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Stochastic and dynamic routing with flexible deliveries for an e-grocer

Pieter S. Bouwstra^a, Gonçalo Correia^b, Peter Bijl^c, Rudy R. Negenborn^a and Bilge Atasoy^a

Abstract—The quality of the delivery service is a crucial asset for an e-grocer to create and maintain a loyal customer-base. With the rapid market growth of e-grocers over the last decade, there is an urgent need for e-grocer specific routing systems. Although stochastic and dynamic routing models are studied for a wide range of applications, e-grocer specific models are missing in the literature. This paper investigates the concept of flexible deliveries, which introduces differentiated time window sizes. This creates the possibility for real-time re-optimization of the sequence of customers in a trip in order to improve the on-time delivery performance. The potential of flexible deliveries is investigated by means of computational experiments in which historic trip instances from the Dutch e-grocer Picnic are used. It is shown that, when re-optimization is activated, on-time delivery performance is improved and this benefit is significant when flexible deliveries represent at least 10% of the deliveries. When 10% of the deliveries are flexible, the number of late deliveries can be reduced by up to 18% and the number of extreme late deliveries (≥ 15 min late) up to 27%. This improved on-time delivery performance comes at the cost of a maximum of 2% increase in the average time spent per delivery.

I. INTRODUCTION

E-grocers offer their customers an online grocery shopping experience by allowing them to order from virtually any location within moments. The groceries are delivered at the customer's front door or even into the kitchen. When compared to other last-mile distribution systems, e-grocers experience large service times relative to their travel times. Moreover, an important requirement for a good customer experience is the interaction between the customer and the driver. The e-grocers' efforts to perform well on this aspect lead to a large uncertainty in these service times.

The first e-grocer businesses emerged in the late 1990s [1] and since the 2010s [2] e-grocers have started to seriously compete with traditional grocery stores. Because of the success of e-grocer start-ups, many traditional food retail market players have started to invest in an online grocery delivery service as well [3]. Since e-grocers have only recently started to gain a significant market share in the total grocery market [4], not much research has been dedicated to the development of routing models for e-grocers specifically. Moreover, using an of-the-shelf routing model does not accurately take into account the specifics of e-groceries, such as the perishable nature and the importance of the customer's trust in the quality of an e-grocer's delivery service [5], [6], [7].

The research community has made rapid developments in the fields of stochastic and dynamic routing models during

the past decade [8], [9]. Results suggest that the use of such models improves the routing performance for a large variety of applications, e.g. [10], [11], [12]. However, such models are not yet investigated and used to improve the last-mile delivery service for e-grocers specifically. This paper addresses this gap with the development of stochastic and dynamic routing models to improve the performance of an e-grocer's routing. For this purpose, the concept of "flexible deliveries" is employed where flexible deliveries have larger time windows than regular ones. This flexibility allows for dynamic re-optimization of the sequence of customers during the trip and therefore creates a potential to mitigate the effects of running early or late (due to various conditions such as weather, road or service specific requirements). The effectiveness of the proposed models is assessed by means of a set of computational experiments that use real data instances from the Dutch e-grocer Picnic¹.

The remainder of this paper is structured as follows. First, related literature on dynamic and stochastic routing models is covered in Section II. Different variants of routing models with flexible deliveries are presented in Section III. Section IV evaluates the performance of these routing model variants with experimental results. The paper is concluded with future research directions in Section V.

II. RELATED LITERATURE

In this section, we review the stochastic and dynamic routing model elements used in the literature that led us to the development of the concept of flexible deliveries. Therefore, it is limited to the most relevant studies.

When it comes to differentiating customer service, [13] study a capacitated vehicle routing problem with stochastic demands (VRPSD) and introduce the concept of premium customers for whom, the probability that their demand is met is larger than other customers. Nevertheless, the concept of flexible deliveries we study in our paper makes a distinction with regards to the delivery time window size rather than the probability of the demand being met.

Re-routing strategies with the reordering of customers in the remaining sequence of a trip are investigated by [14] with a dynamic vehicle routing problem (DVRP). They use real-time traffic data to make these adjustments to the trip planning. However, they do not consider the time window constraints in this adjustment step which is addressed by our study. Furthermore, in the context of an urban freight transport problem, [16] look into the potential of using real-time traffic times for re-optimization of the allocation of

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TABLE I
RELATED LITERATURE

Study	Problem type	Stochastic element(s)	Dynamically re-optimized element(s)	Time windows constraints
Chen et al. (2018) [13]	VRPSD	Demand	None	None
Ng et al. (2017) [14]	DVRP	Travel times	Customer sequence in a trip	None
Errico et al. (2016) [15]	DVRPTW	Service times	Skipping service at a customer	Yes
Taniguchi et al. (2004) [16]	DVRPTW	Travel times	1. Number of trips 2. Trip departure times 3. Customer sequence in each trip	Yes
This study	DTSPTW	1. Service times 2. Travel times	Customer sequence in a trip	Yes

customers to trucks and the sequence of customers in a trip. Their problem includes multiple degrees of freedom which can be re-optimized: the number of trucks, trip departure times and sequence of customers in each trip. In our paper, only the sequence of customers in each trip can be re-optimized which makes the problem more constrained to make a significant impact by means of re-optimization.

[15] investigate dynamic VRP with hard time windows (DVRPTW) and stochastic service times. They investigate two simple recourse actions: skip the service at the current customer or skip the service at the next customer. In the context of e-grocers such recourse actions are not viable. Therefore, in this paper a more sophisticated re-optimization of the sequence of residual customers is investigated.

From Table I it can be concluded that the combination of using a re-optimization model to reconsider the optimal sequence of remaining customers in a trip and the presence of time window constraints is preceded by [16]. However, [16] only considers stochastic travel times while this paper also considers stochastic service times. Moreover, this paper focuses on trip instances specifically encountered by e-grocers. The other studies listed in Table I address a general application of their problem type.

III. PROPOSED MODELS

The VRPs solved by e-grocers are large in size and computational time budget is limited to meet the operations. One solution is to split the traditional VRP into two separate problems: customer-trip assignment and sequencing problem. The customer-trip assignment model needs to be solved the evening before delivery after the orders are collected and therefore has usually very limited computational time. In the sequencing problem, it is feasible to use stochastic travel and service times to optimize the sequence of customers because the size of this problem is small once the customers in a trip are assigned. When the trip begins, re-optimization takes place shortly before the driver departs from a customer. The a-priori sequence of customers should create room for an effective online re-optimization of the customer sequence for improving the performance of the delivery service. The objective for the routing model is therefore to maximize the on-time delivery performance and minimize the operational costs. In this research, the operational costs are represented by the average time spent per delivery.

The considered components of the routing model in the case of an e-grocer are illustrated in Figure 1. The scope

of this study is limited to the a-priori sequencing model and the real-time re-optimization model that adjusts the sequence of the remaining customers. The customer-trip assignment relies on a simple deterministic model to initiate the framework. In order to investigate the performance of different approaches to the concept of flexible deliveries, three different sequencing models and one re-optimization model are designed which are presented in Sections III-A, III-B, III-C and III-D, respectively.

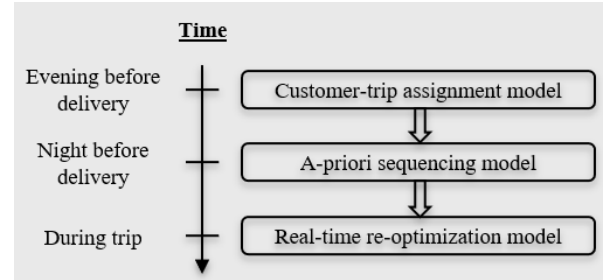


Fig. 1. Models for an e-grocer's routing

A. Benchmark sequencing model

This model is used as the benchmark model due to its simplicity and deterministic nature. The set of all nodes is represented by N where $V \subset N$ represents the customers nodes. All trips depart from and end at the hub which is represented by $starthub$ and the $endhub$ nodes. The routing is given by the binary decision variable $X_{i,j}$ which is 1 if arc (i,j) is included in the trip and 0 otherwise. Additionally, the arrival time at node i is represented by variable T_i .

The optimal sequence of customers is determined based on the minimization of the total trip duration which is the dominant objective function as given in (21) with a large multiplier M . However, a trip can depart from the hub at several moments resulting in the same total trip duration. In this case, the departure time is determined based on the optimization of time-window overlap; the model prefers solutions where the planned moments of delivery lie in the middle of the delivery time windows. The delivery time window (RDW is the size of a regular delivery window) is constrained by the order window (for customer j is given by $(OTWS_j - OTWE_j)$) with SEA_i and SLA_i variables which represent the early and late arrival, respectively as given by Constraints (12) and (13). These are penalized in a quadratic

fashion to penalize larger deviations more.

$$\min M(T_{endhub} - T_{starthub}) + \sum_{j \in V} (SEA_j^2 + SLA_j^2) \quad (1)$$

s.t.

$$\sum_{j \in N} X_{i,j} = 1 \quad \forall i \in V \quad (2)$$

$$\sum_{i \in N} X_{i,j} = 1 \quad \forall j \in V \quad (3)$$

$$\sum_{i \in N} X_{i,starthub} = 0 \quad (4)$$

$$\sum_{j \in N} X_{starthub,j} = 1 \quad (5)$$

$$\sum_{i \in N} X_{i,endhub} = 1 \quad (6)$$

$$\sum_{j \in N} X_{endhub,j} = 0 \quad (7)$$

$$T_j \geq \sum_{i \in N} X_{i,j}(T_i + ST_i + TT_{i,j}) \quad \forall j \in V. \quad (8)$$

$$T_{starthub} \leq \sum_{j \in V} X_{starthub,j}(T_j - TT_{starthub,j}) \quad (9)$$

$$T_{endhub} \geq \sum_{j \in V} X_{i,endhub}(T_i + ST_i + TT_{i,endhub}) \quad (10)$$

$$PTWS_j \leq T_j \leq PTWE_j \quad \forall j \in V \quad (11)$$

$$T_j + SEA_j \geq OTWS_j + 0.5RDW \quad \forall j \in V \quad (12)$$

$$T_j - SLA_j \leq OTWE_j - 0.5RDW \quad \forall j \in V \quad (13)$$

$$X_{i,j} \in \{0, 1\} \quad \forall i, j \in N \quad (14)$$

Constraints (2)-(7) ensure spatial continuity for the routing decisions. Time consistency is guaranteed by Constraints (8)-(10) and Big-M constraints are used to linearize them. ST_i is the planned service time at customer i and $TT_{i,j}$ is the travel time on arc (i, j) . Constraints (11) ensure a planned arrival within the customer's planning window ($PTWS_j$ - $PTWE_j$) which is smaller than the customer's order window ($OTWS_j$ - $OTWE_j$) and used to prevent a planned arrival time in the closing minutes of this order window.

B. Simulation-based sequencing model

The simulation-based sequencing model is developed based on the benchmark sequencing model and it is a stochastic sequencing model since the optimal sequence of customers is selected based on a comparison of performance predictions of multiple solutions. A similar approach in the context of a VRPSD is taken by [17] where they search for the optimal number of trips to deliver uncertain amounts of goods to a set of customers.

This sequencing model completes n runs of the benchmark sequencing model. In each next run, the solution(s) found in the previous run(s) ($s \in S$) are eliminated. This results in a different solution for each run. The mathematical model is similar to that of the benchmark sequencing model (Section III-A). In order to arrive at different solutions, Constraints (15) are added to the model where AS_s represents the

set of arcs used in solution s . The n best solutions are looked for whenever feasible and the best solution among this set is picked based on the a-priori simulated average on-time delivery rate over 25 iterations of the solution. This simulation is performed as explained in Section IV.

$$\sum_{(i,j) \in AS_s} X_{(i,j)} \leq \text{len}(N) - 1 \quad \forall s \in S \quad (15)$$

C. Heuristics-based sequencing model

In order to maximize the potential of real-time re-optimization, it is interesting to investigate how the a-priori sequencing model can contribute to creating possibilities for effective re-optimization. For this purpose, an exploratory research is conducted where flexible deliveries are assigned to specific indices in a trip representing the order of visit. Historical trips from Picnic for one day of operation are used. Next, simulations of those trips are performed, including the re-optimization model (Section III-D). The exploratory research makes use of trip instances which span two one-hour order windows in order to see the impact. By means of this approach insights are gained in effective positions of flexible deliveries within a trip as follows:

- For the first order window (first hour), the possibility for re-optimization is most effective when the flexible delivery is positioned at the middle of that window.
- For the second order window however, the possibility for re-optimization is most effective when the flexible delivery is positioned at the start of that window.

The results from the exploratory research point out that the positions of flexible deliveries within the trip sequence significantly affect the performance of the sequencing model. Therefore, these findings are translated into the formulation of the heuristics-based sequencing model in order to maximize the re-optimization possibilities offered by flexible deliveries. This mathematical model is similar to the benchmark sequencing model (Section III-A). The adapted objective function is given by:

$$\begin{aligned} \min M \sum_{j \in V} OID_j + M/1000(T_{endhub} - T_{starthub}) \\ + \sum_{j \in V} (SEA_j^2 + SLA_j^2) \end{aligned} \quad (16)$$

where the first term is the deviation from the optimal delivery index facilitated by variable OID_j . Note that, the last term is related to the deviation from the communicated delivery time windows (which was the order window in the benchmark sequencing model in Section III-A).

New constraints are given in (17)-(20). Constraints (17) calculate the index of each customer in the trip which is represented by variable I_j . Note that the hub is placed at the initial index (18). Constraints (19) and 20 determine the absolute value of the difference (OID_j) between the optimal index of node j (OI_j) and the index of node j in the solution (I_j). The index of each node in set OD is optimized based

on the heuristics inspired by the exploratory research.

$$\sum_{j \in N} X_{i,j} * (I_j - I_i) = 1 \quad \forall i \in N \quad (17)$$

$$I_{start_hub} = 0 \quad (18)$$

$$OID_j \leq OI_j - I_j \quad \forall j \in OD \quad (19)$$

$$OID_j \leq -OI_j + I_j \quad \forall j \in OD \quad (20)$$

D. Re-optimization model

The re-optimization model is executed when departing from each customer, except the last two customers in the trip (as they do not improve the objective). Inputs are the currently followed trip planning (planned hub departure times, travel times and service times), delivery time windows and the trip progress. The output is the re-optimized customer sequence. Similar to [9], a route-oriented approach is used instead of a customer-by-customer approach such that the remaining part of trip is re-optimized fully instead of merely the next customer in the trip. This remaining part of the trip consists of the current node, all remaining customer nodes, V^R , and the *endhub* node.

The re-optimization model makes use of a smaller delivery time window given by $(TWS_j - TWE_j)$ for regular customers as during the operations it is reduced when communicating to them (see Section IV). When the progress of a trip falls behind the trip planning calculated a-priori, the use of hard time windows constraints might result in an infeasible problem and therefore soft time windows constraints are used. Opposed to other researchers who penalize lateness or earliness in a linear fashion [18], [19], [20], this model includes a quadratic penalty in order to penalize larger deviations more. The adapted objective function is given by:

$$\min \quad dev_opt \sum_{j \in V} D_j^2 + T_{endhub} - T_{currentnode}, \quad (21)$$

where D_j is the total deviation from the optimal arrival window of customer j and dev_opt is the corresponding penalty of this deviation. The value of dev_opt depends on the trade-off between the two elements in the objective.

The routing constraints are similar to the benchmark sequencing model constraints (2)-(10) in Section III-A with a difference that now the decisions can only be changed for the remainder of the trip. In order to optimize the chance that a driver arrives on time, additional constraints (22)-(24) are considered. The optimal arrival window is $(TWS_j + SM - TWE_j - SM)$ where SM is the safety margin. E_j and L_j represent the earliness and lateness with respect to the optimal arrival window, respectively, which then determine the total deviation D_j used in the objective function.

$$E_j \geq (TWS_j + SM) - T_j \quad \forall j \in V \quad (22)$$

$$L_j \geq T_j - (TWE_j - SM) \quad \forall j \in V \quad (23)$$

$$D_j \geq E_j + L_j \quad \forall j \in V \quad (24)$$

IV. CASE STUDY: DUTCH E-GROCER PICNIC

The performances of the proposed models are evaluated through a case-study based on Dutch e-grocer Picnic's operations. In Picnic's operations, the groceries are brought to the

city hubs by trucks and from those hubs they are distributed to the customers by EVs [21]. In this paper, we focus on this distribution from the hub to the customers as the last-mile delivery. The requirements are listed as follows:

- A free one-hour delivery time window needs to be offered to the customers when they place an order (i.e. order time window $(OTWS_j - OTWE_j)$).
- A 20-min delivery time windows $(TWS_j - TWE_j)$ needs to be communicated at the morning of the delivery. If the customer is flexible, this stays as one-hour.
- Maximum computation time for customer-trip assignment is one hour for fleet management the evening before the delivery.
- Maximum computation time for trip planning is six hours to make sure the plan is ready before the morning.

An overview of the experimental method is provided in Figure 2. Benchmark (B), simulation-based (S) and heuristics-based (H) sequencing models are comparatively analyzed with and without re-optimization. Historical data from the e-grocer Picnic is used to sample realizations of travel and service times. For this purpose, trips from a hub in the south of Netherlands are used. 10 repetitions of the simulation experiments are considered in order to generate confidence intervals. The penalty of deviation from the optimal arrival times (dev_opt) is set as 100 and the safety margin (SM) is set as 10 min.

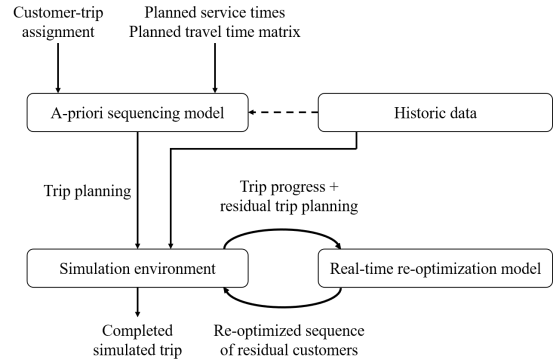


Fig. 2. Overview of the experimental method

The completed simulated trips are analyzed on their on-time delivery rate, rate of extreme late deliveries (≥ 15 min) and average time spent per delivery. Experiments are executed on a personal computer with 16 GB of RAM and a 1.90 GHz processor. Models are programmed in Python 3.7. The a-priori sequencing model and the real-time re-optimization model call the Gurobi MILP solver to find the optimal sequence of customers.

Two types of customer-trip assignments are investigated; for a small and a large vehicle. The large vehicle can carry 33% more groceries than the small vehicle. Figure 3 illustrates the vehicle sizes in comparison to a standard Toyota Prius. Moreover, two different sizes of the flexible delivery time windows are used: 60 min and 75 min.

The results for the different combinations of vehicle sizes and flexible delivery window sizes are presented in Table II



Fig. 3. Scale of the vehicle sizes: small vehicle type (top), large vehicle type (bottom) and a reference car (middle)

with 10% flexible deliveries. Note that the results are provided with the 95% confidence interval. It can be concluded that when re-optimization is activated, on-time delivery performance is improved. However, re-optimization comes at the cost of an increased average time spent per delivery. The simulation-based sequencing model proves to be effective without re-optimization for the large vehicle. For the small vehicle, the simulation-based sequencing model needs the re-optimization model to significantly outperform the benchmark. The simulation-based and heuristics-based sequencing models show similar performance in terms of late deliveries when combined with re-optimization. The heuristics-based approach appears to outperform the simulation-based approach in terms of the rate of extreme late deliveries, especially for the large vehicle. However, the configuration including the simulation-based sequencing model results in a 1% lower average time spent per delivery, irrespective of the vehicle size or the size of the flexible delivery windows.

Extension of the flexible delivery windows to 75 min results in a significant improvement of the on-time performance and is largest for the small vehicle. It is evident that the choice for the routing model has a larger effect for the large vehicle case. The results obtained using small vehicle customer-trip assignments and 75 min windows are presented in Figure 4 in more detail with different fraction of flexible deliveries. The significance of the reduction in the number of late deliveries can be clearly observed when re-optimization is activated especially after 10% of flexibility.

V. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

In this paper, the potential of stochastic and dynamic routing models is shown to improve the on-time delivery performance of an e-grocer's last-mile distribution system. The concept of flexible deliveries was demonstrated to successfully add a degree of freedom to the routing system, which is required to make effective use of real-time re-optimization. The experimental results demonstrate that the use of a stochastic a-priori sequencing model improves the performance as defined by an e-grocer. The effectiveness of the concept of flexible deliveries depends on the type of vehicles used, the fraction of flexible deliveries and the

size of the flexible delivery windows. When compared to the static and deterministic benchmark configuration, the potential gain through the use of a stochastic and dynamic configuration is most profound for the large vehicle. When the fraction of flexible deliveries increases, the effects of re-optimization become more evident. When 10% of deliveries are flexible, real-time re-optimization can significantly improve the performance of the last-mile distribution system. Extending the flexible delivery window results in further improvement of the on-time delivery performance of the routing system, however, it also increases the average time spent per delivery and thereby increases the operational costs.

In this research, the scope is limited to the sequencing and the re-optimization models. For the computational experiments a simplistic customer-trip assignment model was used, which is not optimized for the concept of flexible deliveries. Therefore, it would be interesting to investigate the effects of a customer-trip assignment model when integrated in the framework. A possible research topic would be the design of a customer-trip assignment model that spreads the flexible deliveries over different trips more evenly.

REFERENCES

- [1] N. Saunders and GlobalData, "Online grocery: Lessons from history," May 2019. [Online]. Available: <https://www.onespace.com/blog/2018/10/online-grocery-lessons-history/>
- [2] N. Saunders, "Online grocery: food shopping statistics," May 2019. [Online]. Available: <https://www.onespace.com/blog/2018/08/online-grocery-food-shopping-statistics/>
- [3] M. Mkansi and A. L. Nsankanda, "Leveraging the physical network of stores in e-grocery order fulfilment for sustainable competitive advantage," *Research in Transportation Economics*, 2019.
- [4] J. van der Weerd, T. Leusden, and H. Telling, *Online supermarket omzet op waarde geschat*. Supermarkt en Ruimte, 2019.
- [5] M. Joerss, J. Schröder, F. Neuhaus, C. Klink, and F. Mann, "Parcel delivery: The future of last mile," *McKinsey & Company*, 2016. [Online]. Available: [https://www.mckinsey.com/media/mckinsey/industries/travel transport and logistics/our insights/how customer demands are reshaping last mile delivery/parcel_delivery_the_future_of_last_mile.aspx](https://www.mckinsey.com/media/mckinsey/industries/travel%20transport%20and%20logistics/our%20insights/how%20customer%20demands%20are%20reshaping%20last%20mile%20delivery/parcel_delivery_the_future_of_last_mile.aspx)
- [6] N. Galante, E. G. Lopez, and S. Monroe, "The Future of Online Grocery in Europe," *McKinsey & Company*, pp. 22–31, 2013.
- [7] M. Wilson-Jeanselme and J. Reynolds, "Understanding shoppers' expectations of online grocery retailing," *International Journal of Retail and Distribution Management*, vol. 34, no. 7, pp. 529–540, 2006.
- [8] J. Oyola, H. Arntzen, and D. L. Woodruff, "The stochastic vehicle routing problem, a literature review, part I: models," *EURO Journal on Transportation and Logistics*, vol. 7, no. 3, pp. 193–221, 2018.
- [9] M. W. Ulmer, J. C. Goodson, D. C. Mattfeld, and B. W. Thomas, "Dynamic Vehicle Routing: Literature Review and Modeling Framework," no. June 2018, 2017. [Online]. Available: <http://www.slu.edu/goodson/papers/RouteBasedMDPs.pdf>
- [10] D. Li, C. Antoniou, H. Jiang, Q. Xie, W. Shen, and W. Han, "The Value of Prepositioning in Smartphone-Based Vanpool Services under Stochastic Requests and Time-Dependent Travel Times," *Transportation Research Record*, vol. 2673, no. 2, pp. 26–37, 2019.
- [11] A. Toriello, W. B. Haskell, and M. Poremba, "A dynamic traveling salesman problem with stochastic arc costs," *Operations Research*, vol. 62, no. 5, pp. 1107–1125, 2014.
- [12] C. Archetti, D. Feillet, A. Mor, and M. G. Speranza, "Dynamic traveling salesman problem with stochastic release dates," *European Journal of Operational Research*, vol. 280, no. 3, pp. 832–844, 2020.
- [13] L. Chen, W. C. Chiang, R. Russell, J. Chen, and D. Sun, "The probabilistic vehicle routing problem with service guarantees," *Transportation Research Part E: Logistics and Transportation Review*, vol. 111, no. August 2017, pp. 149–164, 2018.

TABLE II
RESULTS WITH 10% FLEXIBLE DELIVERIES

Late Deliveries					
Models		Small vehicle		Large vehicle	
Sequencing	Re-opt.	Default FTW	Extended FTW	Default FTW	Extended
Benchmark	No	0.0±10 %	-3.1±9.6 %	0.0±9.9 %	-2.8±9.3 %
Simulation-based	No	-1.8±11 %	-4.7±11 %	-7.8±14 %	-11±13 %
Simulation-based	Yes	-3.9±10 %	-12±10 %	-13±13 %	-19±13 %
Heuristics-based	Yes	-4.5±11 %	-14±11 %	-12±10 %	-18±9 %

Extreme Late Deliveries					
Models		Small vehicle		Large vehicle	
Sequencing	Re-opt.	Default FTW	Extended FTW	Default FTW	Extended
Benchmark	No	0.0±38 %	-4±37 %	0.0±54 %	0.0±54 %
Simulation-based	No	-1±23 %	-2±24 %	16±45 %	11±48 %
Simulation-based	Yes	-5±20 %	-23±25 %	4±43 %	4±47 %
Heuristics-based	Yes	-11±25 %	-27±25 %	-19±50 %	-20±54 %

Avg. Time Spent per Delivery					
Models		Small vehicle		Large vehicle	
Sequencing	Re-opt.	Default FTW	Extended FTW	Default FTW	Extended
Benchmark	No	0.0±0.16 %	0.0±0.16 %	0.0±0.56 %	0.0±0.56 %
Simulation-based	No	0.2±0.12 %	0.2±0.12 %	-0.3±0.52 %	-0.3±0.52 %
Simulation-based	Yes	0.4±0.11 %	0.9±0.15 %	-0.1±0.46 %	0.4±0.48 %
Heuristics-based	Yes	1.4±0.20 %	1.9±0.2 %	0.9±0.44 %	1.4±0.45 %

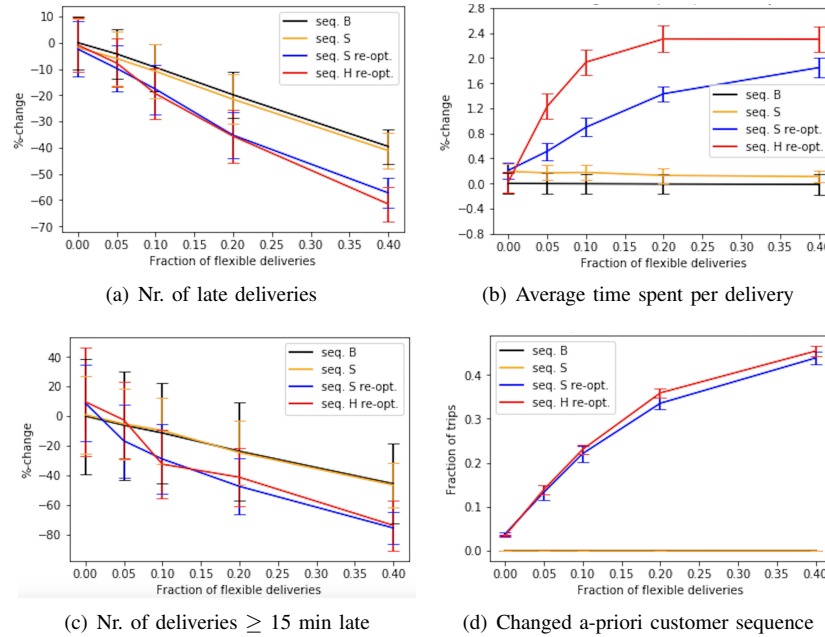


Fig. 4. Results for the small vehicle with extended flexible delivery windows

- [14] K. K. Ng, C. K. Lee, S. Z. Zhang, K. Wu, and W. Ho, "A multiple colonies artificial bee colony algorithm for a capacitated vehicle routing problem and re-routing strategies under time-dependent traffic congestion," *Computers and Industrial Engineering*, vol. 109, pp. 151–168, 2017.
- [15] F. Errico, G. Desaulniers, M. Gendreau, W. Rei, and L.-M. Rousseau, "A priori optimization with recourse for the vehicle routing problem with hard time windows and stochastic service times," *European Journal of Operational Research*, vol. 249, no. 1, pp. 55–66, 2016.
- [16] E. Taniguchi and H. Shimamoto, "Intelligent transportation system based dynamic vehicle routing and scheduling with variable travel times," *Transportation Research Part C: Emerging Technologies*, vol. 12, no. 3–4, pp. 235–250, 2004.
- [17] A. A. Juan, J. Faulin, S. E. Grasman, D. Riera, J. Marull, and C. A. Mendez, "Using safety stocks and simulation to solve the vehicle routing problem with stochastic demands," *Transportation Research Part C: Emerging Technologies*, vol. 19, no. 5, pp. 751–765, 2011.
- [18] N. Balakrishnan, "Simple heuristics for the vehicle routing problem with soft time windows," *Journal of the Operational Research Society*, vol. 44, no. 3, pp. 279–287, 1993.
- [19] D. Taş, N. Dellaert, T. Van Woensel, and T. De Kok, "Vehicle routing problem with stochastic travel times including soft time windows and service costs," *Computers and Operations Research*, vol. 40, no. 1, pp. 214–224, 2013.
- [20] H. I. Calvete, C. Galé, M. J. Oliveros, and B. Sánchez-Valverde, "A goal programming approach to vehicle routing problems with soft time windows," *European Journal of Operational Research*, vol. 177, no. 3, pp. 1720–1733, 2007.
- [21] M. Dalmijn, B. Atasoy, P. Bijl, and R. R. Negenborn, "Charge scheduling of electric vehicles for last-mile distribution of an e-grocer," in *Proceedings of 2020 Forum on Integrated and Sustainable Transportation Systems (FISTS)*, nov 2020, pp. 236–241.