"
- Handbook of Dynamic Game Theory, pp. 1–35, 2017. H. Zhou, C. Liu, B. Yang, and X. Guan, "Optimal dispatch of electric taxis and price making of charging stations using stackelberg game," in *IECON 2015-41st Annual Conference of the IEEE Industrial Electronics Society*. IEEE, 2015, pp. 004 929–004 934.
- M. Koryagin, "Urban planning: A game theory application for the travel demand management," *periodica polytechnica transportation engineering*, vol. 46, no. 4, pp. 171–178, 2018.

 H. Mei, S. Poslad, and S. Du, "A game-theory based incentive
- framework for an intelligent traffic system as part of a smart city initiative," *Sensors*, vol. 17, no. 12, p. 2874, 2017.
- L. Bianco, M. Caramia, S. Giordani, and V. Piccialli, "A gametheoretic approach for regulating hazmat transportation," *Transportation Science*, vol. 50, no. 2, pp. 424–438, 2016.

- [37] D. Fullerton and S. E. West, "Can taxes on cars and on gasoline mimic an unavailable tax on emissions?" *Journal of Environmental Economics and Management*, vol. 43, no. 1, pp. 135–157, 2002.
- [38] K. Iwata and S. Managi, "Can land use regulations and taxes help mitigate vehicular CO2 emissions? An empirical study of japanese cities," *Urban Policy and Research*, vol. 34, no. 4, pp. 356–372, 2016.
- [39] W. Zhang and K. M. Kockelman, "Optimal policies in cities with congestion and agglomeration externalities: Congestion tolls, labor subsidies, and place-based strategies," *Journal of Urban Economics*, vol. 95, pp. 64–86, 2016.
- [40] P. Slowik, S. Wappelhorst, and N. Lutsey, "How can taxes and fees on ride-hailing fleets steer them to electrify," 2019.
- [41] I. V. Chremos and A. A. Malikopoulos, "A socially-efficient emerging mobility market," arXiv preprint arXiv:2011.14399, 2020.
- [42] G. Zardini, N. Lanzetti, M. Salazar, A. Censi, E. Frazzoli, and M. Pavone, "On the co-design of AV-enabled mobility systems," in 2020 IEEE 23rd International Conference on Intelligent Transportation Systems (ITSC), 2020, pp. 1–8.
- [43] S. M. Zoepf, S. Chen, P. Adu, and G. Pozo, "The economics of ride-hailing: driver revenue, expenses, and taxes," *CEEPR WP*, vol. 5, pp. 1–38, 2018.
- [44] M. Ostrovsky and M. Schwarz, "Carpooling and the economics of self-driving cars," in *Proceedings of the 2019 ACM Conference on Economics and Computation*, 2019, pp. 581–582.
 [45] D. Ziemke, I. Kaddoura, and K. Nagel, "The MATSim open Berlin sce-
- [45] D. Ziemke, I. Kaddoura, and K. Nagel, "The MATSim open Berlin scenario: A multimodal agent-based transport simulation scenario based on synthetic demand modeling and open data," *Procedia computer* science, vol. 151, pp. 870–877, 2019.
- [46] K. E. Train, Discrete choice methods with simulation. Cambridge university press, 2009.
- [47] U. S. B. of Public Roads, Traffic assignment manual for application with a large, high speed computer. US Department of Commerce, Bureau of Public Roads, Office of Planning, Urban, 1964, vol. 2.
 [48] A. Marchesi, M. Castiglioni, and N. Gatti, "Leadership in congestion
- [48] A. Marchesi, M. Castiglioni, and N. Gatti, "Leadership in congestion games: Multiple user classes and non-singleton actions." in *IJCAI*, 2019, pp. 485–491.
- [49] A. A. Association, "AAA your driving costs report," American Automobile Association, Tech. Rep., 2019.
- [50] P. M. Boesch, F. Becker, H. Becker, and K. W. Axhausen, "Cost-based analysis of autonomous mobility services," *Transport Policy*, vol. 64, pp. 76–91, 2018.
- pp. 76–91, 2018.

 [51] G. Zardini, N. Lanzetti, A. Censi, E. Frazzoli, and M. Pavone, "Codesign to enable user-friendly tools to assess the impact of future mobility solutions," arXiv preprint arXiv:2008.08975, 2020.
- [52] P. Jochem, S. Babrowski, and W. Fichtner, "Assessing CO2 emissions of electric vehicles in germany in 2030," *Transportation Research Part A: Policy and Practice*, vol. 78, pp. 68–83, 2015.
- [53] D. Schellong, P. Sadek, C. Schaetzberger, and T. Barrack, "The promise and pitfalls of e-scooter sharing," Boston Consulting Group, Tech. Rep., 2019.
- [54] "NATCO Shared Micromobility in the US: 2018," National Association of City Transport Officials, Tech. Rep., 2019. [Online]. Available: https://nacto.org/wp-content/uploads/2019/04/ NACTO_Shared-Micromobility-in-2018_Web.pdf
- [55] The city of Berlin, "Linienuebersicht Stand 2020," 2020. [Online]. Available: bvg.de/de/Fahrinfo
- [56] Z. Wadud, "Fully automated vehicles: A cost of ownership analysis to inform early adoption," *Transportation Research Part A: Policy and Practice*, vol. 101, pp. 163–176, 2017.
- [57] G. Zardini, A. Censi, and E. Frazzoli, "Co-design of autonomous systems: From hardware selection to control synthesis," in 20th European Control Conference (ECC), 2021.
- [58] G. Zardini, D. Milojevic, A. Censi, and E. Frazzoli, "Co-design of embodied intelligence: A structured approach," in *IEEE/RSJ Interna*tional Conference on Intelligent Robots and Systems (IROS), 2021.
- [59] M. Haklay and P. Weber, "Openstreetmap: User-generated street maps," IEEE Pervasive computing, vol. 7, no. 4, pp. 12–18, 2008.
- maps," *IEEE Pervasive computing*, vol. 7, no. 4, pp. 12–18, 2008. [60] VBB, "VBB-Fahrplandaten via GTFS," 2019. [Online]. Available: https://daten.berlin.de/datensaetze/vbb-fahrplandaten-gtfs
- [61] P. Mosendz and H. Sender, "Here's how long it takes to get an Uber in U.S. cities," 2014. [Online]. Available: https://www.newsweek.com/
- exclusive-heres-how-long-it-takes-get-uber-across-us-cities-289133
 [62] TomTom, "Berlin traffic report. TomTom traffic index," 2021.
 [Online]. Available: https://www.tomtom.com/en_gb/traffic-index
- [63] Straßenverkehr in Berlin: Anzahl an Taxis erreicht neuen Rekordwert. [Online]. Available: https://www.berliner-zeitung.de
- [64] T. city of Berlin, "Gettin around in berlin," 2021. [Online]. Available: https://www.berlin.de/en/getting-around/
- [65] T. city of Berlin, "Taxi: Phone numbers, fares, rules," 2021. [Online]. Available: https://www.berlin.de/en/public-transportation/ 1756978-2913840-taxi-phone-numbers-fares-rules.en.html

APPENDIX

Lemma 5. Let Assumptions 1 and 2 hold. Assume $f_j \ge 1$ for all $j \in \{1, ..., N\}$. The Wardrop equilibrium is

$$x_{j}^{*}(p_{j},p_{0}) = \begin{cases} \bar{\ell}_{i}^{-1} \left(\frac{p_{0}-p_{j}}{V_{\Gamma}} + \bar{\ell}_{0} \right) & \text{if } \frac{p_{0}-p_{j}}{V_{\Gamma}} + \bar{\ell}_{0} \geq \bar{\ell}_{j}(0), \\ 0 & \text{else.} \end{cases}$$

Proof. The proof follows directly from the KKT conditions of the optimization problem (1) for $f_i \ge 1$.

Lemma 6. Any non-trivial equilibrium strategy $\gamma_j^* \in \Gamma_j$, $j \in \{1, ..., N\}$, is of the form

$$\gamma_j^*(\gamma_0) = \langle p_j(\gamma_0), x_j^*(p_j(\gamma_0), \gamma_0) \rangle$$
 for $p_j((\gamma_0) \in \mathbb{R}_{\geq 0}$.

Proof. It suffices to observe that (i) $p_j < \tilde{c}_j$ and (ii) $f_j > x_i^*(p_j, p_0)$ are always suboptimal.

Proof of Theorem 2. First, by Lemma 6, there no loss of generality in assuming MSPs only decide on prices. By Lemma 5, x_j^* only depends on γ_j and γ_0 , so the game reduces to N parallel single leader-single follower Stackelberg games, whereby MSPs select the action maximizing their profit. Here, we can without loss of generality assume $\mathcal{U}_j(\gamma_0) = [\tilde{c}_j, \gamma_0 + V_{\mathrm{T}}(\bar{\ell}_0 - \bar{\ell}_j(0))];$ else, the problem is straightforward. Then,

$$U_j(\gamma_j, \gamma_0) = (\gamma_j - \tilde{c}_j) \cdot \bar{\ell}_i^{-1} \left(\frac{\gamma_0 - \gamma_j}{V_T} + \bar{\ell}_0 \right).$$

As U_i is smooth, we can compute its second derivative as

$$\frac{\mathrm{d}^2 U_j}{\mathrm{d} \gamma_j^2} = \frac{\gamma_j - \tilde{c}_j}{V_{\mathrm{T}}^2} \left. \frac{\mathrm{d}^2 \bar{\ell}_j^{-1}}{\mathrm{d} z^2} \right|_{\frac{\gamma_0 - \gamma_j}{V_{\mathrm{T}}} + \bar{\ell}_0} - \frac{2}{V_{\mathrm{T}}} \left. \frac{\mathrm{d} \bar{\ell}_i^{-1}}{\mathrm{d} z} \right|_{\frac{\gamma_0 - \gamma_j}{V_{\mathrm{T}}} + \bar{\ell}_0}.$$

Since $\bar{\ell}_j$ strictly increasing and convex, its inverse is strictly increasing and concave. Thus, $\frac{\mathrm{d}^2 U_j}{\mathrm{d} \gamma_j^2} < 0$, and U_j is strictly concave. So, its maximizer is unique and continuous in the parameters. In particular, $\gamma_0 \mapsto \gamma_j^*(\gamma_0)$ is well-defined and continuous. Hence, $\gamma_0 \mapsto U_0(\gamma_0, \gamma_{-0}^*(\gamma_0))$ is a continuous function, being the composition of continuous functions. Since Γ_0 is compact, Weierstrass' thereom ensures the existence of a maximizer γ_0^* . So, $\langle \gamma_0^*, \gamma_1^*, \dots, \gamma_N^* \rangle$ is an equilibrium.

Proof of Corollary 3. The proof follows directly from the proof of Theorem 2. Indeed, by strict concavity, (2) is a necessary and sufficient condition for optimality. By the intermediate value theorem, (2) admits at least a solution; uniqueness follows again from strict concavity. □

Proof of Corollary 4. With affine delay functions, we have $\bar{\ell}_i^{-1}(z) = (z - \alpha_i)/\beta_i$. Thus, (2) reduces to

$$-\frac{p_j^* - \tilde{c}_j}{V_{\mathrm{T}}} \frac{1}{\beta_j} + \frac{1}{\beta_j} \left(\frac{\gamma_0 + V_{\mathrm{T}} \cdot \bar{\ell}_0 - p_j^*}{V_{\mathrm{T}}} - \alpha_j \right) = 0,$$

which is easily solved to $p_j^* = (\gamma_0 + V_T(\bar{\ell}_0 - \alpha_j) - \tilde{c}_j)/2 > 0$. Then, by Corollary 3, we get (3). To conclude it suffices to observe that social welfare, with $\gamma_{-0} = \gamma_{-0}^*(\gamma_0)$, reduces to

$$U_0(\gamma_0) = k_3 \gamma_0 - k_1 (\gamma_0 + V_T \bar{\ell}_0) - \sum_{i=1}^{N} (k_2 \varepsilon_j + k_3 \gamma_0) \eta_j(\gamma_0)$$

with $\eta_j \coloneqq (\gamma_0 + V_T(\bar{\ell}_0 - \alpha_j) - \tilde{c}_j)/(2V_T\beta_j)$. This is a strongly concave function, whose maximum lies at (4).